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Design of Batteries for a Hybrid Propulsion System of a Training Aircraft

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Abstract: In this article, we propose the parameters of a battery that would be suitable for the conceptual design of a small training aircraft. The mass design of the battery is based on the requirements for real training flights performed by students in pilot training. Such a serial hybrid propulsion aircraft could be used in our UNIZA aviation, training and education center for pilot training. Due to socio-political pressures in reducing emissions generated by vehicles, there has also been massive research in the aviation industry in the field of hybrid and electric aircraft propulsion. In the introduction, the article deals with the energy sources used in aircraft propulsion. In hybrid propulsion, a combination of aviation fuel and electricity is used as the energy source. The required total energy must choose a suitable combination of these two energy sources. The biggest drawback of batteries that can be used in hybrid systems is their low energy density. Low energy density means that larger and heavier batteries need to be used to achieve the required performance, which is their main disadvantage. Therefore, it is necessary to find a suitable compromise between the hybrid's percentage, i.e., the ratio between conventional and electric drive. We applied the hybrid aircraft system's calculations to the real training flights to determine the necessary parameters of the hybrid aircraft suitable for pilot training. This calculation will help in obtaining an idea of the basic parameters of the hybrid drive and the battery parameters, which are necessary for particular applications in the training aircraft. The performed calculations of the hybrid configuration and, especially, the determination of the battery of the hybrid propulsion parameters provide the basic information necessary for the design of the hybrid system of a small training aircraft. These outputs can be used to determine the parameters of batteries that would be used in hybrid systems. A limiting factor to consider with hybrid aircrafts is that the aircraft must be charged on the ground before the flight, which poses interesting logistical and infrastructure problems at the airport.

Keywords: battery; emissions; hybrid propulsion; hybrid variables

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1. Introduction

With the constant growth of air transport, fuel consumption is also growing. The increasing consumption of fuel is also increasing the number of harmful emissions that are produced. Several research studies address the issue of reducing greenhouse gas emissions. When it comes to emissions, carbon dioxide (CO_2) emissions are often mentioned [1]. Therefore, there are great demands for continuous reduction of emissions and the effort to minimize the environment's impact. There is a constant development in the field of construction of modern and efficient aircraft propulsion units. Therefore, manufacturers of current aircraft propulsion units are being pushed to develop aircraft units that will produce as few emissions as possible. The main areas addressed by the aircraft engine manufacturer and aircraft manufacturers to achieve a reduction in carbon dioxide emissions are improvements in jet engine design, improvements in aviation fuels and developments in hybrid and electric propulsion systems.

In the last decade, there have been visible changes in the environment, mainly caused by human activity. These changes have been visible, not only to scientists and experts working on this issue, but have already been visible to the wider public. We are increasingly encountering extreme weather changes, massive fires, floods, etc., which are becoming more and more visible [2]. Therefore, environmental protection should concern each of us, not just a few "Greenpeace" activists. There are attempts to lock in and work together on the conceptual change that is demanded, which should become a top priority for politicians around the world.

This socio-political pressure in Europe has been addressed in two documents: the Advisory Council for Aviation Research and Innovation Agenda in Europe, Flightpath 2050 and the Strategic Research and Innovation Agenda (ACARE) [3,4]. In The United States, the Aeronautics Strategic Implementation Plan has been issued by The National Aeronautics and Space Administration [5].

In recent years, there has been significant development of hybrid and electric passenger cars. Research in the alternative propulsion of automobiles has provided valuable information that can be used in the alternative propulsion of aircrafts [6]. The hybrid-electric drive system is relatively operationally flexible, which is ensured thanks to a larger number of components. The combination of fuel and battery sources offers more options for controlling the propulsion system for different phases of flight [7]. At the same time, this type of drive system reduces energy consumption compared to traditional drive systems. However, reducing energy consumption negatively affects the overall weight of the aircraft and also makes the operation of such a system more complicated. Therefore, proper control of these two hybrid drive subsystems, i.e., electrical and combustion components, is needed to meet environmental requirements and reduce fuel consumption [8]. Non-renewable energy sources have an almost similar negative impact on the environment.

The potential environmental benefits of hybrid electric turboprops have been addressed by Voskuijl et al. [9]. This research provided the determination of range and power limits for a given type of aircraft (70 passengers, range 1528 km).

The potential of electric and hybrid propulsion systems for commercial flying was analyzed in their study [10]. The authors compared different architectures of electric propulsion units with conventional turboprop propulsion of an ATR 72 aircraft.

Research into the effect of an auxiliary electrical system on a jet engine's performance for an A320 aircraft was conducted [11].

Karadotchev et al. worked on the design of the configuration of the electric aircraft, which was based on the Airbus A320 aircraft. In their research, they proposed three different configurations, which included structural power composites, slender wings and distributed propulsion system [12].

Hybrid propulsion applications in automobiles mainly provide knowledge about hybrid propulsion for aviation. The connections that can be used in the automotive and aerospace applications of hybrid drives were presented in research about hybrid-electric propulsion for automotive and aviation applications [13]. Fuel savings of up to 50% and 10% were calculated for the ultralight aircraft and intercity transport aircraft within the mission profiles.

The authors performed a detailed analysis to determine the payload, range, and degree of hybridization in their article "Approach to the Weight Estimation in the Conceptual Design of Hybrid-Electric-Powered Unconventional Regional Aircraft" [14].

The goals and motivations for which we are dealing with the possible introduction of hybrid aircraft (or in the future fully electric aircraft) are such that we want to respond to the current development trend in this area. The enthusiasm for addressing this topic contributes to a cleaner and more efficient air transport and flight training industry in the future. These goals are based on the Strategic Transport Development Plan of the Slovak Republic until 2030, the national goals of the Integrated National Energy and Climate Plan for 2021–2030, and the strategies and long-term goals of the European Union, and are part of them. The transport targets aim to enable the wider use of alternative propulsion for sustainable, green and intelligent transport.

From our point of view, hybrid aircraft have the potential to operate in our university for flight training. It makes sense to introduce a hybrid aircraft configuration in a training aircraft mainly for controlled efficiency output and noise footprint to reduce emissions and operating costs. Thus, as an organization providing flight training, we would be sustainable. Reducing noise emissions at the airport is also important. Žilina Airport is located close to several built-up areas, which are negatively affected by traffic at the airport. During the training flights (especially at night), we encountered several negative experiences from citizens (complaints, dazzling crews with lasers, etc.). The use of hybrid or fully electric aircraft would improve citizens' quality of life around the airport by reducing noise pollution.

Our initial research in this area is therefore focused on analyzing the possible use of hybrid aircraft for training flights. For us, the hybrid aircraft propulsion system provides an interesting range and endurance for flights over longer distances, higher speeds and, thus, longer training flights. In the future, in addition to the use of hybrid aircraft, fully electric aircraft would be used in our institution, which would be used for shorter training flights around the airport.

Therefore, this article represents initial research in this area, which we would like to aim for in the future to align with the mentioned goals based on the Strategic Development Plan of the Slovak Republic.

2. Energy Sources Used for Aircraft Propulsion

The most commonly used aviation fuels include AVGAS 100 LL and Jet A-1. AVGAS 100 LL aviation gasoline is used for petrol piston engines. Jet A-1 kerosene is the most widespread aviation fuel intended for turbine-type jet engines. Both of these aviation fuels reliably provide the predictable and constant amount of energy needed to fly. The specific energy of AVGAS 100 LL aviation gasoline is approximately 12.22 kWh/kg, and the Jet A-1 has a specific energy of 11.99 kWh/kg. Specific energy expresses how much energy can be obtained from a source at a certain weight. Higher specific energy means that it is possible to obtain more energy from this source or that the source's weight will be less with the exact energy requirement [15].

What about the specific energy of the batteries that could be used in a hybrid aircraft? The specific energy of a Ni-MH battery ranges from 0.06 to 0.12 kWh/kg. With a NiCd-based battery, the specific energy is even lower in values from 0.04 to 0.06 kWh/kg. The highest specific energy is achieved by Li-ion batteries, reaching values of 0.15 to 0.25 kWh/kg. Individual cells typically operate in the range of 2.5 V to 4.2 V, which is approximately three times the NiCd or NiMH cells, requiring fewer cells for a battery with the same voltage. The disadvantages of these cells are the requirements for accurate charging (if the charging and discharging values are not observed, the cell may be destroyed), the Li-pol cell packaging is in the form of a bag made of relatively fragile aluminum foil, which can be easily damaged (fire hazard), degradation and loss of capacity at high temperature and when discharging below 2 V (or below 2.8 V—depending on the type of materials used) [16].

From the above findings, it can be said that the specific energy of the batteries is about two orders of magnitude lower than that of aviation fuels. It is also necessary to consider the parameter of batteries that affect the specific energy of batteries, such as the chemical composition of the battery, state of charge, charging and discharging rate, and the number of charging cycles. Due to the amount of energy required and the weight, it is currently more advantageous to use aviation fuel. Thus, unless there is some unexpected rapid development of the batteries, it is improbable that the Li-ion battery will reach a specific energy value close to aviation fuel [17,18].

3. Hybrid Aircraft Propulsion

Hybrid electric propulsion systems can be included in the field of alternative propulsion technologies. Alternative propulsion technologies also include other aircraft propulsion methods that are different from conventional propulsion. In this category, we can include propulsion systems using alternative fuels or using solar energy [19].

Thus, an electric hybrid propulsion system can be called a propulsion system that uses a conventional internal combustion engine and an electric motor for propulsion. In a hybrid-electric propulsion system, aviation fuel or batteries are used as the energy source. The most commonly used hybrid propulsion systems are parallel and series systems [20,21].

A parallel hybrid system uses a combustion engine operating in parallel with an electric motor, while a series hybrid system consists of a co-operating internal combustion engine and an electric motor arranged in series.

3.1. Parallel Hybrid Propulsion Architecture

The parallel architecture of a hybrid electric drive is one in which the propeller can be driven by either an electric motor or an internal combustion engine. A parallel hybrid system provides more complex options than a serial system. For this drive architecture, a gearbox must be included in the system. This transmission will allow the aircraft propeller to be powered simultaneously by an internal combustion engine or an electric motor. The internal combustion engine works at the point of its maximum efficiency, but only in a certain mode. Therefore, with such a hybrid system, it is desirable for the internal combustion engine to operate at its maximum efficiency point and the electric motor to supply the remaining energy required for the current power requirements of the system. Thus, a parallel hybrid system (see Figure 1) provides the advantage that the electric motor can be less powerful (i.e., also smaller and lighter) than with a series architecture. Thus, the electric motor itself does not have to provide the required maximum power to drive the propeller. An electric generator is not necessary in such an architecture, because it is not necessary to convert the mechanical energy of the internal combustion engine into electrical energy. However, the mentioned (more complex) mechanical transmission is necessary, which is necessary for the distribution of mechanical energy between the internal combustion engine, the electro-motor, and the propeller.

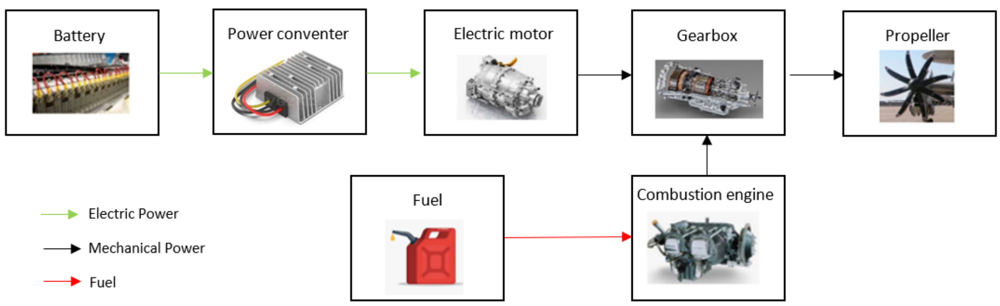

Figure 1. Parallel hybrid architecture. Source: Processed by authors.

3.2. Serial Hybrid Propulsion Architecture

In the series hybrid drive architecture, the internal combustion engine operates at a constant speed and drives an electric generator to which it is mechanically connected (see Figure 2). The electric generator then supplies electricity to the system. The series hybrid system also contains batteries, which also provide electricity to the electric motor. Thus, electric energy from batteries and/or a generator is used to supply an electric motor (AC), which must be converted to an alternating current. This is achieved by using a converter, sometimes referred to as inventories. The thus-adjusted alternating electric current supplies the electric motor, which subsequently drives the propeller.

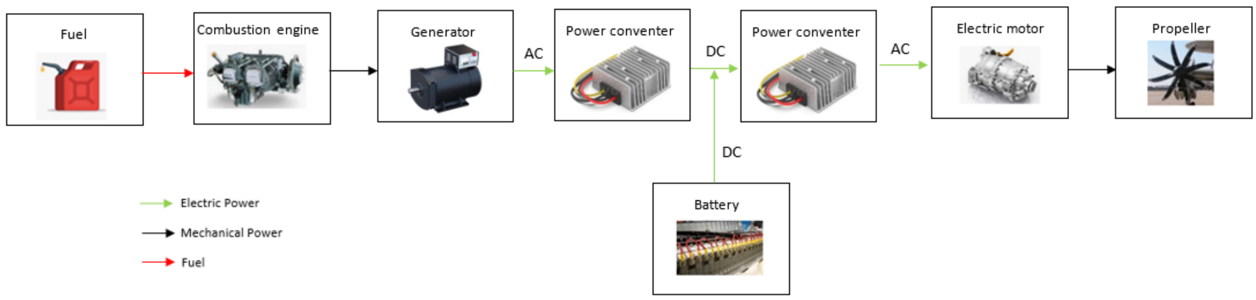

Figure 2. Serial hybrid architecture. Source: Processed by authors.

Series hybrid-electric architecture features great flexibility that enables different operation strategies. This means that, for example, when an aircraft takes off when a lot of energy is needed to drive an electric motor, it is supplied by an internal combustion engine and also by batteries. Conversely, when a large amount of energy is not required, the electric motor can be powered either from a battery (in this case, the aircraft operates in full electric mode) or from a generator system powered by an internal combustion engine. If the power consumption is even lower, the battery can be recharged with excess generator power.

One of the main advantages of the series system is that the internal combustion engine can operate at a constant load, when maximum thermodynamic efficiency can be achieved (compared to the traditional mode of operation of an internal combustion engine). In this way, the internal combustion engine can be optimized for a given fraction of the total required maximum rated power.

The advantage of the serial hybrid system is also that its architecture is very simple, the propeller is driven by an electric motor, and it can be designed to have a low operating speed. The electromotor designed in this way frees the drive system of complex, heavy and inefficient reduction gears. The standard configuration of the drive also allows the placement of electric motors at different locations on the aircraft (for example, along the wingspan). Such a layout of the hybrid propulsion system allows only this type of hybrid architecture.

However, in the case of a serial hybrid system, it is also necessary to mention its disadvantages. Perhaps the biggest disadvantage is the fact that it is necessary to have a generator in the system to convert the mechanical energy of the internal combustion engine into electrical energy. This fact has a negative effect mainly on the overall volume, weight and efficiency of the hybrid system.

3.3. Series-Parallel Architecture

The third drive architecture we will mention is the series-parallel architecture, which is also called the mixed architecture. As the name implies, this architecture combines serial and parallel architecture. Figure 3 shows a possible solution for a serial-parallel architecture. However, there are several technical possibilities for arranging such a drive architecture, which are used in particular in the automotive field. The main advantage of such an architecture is that it allows having a smaller internal combustion engine that operates in optimal mode and directly during the air mission allows charging batteries without traction transfer, which traditional parallel architectures do not allow. Such a drive architecture is widely used in the automotive industry, but due to its complexity and weight, it is not possible to apply in the field of aviation.

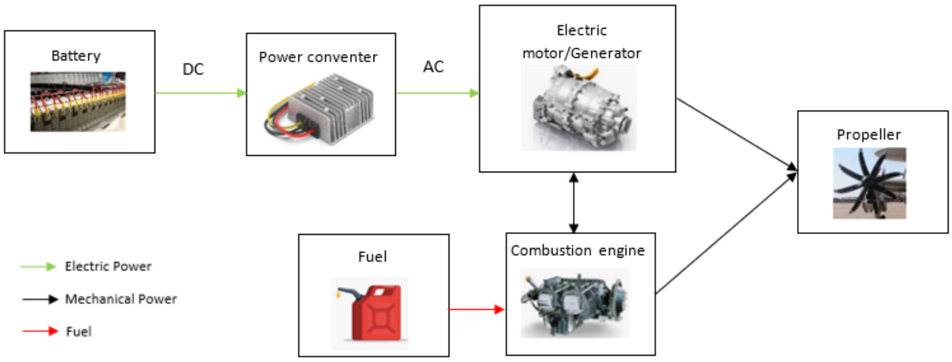

Figure 3. Series-parallel architecture. Source: Processed by authors.

4. Methodology of Calculation of a Hybrid Propulsion System

The hybrid propulsion system calculation is based on the design developed according [22,23].

4.1. Definition of Hybrids Variables

The first variable in hybrid propulsions is energy mass. The energy mass of a non-hybrid aircraft is equal to the mass of the fuel. All of the energy to power the aircraft, therefore, comes from the fuel. However, in a hybrid electric aircraft, the energy mass is the fuel's energy mass and the energy-mass of the battery. In the case of hybrid aircraft, it is necessary to define the variable so-called energy weight fraction β, is defined as the ratio between the energy weight and the aircraft weight to quantify the total amount of energy onboard, as follows:

$$\beta = \frac{W_{energy}}{W} = \frac{W_{gas} + W_{battery}}{W} \quad [-] \tag{1}$$

The ratio between the battery's weight and the energy weight is used to quantify the battery's weight concerning the % of the hybrid (x). The relationship will then look like this:

$$x = \frac{W_{battery}}{W_{energy}} \quad [-] \tag{2}$$

Determining the percentage of a hybrid provides a comparison between two designs of hybrid aircraft having different weights. The expression of the percentage of the hybrid better describes the energy conditions on board the aircraft.

The amount of fuel required to charge the batteries is defined as the fuel weight fraction α.

$$\alpha = \frac{W_{gas,charge}}{W_{gas}} \quad [-] \tag{3}$$

The weight fraction of the fuel component makes it possible to analyze the relationships between the range and the percentage of hybrids from an energy point of view. The calculations will assume that the systems will not operate simultaneously, i.e., it is assumed with α = 0. It is assumed that all power at any point of flight will be provided by either an internal combustion engine or a battery-powered electric motor (these two systems will never co-operate simultaneously). All of the hybrid variables (see Table 1) listed are weight fractions, so their values may range from 0 to 1 [23].

Table 1. Values of hybrid variables. Source: [23].

Variable	Value 0	Value 1
Energy weight fraction (β)	No weight on the aircraft is dedicated towards the battery and/or liquid fossil fuels.	The entire aircraft weight consists only of battery and/or liquid fossil fuels.
Percent hybrid (x)	Aircraft is fully powered by gasoline.	Batteries fully power the aircraft.
Charging fuel weight fraction (α) fraction	All the fuel onboard is used for propulsion.	All the fuel on board is used to charge the battery.

The approximate values of the efficiencies of the individual components of the hybrid drive, which we have determined and will use in the calculations, are [23–25]:

$$\begin{gathered}
\eta_{eng} = 0.3\ (Gas\ engine) \\
\eta_{gen} = 0.9\ (Generator) \\
\eta_{PE} = 0.95\ (Power\ electronics) \\
\eta_{batt} = 0.9\ (Battery) \\
\eta_{EM} = 0.9\ (Electric\ motor) \\
\eta_{prop} = 0.8\ (Propeller)
\end{gathered} \tag{4}$$

Approximate values of the efficiencies of the individual components of the hybrid system were determined on the basis of expert sources related to the given issue.

The propeller efficiency η occurs in the range of a good performance light aircraft efficiency, which is from 0.8 to 0.85 [26]. However, other authors indicate the efficiency of the propeller as 0.75 until 0.9 [27].

This design was based on the design of the Hybrid Electric Aircraft Propulsion Case Study for the Skydiving Mission, with the efficiencies of the individual components, such as the efficiency of the electric motor at 0.9; power electronics efficiency at 0.9 and battery efficiency at 0.9 [28].

The efficiency of a hybrid propulsion system will be determined as the product of the individual components' efficiencies, which will be arranged in series. The individual efficiencies will be determined as follows:

$$\begin{gathered}
\eta_{fuel \to prop} = \eta_{eng}\eta_{gen}\eta_{PE}\eta_{EM}\eta_{prop} \\
\eta_{batt \to prop} = \eta_{batt}\eta_{PE}\eta_{EM}\eta_{prop} \\
\eta_{charge} = \eta_{eng}\eta_{gen}
\end{gathered} \tag{5}$$

5. Calculation of Battery Parameters for a Small Training Aircraft

For the following calculations, we will use the parameters described in Table 2. The range of the aircraft is determined to be 407 km, and the battery will be required to provide 20 HP for 5 min.

The range value was determined based on an accurate training flight, which was performed as part of the pilot student's training. The training flight route is shown in Figure 4. The total flight time was 2 h and 16 min. The flight was performed on the aircraft type Zlín 142. The calculations consider using a serial hybrid system with the efficiencies of the individual components described in the previous chapter of the article.

Table 2. Determined basic output parameters for the calculation of the hybrid propulsion system. Source: [23] processed by authors.

Parameter	Symbol	Value	Unit
Aircraft weight	W	1098	kg
Energy weight fraction	β	0.3	-
Charging fuel weight fraction	α	0	-
Lift-to-drag ratio	L/D	7.38	-
Average power (combustion engine + electric motor)	P_{avg}	200	HP
Endurance	t	2.16	h
Motor voltage	V_{motor}	400	V
Battery knockdown value	W_c	0.5	-
Cell capacitance	Q_{cell}	3.3	Ah
Nominal cell voltage	V_{cell}	3.6	V
Minimum cell voltage	V_{min}	2.5	V
C-rate discharge	C_{dis}	1	C
Cell battery weight	W_{cell}	0.05	kg

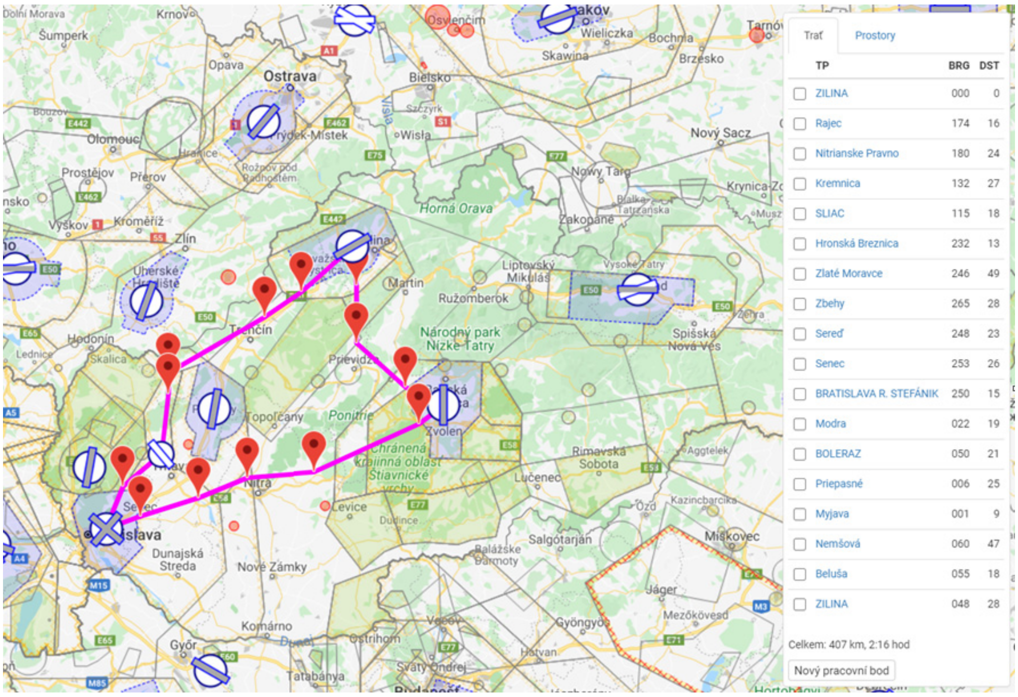

Figure 4. The training flight route for the considered theoretical design of the batteries of the aircraft's serial hybrid system. Source: Authors.

When designing the training aircraft with a serial hybrid drive, we used the parameters of the Zlín aircraft, which is currently used for pilot training. Such a type of propulsion does not exist in said aircraft. For our hybrid aircraft design, it was necessary to determine the maximum takeoff weight of the aircraft and the lift-to-drag ratio. These two essential parameters were for calculating the hybrid's percentage based on the determined range that we determined according to the operating manual of the Zlín aircraft.

The MTOW of the theoretical aircraft was determined based on the MTOW of the considered theoretical hybrid aircraft Zlín, in which such a hybrid propulsion system would be implemented. For this theoretical aircraft, we considered the same power as the conventional internal combustion engine used to power this aircraft. These design parameters were then used to calculate the percentage of the hybrid of such a theoretical hybrid aircraft. When designing a theoretical hybrid aircraft, we could also proceed in such a way that according to the design of the hybrid aircraft, the percentage of the hybrid would be determined, and the corresponding weight would then be calculated from the relation [23]:

$$W = \frac{P_{avg} * t}{x\beta\rho_b\eta_{b\,\rightarrow p} + (1-x)\beta\rho_b\left[(1-\alpha)\eta_{g\rightarrow p} + \alpha\eta_{ch}\,\eta_{b\,\rightarrow p}\right]} \tag{6}$$

where:

- W—weight
- P_{avg}—average power
- t—time/endurance
- x—percent hybrid
- β—energy weight fraction
- α—charging fuel weight fraction
- ρ_b—specific energy of battery
- ρ_b—specific energy of fuel
- $\eta_{b\,\rightarrow p}$—transmission efficiency between battery and propeller
- $\eta_{g\,\rightarrow p}$—transmission efficiency between fuel and propeller
- η_{ch}—charging efficiency

Equation (6) is based on the sum of the energy provided by the two hybrid drive sources, which must equal the given energy demand [23]:

$$E_{req} = E_{batt} + E_{gas} \tag{7}$$

The amount of energy of individual sources is defined by the respective weight of the source and its specific energy. After taking into account the losses of the conversion of the energy of sources into mechanical energy, we obtain the equation [23]:

$$E_{req} = W_{batt}\rho_{batt}\eta_{b\,\rightarrow p} + W_{gas}\rho_{gas}\eta_{g\,\rightarrow p} \tag{8}$$

The energy contained in the gasoline can be used to charge the battery or directly to drive the propeller. The energy required to perform a flight can be calculated as the average required power multiplied by the flight duration [23].

$$P_{avg} * \ t == W_{batt}\rho_{batt}\eta_{b\,\rightarrow p} + W_{gas}\rho_{gas}\left[(1-\alpha)\eta_{g\rightarrow p} + \alpha\eta_{ch}\,\eta_{b\,\rightarrow p}\right] \tag{9}$$

Finally, the fuel and battery weights can be related to the aircraft weight in terms of percent hybrid and energy weight fraction, resulting in the follow expression relating the aircraft weight and percent hybrid [23]:

$$P_{avg} * \ t = x\beta\rho_b\eta_{b\,\rightarrow p} + (1-x)\beta W\rho_b\left[(1-\alpha)\eta_{g\rightarrow p} + \alpha\eta_{ch}\,\eta_{b\,\rightarrow p}\right] \tag{10}$$

Equation (10) is a linear equation that relates to the percentage of the hybrid and the weight of the aircraft; its solution is possible for each variable explicitly. If a maximum target weight limit is set, the required hybrid percentage is as follows [23]:

$$x = \frac{P_{avg} * \ t - \beta W\rho_b\left[(1-\alpha)\eta_{g\rightarrow p} + \alpha\eta_{ch}\,\eta_{b\,\rightarrow p}\right]}{\beta W\left[\rho_b\eta_{b\,\rightarrow p} - \rho_b\left((1-\alpha)\eta_{g\rightarrow p} + \alpha\eta_{ch}\,\eta_{b\,\rightarrow p}\right)\right]} \tag{11}$$

If we know the percentage of the hybrid, the corresponding weight can be calculated from Equation (6).

Therefore, in the calculations, we considered the maximum takeoff weight of the theoretical hybrid aircraft of 1090 kg (including the total weight of the hybrid aircraft) and lift-to-drag ratio of 7.38. Lift-to-drag ratio was determined from the flight manual of the aircraft Zlín 142 under the conditions: engine mode idling and flaps closed [29].

We applied the calculation methodology described in Section 4.1 to the input the parameters of the aircraft's serial hybrid system determined by us [30].

The range that can be obtained by burning gasoline is obtained from the Breguet Range Equation. The derivation is based on the definition of specific fuel consumption (PSFC), as follows [23]:

$$PSFC = \frac{dW_{fuel}\ /dt}{P} \tag{12}$$

Assuming an equilibrium cruise condition, where lift generated by the aircraft is the same as the total weight, and power is the product of drag and velocity, substituting and rearranging Equation (1) gives:

$$dt = \frac{1}{PSFC * g} \frac{dW_f}{D * V} \frac{L}{W} \tag{13}$$

To express the range from Equation (2), it is necessary to multiply both sides of the equation by the velocity and then integrate (note: the change in fuel weight is the same as the change in aircraft weight):

$$R_{gas} = V * \int dt = \frac{1}{PSFC * g} \frac{L}{D} \int \frac{1}{W} dW \tag{14}$$

For simplicity, it was assumed that the lift-to-drag ratio, which relates to the aircraft's angle of attack, is constant regardless of weight. This means that the aircraft is climbing as the aircraft weight decreases over time. To obtain the range given by the amount of fuel burned, we integrate from the final weight ($W2$) to the initial weight ($W1$) [23]:

$$R_{gas} = \frac{1}{PSFC * g} \frac{L}{D} * \int_{W_2}^{W_1} \frac{1}{W} dW = \frac{1}{PSFC * g} \frac{L}{D} \, ln \left(\frac{W_1}{W_2} \right) \tag{15}$$

Since the final weight is simply the initial weight of the aircraft subtracting the fuel burned for propulsion, it can be expressed as such:

$$\begin{gathered} W_2 = W_1 - W_{gas,propulsion} \\ W_2 = W_1 - (1-\alpha)(1-x)\beta W_1 \\ W_2 = [1-(1-\alpha)(1-x)\beta] \end{gathered} \tag{16}$$

Finally, substituting Equation (16) into Equation (15) gives the gas range as a function of PSFC and several nondimensionalized variables [23]:

$$R_{gas} = \frac{1}{PSFC * g} \frac{L}{D} \, ln \left(\frac{1}{1-(1-\alpha)(1-x)\beta} \right) \tag{17}$$

To simplify further, PSFC is a measure of how efficiently the gas engine can convert chemical energy from the gasoline to mechanical energy. Therefore, it is possible to express it as a power specific energy consumption (PSEC) as [23]:

$$PSEC = PSFC * \rho_{gas} = \frac{1}{\eta_{gas \to prop}} \tag{18}$$

Substituting Equation (18) into Equation (17) and rearranging gives:

$$R_{gas} = \eta_{gas \to prop} \frac{\rho_{gas}}{g} \frac{L}{D} \, ln \left(\frac{1}{1-(1-\alpha)(1-x)\beta} \right) \quad [m] \tag{19}$$

This equation which shows an explicit relationship between percent hybrid and the range provided by gasoline for a given efficiency and aircraft design.

Energy in the battery can come from two sources: when it is initially charged prior to the flight (ground-charge) or part of the gas is used to charge the battery during the flight (air-charge). The energy due to ground-charge can be calculated using the battery specific energy as shown [23]:

$$E_{ground} = \rho_{batt}\, W_{batt} = \rho_{batt}\, x\beta W \tag{20}$$

Similarly, the energy due to air-charge can be expressed as the product of fuel specific energy and the fuel mass onboard for charging. Accounting for the loss during charging, this can be expressed as [23]:

$$E_{air} = \eta_{charge}\, \rho_{gas}\, W_{gas,charge} = \eta_{charge}\, \rho_{gas}\, \alpha(1-x)\beta W \tag{21}$$

The total energy from the battery is, therefore, the sum of Equations (20) and (21), which simplifies to:

$$E_{batt} = \beta W \left[\rho_{batt}\, x + \eta_{charge}\, \rho_{gas}\, \alpha(1-x) \right] \tag{22}$$

With the energy known, it is then possible to calculate the time it takes to fully discharge the battery for a given power:

$$t_{discharge} = \frac{E_{batt}}{P} \tag{23}$$

Assuming an equilibrium flight condition, Equation (12) can be rewritten as:

$$t_{discharge} = \frac{E_{batt}}{D * V}\, \frac{L}{W} \tag{24}$$

Lastly, multiplying the time to discharge with the aircraft velocity to obtain the range due to battery, and substituting Equation (22) in Equation (24) gives:

$$R_{batt} = \eta_{batt\, \rightarrow prop}\, \beta \frac{L}{D} \left[\rho_{batt}\, x + \eta_{charge}\, \rho_{gas}\, \alpha(1-x) \right] \tag{25}$$

To Equation (24) an efficiency factor enters into the account, which takes into account the inefficiency of the conversion of electricity into mechanical energy.

With the range equations derived for burning gasoline and discharging the battery as shown in Equations (19) and (25) respectively, the total range is simply the sum of the two:

$$\begin{aligned} R = \quad & \tfrac{L}{D}\tfrac{1}{g}\Big\{ \eta_{gas\, \rightarrow prop} \tfrac{\rho_{gas}}{g} ln \left(\tfrac{1}{1-(1-\alpha)(1-x)\beta} \right) \\ & + \eta_{batt\, \rightarrow prop}\, \beta \left[\rho_{batt}\, x + \eta_{charge}\, \rho_{gas}\, \alpha(1-x) \right] \Big\} \end{aligned} \tag{26}$$

Although Equation (14) provides a direct relationship between the range and percent hybrid, due the equation's nonlinearity, it is difficult to explicitly solve for percent hybrid for a given range requirement. For that, a linearization process is done to provide an approximated close form solution [23].

First, recall that the natural logarithmic function can be approximated using the Taylor Series for all $|z| \leq 1$ and $z \neq 0$, as follows [23]:

$$\ln(z) = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^2}{3} \pm \ldots \tag{27}$$

Therefore, for a first-degree approximation, the logarithmic part in Equation (14) can be approximated as [23]:

$$\ln\left(\frac{1}{1-(1-\alpha)(1-x)\beta} \right) = -\ln(1-(1-\alpha)(1-x)\beta) \approx (1-\alpha)(1-x)\beta \tag{28}$$

Substituting Equation (28) into Equation (26) gives:

$$R = \frac{L}{D}\frac{1}{g}\left\{\eta_{gas\ \to prop}\ \rho_{gass}\ (1-\alpha)(1-x)\beta + \eta_{batt\ \to prop}\ \beta\left[\ \rho_{batt}\ x + \eta_{charge}\ \rho_{gas}\ \alpha(1-x)\right]\right\} \quad (29)$$

which is now a linear equation. Then, expanding Equation (29) and solving for the percent hybrid yields [23]:

$$x = \frac{R - \frac{L}{D}\beta\frac{1}{g}\left[\eta_{b\ \to p}\ \eta_{ch}\rho_g\alpha + (1-\alpha)\eta_{g\to p}\rho_g\right]}{\frac{L}{D}\beta\frac{1}{g}\left[\eta_{b\ \to p}\ (\rho_b - \eta_{ch}\rho_g\alpha) - (1-\alpha)\eta_{g\to p}\rho_g\right]} \quad (30)$$

$$x = 0.84\ [-]$$

where:

- R [m]
- L/D [-]
- β [-]
- g [m/s^2]
- all $\eta \ldots$ in Equation (18) [-]
- α [-]
- ρ_{batt} [m^2/s^2]
- ρ_{gass} [m^2/s^2]

Equation (30) represents an explicit solution for the maximum percentage of hybrids as a function of the required range. If the percentage of hybrid propulsion is outside the range of 0 to 1, it means that the specified aircraft configuration has unsatisfactory range requirements. Equation (30) is the sum of the range that can be obtained by burning gasoline (obtained from the Breguet range) and the range that can be obtained from the battery. Yeung and Artur Bensel provide a more detailed derivation of the overall scope in their works [23,31].

Substituting and solving the above Equation (30), the hybrid's maximum value is determined to be 84%. Therefore, the calculated value is the maximum permitted percentage of the hybrid for a specified range of 407 km.

The next step is to determine the necessary parameter of the batteries used to drive the electric motor. Important parameters that need to be determined are battery performance and endurance. The battery must meet both requirements, but we will consider a higher calculated value in the following calculations. For the battery power requirement, the following equation shall be used to calculate the number of battery cells [23]:

$$N_s = \frac{V_{motor}}{V_{cell}} \quad (31)$$

$$N_s = 111.11 \implies 112$$

$$N_p = \frac{20\ HP}{V_{min}\eta_{EM}N_sQ_{cell}C_{dis}} \quad (32)$$

$$N_p = 17.68 \implies 18$$

For the endurance requirement, Equations (7) and (8) are used:

$$N_s = \frac{V_{motor}}{V_{cell}} \quad (33)$$

$$N_s = 111.11 \implies 112$$

$$N_p = \frac{P * t}{V_{nom}\eta_{EM}N_sQ_{cell}} \quad (34)$$

$$N_p = 1.27 \implies 2$$

Since we have calculated a more significant number of parallel cells in the battery's performance requirements, we will continue to consider this higher value. The battery weight can, therefore, be calculated as follows:

$$W_{batt} = \frac{N_s * N_p * W_{cell}}{W_c} \tag{35}$$

$$W_{batt} \cong 183 \text{ kg}.$$

The corresponding percentage of the hybrid propulsion system concerning the calculated battery weight can be determined as:

$$x_{batt} = \frac{W_{batt}}{\beta W} \tag{36}$$

$$x_{batt} = 0.56$$

The calculated emergency energy and battery life are the minimum battery requirements, so the percentage hybrid must be more significant than 56%. Using these two constraints, the upper and lower percentages of the hybrid system are determined and summarized in Table 3.

Table 3. Determined values of the upper and lower limits of the percentage of the hybrid system. Source: According [23] processed by authors.

#	Constraint	Percentage Hybrid Limit
1	Range > 407 km	x < 84%
2	Emergency 20 HP for 5 min.	x > 56%

The calculated hybrid percentages of 84% in the previous chapter represent optimal values for minimizing fuel and electricity costs for a given flight. Fuel and electricity costs can be calculated according to Equation (37) to calculate fuel and electricity costs for a given flight. This equation does not include costs other than aircraft acquisition or maintenance costs, depreciation, interest, insurance, maintenance, instructor, fees and charges.

In the aircraft hybrid system under consideration, in which the batteries are not expected to be charged during the flight, the operating costs are the sum of the fuel and electricity costs used to charge the batteries [23].

$$C_{ost} = x\beta W\rho_{batt}C_{batt} + (1-x)\beta W\rho_{gas}C_{gas} \tag{37}$$

$$C_{ost} = 131.18 \text{ €}$$

C_{batt} a C_{gas} represent the cost of electricity and fuel per unit of energy. The values calculated in Equation (37) were the average prices for fuel and electricity at the writing time.

We applied the same battery calculation principle for the aircraft's serial hybrid system to nine training flights. A description of the flights is given in Table 4.

Table 4. Data on training flights used in serial hybrid system calculations. Source: Authors.

Flight	Length [km]	Time [hod]	Route
1	238	1.19	Žilina-Beluša-Nemšová-Bobot-Topoľčany-Bojná-Piešťany-Priepasné-Stará Turá-Nemšová-Beluša-Žilina
2	217	1.12	Žilina-Kysucký Lieskovec-Oravská Lesná-Námestovo-Zuberec-Liptovský Mikuláš-Ružomberok-Martin-Slovenské Pravno-Rajec-Žilina
3	301	1.4	Žilina-Beluša-Nemšová-Trenčín-Stará Turá-Myjava-Holíč-Senica-Priepastné-Piešťany-Bojná-Malé Bielice-Nováky-Nitrianske Pravno-Rajec-Žilina
4	271	1.3	Žilina-Beluša-Nemšová-Beckov-Stará Turá-Myjava-Senica-Priepastné-Piešťany-Bojná-Malé Bielice-Nováky-Slovenské Pravno-Rajec-Žilina
5	352	1.57	Žilina-Beluša-Trenčín-Beckov-Piešťany-Leopoldov-Senec-Bratislava-Senec-Sereď-Zbehy-Zlaté Moravce-Nitrianske Pravno-Rajec-Žilina
6	338	1.53	Žilina-Strečno-Martin-Ružomberok-Liptovský Mikuláš-Východná-Poprad-Vernár-Telgárt-Polomka-Ľubietová-Sliač-Hronská Breznica-Žiar nad Hronom-Handlová-Nitrianske Pravno-Rajec-Žilina
7	216	1.12	Žilina-Beluša-Nemšová-Trenčín-Stará Turá-Myjava-Priepasné-Piešťany-Beckov-Trenčianske Teplice-Beluša-Žilina
8	190	1.03	Žilina-Kysucký Lieskovec-Oravská Lesná-Námestovo-Zuberec-Liptovská Sielnica-Ružomberok-Martin-Strečno-Žilina
9	201	1.07	Žilina-Strečno-Martin-Slovenské Pravno-Nováky-Malé Bielice-Bánovce nad Bebravou-Trenčianske Jastrabie-Trenčín-Nemšová-Beluša-Žilina

6. Results and Discussion

Hybrid electric systems provide an exciting alternative for propulsion to conventional fuel-burning engines. These systems make it possible to reduce fuel consumption and emissions and reduce noise pollution around airports.

To calculate the parameters of a hybrid aircraft battery, which could be used to train future pilots, we used the characteristics of the Zlín aircraft. In this type of aircraft, we theoretically considered the application of a serial hybrid system, in which it was necessary to determine the input values of the system for the subsequent calculation of such a system. The scheme of such a hybrid system is mathematically defined as the product of the efficiencies of the individual components connected in series. The considered efficiencies of the individual components of the hybrid system are given in Section 4.1.

In the design, we considered a hybrid system in which the propulsion energy would be used either from fuel or batteries.

This means that the combustion engine and the electric motor would never work at the same time. The energy required to power the aircraft at any stage of the flight is supplied by either an internal combustion engine or an electric motor. All other input variables needed for the calculation are listed in Table 2.

The parameters of the performed training flight, which is shown in Figure 4, were used for the model calculation. The total flight length was 407 km, and the flight time was 2 h and 16 min. After substituting the hybrid system's specified input parameters and flight length, we determined the hybrid's maximum percentage concerning the specified required range. The maximum percentage of the hybrid thus determines the upper limit of the aircraft's hybrid system's percentage for the input parameters determined by us. After determining the maximum percentage of the hybrid, it was necessary to determine the battery's parameters, which will meet the requirements for our simulated training flight. In determining the battery requirements, we used study according to which the battery must meet a specified set of equations to determine the number of serial and parallel battery cells. The number of battery cells was determined by calculation: serial cells 112 and parallel cells 18. The parameters of the hybrid drive's specified battery are a voltage of 403.2 V, and a battery capacity of 59.4 Ah.

Furthermore, they were weighing approximately 183 kg. Concerning the weight of the calculated battery, it is then possible to determine the hybrid's corresponding percentage, namely 84%. Due to the limitations that the battery must meet, this value is the minimum percentage of the aircraft's hybrid system.

At direct operating costs, the optimal percentage hybrid is 1, or 100%. In our case, the value of the hybrid, at 84%, expresses the optimal cost. The minimum operating costs were set at 131.18 € in the calculation. By changing the hybrid percentage in the specified calculated range of 56% to 84%, these operating costs can be adjusted. The calculated range of percentages of the hybrid system in a training aircraft allows us to determine the required range of the aircraft. All the results are included in Table 5.

Table 5. Data on training flights used in serial hybrid system calculations. Source: Authors.

Flight	x [-]	N_p [-]	N_s [-]	W_{batt} [kg]	X_{batt} [-]	C_{ost} [€]
1	0.932	112	18	183	0.56	74.49
2	0.944	112	18	183	0.56	67.48
3	0.895	112	18	183	0.56	89.16
4	0.912	112	18	183	0.56	85.58
5	0.864	112	18	183	0.56	112.74
6	0.873	112	18	183	0.56	108.05
7	0.945	112	18	183	0.56	67.14
8	0.959	112	18	183	0.56	58.41
9	0.953	112	18	183	0.56	62.11

The percentage of the hybrid for the considered training flights is shown in the graph in Figure 5. For the considered range of the training flights from 190 km to 407 km, the calculations determine the approximate value of the percentage of the hybrid 100%. Theoretically, the entire training flight could be performed on electric propulsion for these specified aircraft range values. Of course, with increasing range, the percentage of hybrid decreases when part of the training flight would be performed on electric propulsion and conventional propulsion.

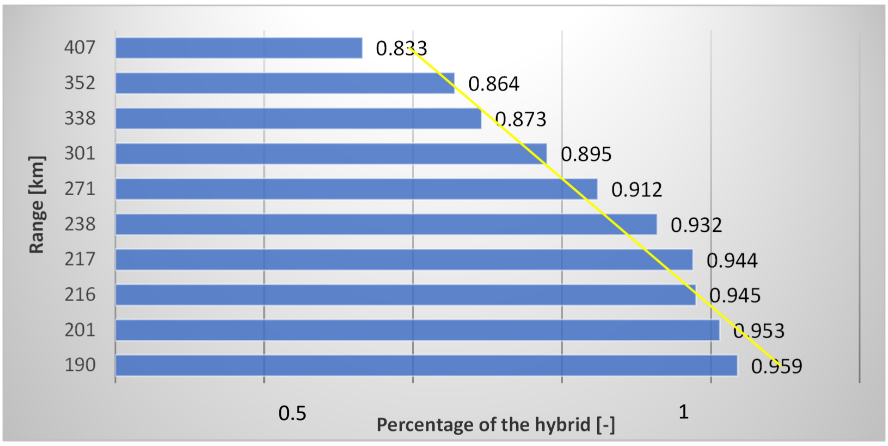

Figure 5. Percentage values of the hybrid drive for individual range distances. Source: Authors.

The graph in Figure 6 shows the direct operating costs as a function of the range. The costs for ranges from 190 to 407 km he calculations determine the approximate percentage of the hybrid from 96 to 83%. With the increasing range, direct operating costs also increase when, in addition to electricity costs, fuel costs must also be considered. In the calculations, the price of electricity was considered to be 0.1251 €/kWh, and the price of aviation fuel was 0.046 €/kWh.

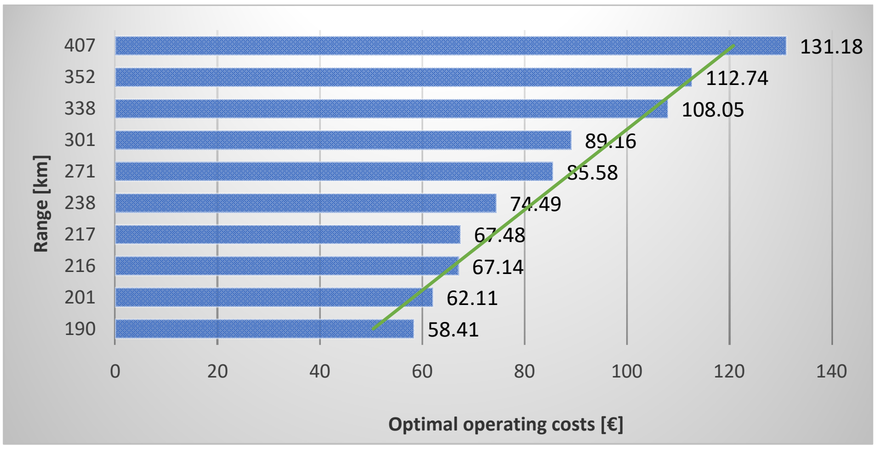

Figure 6. Values of operating costs for individual range distances. Source: Authors.

In the case of an aircraft used for training flights around the airport, for takeoff and landing training, the percentage of the hybrid could be higher at the expense of a lower range. This would be more advantageous from the point of view of electric aircraft operation, which would reduce emissions and noise around the airport.

The hybrid propulsion design's performed calculations provided us with the basic parameters necessary for the hybrid propulsion of a small aircraft. Further direction of the research and the described results could be directed to creating a specific theoretical model of a hybrid-powered aircraft. In pilot training at the Air Transport Department, hybrid propulsion of training aircraft would find the most appropriate application in training flights near the airport. In these cases, the hybrid drive would benefit frequent takeoffs and landings where battery power would be used. In such situations, it would be possible to reduce emissions and noise around the airport. It would also be possible during night flights to reduce the noise level of aircraft around the airport and thus improve residents' quality of life on the outskirts of the airport. A possible direction of future research could be focused on the design of a hybrid propulsion aircraft suitable for flight training. The knowledge from the calculations presented in this article could design the optimal hybrid drive for specific training missions. As emissions from batteries and electricity generation must also be considered in hybrid propulsion, further research in this area would be appropriate. A limiting factor in hybrid propulsion would undoubtedly be the time required to charge the battery, i.e., when the aircraft would have to remain on the ground to recharge the batteries. Extensive research is needed for these shortcomings, as long as this technology is reliably implemented in aviation practice.

7. Conclusions

The presented article provided knowledge about using a hybrid propulsion system of small aircraft, which would use a combination of internal combustion and electric engine for propulsion. At present, there is more and more socio-political pressure on any "dirty industry" (an industry with high CO_2 emissions). One of the industries where we are always trying to reduce emissions is the aviation industry. Social movements have a significant impact on political bodies, such as the European Commission or NASA. Flightpath 2050 and SRIA presented bold goals that require companies to invest money in a new, more environmentally friendly way in aviation and, thus, in unconventional ways of propelling aircraft.

The technical differences between an internal combustion engine and an electric motor are relatively significant, and electric motors have an advantage in control, are almost noiseless and do not require extensive maintenance. However, electric motors' advantage is limited by batteries, which currently have a low energy density. The low energy density of batteries is why hybrid drives are used very little in aviation. Problems with heavy batteries that need to be used due to their low energy density can be solved using internal combustion an and electric engine (so-called hybrid propulsion system).

The article describes the calculation for the optimal percentage of the hybrid propulsion system. Determining the optimum percentage of the hybrid propulsion system (the ratio between a conventional internal combustion engine and an electric engine). Several limitations and simplifications are set out in the calculations, which more or less affect the permissible percentage of the hybrid drive. The main parameter that is crucial in the design of the hybrid aircraft propulsion is range. In the calculations, we set a maximum range of 407 km, for which we got a percentage of the hybrid at 84%, i.e., the maximum percentage of a hybrid drive (0% means no hybrid drive and 100% means pure electric drive). This value can be further worked on and can be used to find optimal operating costs and emissions. The calculations also consider that the battery will be able to supply the theoretical maximum amount of energy. In reality, however, the amount of power that a battery can supply depends on the state of charge of the battery, and the type of battery. As such, the detailed calculations also need to focus on the value of the variable energy supply from the barrel. A compromise must be found between range, weight, cost, and emissions by designing a hybrid aircraft. An important factor is the weight of the battery, i.e., the ratio between the battery's weight and the weight of the fuel. By changing the ratio of these two weights, it is possible to achieve significant changes in a hybrid system's design.

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