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Review

Modern Techniques for the Optimal Power Flow Problem: State of the Art

Benedetto-Giuseppe Risi [1,2], Francesco Riganti-Fulginei [1,*] and Antonino Laudani [1]

1 Department of Industrial, Electronic and Mechanical Engineering, Roma Tre University, Via Vito Volterra 62, 00146 Rome, Italy
2 Enel Grids, Via Mantova 24, 00198 Rome, Italy
* Correspondence: riganti@uniroma3.it

Abstract: Due to its significance in the operation of power systems, the optimal power flow (OPF) problem has attracted increasing interest with the introduction of smart grids. Optimal power flow developed as a crucial instrument for resource planning effectiveness as well as for enhancing the performance of electrical power networks. Transmission line losses, total generation costs, FACTS (flexible alternating current transmission system) costs, voltage deviations, total power transfer capability, voltage stability, emission of generation units, system security, etc., are just a few examples of objective functions related to the electric power system that can be optimized. Due to the nonlinear nature of optimal power flow problems, the classical approaches may become locked in local optimums, hence, metaheuristic optimization techniques are frequently used to solve these issues. The most recent optimization strategies used to solve optimal power flow problems are discussed in this paper as the state of the art (according to the authors, the most pertinent studies). The presented optimization techniques are grouped according to their sources of inspiration, including human-inspired algorithms (harmony search, teaching learning-based optimization, tabu search, etc.), evolutionary-inspired algorithms (differential evolution, genetic algorithms, etc.), and physics-inspired methods (particle swarm optimization, cuckoo search algorithm, firefly algorithm, ant colony optimization algorithm, etc.).

Keywords: RES (renewable energy systems); DG (distributed generation); OPF (optimal power flow); NN (artificial neural networks)

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1. Introduction

With the advent of smart grids, the optimal power flow (OPF) problem has gained more and more attention due to its importance in power system operation. Optimal power flow developed as a crucial instrument for resource planning effectiveness as well as for enhancing the performance of electrical power networks. Through the use of control variables, starting from predetermined objective functions, and adhering to predetermined limitations, optimum power flow seeks to find the best or the most secure operating point. Transmission line losses, total generating costs, FACTS costs, voltage deviations, total power transfer capabilities, voltage stability, emission of generation units, system security, etc., are only a few examples of objective functions connected to the electric power system that can be optimized. The generated active powers, the voltage of the generation buses, the settings for the transformer taps, and the size of the FACTS devices are some of the system control variables that can be changed. To solve the optimal power flow problem, several traditional (deterministic) and modern (nondeterministic) heuristic optimization strategies have been put forth. Sensitivity analyses and gradient-based techniques are used in most traditional optimization strategies. Due to the nonlinear nature of optimal power flow problems, the classical approaches may become locked in local optimums, hence metaheuristic optimization techniques are frequently used to solve these issues. New methods have been developed to avoid these problems. In this review paper, metaheuristics and probabilistic methods proposed in literature will be analyzed and exposed. Furthermore, these methods allow load flow to perfectly fit new grid changes, such as DG penetration. Electricity in recent years, moved away from centralized production and increasingly towards DG. Therefore, it is easy to see that the current electricity grid is undergoing a period of great change and transformation. Traditional distribution grids are designed for "passive" operation and are characterized by a generally radial topology and unidirectional power flows towards the transformer. Therefore, the distribution grid is not designed to host significant amounts of distributed generation. From this knowledge, it is legitimate to think that much of this additional demand will be met with the help of DG, but only if the grid is ready, i.e., infrastructure upgrades are needed and are combined with the application of new technologies for monitoring and regulating the grid. The transition to the grid of the future, however, depends on quantifying how to optimize investments according to the maximum use of the current grid in terms of new RES and new installable loads. In this challenging context, the optimal load flow problem aims to solve the grids to show the energy distribution companies how best to invest money. The time-to-market strategy imposes a constraint on getting the fastest load flow result, which is also lead by real-time energy market regulation. This paper aims to give the reader an overview mainly of the LF and OPF methods found in recent literature, with a global focus on the best-fit method over grid topology as an innovation aspect. Within several methods deployed in the last half century, this paper will report on only the ones mentioned in at least several papers so that the most relevant papers will be mentioned. The paper will first take into account the load flow theory, and then focus on classical and new methods (probabilistic and metaheuristic) with a special analysis of specific grid topology-related applications.

2. Load Flow Problem Theory

The fundamental analysis in power systems is load flow analysis. It entails examining the grid from the standpoint of electricity flows. Loads and generators, we know, have time-varying diagrams. Load flow analysis is used to determine the nodal voltages and electricity flowing in the lines that connect these nodes. We evaluate the loads or generators at a certain time instant for this purpose. This moment will pass quickly (3–4 min). We consider a state to be a stationary condition if we observe a load that remains constant during such a time frame. Two stationary states can be thought of as two sequential instants (steady state operation). In these circumstances, we consider that the system is working.

The idea of load flow analysis is to find the voltage on each busbar and the power flows on the various lines, because we need to contain the voltage within certain limits, also not to overload the lines themselves. Consider, for example, busbar 2 in Figure 1. If we analyze it in more detail, we find the following diagram shown in Figure 2:

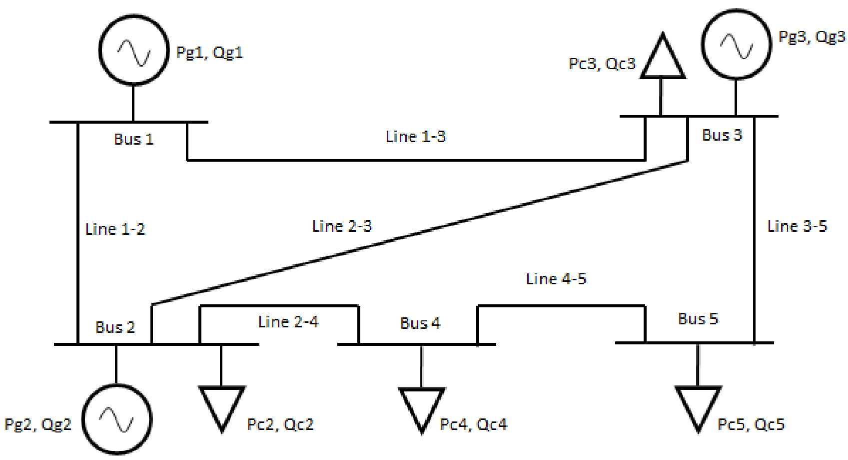

Figure 1. Typical scheme of a load flow power grid.

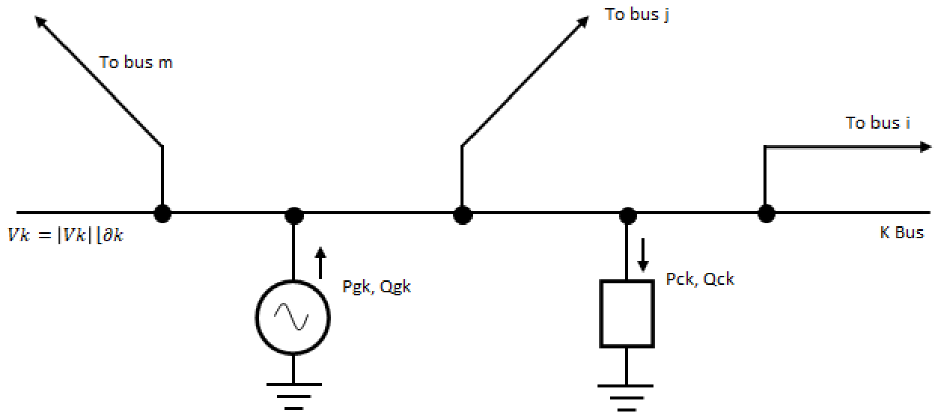

Figure 2. Detail of k-bus of Figure 1 generic grid.

The P_k and Q_k of the k busbar will be expressed in Equation (1):

$$\begin{aligned} P_k &= P_{gk} - P_{ck} \\ Q_k &= Q_{gk} - Q_{ck} \end{aligned} \tag{1}$$

where:

$$S_k = V_k \cdot I_k{}^* = P_k + jQ_k \tag{2}$$

Considering the transmission lines equivalent model, then we get Figure 3:

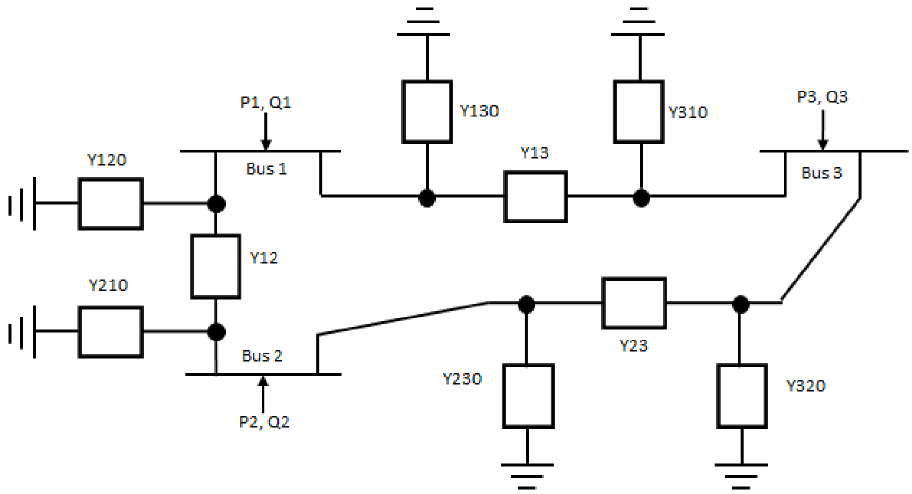

Figure 3. A three bus power grid, considering complete lines model.

The bus admittance matrix (Y_{bus}) can be written for simple inspection. The elements of the diagonal Y_{ii} are called self-admittances while the other elements Y_{ij} are called mutual admittances. Therefore, it is possible to write Equation (3) in compact form:

$$I_{bus} = Y_{bus} * V_{bus} \tag{3}$$

where the nodal voltages can be found solving linear algebraic equations in matrix form. Let us consider Equation (3), which can be expressed in a less compact form:

$$I_k = \sum_{n=1}^{N} Y_{kn} \cdot V_n \rightarrow I_1 = Y_{12} * V_1 + Y_{12} * V_2 + Y_{13} * V_3 \tag{4}$$

for each element I_k of the I_{bus} vector. The power at a given bus results in:

$$S_k = P_k + jQ_k = V_k \cdot I_k^* \tag{5}$$

Substituting (4) in (5) we get:

$$P_k + jQ_k = V_k \cdot \left[\sum_{n=1}^{N} Y_{kn} \cdot V_n \right]^* \tag{6}$$

with k = 1, 2, . . . , N nodes. Now, considering that:

$$\begin{aligned} V_n &= |V_n| e^{j\partial n} \\ Y_{kn} &= |Y_{kn}| e^{j\theta kn} \end{aligned} \tag{7}$$

substituting (6) in (7), we get:

$$P_k + jQ_k = V_k \cdot \sum_{n=1}^{N} Y_{kn} \cdot V_n \cdot e^{j(\partial n - \partial k - \theta kn)} \tag{8}$$

That is, applying Euler's equality in Equation (8):

$$\begin{aligned} P_k &= V_k \cdot \sum_{n=1}^{N} Y_{kn} \cdot V_n \cdot cos(\partial_n - \partial_k - \theta_{kn}) \\ Q_k &= V_k \cdot \sum_{n=1}^{N} Y_{kn} \cdot V_n \cdot sin(\partial_n - \partial_k - \theta_{kn}) \end{aligned} \tag{9}$$

We then have (9) which represent the power flow equations. These two equations expressed in P (active power) and Q (reactive power) tie together the active and reactive power of each bus to the module and the voltages of the buses via the admittances (self or mutual) of the grid.

These two equations are:

1. Algebraic (no derivatives), since we are in stationary conditions.
2. Nonlinear, due to the presence of trigonometric functions and nodal voltages.

To solve these equations, we must, therefore, make use of iterative numerical computation.

3. Principal Load Flow Methods: Clusters and Descriptions

Load flow methods essentially consist of an algorithm to iteratively solve nonlinear power flow Equation (9). According to the most recent review papers, load flow methods can be classified as conventional (also referred as classic) and non-conventional ones. Non-conventional ones are best projected in order to achieve optimal power flow problem solutions. Then, some methods can be better applied to certain grid topologies than others, thus, grids can be classified as classical unidirectional grids or RES penetrated grids (referred to as bidirectional grids due to the power inversion phenomenon). For the sake of completion, it is worthwhile mentioning that power flow problems can be solved in centralized and distributed/decentralized settings [1,2]. Then, typical grid topology is radial grids, linear grids, and rural grids. Now let us see how different methods work.

3.1. Classical Methods

Within classical methods we can find NR and GS method referenced in literature. Let us study how they work.

3.1.1. Newton—Raphson Method

Newton—Raphson's method, also known as the method of tangents, is one of the methods for the approximate calculation of a solution to an equation of the form $f(x) = 0$. It is applied after determining an interval [a,b] that contains a single root. The method consists of substituting the curve $y = f(x)$ for the tangent to the curve itself, starting from any point; for simplicity, one can start from one of the two points whose abscissa is the extreme of the interval [a,b] and take, as the approximate value of the root, the abscissa X_t of the point where the tangent intersects the x-axis within the interval [a,b]. Proceeding iteratively, it is shown that the recurrence relation of the method is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{10}$$

which makes it possible to determine successive approximations of the root of the equation $y = f(x) = 0$. With the hypotheses posed, it is shown that the succession of X_n converges to the root rather quickly. In more detail, it is shown that if one is in an opportune neighbourhood of the solution, convergence will be quadratic, otherwise it will be linear (slower). In practice, given a permissible approximation tolerance of tau, the iterative procedure is terminated when the following condition is fulfilled:

$$|x_{n+1} - x_n| < \tau \cdot |x_{n+1}| \tag{11}$$

Sometimes N-R may even fail to converge if one is looking for a solution that is too far away from the considered interval. In order to determine the interval in question in a more approximate but still effective manner, one usually precedes N-R with the bisection method to calculate the interval in which to operate.

Let us then consider solving the nonlinear equations of the LF problem. These equations express the relationship between P_k, Q_k, and V_k. We can then write these equations in general form according to the more extended notation that takes into account all the nodes from 2 to N, without considering the balance node that we have assumed to be number 1:

$$y = f_k(x) = \begin{bmatrix} P_k(x) \\ Q_k(x) \end{bmatrix} = \begin{bmatrix} P_2 \\ P_N \\ \dots \\ Q_2 \\ Q_N \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \dots \\ f_N(x) \end{bmatrix} \tag{12}$$

The unknowns can also be made explicit according to the same notation:

$$x = \begin{bmatrix} \partial_k \\ V_k \end{bmatrix} = \begin{bmatrix} \partial_2 \\ \partial_N \\ \dots \\ V_2 \\ V_N \end{bmatrix} \tag{13}$$

Now, making an initial assumption of the solution (called start) equal to $x = x_0$, we can then expand the generic $f(x)$ in Taylor series around x_0, thus finding that:

$$y = f(x_0) + \frac{df(x)}{dx}|_{x=x_0} \cdot (x - x_0) + \dots R(x) \tag{14}$$

By stopping the development at first order, we then solve for x and obtain:

$$x = x_0 + \left[\frac{df(x)}{dx}|_{x=x_0} \right]^{-1} \cdot [y - f(x_0)] \tag{15}$$

Having a system of equations and implementing a method based on iterative calculation, the previous equation can be rewritten as:

$$x^{i+1} = x^i + J^{-1(i)} \cdot \left\{ y - f\left[x^i \right] \right\} \tag{16}$$

where the J represents the Jacobian, defined as:

$$J^{(i)} = \frac{df(x)}{dx}|_{x=x^i} = \begin{bmatrix} \frac{df_1(x)}{dx_1} & \cdots & \frac{df_1(x)}{dx_N} \\ \vdots & \ddots & \vdots \\ \frac{df_N(x)}{dx_1} & \cdots & \frac{df_N(x)}{dx_N} \end{bmatrix} \quad (17)$$

i represents the current number of iterations reached. The calculation stops when x^{i+1} varies within the set tolerance limit and convergence is, therefore, reached. Now, the fact that it is necessary to calculate the inverse of the Jacobian is computationally disadvantageous, as it is a very expensive operation. Calculating an inverse matrix from an $N \times N$, in fact, increases almost linearly with N. In addition, we will also have $2N - 1$ equations to solve. Generally, one does not calculate the inverse of the Jacobian but resorts to other means. To this end, we consider the increment on x and y that each iteration produces as: $\Delta x^{i+1} = x^{i+1} - x^i$

$$\Delta y^{i+1} = y^{i+1} - y^i \quad (18)$$

Therefore, the following equation is obtained in which the inverse of the Jacobian appears:

$$x^{i+1} = x^i + J^{-1(i)} \cdot \left\{ y - f\left[x^i\right] \right\} \quad (19)$$

It can be rewritten as:

$$J^i \cdot \Delta x^i = \Delta y^i \quad (20)$$

This, in essence, is a system that is in the form Ax = y and can be solved by the known methods of linear algebra. In fact, in this way we have linearized a nonlinear problem, i.e., from a set of nonlinear algebraic equations we have obtained a set of linear equations.

3.1.2. Gauss—Seidel Method

An alternative method to Newton—Raphson's method is the Gauss—Seidel method. In numerical analysis, the Gauss—Seidel method is an iterative method, similar to the Jacobian method, for solving a linear system written in the matrix form Ax = b. Let us try to linearize the LF equations. To this end, we recall that:

$$I_{bus} = Y_{bus} * V_{bus} \text{ [A]} \quad (21)$$

We know that the k-th current I_k of the I_{bus}, written in a less compact form, will be equal to:

$$I_k = \sum_{n=1}^{N} Y_{kn} \cdot V_n \text{ [A]} \quad (22)$$

In extended form:

$$I_k = \sum_{n=1}^{N} Y_{kn} \cdot V_n = Y_{k1} * V_1 + Y_{k2} * V_2 + \ldots + Y_{kk} * V_k + \ldots + Y_{kN} * V_N \text{ [A]} \quad (23)$$

With k = 1, 2, . . . , N nodes. The complex power can be written as:

$$S_k = P_k + jQ_k = V_k \cdot I^*{}_k \text{ [VA]} \quad (24)$$

Thus, expressing the current as a function of I_k, we have:

$$I_k = \frac{Pk - jQk}{Vk^*} \text{ [A]} \quad (25)$$

Equalizing the two expressions in I_k we have:

$$\frac{P_k - jQ_k}{V_k{}^*} = Y_{k1} * V_1 + Y_{k2} * V_2 + \ldots + Y_{kk} * V_k + \ldots + Y_{kN} * V_N \quad (26)$$

The unknown is V_k, with k being the number of the node under consideration. We can rewrite the expression as:

$$\frac{P_k - jQ_k}{V_k{}^*} = \sum_{n=1}^{k-1} Y_{kn} \cdot V_n + Y_{kk} * V_k + \sum_{n=k+1}^{N} Y_{kn} \cdot V_n \quad (27)$$

Hence, finally:

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k{}^*} - \left(\sum_{n=1}^{k-1} Y_{kn} \cdot V_n + \sum_{n=k+1}^{N} Y_{kn} \cdot V_n \right) \right] [\mathrm{V}] \quad (28)$$

with $k = 1, 2, \ldots, N$ nodes. This expression links V_k ($V_{kr} + jV_{ki}$) with the active and reactive power of the individual bus and is the solving equation underlying the Gauss—Seidel method.

This equation is:

– algebraic (no derivatives) and
– nonlinear.

It can be also solved numerically by iteration, considering V_k at the first member as the result of a (linear) calculation that sees V_k at the second member as the current one and V_k at the first member as that of the next iteration. In formula:

$$V_k^{i+1} = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^{(i)*}} - \left(\sum_{n=1}^{k-1} Y_{kn} \cdot V_n^i + \sum_{n=k+1}^{N} Y_{kn} \cdot V_n^i \right) \right] [\mathrm{V}] \quad (29)$$

Let us, therefore, consider a network with a slack bus and only PQ buses; we will deal with the PV buses later. The solving equation obviously applies to the PQ buses only as the slack bus is not part of the LF calculations. Let us, therefore, see how to set up an iterative method. To start the first account, we can also make the initial flat start assumption here, i.e., all voltages of the various busbars equal 1 + j0 p.u. From this assumption, the nodal voltages (i + 1) are calculated with reference to the formula above. From the calculation in step 2, a more accurate estimate is obtained, and so on, until $V_k(i + 1) - V_k(i)$ are less than a set tolerance. Therefore, to summarize, the steps of the algorithm are:

- the generation and absorption powers on each busbar are known, the total powers at each bus are calculated,
- the Y_{bus} is created,
- the nodal voltages (also in polar coordinates if desired) are set to the flat start value, and
- the solving equation is solved iteratively until convergence below a set tolerance is achieved.

One computation aspect of the G—S method is that it has a rather slow convergence, as one has linearised the problem and the speed of convergence has the same order of magnitude as the number of busbars. This is a disadvantage in networks with hundreds of busbars.

It is possible, however, to speed up convergence, with an alpha-factor. This guarantees faster convergence from the case where alpha is unitary. The value of this parameter can vary between 1.5 and 1.7. However, G—S is poorly suited for networks with many known factors where, instead, N-R requires a fixed number of iterations approximately independent of the number of nodes.

3.2. About Optimal Power Flow

A power system optimization challenge is optimal power flow (OPF) [3–13]. Carpentier [14] first raised this issue in 1962. The objective of OPF is to identify a steady-state operating point that meets operating constraints and demands while minimizing the cost of power generation. A nonlinear programming (NLP) problem can be used to formulate the issue since some restrictions and potential objective functions are nonlinear. Kirchhoff's voltage law (KVL), which describes the power flow in the transmission line as a nonlinear function of bus voltage and phase angle, contains limitations that are the major source of nonlinearity in the standard OPF polar formulation. The feasible region of the OPF issue is nonconvex due to the presence of nonlinear equality constraints [15], increasing the likelihood that local OPF solutions exist. However, one of the authors in the 1997 work [16] made the observation that OPF solutions are distinct in practice, which is still a widely held belief. Dommel and Tinney [17] offered the first solution to the OPF issue in 1968, and several others have subsequently been put forth. The authors of [18,19] provide an excellent summary of the literature on traditional optimization techniques applied to OPF during the past 30 years. If there is a local minimum, none of these methods will necessarily uncover the global minimum. The possibility that the OPF problem has local optimum states is a significant but poorly discussed issue in the literature. Both traditional local nonlinear programming and evolutionary techniques are included in a recent literature review [20] of OPF. Evolutionary algorithms are global optimization techniques that attempt to find global optima. Global techniques are much slower than the local ones so should only be used for problems where local optima may exist. The possibility of convergence to local solutions is cited by the authors of [21] as a significant shortcoming of classical optimization approaches used to tackle the OPF problem. However, none of these surveys mention any instances of local OPF optimums or provide an estimation of how frequently they occur in actual usage. Numerous approaches have been found to expand the OPF issue. For instance, risk-based OPF and security-constrained OPF address the issue of ensuring the network works securely. Nevertheless, if local optimum solutions exist for the original OPF issue, they should also exist for extensions, so in this paper we restrict attention to the standard OPF problem. The authors of [22] provide an intriguing semi-definite program (SDP) formulation of an OPF issue dual. They demonstrate that if there is no duality gap, a globally optimal solution to the OPF can be recovered from the SDP dual, and they provide a testable condition that ensures there is no duality gap after solving the SDP dual. However, it is not clear whether there will be a duality gap solely from the qualities of a general network, and it is unclear how frequently the strategy actually works in practice. In [23,24], sufficient requirements for the absence of a duality gap that solely depend on network features are provided. These are applicable to tree networks and lossless loop networks and call for set voltage magnitudes, restrictions on the angle difference across lines, and/or a great deal of freedom in the real and reactive power balance at buses. However, neither universal networks nor any of the examples in this paper satisfy these requirements. The SDP technique has some drawbacks, which are explored in [25]. Examples of modified IEEE test situations where the SDP recovery approach fails are presented in [26], and a branch and bound method that finds the global optimum in these cases is proposed. This technique makes use of the SDP formulation for the relaxations. However, there were no local OPF solutions in any of these circumstances.

3.3. Non-Classical Methods

In this paragraph, principals non-classical methods referred to in the literature will be presented.

3.3.1. Probabilistic Load Flow Methods

The deterministic load flow problem ignores uncertainties such as the variation of load requirements and the power oscillations brought on by renewable generation, which might be of utmost significance in the evaluation of distribution systems with or without DR penetration. It is necessary to use a different mathematical strategy to account for these uncertainties. The solutions up to this point have a probabilistic foundation—approach, fuzzy logic application, or interval analysis. The probabilistic load flow [12] was first proposed in 1974 [27], and it has since been improved and used in the study and design of power systems. To derive system states and power flows in terms of a density function and to incorporate system uncertainties in the result, inputs must be provided using their probability density function (or alternatively, the cumulative density function). The load uncertainty is typically not very high, and a normal distribution can be used to model it in most cases [9]. The expansion of renewable resources, however, brings with it additional difficulties. For instance, wind variability is quite large and cannot be modeled by a normal distribution. Either a numerical or an analytical method can be used to solve the probabilistic load flow. Below is an overview of the fundamental methods employed in the probabilistic load flow solution and how it is applied to distribution system analysis [10].

Numerical Solution Methods

Random number generation and random sampling are the two primary components of a Monte Carlo simulation approach. A probabilistic load flow based on this method basically entails repeatedly solving a deterministic power flow with inputs in various combinations while utilizing the nonlinear version of the load flow equations. Results from this approach are typically used as a reference to the results from other probabilistic load flow solutions to check their accuracy because the precise load flow equations are used. Due to the potential need for a high number of load flow solutions, this method's principal drawback is the lengthy computation time that may be involved.

Analytical Solution Methods

The fundamental principle of the analytical approach is to perform algebraic operations on density functions of random input variables in order to derive density functions for random state variables and line flows. Input power variables might not be entirely independent of one another, and load flow equations are nonlinear, which make it challenging to solve probabilistic load flow equations. Several simulations are typically included to perform the probabilistic load flow using an analytical approach: power variables are completely independent or linearly coupled; normal distribution and discrete distribution are typically assumed for both load and generation; and network setup and parameter are constant. These assumptions are all made in the load flow equations. Different methods have been proposed to mitigate the error caused by the linearization of the load flow equations. Two typical solutions consist of using multilateralization or the quadratic probabilistic load flow. Point estimation methods can be included into the group of approximate methods. Since then, a number of works have addressed the shortcomings of the initial strategy, for instance, by utilizing correlated input variables or lowering the quantity of input random variables. Cumulants and Gram—Charlier expansion theory can be combined, as suggested in [12], to increase the effectiveness of a probabilistic load flow method. The fundamental benefit of utilizing Gram series is that the density function of any type of standardized variable may be expressed as the sum of weighted standard normal distributions at various derivative orders. The weights can be determined using several cumulant orders. It is possible to obtain cumulative density functions directly without integrating probability density functions. Although it has the drawback of requiring linearization, it can still be used to independent random variables. This method may have convergence issues for non-normal distributions; a less computationally intensive alternative is the Cornish—Fisher expansion.

3.3.2. Typical Grids Applications for Probabilistic Methods

The connection of generating and energy storage units to distribution systems presents several challenges for probabilistic calculations: wind forecast cannot be based on a normal distribution; there can be power generation interdependence between different DG units (e.g., correlated wind power generation) [11]; and the continuous and typically large variation of the power generation from DG may result in a large error if linearized load flow equations around the mean values are used. Due to the substantial correlation between generations from wind turbines in surrounding wind farms, modeling the interdependence of wind power generation is crucial (see [28,29]). The analytical probabilistic load flow method needs to incorporate theoretical expressions that take into consideration the relationship between stochastic generations and loads, as shown in [28]. A linearized version of the load flow equations was utilized in [30], one of the earliest papers on probabilistic load flow analyses in radial distribution systems [4] with stochastic wind generation. By including the adjustment of voltage and reactive power control devices, the authors improved the approach [31]. To account for uncertainties in time-varying loads, voltage regulators, switching capacitors, and network reconfiguration, Su [32] employed a single-phase version with the Monte Carlo method. The Monte Carlo approach and a single-phase distribution model were also applied in [28,33]. The authors of the first research examined the effects of stochastic wind generation, while the authors of the second presented the effects of solar generation. The influence of wind farms [6] was examined using a probabilistic three-phase load flow based on the Monte Carlo method in [34]. This article proposed a novel way for simulating induction generators that could take into consideration system imbalance. Distribution systems with load and generation uncertainties have also been studied using fuzzy set theory [35–37]. The topic of [38] was the application of the interval arithmetic technique to the load flow problem in distribution systems with uncertain input parameters. Distribution systems with energy storage units have been examined in very few studies, the test cases have been rather straightforward, and the major objective has not been to determine how energy storage affects voltages, power flows, and losses [39–41].

3.3.3. Metaheuristic Methods for OPF

In general, the exact method and approach method can be used to solve the optimization problem. The words meta and heuristic are the roots of the word metaheuristic [5]. Meta refers to a high-level methodology, whereas heuristics refers to the art of developing novel problem-solving techniques. A metaheuristic is a heuristic approach that does not depend on the nature of the problem. There are two types of metaheuristic methods: metaheuristics based on single-solution local searches and metaheuristics based on population data (random search). ACO, GA, and PSO are a few instances of metaheuristic techniques based on population. ACO, GA, and PSO are the three metaheuristic techniques used in this study.

Genetic Algorithm (GA)

The widely used stochastic optimization technique known as "genetic algorithms" (GA) draws its inspiration from the mechanics of genetics and natural selection as shown in Figure 4. They operate under the tenet of best-adapted individuals prevailing. GA was utilized by A-F. Attia et al. [42] to resolve the OPF issue. The IEEE-30 bus system has been used to create and implement the suggested solution. Results are evaluated in comparison to other techniques in the literature. The efficiency of the suggested strategy was demonstrated through comparison. Three optimization techniques, including GA, were employed by S. Kahourzade et al. [43] to address single- and multi-objective OPF problems. On the IEEE-30 bus system, the proposed technique has been developed and used.

Programming in Evolution (EP)

The focus of evolutionary programming (EP), a stochastic optimization, is on the behavioral relationships between parents and children [44]. Three optimization techniques, including EP, were employed by S. Kahourzade et al. [43] to address single- and multi-objective OPF problems. On the IEEE-30 bus system, the proposed technique has been developed and used. Results for the single- and multi-objective strategies were superior to those of other techniques. EP was employed by P. Somasundaram et al. [44] to address the OPF issue. The approach was created and used with the IEEE-30 bus system. The obtained results are contrasted with those obtained using the traditional security-constrained optimal power flow (SCOPF).

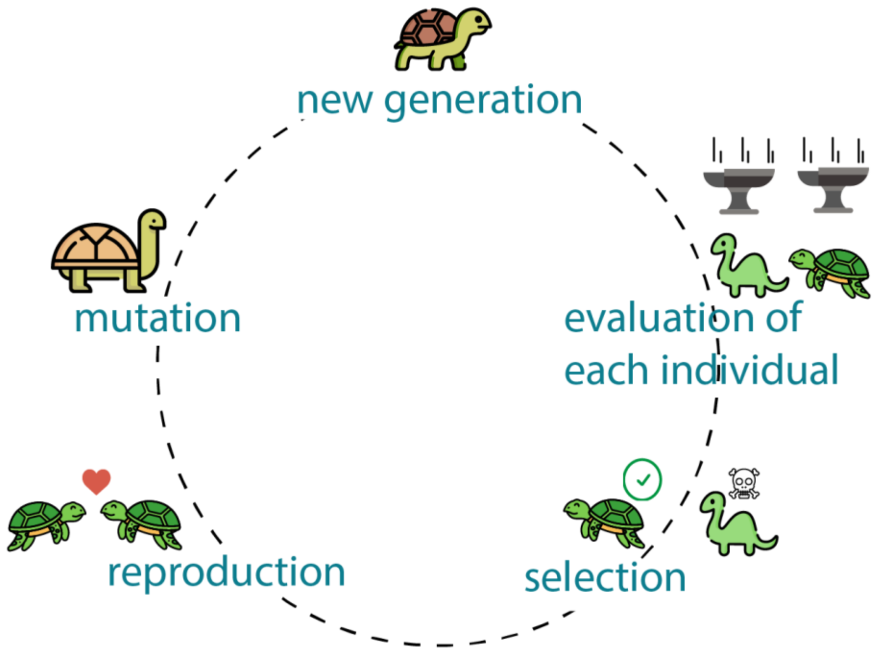

Figure 4. Genetic Algorithm simple illustration.

Particle Swarm Optimization (PSO)

Kennedy and Eberhart proposed the particle swarm optimization (OPF) technique in 1995 as an alternative to conventional genetic algorithms (GA). These algorithms were influenced by the organized movements of swarms of insects as shown in Figure 5. Animals move in groups to find food or to defend themselves from predators, according to observation. PSO was employed by N. Kalfallah et al. [45] to regulate reactive power in the western Algerian power system. Utilizing the approach has improved the voltage profile and decreased power losses. It also demonstrated how installing SCV devices can dramatically reduce power losses and keep voltages within safe ranges. Three optimization techniques, including PSO, were employed by S. Kahourzade et al. [44] to solve the single- and multi-objective OPF issue. On the IEEE-30 bus system, the proposed technique has been developed and used.

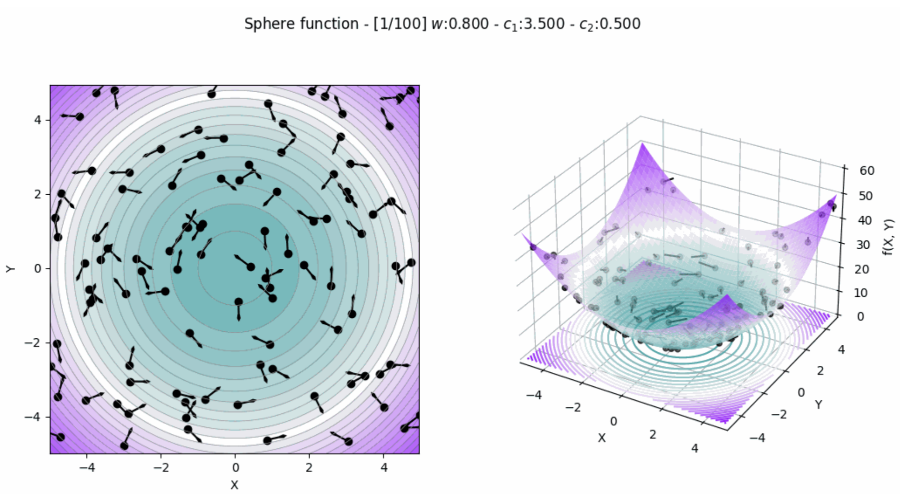

Figure 5. PSO algorithm simple illustration.

Differential Evolution (DE)

The differential evolution (DE) method draws inspiration from GA and evolutionary tactics. Problems with continuous variables are addressed by it. DE was employed by M.

Varadarajan and K.S. Swarup [46] to address the OPF issue. The technique's performance was evaluated on the IEEE-14, 30, 57, and 118 bus systems, and its outcomes were contrasted with those of traditional approaches. It has been found that using the DE approach to solve OF problems with nonlinear objectives and constraints is a straightforward but effective technique. DE was employed by M. Varadarajan and K.S. Swarup [47] to address the OPF issue. The technique was evaluated using the IEEE-14, 30, and 118 bus systems, and the findings are then contrasted with those of a traditional method and, subsequently, with those of the PSO. It has been observed that results obtained with the DE method were more robust than those of PSO and sequential quadratic programming (SQP) algorithms.

Artificial Bee Colony (ABC)

The artificial bee colony is based on the unique manner in which bees communicate as shown in Figure 6. In order to find the best solutions to various optimization problems, the ABC algorithm does, in fact, imitate the foraging and dancing behaviors of honey-bees [48]. ABC was utilized by M. R. Adaryani and A. Karami [49] to solve a multi-objective problem. The method's efficacy and validity were tested on the IEEE-9. 30 and 57 bus systems. Results have been compared with those obtained using other techniques described in the literature. The method can be used to solve a variety of problems on a wide scale, as evidenced by its effectiveness and the caliber of its outcomes. Chaotic ABC was utilized by K. Ayan et al. [48] to address the STSCOPF (security and transient stability constrained optimum power flow) problem. The IEEE-30 and 39 bus systems have been used to develop and implement its method. Results were compared to those found in the literature and demonstrated the validity and efficacy of the suggested method.

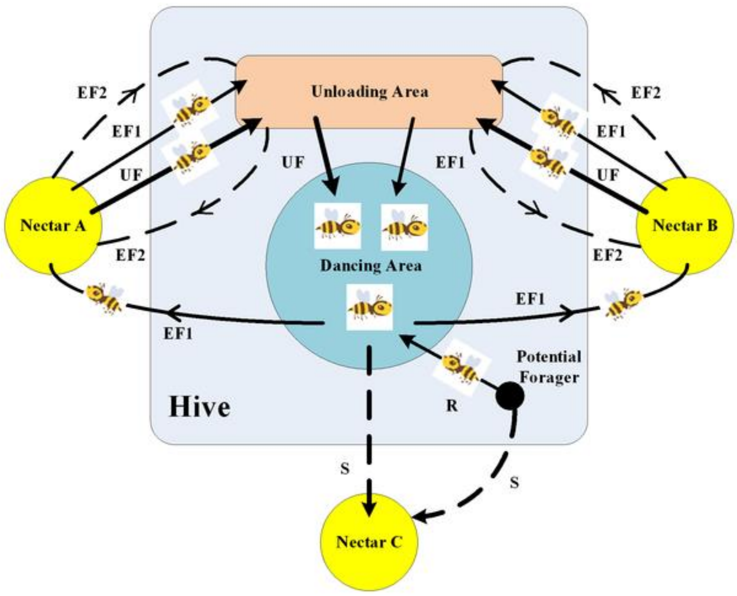

Figure 6. ABC algorithm simple illustration.

Gravitational Search Algorithm (GSA)

The Newtonian theory of gravity served as the inspiration for the gravitational search algorithm (GSA) in optimization techniques [50] as shown in Figure 7. GSA was utilized by S. Duman et al. [51] to resolve OPF issues. The technique was created and examined on the IEEE-30 and 57 bus systems. Results from GSA are contrasted with those from other metaheuristic techniques that have been published in the literature. It has been noted that outcomes are robust, efficient, and high quality for resolving OPF issues. The allocation of the thyristor-controlled series compensator (TCSC) utilizing the congestion rent contribution methodology based on the location of the marginal price was solved by A. Sharma and S. K. Jain [52] using GSA. The technique was created and examined on the

IEEE-30 and 57 bus systems. Other heuristics methods reported in the literature are used to compare GSA's performance.

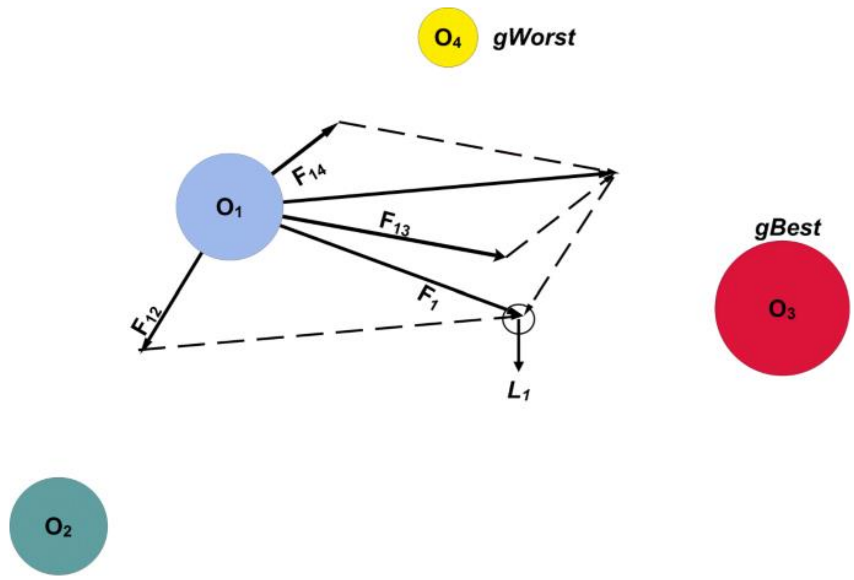

Figure 7. GSA algorithm simple illustration.

Wolf Optimization Algorithm

Mirjalili et al. introduced the grey wolf optimizer (GWO) algorithm in 2014 [53]. The leadership structure and hunting strategy employed by grey wolves to capture their prey until it stops moving are mimicked by this algorithm. By mimicking the social behavior of grey wolves in their social life when seeking food, GWO is similar to other population-based metaheuristic algorithms; they adhere to hierarchy in the group. The hierarchy of grey wolves is divided into two levels as shown in Figure 8, with (alpha) symbolizing the group's leaders and (beta), which assists alpha in making choices [54]. Delta and omega are the next levels; they are the lowest ranks in the group, and they must eat after every level. In actuality, these wolves hunt in packs and engage in a three-step process that includes chasing, encircling, and attacking. The method begins with a predetermined number of randomly positioned wolves. The hierarchy of grey wolves' leadership can be simulated using four different wolf group types [55,56]. This hierarchy respects the social dominant degree in which the high class is named alpha (α) and is mostly responsible for making decisions about hunting and order the other wolves in the pack. The steps are reported below:

1. Initializing the search agents.
2. Assigning alpha, beta and gamma by fitness.
3. Encircling the prey: represents the circular area around the best solution (prey).
4. Hunting step: the encircling process comes to the second step involving hunting guided by the alpha wolf group.
5. Attacking the prey.
6. Steps 2 to 5 are then repeated until the maximum number of iterations is reached.

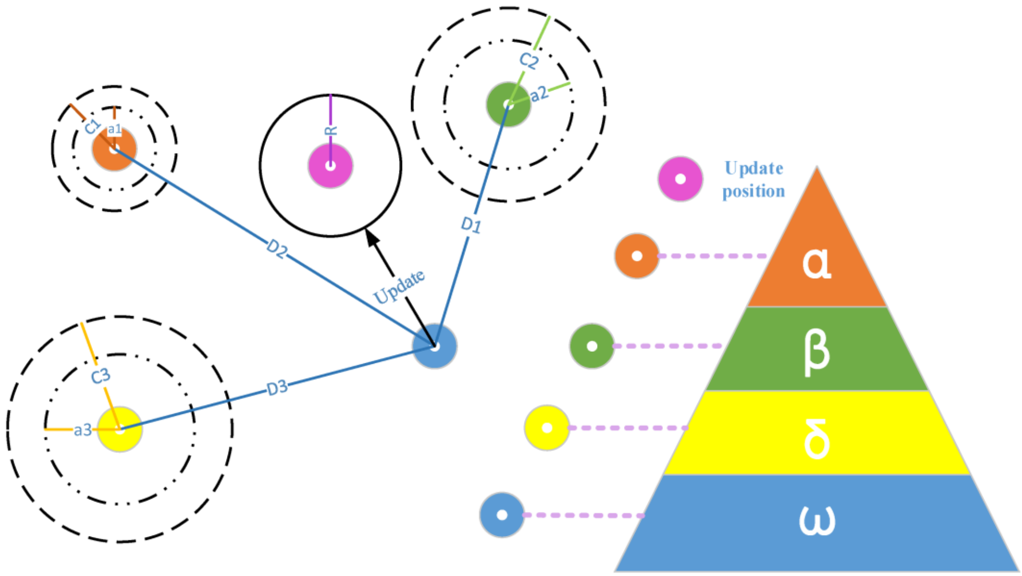

Figure 8. Grey Wolf algorithm hierarchy simple illustration.

Cuckoo Optimization Algorithm

The cuckoo optimization algorithm (COA) is a metaheuristic algorithm that draws its inspiration from nature and the cuckoo family of birds [57]. The primary drivers behind the creation of this new evolutionary optimization algorithm were the unique lifestyle of these birds and their traits in egg laying and breeding as shown in Figure 9. It is a brand new evolutionary algorithm that works well for problems involving continuous nonlinear optimization. Like other evolutionary techniques, COA begins with a starting population. The cuckoo optimization algorithm's foundation is the struggle to live among cuckoos [7]. Some of the cuckoos or their eggs may not survive the competition. The cuckoo groups that have survived travel to a more favorable environment where they begin to reproduce and lay eggs. The goal of the cuckoos' survival strategy is to eventually unite them into one society with the highest profit margins. The COA algorithm's capacity to resolve challenging, nonlinear, and nonconvex optimization problems has been demonstrated through the application to a variety of benchmark functions and a real-world scenario [58,59]. Faster convergence rates and a decrease in computing complexity are the COA's major characteristics [60]. It is a population-based algorithm, just like other evolutionary algorithms. Within the boundaries of the control parameter, the initial population is created at random. Then, until a stopping requirement is achieved, the levy flight operator is performed on each person.

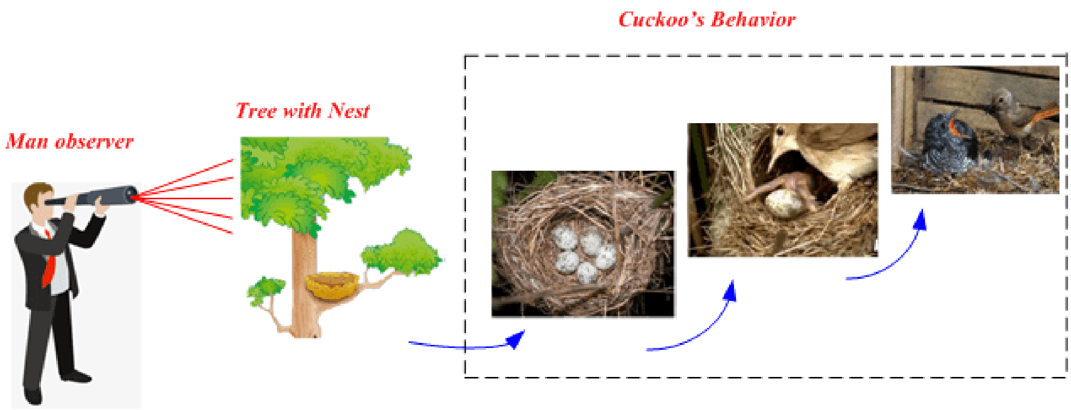

Figure 9. Cuckoo algorithm simple illustration.

3.3.4. Application of Heuristic Methods on AC MV Grids

Heuristic methods answer the best way to expand AC medium-voltage distribution networks for rural applications [8,9]. By limiting the number of branches that are linked to a substation, the notion of a minimal spanning tree is applied to choose the best distribution feeder routes (mixed-integer linear programming formulation). The maximum anticipated current that is absorbed by the loads is computed in order to determine the lowest thermal bound of the conductor caliber for the selected feeder routes. Networks of rural, residential, commercial, and industrial grid areas are primarily grouped in metropolitan regions, and electrical distribution networks are a part of the power system responsible for supplying electrical services to these areas [61]. In order to reduce investment costs, these grids are often designed with a radial configuration [13]. However, this configuration causes these grids to create large power losses (between 6 and 18 percent in the case of Colombia, for example) [62]. Due to the dispersion of the loads and low power consumption, this situation is less promising when rural distribution networks are examined [63]. Given that electricity services are globally viewed as public services, the regulatory bodies in charge of managing the electricity market have given distribution companies benefits, such as tax breaks or the addition of new factors to the formula for calculating energy prices, to encourage the provision of electrical services to rural areas. The objective is to raise the standard of living in rural communities [62]. An example of a mixed integer nonlinear programming (MINLP) problem is the challenge of electrical distribution expansion planning that is optimal [64]. It entails making the best choices for distribution line routes, choosing the size (caliber) of conductors, locating and sizing existing and new substations, and calculating the operational expenses of the grid over the planning horizon [65,66]. The distribution system planning problem has been addressed by a number of optimization methodologies; the majority of these approaches concentrate on metropolitan network applications, some of which are briefly discussed here. In [64], a model with an explicit radiality restriction discusses the subject of optimal growth planning in distribution networks. The resulting MINLP model was solved in the A mathematical programming language (AMPL) environment using a test feeder of 54 buses. The authors did not compare the reported numerical results to other optimization method-ologies based on exact models or metaheuristics, and the optimization model does not include different caliber options, which reduces complexity due to a smaller size of the solution space. Nevertheless, the reported numerical results satisfy the planning requirements. The authors of [67] recommended utilizing a genetic algorithm and an ant-colony optimizer to create distribution networks for medium-voltage applications. The methodology is tested on a small test feeder with 10 nodes, accounting for different diameter sizes. To demonstrate the methodology's efficacy, no comparison to precise or metaheuristic methodologies was undertaken. The authors of [68] describe the application of a genetic algorithm to the growth of medium-voltage distribution networks. Numerical results came from two test feeds with 13 and 30 buses, respectively. The grid topology has been set by the utility, which restricts the size of the solution space and transforms the original problem of planning distribution systems into a problem of choosing conductors in distribution networks in the most efficient manner. To design distribution networks taking into account a variety of objective functions, including investment costs, operating costs, and the amount of unsupplied energy, the authors introduced a multi-objective optimization model in [66]. A distribution network of 54 buses was utilized to numerically confirm the authors' theory; although, they only used the single-phase equivalent of the network in their formulation and did not compare it to exact or metaheuristic approaches. The authors of [69] developed an optimization methodology for planning distribution networks using a three-phase description of the network by combining a genetic algorithm with the tabu search algorithm (TSA). The provided numerical results demonstrated the usefulness of the TSA approach in determining the set of calibers that must be assigned to the network after the grid topology was defined using a heuristic graph generator. [63] provides a thorough analysis of the distribution network expansion planning problem, including a discussion of several optimization approaches and models. The authors examined the integration of distributed generation and energy storage technologies in the context of the conventional issue of expansion planning. The validation of numbers in a 54-bus system shows that the variances in the various parts of the final solution plan and energy storage technologies, such as generators, were considered. Using the earlier optimization models simplifies the planning of the distribution system. This reduction decreases the complexity of the optimization issue for the assessment of the power flow problem [12] by ignoring possible load and impedance imbalances as well as loads with single-phase, two-phase, and three-phase connections, including X- and Y-connections. In order to go beyond the prior research that is currently available in the literature, we provide an easily implementable optimization technique to create rural distribution grids that is composed of three steps and that can be used to single-phase or three-phase distribution grids [70]. The first step handles the route selection problem by articulating the problem of least spanning tree (linear integer programming model) to reduce the total length of the conductors [71,72].

The grid topology identified in the first stage is used in the second phase to compute the predicted current in each distribution line route, allowing the initial conductor calibers to be determined. The third stage establishes a TSA with the calibers selected in the previous stage in order to determine the best grid calibers that minimize the costs of purchasing conductors and the costs of energy losses over the anticipated horizon [72]. The main advantage of this heuristic approach is that the initial solution for a rural distribution grid obtained by applying the first two stages can be regarded as the upper bound of the objective function, i.e., the maximum admissible grid costs, which implies that the TSA seeks to reduce this function by utilizing and exploring the solution space. The TSA, in conjunction with a three-phase power-flow technique, is used to determine the root-mean-square (RMS) of the current in lines and the voltages in nodes, as well as the operational costs associated with power losses across the planning horizon [73]. In Figure 10, it is than reported the structure of 30 bus typical grid with nodes labeled in numbers.

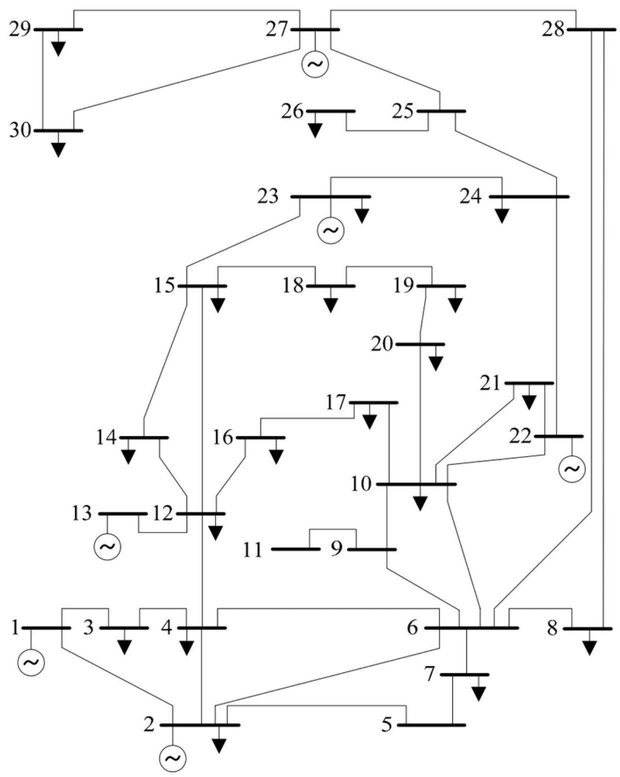

Figure 10. IEEE typical 30 bus rural grid.

4. Conclusions

In recent years, centralized production of electricity has decreased and decentralized generation (DG) has increased. Thus, the current power system is going through a time of significant change and development. Traditional distribution grids have a usually radial layout, unidirectional power flows towards the transformer, and are designed for "passive" operation. As a result, a sizable percentage of dispersed power is not intended for the distribution grid. According to what has been revealed, it is reasonable to assume that DG will assist in meeting a large portion of this increased demand, but only if the grid is ready, which means infrastructure upgrades are required in addition to the use of new technology for grid monitoring and regulation. However, determining how to optimize investments in accordance with the present grid's maximum utilization in terms of new RES and new installable loads is essential for the transition to the grid of the future. The optimal load flow problem seeks to resolve the grids in this difficult situation in order to instruct energy distribution businesses on how to make the greatest financial investments. Additionally, the time-to-market strategy places a restriction on obtaining the quickest load flow results, which is also driven by real-time energy market regulation. Several techniques have been presented in this review paper to show how much interest there is on this topic and what are the opportunities of research in this field. In our future work, we intend to investigate the development of a new load flow method called compensation admittance load flow (CALF) which aims to combine NR and GS advantages with a simple model of implementation. Furthermore, we intend to consider the possibility of developing more effective and efficient solution methods, relying on hybridization of metaheuristics, for optimal power flow [74–89].

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