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Back-Calculation Method for Estimation of Geomechanical Parameters in Numerical Modeling Based on In-Situ Measurements and Statistical Methods

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Abstract: An important element of numerical modeling for specific mining issues is the selection of model parameters. The incorrect determination of geomechanical parameters can result in significant calculation errors carried throughout the entire problem. This paper presents a method for determining effective geomechanical parameters for technological and residual pillars through the use of numerical modeling, specifically, back-calculation. This is based on the results of numerical simulations, measurement data (e.g., excavation convergence measurements), and statistical methods (a non-linear regression model with "dummy" variables). The result is that appropriate parameters of pillars are set out iteratively so that the displacements of selected points in the numerical model correspond (with some approximation) to the results of mine measurements. The procedure of determining pillar parameters is presented using a case study of one mining field in an underground copper mine, where the deposit is mined using the room and pillar system. Numerical calculations were performed using a Phase2 v. 8.0 program (Rocscience, Toronto, Canada), while statistical calculations used a Statistica computer program. The results of excavation convergence measurements performed in the analyzed mine have been applied. This paper shows that for the presented method, the resulting matching of theoretical values of convergence determined numerically for specified pillar parameters to in-situ results of convergence measurements, is very good (R^2 = 0.9896). This work exemplifies a set of the parameters of pillars for an elastic model of rock mass, but this method can also be applied to other models.

Keywords: back-calculation in geomechanics; rock mechanics; numerical modeling; room and pillar mining system; non-linear regression

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1. Introduction

In the past, rock mechanics was primarily described using empirical methods, based on experience and analytical methods giving a closed form of solution. As a result of the development of computer technology and computational methods, the most common methods used to solve geomechanics problems are numerical methods, which allow for the analysis of complex geometry (in close to real conditions) and different material behaviors. Both two- and three-dimensional tasks are solved. The most commonly used numerical calculation methods in geomechanics are the finite difference method, finite element method, and the boundary element method; though the discrete element method or hybrid methods (hybrid continuum/discrete methods) [1,2] are quite often used. Numerical modeling plays a very important role in the design of underground mines. It allows assessment of the current mining situation and predicts the behavior of rock mass along with the progress of work. Numerical methods in underground mining are used for, among others, stability analysis of mining excavations and the design of their support [3–10], to compare different types of support [11], for simulation of deposit excavation using various mining systems [12–20], for studies of the behavior and stability of pillars and remnants [21–29], for analysis of mining salt deposits with backfill and the determination of the backfilling

influence on the dynamics of deformation of the undermined rock mass [30], as well as for the assessment of seismic and rockburst hazard [20,22,27,28,31–35], etc.

A key element of any rock mass behavior modeling is the correct determination of its parameters and verification of obtained results. The introduction of incorrect parameters into the model will result in incorrect results. These parameters should be chosen so that the values set out as a result of numerical calculations correspond to actual values obtained on the basis of observations and mine measurements. The main source of input data for a numerical model are laboratory tests of rock samples and field tests. The impact of the sample size on the mechanical characteristics of rock samples is referred to as the "size effect" and is a significant issue [36,37]. The values of strength and deformation parameters set out by laboratory tests are often reduced to take into account the "size effect" between the sample (micro scale) and the rock mass (macro scale). The parameters of rock mass for numerical calculations can be set out using rock mass weakening factors chosen depending on the structure of the rock mass and the compressive strength of the rock samples or by using Hoek–Brown classification [38–40].

In practice, however, it happens that the obtained calculated values (e.g., of rock mass displacements) differ significantly from the values measured in mining conditions. The choice of suitable geomechanical parameters for numerical calculations is a particularly important issue in the case of technological and residual pillars in room and pillar mining systems and when considering their progressive destruction, when the mining front is moved. The yielding of pillars in numerical calculations is introduced by reducing their strength and deformation parameters. In order to make the best projection of the actual working conditions of pillars, the strength and deformation parameters are chosen most often using a method called "back-calculation". The required parameters are set out based on in-situ measurements in such a way, that a numerical model should fit best to the actual behavior of the object. Back-calculation based on measurements of displacements was initiated by Sakurai in 1981 [41] and is widely applied in geomechanics.

This paper presents a method for determining effective geomechanical parameters for technological and residual pillars used in numerical calculations for room and pillar mining systems using back-calculation, which is based on the results of numerical simulations, measurement data (e.g., excavation convergence measurements), and statistical methods. Numerical calculations have been performed as a plane strain problem using Phase2 v. 8.0 program (Rocscience, Toronto, Canada), which uses the finite element method, while the statistical analysis was done using Statistica. The above method is applied to one of the mining fields of the underground copper mines belonging to KGHM Polska Miedz SA. The analysis covered the D-IE mining field located in the Polkowice-Sieroszowice mine in south-western Poland. Copper ore deposit in the analyzed field was mined using the room and pillar system with a roof deflection. This work exemplifies how to set out the parameters of pillars for an elastic model of rock mass, but this method can also be used for other models.

2. The Method of Determination of Effective Geomechanical Parameters for Technological and Residual Pillars for Numerical Modeling Using Back-Calculation Based on the Results of Numerical Simulations, Measurement Data, and Statistical Methods

For the best projection of reality to determine the effective geomechanical parameters of technological and residual pillars applied to numerical calculations for room and pillar mining systems, back-calculation has been suggested based on the results of numerical simulations and measurement data, in which a non-linear regression model was applied with so-called dummy variables. This allows determining the values of the pillars parameters, for which the values of excavation convergence calculated in the numerical model will be most closely related to the results of convergence measured is-situ in this excavation. The model of non-linear regression with "dummy" variables is described by the formula:

$$Y = P_1Z_1 + P_2Z_2 + P_3Z_3 + \ldots + P_kZ_k, \quad (1)$$

where:

Y—explained variable (results of in-situ measurements of convergence for a chosen excavation),
$Z_1, \ldots, Z_k$—so-called "dummy" variables (with a value of 1 or 0),
$P_1, \ldots, P_k$—quadratic functions acting as parameters of a regression model with "dummy" variables.

The established effective geomechanical parameters of technological and residual pillars are independent variables of quadratic functions $P_1, \ldots, P_k$. The number of variables in a quadratic function depends on the number of determined pillar parameters. Therefore, the actual determined parameters of a regression model with dummy variables are independent variables of the quadratic function being the parameters of pillars. In a case when all technological and residual pillars are characterized by one parameter such as E then functions $P_1, \ldots, P_k$ take the form of a quadratic function of one variable:

$$P = \beta_0 + \beta_1 E + \beta_{11} E^2 \quad (2)$$

If we use more parameters to characterize the work of technological and residual pillars, the Formula (1) takes the form:

- for two parameters (e.g., E, c):

$$P = \beta_0 + \beta_1 E + \beta_2 c + \beta_{11} E^2 + \beta_{12} Ec + \beta_{22} c^2, \quad (3)$$

- for three parameters (e.g., E_1, E_2, c):

$$P = \beta_0 + \beta_1 E_1 + \beta_2 E_2 + \beta_3 c + \beta_{11} {E_1}^2 + \beta_{12} E_1 E_2 + \beta_{22} {E_2}^2 + \beta_{13} E_1 c + \beta_{33} c^2 + \beta_{23} E_2 \mathrm{c} \quad (4)$$

The parameters of quadratic functions: β_0, β_1, β_2, β_3, β_{11}, β_{12}, β_{13}, β_{11}, β_{22}, and β_{33} are set out by the least square method on the basis of convergence values defined in numerical simulations for different combinations of established values of the parameters characterizing the pillars (the matrix of the numerical experiment). The matrix of the numerical experiment is selected iteratively in such a way that the calculations are based on interpolation, i.e., the calculated parameters of pillars should be within a space defined by the matrix of the numerical experiment.

In order to match the results of the numerical experiment to measurement data, the regression model parameters are set out using backward stepwise regression. This involves eliminating from the model the effective geomechanical parameters of pillars, which had no significant effect on the optimal adjustment of convergence values (identified on the basis of numerical modeling) to the measurement data. The statistical significance of respective parameters is evaluated on the basis of confidence intervals or p-values for the assumed level of significance α. The regression model with dummy variables, which incorporates all the selected parameters of pillars, is individually deprived of those where $p > \alpha$ (and therefore those for which there was no reason to reject the hypothesis, that the established parameter is equal to 0). In the next step, a new regression model is set out with a number of parameters reduced by 1. Finally, when all parameters of the pillars are considered statistically significant (significantly different from 0), the process of elimination is interrupted and matching of the results of the predicted excavation convergence to measurement data is estimated based on the coefficient of determination R^2. On this basis, a decision is made about the values of the geomechanical parameters for technological and residual pillars adopted for further numerical calculations.

The procedure of determining effective geomechanical parameters for technological and residual pillars based on the results of numerical simulations and data from mine measurements using statistical methods is presented in Figure 1. According to Figure 1, the first step (1) includes the choice of strength and deformation parameters of pillars, which have a decisive impact on the value of calculated vertical displacements for a given model of rock mass. Then, after construction of a numerical model of the selected mining field, (2) one should specify the number of quested effective geomechanical parameters charac-

terizing the work of technological and residual pillars (3) and select the values of these parameters, for which numerical simulations will be carried out (construction of the numerical experiment matrix) (4). In the case of a numerical experiment matrix consisting of three parameters describing the work of the pillars (e.g., E_1, E_2, and c) and three different values adopted for each parameter, the number of numerical simulations will be 27. Simulations performed for a numerical experiment matrix are a basis for determining a convergence graphs model in a chosen excavation in the following steps of mining excavation (5). On the established convergence curves, one should choose representative points for the regression analysis (6) and determine the quadratic functions of an appropriate number of variables depending on the number of sought geomechanical parameters of the pillars (7). Knowing the quadratic functions serving as model parameters, one can specify the non-linear regression model with "dummy" variables (8) and determine the values of effective geomechanical parameters for the technological and residual pillars (9). The next step is to examine whether the received parameters have physical meaning (e.g., $c \geq 0$, $E \geq 0$, $\varphi \geq 0$) (10). If not, calculations should be terminated (11), as this may mean that the numerical model has not been properly constructed: too small density of the grid, insufficient accuracy of calculations, etc. When the calculated parameters have physical meaning, it should be checked whether they belong to one type of parameter (12). If not, then one should check whether the set parameters are statistically significant (14) and if so, it should be checked whether the parameters belonging to one type are significantly different from each other (13). If among the established pillar parameters are those for which there are no grounds to reject the statistical hypothesis that they are equal, one should eliminate one of them, reduce the number of sought pillar parameters, (15) and repeat the procedure. If all set-out parameters are significantly different from each other and in the case of parameters not belonging to one type, one should check their statistical significance (14). In a case when among the established parameters are those not statistically significant, one of them must be eliminated (15), and following the steps of the procedure should be repeated. However, when all pillar parameters have been considered statistically significant, one should check whether they are in a space defined by the experiment matrix (16). If the set parameters do not belong to this space, one should return to the construction of the experiment matrix (4) and change the range of selected values. Otherwise, the set of effective geomechanical parameters for technological and residual pillars can be applied for further numerical modeling, evaluating the matching of numerical experiment data to measurement data on the basis of the coefficient of determination R^2 (17). The procedure of selecting pillar parameters based on numerical simulations and measurement data can be performed repeatedly for different models of rock mass, searching for optimal parameters that will produce the best match of numerical experiment data to measurement data.

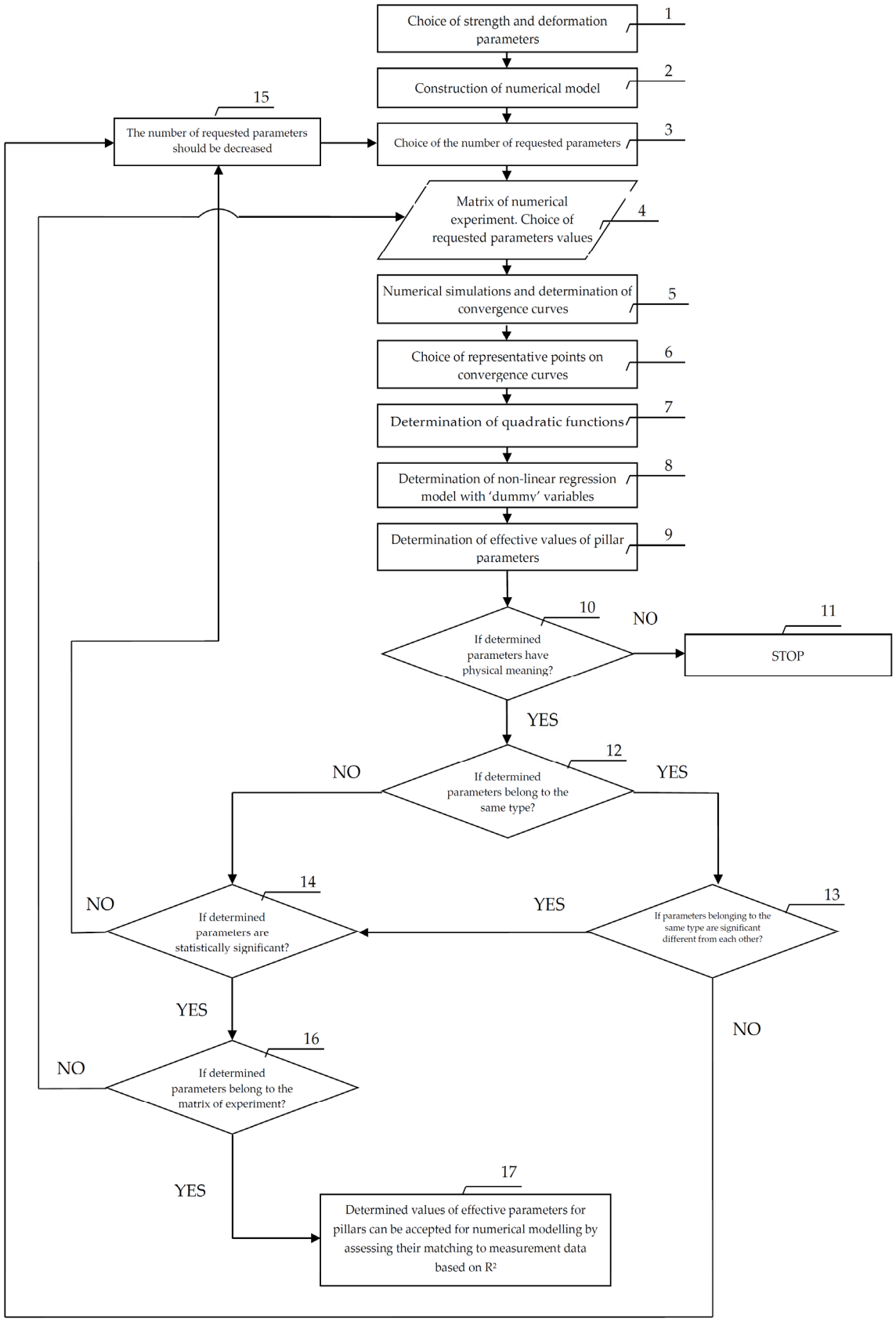


Figure 1. Method of determination of effective geomechanical pillar parameters on the basis of numerical simulations and measurement data using statistical methods.

3. Case Study in Polkowice-Sieroszowice Mine

3.1. Characteristics of Research Area

The method of determining effective geomechanical method parameters for technological and residual pillars for numerical modeling using back-calculation based on the results of numerical simulations, measurement data, and statistical methods are presented in the example of one mining field (D-IE) in Polkowice-Sieroszowice underground copper mine. The Polkowice-Sieroszowice mine belongs to KGHM Polska Miedz SA. It is located in the south-western part of Poland and mines the copper ore deposit which covers the central part of a geological unit known as Sudetic Monocline. Sudetic Monocline falls gently towards the northeast. It is constructed of Permian and Triassic sediments, which have a base made from Proterozoic crystalline rocks and Carboniferous sedimentary rocks. The deposit occurs in Permian formations, contacted by a dolomite limestone series, red sandstone, and Lower Permian limestone. Shaped in the form of a pseudobed of variable thickness (from 0.4 to approx. 20 m) and low gradient (approx. 4°), it lies at great depth (from 600 to 1400 m). The copper ore deposit is formed by an accumulation of sulfides, mainly chalcocite, bornite, and chalcopyrite. Sulfide mineralization occurs at the contact of red sandstone and Permian limestone layers. It includes carbonate rocks (dolomites and limestones), copper-bearing shales in the bottom part of the Permian limestone, and white sandstones. The deposits of the Polkowice-Sieroszowice mine subject to ore mineralization are mainly carbonate rocks and shales. Mining of the deposit is performed using a variety of room and pillar systems, depending on geological and mining conditions in a given mining field [29].

D-IE mining field was located in the mining area Sieroszowice I, in the Polkowice-Sieroszowice mine. It was a closing field explored since March 2005, where mining works were carried out in the vicinity of gobs. In February 2008, due to problems with maintaining roof stability, the remnant was left behind on the right side of the D-IE mining field (Figure 2).

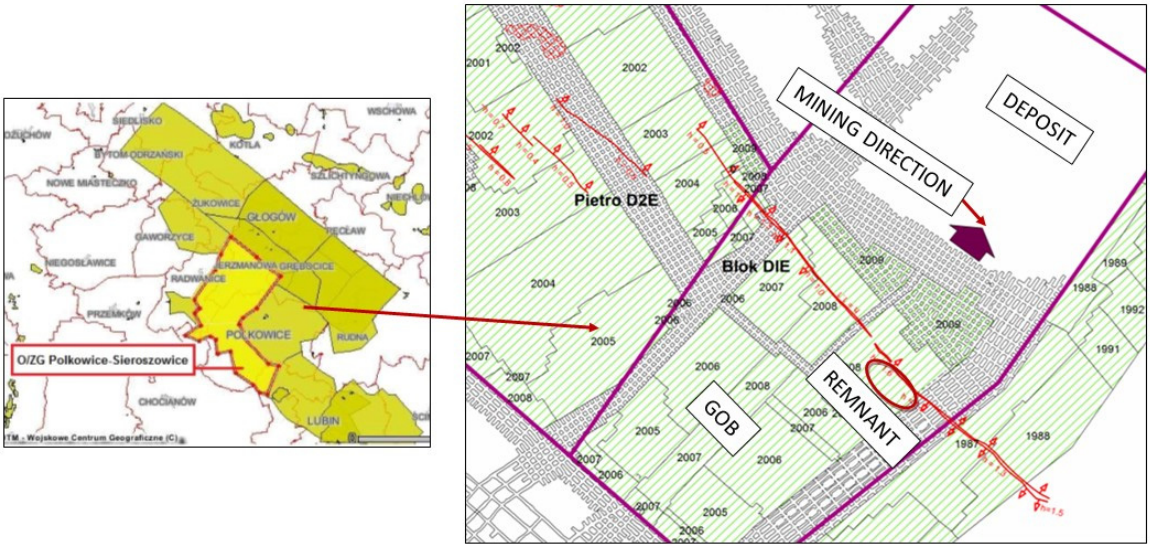


Figure 2. Mining field in the Polkowice-Sieroszowice mine where remnant has been left.

In the D-IE mining field, the deposit balance occurs in the lower part of a carbonate series of Permian limestones and the roof part of new red sandstone; it is comprised of grey quartz, fine-grained sandstone, loamy copper-bearing shale, and dolomite loamy shale, as well as streaked, dark grey, crypto-crystalline dolomite. The roof is made of rock layers, being part of a Permian limestone carbonate series, namely of calcareous dolomites with clear divisibility of bed (occurring in intervals of 0–2 m above the excavation roof), of concise calcareous dolomites with quite clear divisibility (occurring in intervals of 2–5 m above the excavation roof), and calcareous dolomites and dolomitic limestones with a bed structure (that occur in intervals of more than 5 m above the excavation roof). The carbonate

series is directly covered by anhydrites. The direct floor is built of grey sandstones of red Permian sediment rock. These are fine-grained quartz sandstones with a loamy bond, carbonate-loam bond, and locally anhydrite bond (in the eastern part of the area). The roof part of the sandstones, due to the larger amount of carbonate bond, is harder and more concise. The deposit is oriented towards NW-SE and its decline (2–3°) towards NE. The rock formation has marginal tectonic sensitivity. The height of the mined deposit is 2.0–2.8 m.

Until 2008 mining of the deposit in the discussed area was conducted using a room and pillar system with roof deflection and closing pillar (J-UGR-PS), while in 2008 the closing pillar was liquidated, and further works were performed using a room and pillar system with roof deflection (J-UG-PS). Exploration using the room and pillar system consists in cutting the deposit with rooms and strips with separation of technological pillars of a certain geometry, which protect the roof over the working area. The size of the pillars is chosen to provide its work in the post-critical state. In the D-IE mining field, the cutting work was carried out using technological pillars situated perpendicular to the mining front, with basic dimensions of 6 × 8 m (J-UGR-PS and J-UG-PS). In the discussed mining systems, the height of excavation in the cutting phase depends on the thickness of the deposit and the requirements of working machines and is not more than 4.5 m. The width of excavations does not exceed 7 m. The minimum size of the opening face of the mining front is equal to the sum of two strips and the length of two rows of pillars into undisturbed rock. In the D-IE mining field, the width of the opening was 4–5 strips. Along with the progress of the mining front, the technological pillars from the last row before gobs, depending on the degree of their disintegration, are adjusted or cleft into smaller ones. The resultant support pillars are adjusted to residual dimensions in elementary plots and then left in gobs. They work as supports to mitigate the deflection of roof layers. For D-IE field size of residual pillars left in the gobs amounted to approx. 5.2 m^2. In a room and pillar system with roof deflection and a closing pillar, the technological pillars are left in a separated part of the field, creating a gradually lengthening closing pillar. In one of the excavations of this pillar, a conveyor belt is assembled, which successively extends along with the progress of cutting. The width of a closing pillar depends on local geological and mining conditions and is generally 40–120 m [39]. The work of mining systems J-UGR-PS and J-UG-PS are shown in Figures 3 and 4.

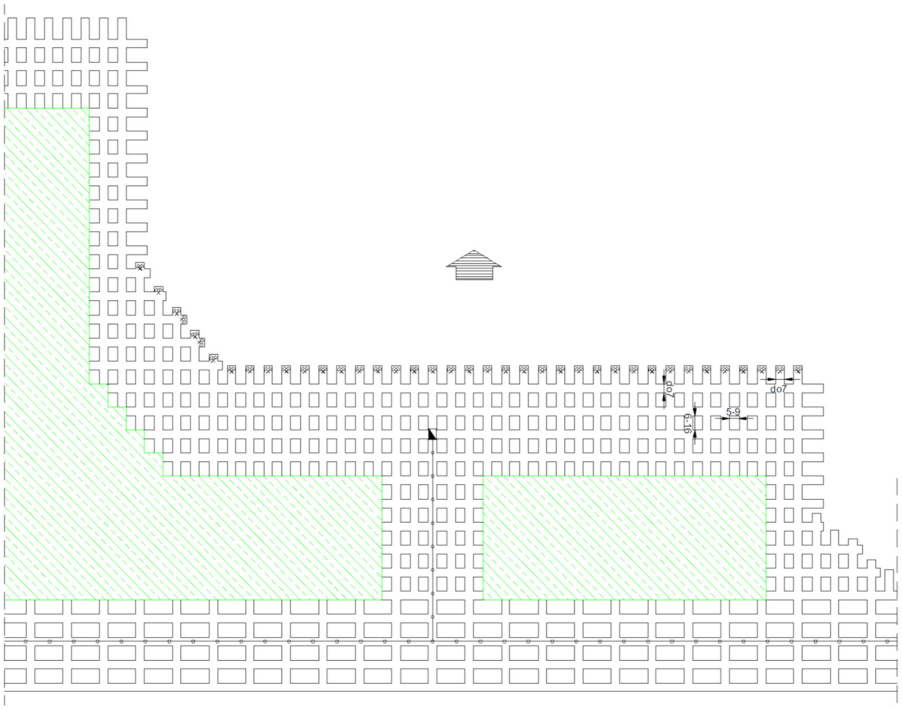


Figure 3. Room and pillar mining system with closing pillar J-UGR-PS [42].

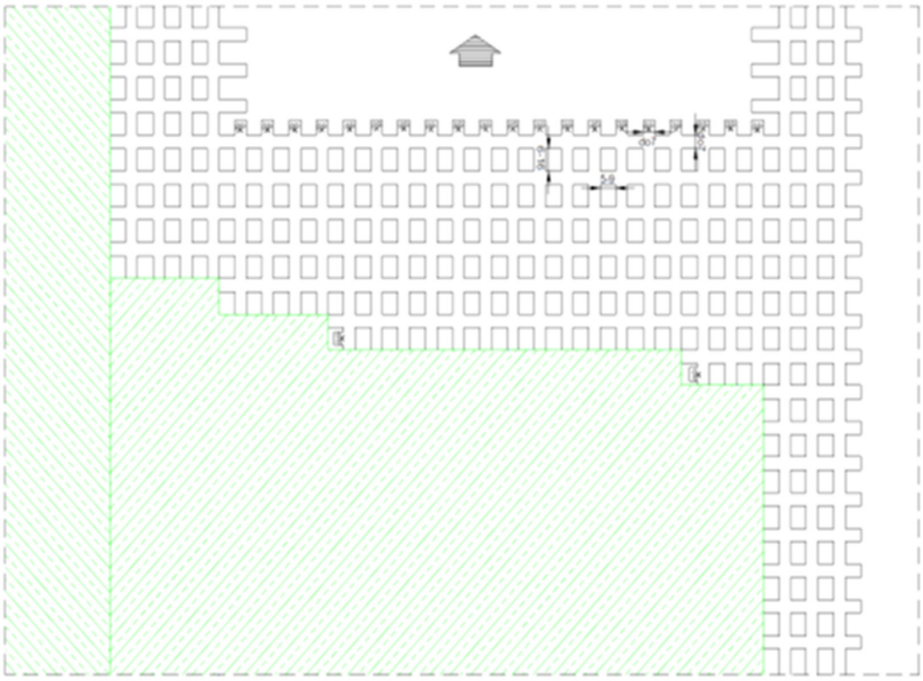


Figure 4. Room and pillar mining system with roof deflection J-UG-PS [42].

3.2. Characteristics of Numerical Modeling

Numerical calculations were performed in a plane strain state by a computer program Phase2 v. 8.0 (Rocscience, Toronto, ON, Canada) [43]. The computational model was a plate, which comprises the rock layers creating the rock mass (Figure 5). Construction of the mass resulted from geological recognition conducted in the analyzed field. The upper edge of the model was loaded with vertical pressure, replacing the influence of overlaying rocks. It was assumed that at the upper edge of the plate the stress should be equal to 17.657 MPa, corresponding to the value of the vertical stress set out for the D-IE mining field on the basis of data from the borehole S-294. The calculations considered the deadweight of rock layers. Horizontal stress values were determined on the basis of Poisson's ratio υ of a given rock layer. For the edges of the plate, the displacement edge conditions were assumed. At the lower edge of the model—lack of vertical displacements while at the side edges—no displacements in directions perpendicular to the surface of the edge. An applied grid of finite elements was composed of three nodal elements of triangular shape. In the central part of the plate, adjacent to the excavations, the grid was densified to improve the accuracy of numerical calculations.

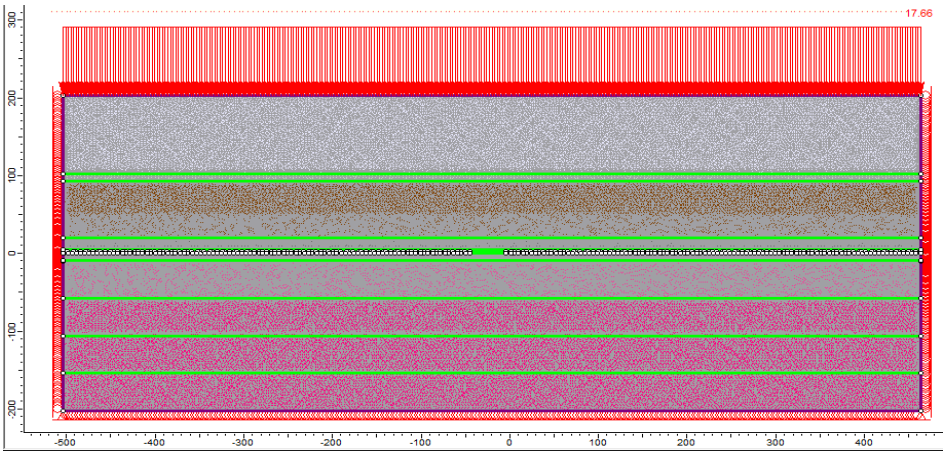


Figure 5. Calculation scheme for D-IE mining field.

The calculations were performed stepwise, simulating the mining carried out using a room and pillar system with parameters characteristic of an analyzed mining field (64 calculation steps). The first step covered the situation in the rock mass before the creation of mine excavations (Figure 6a). The second step consisted of the cutting of undisturbed rock into technological pillars (8 m in width) (Figure 6b). In the next steps, the size of the technological pillars was reduced to residual size (3 m width) and further technological pillars were cut out (Figure 6c). Cutting of the deposit was carried out with strips having dimensions 6 m width under the roof. Numerical simulations considered the width of the working area opening consisting of 5 strips.

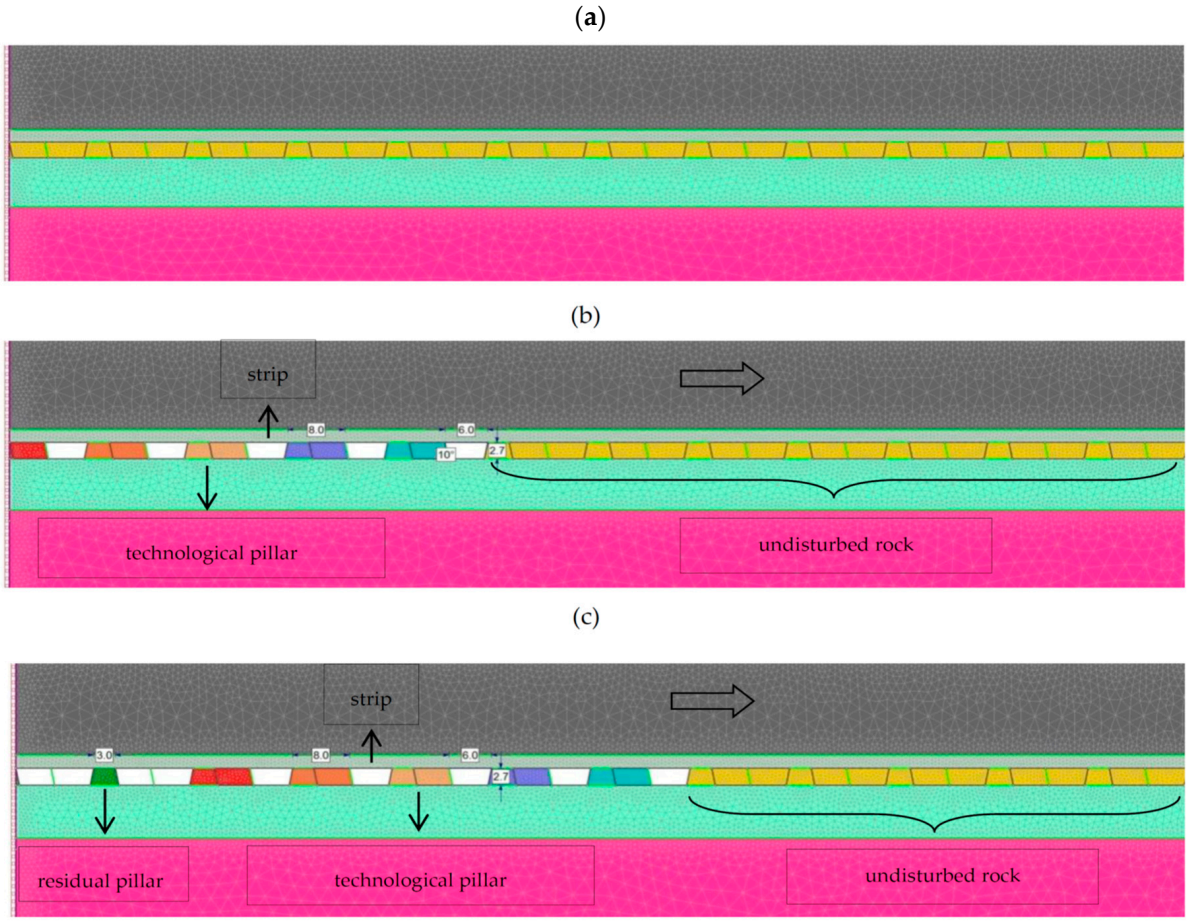


Figure 6. Simulation of room and pillar mining in the analyzed field: (**a**) step 1, (**b**) step 2, (**c**) step 4.

3.3. Determination of Parameters for Rocks and Rock Mass

The parameters of rock mass, which were assumed for numerical modeling, were determined based on Hoek–Brown classification. The results of laboratory tests of rock samples taken from geotechnical boreholes located in the analyzed mining field were applied. The averaged parameters of the rocks, which were designated in the laboratory for the D-IE field, are shown in Table 1, while the parameters of the rock mass are shown in Table 2.

Table 1. Averaged geomechanical parameters of rock.

Location	Name of Rock	h [m]	ρ [kg/dm^3]	R_c [MPa]	R_r [MPa]	E_s [MPa]	v [-]
ROOF	Main anhydrite	100.0	2.90	93.1	6.4	56,100	0.24
	Loamy anhydrite breccia	10.0	2.25	36.0	1.7	13,650	0.18
	Basic anhydrite	73.0	2.90	95.5	5.5	54,600	0.25
	Calcareous dolomite I	15.0	2.53	132.5	8.3	51,090	0.24
	Calcareous dolomite II	2.0	2.74	213.0	16.0	99,320	0.27
MINED DEPOSIT	Mined deposit	2.7	2.63	110.9	7.4	34,450	0.21
BOTTOM	Quartz sandstone I	8.2	2.12	22.1	1.4	8190	0.15
	Quartz sandstone II	194.5	1.95	16.7	0.7	6190	0.13

Table 2. Parameters of rock mass assumed for numerical modeling.

Location	Name of Rock	h [m]	E_s [MPa]	v [-]	σ_t [MPa]	c [MPa]	φ [°]
ROOF	Main anhydrite	100.0	41,110	0.24	0.746	6.967	38.66
	Loamy anhydrite breccia	10.0	7100	0.18	0.093	2.507	39.06
	Basic anhydrite	73.0	40,010	0.25	0.765	7.146	38.66
	Calcareous dolomite I	15.0	44,980	0.24	2.933	12.085	39.00
	Calcareous dolomite II	2.0	87,440	0.27	4.715	19.895	39.00
MINED DEPOSIT	Mined deposit	2.7	25,240	0.21	0.825	8.424	39.31
BOTTOM	Quartz sandstone I	8.2	4260	0.15	0.057	1.538	39.06
	Quartz sandstone II	194.5	3220	0.13	0.043	1.160	39.06

Marking in the above tables: h—thickness of rock layers, ρ—volume density, R_c—rock sample uniaxial compression strength, R_r—rock sample tensile strength, E_s—longitudinal elasticity modulus, v—Poisson coefficient σ_t—tensile strength of rock mass, c—coefficient of cohesion, ϕ—angle of internal friction.

3.4. Determination of Effective Geomechanical Parameters for Technological and Residual Pillars by Numerical Modeling Using Back-Calculation Based on the Results of Numerical Simulations, Measurement Data and Statistical Methods

The parameters of the technological and residual pillars described by an elastic model were determined using the procedure presented in the article. On the basis of in-situ tests carried out in the ZG Polkowice-Sieroszowice mine in 2002–2007, the course of the excavation convergence in time was determined for room and pillar mining systems with roof deflection and closing pillar. The pattern convergence curve, which was adopted as a reference for numerical calculations performed for the D-IE field, is shown in Figure 7. Each measurement of convergence made on a chosen test post was referred to as the mining step and the position of the working front line on a given day of measurement. This enabled the construction of a model that would fit the actual situation in the field and allow the determination of the effective geomechanical parameters of the technological and residual pillars.

The values of longitudinal elasticity modulus E were determined for an elastic model, (the parameter having a decisive impact on the numerically calculated values of displacements). Depending on the distance from the face of the mining front, the degree of pillar disintegration is varied. In the first step, the pillars were divided into three groups. Three technological pillars (located close to undisturbed rock) were characterized by longitudinal elasticity modulus E_1, the other two technological pillars with modulus E_2, and the residual pillars with modulus E_3 (Figure 8). It was assumed that $E_1 = E_2 + a$. The following values of E_1, E_2, and E_3 were adopted for the matrix of the numerical experiment:

E_1	E_2	E_3
2000 MPa	500 MPa	100 MPa
6000 MPa	1500 MPa	150 MPa
10,000 MPa	2500 MPa	200 MPa

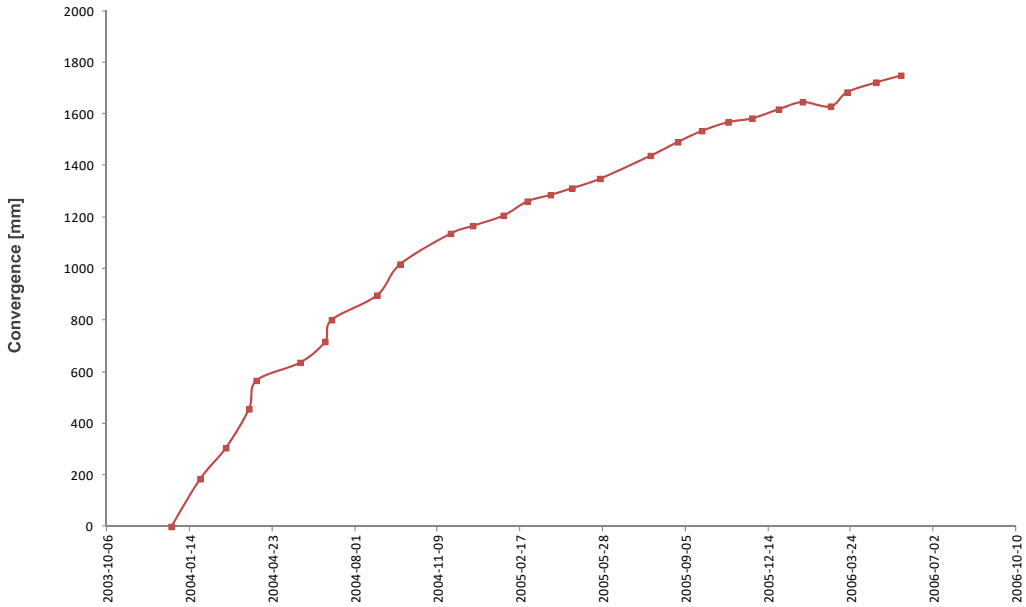


Figure 7. The results of excavation convergence measurements for J-UGR-PS mining system in Polkowice-Sieroszowice mine.

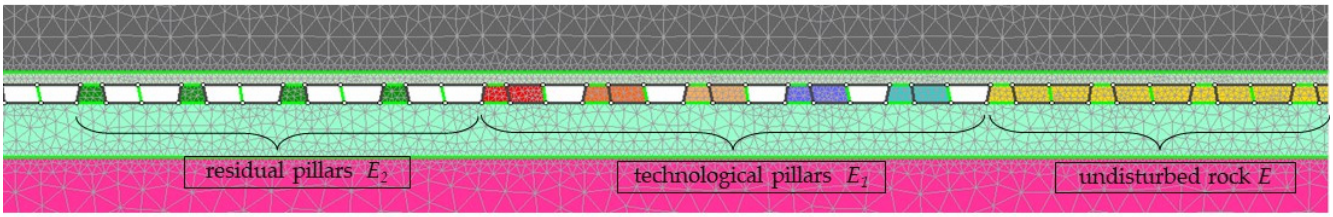


Figure 8. Fragment of the model of analyzed exploration field with marked division of pillars.

27 numerical simulations were performed for all combinations of assumed values of Young's modulus E. The convergence of the selected excavation in subsequent steps of the executed mining for a few chosen cases is presented in Figure 9.

Using the method of surface regression with Statistica v. 10 program, the quadratic function parameters of three variables were determined for selected points that are included in a non-linear regression model with dummy variables:

$$\begin{gathered} Y = Z_1(-145.75967 + 0.01(E_2 + a) - 0.00000046(E_2 + a)^2 + 0.05718E_2 - 0.00001043{E_2}^2 - 0.1659E_3 + 0.00054{E_3}^2 \\ 0.00000100(E_2 + a)E_2 + 0.00000087(E_2 + a)E_3 + 0.00000142E_2E_3) + \ldots + Z_9(-4620.2027 + 0.0405(E_2 + a) - \\ 0.00000188(E_2 + a)^2 + 0.36882E_2 - 0.00006627{E_2}^2 + 22.9864E_3 - 0.04617{E_3}^2 - 0.00000146(E_2 + a)E_2 + \\ 0.00002395(E_2 + a)E_3 - 0.00019174E_2E_3) \end{gathered} \quad (5)$$

The values of a, E_2, and E_3 calculated using Statistica v. 10 are shown in Table 3.

$$E_1 = E_2 + a = 725.066 + 5541.145 = 6266.211 \text{ MPa}$$

Based on confidence intervals, statistical inference was performed to check whether E_1 and E_2 are significantly different from each other, namely if the added value of a is different from zero. The assumed level of significance $\alpha = 5\%$.

H_0: $a = 0$

H_1: $a \neq 0$

Table 3. Parameters of pillars determined for the elastic model using Statistica v. 10.

Parameter	Estimation	*p*	Lower Confidence Limit	Upper Confidence Limit
E_2	725.066	0.739776	−4582.5	6032.6
a	5541.145	0.893210	−95,323.3	106,405.6
E_3	135.808	0.000182	122.6	209.0

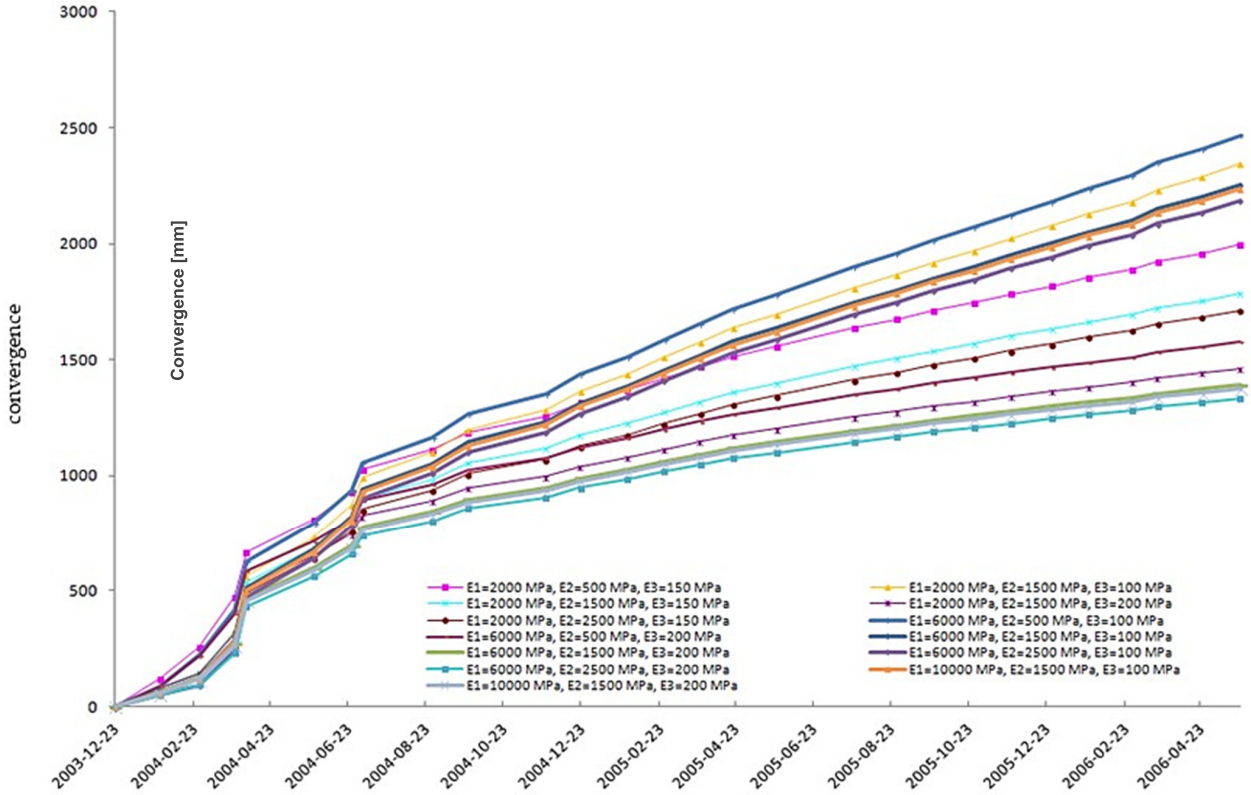


Figure 9. Chart of convergence determined numerically for chosen excavation in D-IE mining field for different combinations of parameter values assumed for pillars.

The $a = 0$ parameter is within 95% of the confidence interval, which indicates that there is no basis to reject the zero hypothesis H_0 (Figure 10). Therefore, it cannot be stated that at the level of significance $\alpha = 5\%$ $E_1 \neq E_2$. In addition, statistical inference based on test probability of p-value ($p = 0.893210 > \alpha = 0.05$) also indicates that there is no basis to reject H0: $a = 0$ hypothesis. In such a situation, it is assumed that $E_1 = E_2$ and the number of sought parameters of pillars was reduced to two. Technological pillars were characterized by longitudinal elasticity modulus E1, while residual pillars with longitudinal elasticity modulus E_2 (Figure 11). It was assumed, that $E_1 = E_2 + a$ and the following values of E_1 and E_2 were applied to the matrix of the numerical experiment:

E_1	E_2
200 MPa	80 MPa
400 MPa	120 MPa
600 MPa	160 MPa

There were nine numerical simulations. Convergence in the selected excavation is shown in Figure 12 in subsequent steps.

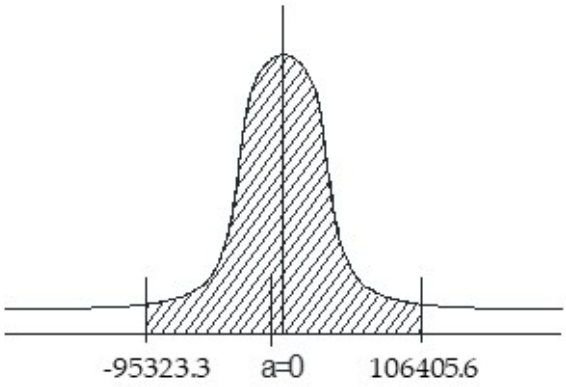


Figure 10. Intervals of confidence for a.

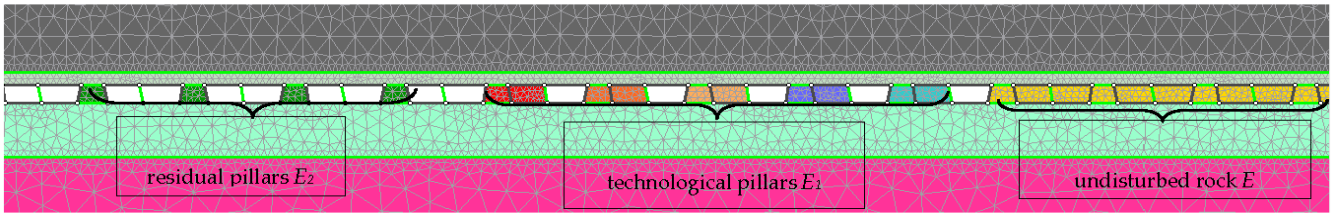


Figure 11. Fragment of the model of analyzed mining field with marked division of pillars.

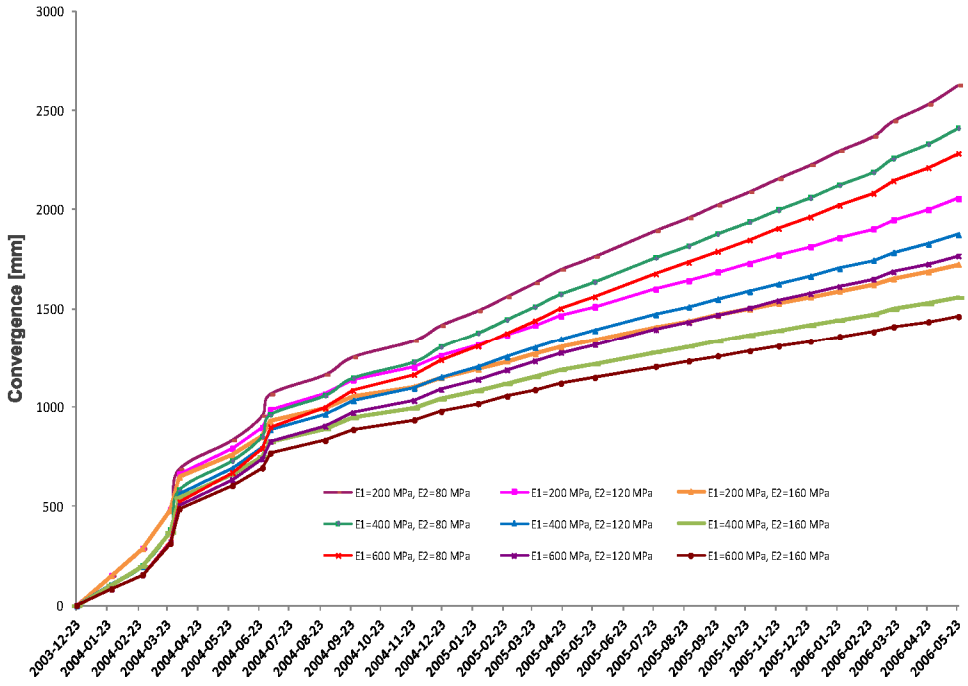


Figure 12. Results of convergence calculated numerically for the matrix of the experiment.

Using the method of surface regression with Statistica v. 10 program, the quadratic function parameters of two variables were determined for selected points that are included in the non-linear regression model with dummy variables:

$$Y = Z_1(-220.077 + 0.424(E_2 + a) - 0.000322(E_2 + a)^2 - 0.0178E_2 + 0.00005993{E_2}^2 + 0.00000610(E_2 + a)E_2) + \ldots + Z_{16}(-4746.12 + 1.84(E_2 + a) - 0.000983(E_2 + a)^2 + 28.539E_2 - 0.06987090{E_2}^2 - 0.00252617(E_2 + a)E_2) \quad (6)$$

The values of a and E_2 *were* determined using Statistica v.10 presented in Table 4.

$$E_1 = E_2 + a = 136.931 + 174.424 = 311.355 \text{ MPa}$$

Table 4. Pillar parameters set for the elastic model using the Statistica v. 10 program.

Parameter	Estimation	p	Lower Confidence Limit	Upper Confidence Limit
E_2	136.931	0.0000000000119	122.134	151.729
a	174.424	0.0000162422638	238.441	391.832

Statistical inference carried out based on confidence intervals was performed to check whether E_1 and E_2 are significantly different from each other, namely if the added value of a is different from zero. The assumed level of confidence α = 5%.

H_0: $a = 0$

H_1: $a \neq 0$

The parameter $a = 0$ is located outside the 95% of confidence interval, which leads to rejection of zero hypothesis H_0 (Figure 13). It can therefore be stated that at significance level α = 5% $E_1 \neq E_2$. Statistical inference based on test probability of p-value ($p = 0.010470 < \alpha = 0.05$) also suggests rejection of hypothesis H_0: $a = 0$.

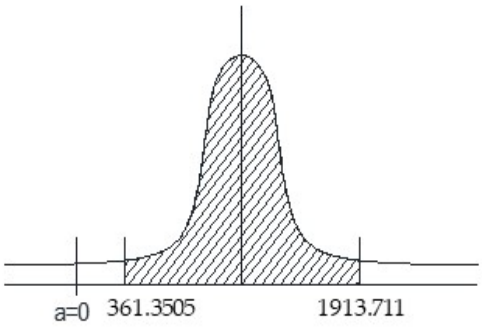


Figure 13. Confidence intervals for a.

The final values of E_1 and E_2 parameters determined using Statistica v.10 are shown in Table 5.

Table 5. Pillar parameters set for the elastic model using the Statistica v. 10 program.

Parameter	Estimation	p	Lower Confidence Limit	Upper Confidence Limit
E_1	311.355	0.0000364562557	198.823	423.887
E_2	136.931	0.0000000000119	122.134	151.729

On the basis of p-values it can be concluded that E_1 and E_2 parameters are statistically significant (E_1: $p = 0.0000364562557 < \alpha = 0.05$; E_2: $p = 0.0000000000119 < \alpha = 0.05$). The effective parameters of pillars E_1 = 311.355 MPa and E_2 = 136.931 MPa are also located in the space defined by the matrix of the numerical experiment. The resulting matching of theoretical values of convergence determined for specified pillar parameters, to in-situ results of convergence measurements is shown in Figure 14.

Reduced values of elasticity modulus for technological and residual pillars were adopted for further numerical modeling of the geomechanical situation in the D-IE mining field. The final validation of the numerical models was based on the results of convergence measurements of excavations carried out in this field.

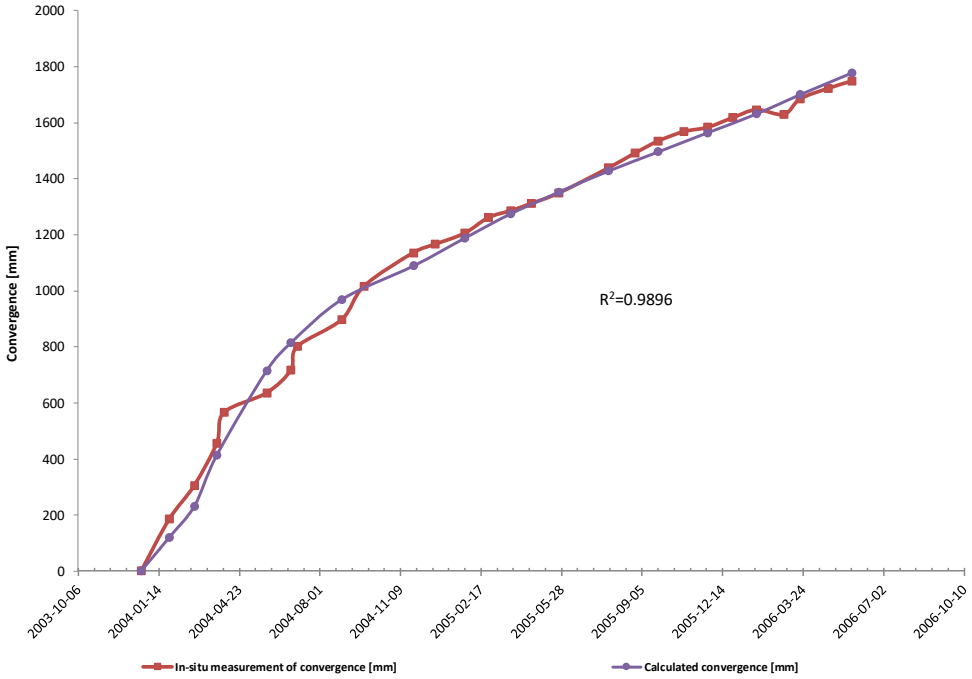


Figure 14. Matching of results of in-situ convergence measurements in Polkowice-Sieroszowice mine to convergence values obtained for effective parameters of pillars determined using statistical methods (elastic model).

4. Conclusions

In this paper, the innovative method for determining the geomechanical parameters of technological and residual pillars by numerical calculations using back-calculation based on the results of numerical simulations, in-situ measurement data, and a non-linear regression model with dummy variables, has been presented. The method has the potential to be useful in numerical modeling for specific mining issues due to the importance of model parameter selection. Incorrect determination of the geomechanical parameters results in significant calculation errors throughout the entire analyzed problem. These parameters should be chosen so that the values set out as a result of numerical calculations correspond to actual values obtained on the basis of observations and mine measurements.

In the case of room and pillar mining system modeling, a particularly difficult issue is the choice of parameters for technological and residual pillars (considering the progress of their destruction when a mining front is moved). Incorrect determination of the geomechanical parameters of pillars results in significant calculation errors throughout the whole analyzed mining field. The presented method enables the determination of pillar parameter values, for which the calculated numerical values of convergence of a chosen excavation in the model would be most closely related to the results of excavation convergence measurements carried out in mining conditions. This method facilitates estimation of the matching of predicted excavation convergence results to measurement data (based on the coefficient of determination, R^2. The case study presented in this paper proved that by using the described method, the matching between results of in-situ convergence measurements in mines to convergence values obtained for effective parameters of pillars determined using statistical methods is very good, with R^2 is equal to 0.9896.

The method has a lot of advantages and can be used to calibrate the numerical models of other engineering problems, e.g., for the determination of gob parameters for different mining systems. The method allows the determination of pillar parameters for different models of rock mass. In the case of an elastic-plastic model, it enables the description of the post-destructive part of stress-strain characteristics using the appropriate values of strength parameters such as the Coulomb–Mohr hypothesis. This is of vital importance for the accuracy of obtained results of numerical modeling and thus for the degree of real

situation representation (e.g., a real situation in a modeled mining area). An appropriately verified numerical model allows for accurate analysis of the current situation in an analyzed region, for forecasting rock mass behavior during the progress of work, and for identification of dangerous phenomena which might create a threat to people working in a particular area. Future research will include the application of the method described in this manuscript to the numerical analysis of different mining situations.

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