Article

A Full-Newton AC-DC Power Flow Methodology for HVDC Multi-Terminal Systems and Generic DC Network Representation

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Abstract: The use of Direct Current (DC) transmission links in power systems is increasing continuously. Thus, it is important to develop new techniques to model the inclusion of these devices in network analysis, in order to allow studies of the operation and expansion planning of large-scale electric power systems. In this context, the main objective of this paper is to present a new methodology for a simultaneous AC-DC power flow for a multi-terminal High Voltage Direct Current (HVDC) system with a generic representation of the DC network. The proposed methodology is based on a full Newton formulation for solving the AC-DC power flow problem. Equations representing the converters and steady-state control strategies are included in a power flow problem formulation, resulting in an expanded Jacobian matrix of the Newton method. Some results are presented based on HVDC test systems to confirm the effectiveness of the proposed approach.

Keywords: high-voltage direct current; multi-terminal HVDC systems; control modes on multi-terminal HVDC system; power flow; Newton method

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1. Introduction

With the prospect of the increasing use of Direct Current (DC) transmission links in power systems, it has become increasingly important to have techniques to model the devices responsible for the AC-DC interconnection in power system analysis software, particularly in the power flow (PF), to allow correct network modeling as a whole to improve the quality of simulation results that could be used in studies of operation and transmission expansion planning of electric power systems.

DC transmission has become an alternative that is technically and economically competitive in the transport of large amounts of active power over long distances, in underwater crossings with the use of cables, and in asynchronous connections within a large variety of lengths, including zero, between two areas [1–5].

The DC transmission links generally are characterized by the interconnection of two systems of Alternating Current (AC) by two converter stations: a rectifier and an inverter. The connection between these stations is established by one or more DC transmission lines: single or double polarity. The rectification and inversion are performed in the converter bridges composed by semiconductor elements.

High Voltage Direct Current (HVDC) transmission technology was used for the first time in Brazil in the 1980s to integrate the energy generated by the Itaipu Hydroelectric Power Plant, connecting the Foz do Iguaçu and Ibiúna converter stations, utilizing two

dipoles at ± 600 kV, over about 800 km. In 2009, Brazil resumed the use of HVDC transmission technology with the construction of the ± 600 kV transmission system associated with the Rio Madeira hydroelectric complex (Santo Antônio and Jirau hydroelectric power plants), composed of two bipoles designed to transmit 6300 MW over approximately 2400 km, connecting Coletora Porto Velho and Araraquara stations. By the year 2020, an important initiative to consider is the establishment of the Belo Monte HVDC links (2539 km-long transmission), with the construction of the ± 800 kV transmission system associated with the Belo Monte hydroelectric power plant.

From this context, it appears that the study of severe operating conditions is important to support studies on the reliability of hydroelectric plants. These studies can assist in assessing the Availability Factor (AFA) [6]. Thus, modeling the electrical system as close as possible to reality is of fundamental importance. This is the case of the Santo Antônio hydroelectric plant, which may be impacted by the behavior of the HVDC transmission system.

The power flow software is the most frequently used tool in both operation and expansion planning studies of electrical power systems. It is used to determine the equipment rating, electrical equipment loading and system losses, bus voltage magnitudes and angles, and reactive power support requirements to maintain voltages within limits for a given scenario and a contingency list. For each case study, the network configuration and parameters, the bus active and reactive load, and MW generation must be specified. The increasing complexity of power systems, introduced by new large-scale AC and HVDC interconnections and by the application of Flexible AC Transmission System (FACTS) devices in such systems, have imposed new challenges to power system engineers and software developers. The reference [7] describes a generic methodology for representing control devices and the associated challenges.

The commonly used techniques in studies of power flow alternate the convergence process between AC and DC system models. This method is carried out until the global convergence of the two systems is obtained. It is important to point out that the inclusion of DC links can cause, in general, a significantly slower convergence of power flow. One of the main reasons for this slow convergence is the different rates of convergence of the DC and AC systems, which interfere with each other, slowing system global convergence.

In reference [8], a simultaneous (full Newton) methodology for the representation of DC transmission links based on the DC transmission link of the hydro electrical unit Itaipu (Brazil) was proposed by including equations that model the converters, the DC network (considering only two DC bus), and control modes to the set of equations that model the power system. The proposed model aims to (i) improve the convergence characteristics of systems with coupling between the AC-DC set of equations in the Newton-Raphson method and (ii) allow the adjustment of the control modes automatically during the iterative process.

In [9], the authors present a versatile approach to AC/DC system representation, which is able to model several types of converter structures, including the conventional two-terminal AC-DC power flow, and to connect to more DC terminals. The sequential (alternate) method is used to solve the AC-DC power flow, in which the AC network solution is carried out through the conventional Newton-Raphson method, and in the DC network model, the system presents five variables per terminal. It is also proposed that the variation of taps be limited, which is an improvement compared to other methodologies. Another advantage is the possibility of solving DC systems connected in series by adjusting some equations of the DC network.

Another study of DC transmission links [10] uses the eliminated variable method, which is a simple and reliable tool for the study of DC transmission links in multi-terminal systems, in which the DC and AC variables are treated separately. In this work, the active and reactive powers demanded by inverters are treated as voltage-dependent loads. Thus, the DC steady-state equations are solved, and the DC variables are eliminated from the PF equations. Voltage-dependent PQ buses were adopted in [11] to represent the HVDC system in the power flow problem. The work [12] proposed a modified power flow

formulation considering the HVDC system as a voltage-dependent load on the AC side at both DC system sides in two-terminal systems.

In [13], a nodal voltage-based universal steady-state PF formulation is proposed. The main goal is to consider the bipolar VSC-MTDC (Voltage Source Converter Multi-Terminal HVDC). A power flow alternating iterative method is proposed to obtain the positive/negative-pole DC network power flow. A series of nodal equivalent methods involving various control strategies are also proposed for the power flow algorithm. A comparative study of full Newton and sequential AC-DC power flow formulations for a VSC-MTDC is presented in the paper [14].

In [15], the authors use a sequential methodology to present a study where the solution of the AC-DC power flow is intrinsically related to the characteristics of a linear coefficient matrix G , which integrates the information of the network and HVDC control modes. The main contribution of this paper is to demonstrate that a necessary condition to solve the HVDC system is that the coefficient matrix G must be nonsingular. In the study, the conditions for the characteristic of a G matrix under feasible combinations of control modes and the parameters of the HVDC systems are introduced. The reference [16] proposed an alternative approach also based on a sequential method for the AC-DC power flow solution to handle multi-infeed DC systems.

The authors of [17] present some methods based on the correlation analysis between the steady-state security region and operational constraints. The reference [18] proposes an operation strategy of the hybrid multi-terminal high voltage DC to increase the utilization of the AC transmission corridors in parallel with HVDC systems considering the available transmission capacity (ATC) between two electrical areas. An analysis of distribution networks is carried out in the reference [19]. This work proposed an integrated load flow approach for AC-DC distribution systems.

In this context, the main objective of this paper is to propose a new methodology for a full Newton AC-DC power flow formulation for an MTDC system with a generic methodology for representing the DC network in stationary studies. The main motivation is the increasing interest in the operational feasibility and potential application of multi-terminal HVDC systems. It is sought to demonstrate that the proposed methodology provides an efficient and generic way to represent any DC network in HVDC multi-terminal systems through a system of equations. Finally, some results are presented based on HVDC test systems to validate the effectiveness and robustness of the proposed methodology.

From the above, the main contributions of this paper are

- to present an alternative power flow formulation based on a full Newton methodology for MTDC transmission systems, where all DC variables are considered in the formulation, allowing for a generic representation of HVDC control modes;
- to describe the AC/DC base transformation in the Jacobian matrix calculation;
- to propose a formulation that represents the generic DC network.

2. Modeling Background

The steady-state model for power flow studies of the HVDC utilized in this work is constituted by three subdivisions: the converter model itself, more specifically the rectifier and inverter model, the DC transmission line/network model, and the control system models for the Multi-Terminal HVDC (MTDC).

- converter model;
- DC transmission line/network model;
- control model in Multi-Terminal HVDC (MTDC) systems.

2.1. Mathematical Model of the Converter

The model for the converter used in Multi-Terminal HVDC systems can be conveniently represented by Figure 1. The variables are adequately represented in a p.u. system.

Figure 1. Converter model representation.

The values of rectifier and inverter commutation resistors, R_r e R_i , are given by Equations (1) and (2). Note that R_i is conveniently adopted as a negative value.

$$R_r = \frac{3.X_r}{\pi} \quad (1)$$

$$R_i = -\frac{3.X_i}{\pi} \quad (2)$$

The representation of the rectifiers and inverters is based on the following premises [20]:

- the current I_c is ripple-free;
- the AC systems consist of constant-frequency, perfectly sinusoidal, balanced voltage sources behind balanced impedances, which assumes that all harmonic currents and voltages introduced by the commutation system do not propagate into the AC system because of filtering;
- the converter transformers do not saturate.

The equations that model the rectifier and inverter for MTDC systems are the same as those used in the DC link models for systems with only two DC terminals [8].

2.1.1. Mathematical Model of the Rectifier

The equations that model the behavior of the rectifier in the DC link are given by [1,20,21]:

$$Vd_r - k_r.a_r.V_r.\cos(\alpha) + \frac{3}{\pi}.X_r.I_r = 0 \quad (3)$$

$$\frac{2.\mu_r + \sin(2.\alpha) - \sin(2.(\alpha + \mu_r))}{\cos(2.\alpha) - \cos(2.(\alpha + \mu_r))} - \tan(\phi_r) = 0 \quad (4)$$

$$\cos(\alpha) - \cos(\alpha + \mu_r) - \frac{2.R_r.I_r}{k_r.a_r.V_r} = 0 \quad (5)$$

2.1.2. Mathematical Model of the Inverter

Similarly, the equations that model the behavior of the inverter in the DC link are given by [1,20,21] are

$$Vd_i - k_i.a_i.V_i.\cos(\gamma) - \frac{3}{\pi}.X_i.I_i = 0 \quad (6)$$

$$\frac{2.\mu_i + \sin(2.\gamma) - \sin(2.(\gamma + \mu_i))}{\cos(2.\gamma) - \cos(2.(\gamma + \mu_i))} - \tan(\phi_i) = 0 \quad (7)$$

$$\cos(\gamma) - \cos(\gamma + \mu_i) - \frac{2.R_i.I_i}{k_i.a_i.V_i} = 0 \quad (8)$$

2.2. DC Transmission Network Model

Considering a circuit that contains N nodes with only independent current sources, the simplified nodal equations have the following form:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{N-1} \end{bmatrix} = \begin{bmatrix} G_{11} & -G_{12} & \cdots & -G_{1,N-1} \\ -G_{21} & G_{22} & \cdots & -G_{2,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ -G_{N-1,1} & -G_{N-1,2} & \cdots & G_{N-1,N-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \end{bmatrix} \quad (9)$$

where V_1 and V_{N-1} denote the voltages of $N-1$ nodes with respect to the reference node.

The proposed methodology provides an efficient and generic way to represent any DC network through the system of equations presented in (9). This approach has become useful in some studies, for example, in research related to “Tap HVDC”, which seeks to disfigure the DC link as a transmission point-to-point system and make it flexible regarding the extraction or injection along the transmission line.

2.3. Control Model in MTDC Systems

The basic control principle for MTDC systems is a generalization of the one used in two-terminal systems [8]. As is shown in [20], the control characteristic for each converter is composed of piece-wise functions representing constant-current control (CC) and constant-firing angle control (Constant Extinction Angle (CEA) for inverters and Constant Ignition Angle (CIA) for rectifiers). In addition, an optimal constant-voltage segment may be included. The following is a general discussion of significant aspects related to the control of parallel-connected and series-connected systems.

The equations that model the controls are as follows:

- CIA/CEA Control with Constant-Voltage (CV):

$$\cos(\delta_c^{sch}) = \cos(\delta_c) \quad (10)$$

$$Vd_c^{sch} = Vd_c \quad (11)$$

The use of the variable $\cos(\delta_c)$ instead of δ_c improves the linearity of the problem [9].

- Constant-Current Control (CC):

$$I_c^{sch} = I_c \quad (12)$$

$$Vd_c + 0.97 \cdot (-k_c \cdot a_c \cdot V_c \cdot \cos(\delta_c^{min}) + R_c \cdot I_c) = 0 \quad (13)$$

- Constant-Power Control (CP):

$$P_c^{ccsch} = P_c^{cc} \quad (14)$$

$$Vd_c + 0.97 \cdot (-k_c \cdot a_c \cdot V_c \cdot \cos(\delta_c^{min}) + R_c \cdot I_c) = 0 \quad (15)$$

where:

$$k_c = \left(\frac{3\sqrt{2}}{\pi} \right) \cdot n_b$$

For the converter operating with current or power control mode, it is common to adjust the transformer tap control with the phase control. Consequently, the converter will operate at some DC voltage below its minimum extinction or ignition angle characteristic to prevent mode shifts from occurring with normal AC voltage changes. Typically, a 3% voltage margin is provided, with the normal α^{min} or γ^{min} given in (13) and (15), and typical values of the control angles α and γ are 15° and 20°, respectively, for those DC terminals with a scheduled current or power control. From a practical point of view, this common voltage margin of 3% can be considered in the PF computation by adjusting the DC voltage equations for such converters with a coefficient of 0.97 [9,22].

3. Proposed Methodology

The proposed method of solution consists of including the equations that model the HVDC systems in the conventional power flow formulation. In order to achieve this goal, six new state variables will be included for each converter: ϕ_c , I_c , μ_c , a_c , δ_c , and Vd_c . The assumptions considered for Newton's method to solve simultaneously the AC-DC PF, based on the system of equations $\Delta y = J \Delta x$, are presented below.

Considering the equations described in Section 2, the system of equations related to the HVDC system (Δy) is structured as shown in Table 1.

Table 1. Mismatch vector of equations in AC-DC Power Flow (Δy).

		Mismatch	Equation
Converter equations	Converter 1	$\Delta y_1^{conv.1}$	Equation (3) or (6)
		$\Delta y_2^{conv.1}$	Equation (4) or (7)
		$\Delta y_3^{conv.1}$	Equation (5) or (8)
	$\vdots$	$\vdots$	$\vdots$
	Converter n	$\Delta y_1^{dc,conv.n}$	Equation (3) or (6)
		$\Delta y_2^{conv.n}$	Equation (4) or (7)
Control equation	Converter 1	$\Delta y_3^{conv.n}$	Equation (5) or (8)
		$\Delta y_1^{cont.1}$	Equation (10) or (12) or (14)
		$\Delta y_2^{cont.1}$	Equation (11) or (13) or (15)
	$\vdots$	$\vdots$	$\vdots$
	Converter n	$\Delta y_1^{conv.n}$	Equation (10) or (12) or (14)
		$\Delta y_2^{cont.n}$	Equation (11) or (13) or (15)
DC network equations	—	Δy_G	Equation (9)

Based on Table 1, the vector of state variables related to the HVDC system (Δx) will be updated according to the order presented in Table 2.

The general linear system to be solved at each iteration in the solution process is shown in Equation (16).

$$\begin{bmatrix} \vdots \\ \Delta P'_c \\ \Delta Q'_c \\ \vdots \\ ine\Delta y_1^{conv.1} \\ \Delta y_2^{conv.1} \\ \Delta y_3^{conv.1} \\ \vdots \\ ine\Delta y_1^{cont.1} \\ \Delta y_2^{cont.1} \\ \vdots \\ ine\Delta y_G \end{bmatrix} = \begin{bmatrix} J_{AC-AC} & J_{AC-DC} \\ ineJ_{DC-AC} & J_{DC-DC} \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ \Delta \theta_c \\ \Delta V_c \\ \vdots \\ ine\Delta x_1^{conv.1} \\ \Delta x_2^{conv.1} \\ \Delta x_3^{conv.1} \\ \vdots \\ ine\Delta x_1^{cont.1} \\ \Delta x_2^{cont.1} \\ \vdots \\ ine\Delta x_G \end{bmatrix} \quad (16)$$

where:

$$[J_{AC-AC}] = \begin{bmatrix} \ddots & \vdots & \vdots & \dots \\ \dots & \frac{\partial P'_c}{\partial \theta_c^{conv.1}} & \frac{\partial P'_c}{\partial V_c^{conv.1}} & \dots \\ \dots & \frac{\partial Q'_c}{\partial \theta_c^{conv.1}} & \frac{\partial Q'_c}{\partial V_c^{conv.1}} & \dots \\ \dots & \vdots & \vdots & \dots \end{bmatrix}$$
$$[J_{AC-DC}] = \left[\begin{array}{ccc|cc|c} \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ \frac{\partial P'_c}{\partial x_1^{conv.1}} & \frac{\partial P'_c}{\partial x_2^{conv.1}} & \frac{\partial P'_c}{\partial x_3^{conv.1}} & \dots & \frac{\partial P'_c}{\partial x_1^{cont.1}} & \frac{\partial P'_c}{\partial x_2^{cont.1}} & \dots \\ \frac{\partial Q'_c}{\partial x_1^{conv.1}} & \frac{\partial Q'_c}{\partial x_2^{conv.1}} & \frac{\partial Q'_c}{\partial x_3^{conv.1}} & \dots & \frac{\partial Q'_c}{\partial x_1^{cont.1}} & \frac{\partial Q'_c}{\partial x_2^{cont.1}} & \dots \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \dots \end{array} \right] \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$
$$[J_{DC-AC}] = \left[\begin{array}{cc|cc|c} \dots & \frac{\partial y_1^{conv.1}}{\partial \theta_c^{conv.1}} & \frac{\partial y_1^{conv.1}}{\partial V_c^{conv.1}} & \dots & \vdots \\ \dots & \frac{\partial y_2^{conv.1}}{\partial \theta_c^{conv.1}} & \frac{\partial y_2^{conv.1}}{\partial V_c^{conv.1}} & \dots & \vdots \\ \dots & \frac{\partial y_3^{conv.1}}{\partial \theta_c^{conv.1}} & \frac{\partial y_3^{conv.1}}{\partial V_c^{conv.1}} & \dots & \vdots \\ \dots & \vdots & \vdots & \dots & \vdots \\ ine \dots & \frac{\partial y_1^{cont.1}}{\partial \theta_c^{conv.1}} & \frac{\partial y_1^{cont.1}}{\partial V_c^{conv.1}} & \dots & \vdots \\ \dots & \frac{\partial y_2^{cont.1}}{\partial \theta_c^{conv.1}} & \frac{\partial y_2^{cont.1}}{\partial V_c^{conv.1}} & \dots & \vdots \\ \dots & \vdots & \vdots & \dots & \vdots \\ ine \dots & \frac{\partial y_G}{\partial \theta_c^{conv.1}} & \frac{\partial y_G}{\partial V_c^{conv.1}} & \dots & \vdots \end{array} \right]$$
$$[J_{DC-DC}] = \left[\begin{array}{cccc|cc|c} \frac{\partial y_1^{conv.1}}{\partial x_1^{conv.1}} & \frac{\partial y_1^{conv.1}}{\partial x_2^{conv.1}} & \frac{\partial y_1^{conv.1}}{\partial x_3^{conv.1}} & \dots & \frac{\partial y_1^{conv.1}}{\partial x_1^{cont.1}} & \frac{\partial y_1^{conv.1}}{\partial x_2^{cont.1}} & \dots \\ \frac{\partial y_2^{conv.1}}{\partial x_1^{conv.1}} & \frac{\partial y_2^{conv.1}}{\partial x_2^{conv.1}} & \frac{\partial y_2^{conv.1}}{\partial x_3^{conv.1}} & \dots & \frac{\partial y_2^{conv.1}}{\partial x_1^{cont.1}} & \frac{\partial y_2^{conv.1}}{\partial x_2^{cont.1}} & \dots \\ \frac{\partial y_3^{conv.1}}{\partial x_1^{conv.1}} & \frac{\partial y_3^{conv.1}}{\partial x_2^{conv.1}} & \frac{\partial y_3^{conv.1}}{\partial x_3^{conv.1}} & \dots & \frac{\partial y_3^{conv.1}}{\partial x_1^{cont.1}} & \frac{\partial y_3^{conv.1}}{\partial x_2^{cont.1}} & \dots \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \dots \\ ine \frac{\partial y_1^{cont.1}}{\partial x_1^{conv.1}} & \frac{\partial y_1^{cont.1}}{\partial x_2^{conv.1}} & \frac{\partial y_1^{cont.1}}{\partial x_3^{conv.1}} & \dots & \frac{\partial y_1^{cont.1}}{\partial x_1^{cont.1}} & \frac{\partial y_1^{cont.1}}{\partial x_2^{cont.1}} & \dots \\ \frac{\partial y_2^{cont.1}}{\partial x_1^{conv.1}} & \frac{\partial y_2^{cont.1}}{\partial x_2^{conv.1}} & \frac{\partial y_2^{cont.1}}{\partial x_3^{conv.1}} & \dots & \frac{\partial y_2^{cont.1}}{\partial x_1^{cont.1}} & \frac{\partial y_2^{cont.1}}{\partial x_2^{cont.1}} & \dots \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \dots \\ ine \frac{\partial y_G}{\partial x_1^{conv.1}} & \frac{\partial y_G}{\partial x_2^{conv.1}} & \frac{\partial y_G}{\partial x_3^{conv.1}} & \dots & \frac{\partial y_G}{\partial x_1^{cont.1}} & \frac{\partial y_G}{\partial x_2^{cont.1}} & \dots \end{array} \right] \begin{bmatrix} \frac{\partial y_1^{conv.1}}{\partial x_G} \\ \frac{\partial y_2^{conv.1}}{\partial x_G} \\ \frac{\partial y_3^{conv.1}}{\partial x_G} \\ \vdots \\ \frac{\partial y_1^{cont.1}}{\partial x_G} \\ \frac{\partial y_2^{cont.1}}{\partial x_G} \\ \vdots \\ \frac{\partial y_G}{\partial x_G} \end{bmatrix}$$

Table 2. Vector of state variables related to the High Voltage Direct Current (HVDC) system (Δx).

		Δx	
State variables related to the converter equations	Converter 1	$\Delta x_1^{conv.1}$	$\Delta \phi_c^{conv.1}$
		$\Delta x_2^{conv.1}$	$\Delta I_c^{conv.1}$
		$\Delta x_3^{conv.1}$	$\Delta \mu_c^{conv.1}$
	$\vdots$	$\vdots$	$\vdots$
	Converter n	$\Delta x_1^{conv.n}$	$\Delta \phi_c^{conv.n}$
		$\Delta x_2^{conv.n}$	$\Delta I_c^{conv.n}$
$\Delta x_3^{conv.n}$		$\Delta \mu_c^{conv.n}$	
State variables related to the Control equations	Converter 1	$\Delta x_1^{cont.1}$	$\Delta a_c^{conv.1}$
		$\Delta x_2^{cont.1}$	$\Delta \delta_c^{conv.1}$
	$\vdots$	$\vdots$	$\vdots$
	Converter n	$\Delta x_1^{cont.n}$	$\Delta a_c^{conv.n}$
		$\Delta x_2^{cont.n}$	$\Delta \delta_c^{conv.n}$
	State variables related to the DC network equations	Converter 1	Δx_G
$\vdots$		$\vdots$	
Converter n		ΔVd_n	

By observing Equation (16), the original Jacobian matrix of the problem is preserved. The new derivatives are located in additional rows and columns. The AC-AC block is the original Jacobian matrix and contains the derivatives of the equations of MW and MVar power from the AC system in relation to the variables of the original system state. In the AC interface buses, injections of active and reactive power regarding the DC system should be considered. These injections are given by [1]

$$P_r^{dc} = Vd_r \cdot I_r \quad (17)$$

$$Q_r^{dc} = Vd_r \cdot I_r \cdot \tan(\phi_r) \quad (18)$$

$$P_i^{dc} = Vd_i \cdot I_i \quad (19)$$

$$Q_i^{dc} = -Vd_i \cdot I_i \cdot \tan(\phi_i) \quad (20)$$

From there, the residuals of the interface buses are given by

$$\Delta P'_c = P_c^{sch} - P'_c \quad (21)$$

$$\Delta Q'_c = Q_c^{sch} - Q'_c \quad (22)$$

where:

$$P'_c = P_{calc} + P_c^{dc} \cdot MVA_{ac}^{dc} \quad (23)$$

$$Q'_c = Q_{calc} + Q_c^{dc} \cdot MVA_{ac}^{dc} \quad (24)$$

$$MVA_{ac}^{dc} = \frac{MVA_{base}^{dc}}{MVA_{base}^{ac}} \quad (25)$$

Tutorial Example

This tutorial example shows with more details the basic principles of the proposed AC-DC power flow with a HVDC multiterminal system with three converter terminals

(connected to infinity buses), one intermediate DC bus (called X in this example), and three HVDC lines, shown in Figure 2, which is composed of a DC network (represented by a generic methodology), with control modes that are shown below.

Figure 2. Tutorial Example: System test multiterminal HVDC.

For the case in study, the converters are working under the control modes shown in Table 3.

Table 3. Control mode configuration for the HVDC multi-terminal tutorial example test system.

	Rectifier 1	Rectifier 2	Inverter 1
Control modes	Constant-Power (CP)	Constant-Current (CC)	CEA with Constant-Voltage (CV)

The system of equations that models, in a generic form, the DC network in this case is

$$\begin{bmatrix} I_{r1} \\ I_{r2} \\ I_{i1} \\ I_x \end{bmatrix} = \begin{bmatrix} \frac{3}{R_{dc}} & -\frac{1}{R_{dc}} & 0 & -\frac{2}{R_{dc}} \\ -\frac{1}{R_{dc}} & \frac{2}{R_{dc}} & -\frac{1}{R_{dc}} & 0 \\ 0 & -\frac{1}{R_{dc}} & \frac{3}{R_{dc}} & -\frac{2}{R_{dc}} \\ -\frac{2}{R_{dc}} & 0 & -\frac{2}{R_{dc}} & \frac{4}{R_{dc}} \end{bmatrix} \cdot \begin{bmatrix} V_{dr1} \\ V_{dr2} \\ V_{di1} \\ V_{dx} \end{bmatrix} \quad (26)$$

The residuals equations related to the DC system (Δy^{dc}) are

$$\Delta y_1^{dc} = -V_{dr1} + k_r \cdot a_{r1} \cdot V_{r1} \cdot \cos(\alpha_1) - \frac{3}{\pi} \cdot X_{r1} \cdot I_{r1} \quad (27)$$

$$\Delta y_2^{dc} = -\frac{2 \cdot \mu_{r1} + \sin(2 \cdot \alpha_1) - \sin(2 \cdot (\alpha_1 + \mu_{r1}))}{\cos(2 \cdot \alpha_1) - \cos(2 \cdot (\alpha_1 + \mu_{r1}))} + \tan(\phi_{r1}) \quad (28)$$

$$\Delta y_3^{dc} = -\cos(\alpha_1) + \cos(\alpha_1 + \mu_{r1}) + \frac{2 \cdot R_{r1} \cdot I_{r1}}{k_r \cdot a_{r1} \cdot V_{r1}} \quad (29)$$

$$\Delta y_4^{dc} = -V_{dr2} + k_r \cdot a_{r2} \cdot V_{r2} \cdot \cos(\alpha_2) - \frac{3}{\pi} \cdot X_{r2} \cdot I_{r2} \quad (30)$$

$$\Delta y_5^{dc} = -\frac{2 \cdot \mu_{r2} + \sin(2 \cdot \alpha_2) - \sin(2 \cdot (\alpha_2 + \mu_{r2}))}{\cos(2 \cdot \alpha_2) - \cos(2 \cdot (\alpha_2 + \mu_{r2}))} + \tan(\phi_{r2}) \quad (31)$$

$$\Delta y_6^{dc} = -\cos(\alpha_2) + \cos(\alpha_2 + \mu_{r2}) + \frac{2 \cdot R_{r2} \cdot I_{r2}}{k_r \cdot a_{r2} \cdot V_{r2}} \quad (32)$$

$$\Delta y_7^{dc} = -Vd_{i1} + k_i \cdot a_{i1} \cdot V_{i1} \cdot \cos(\gamma_1) + \frac{3}{\pi} \cdot X_{i1} \cdot I_{i1} \quad (33)$$

$$\Delta y_8^{dc} = -\frac{2 \cdot \mu_{i1} + \sin(2 \cdot \gamma_1) - \sin(2 \cdot (\gamma_1 + \mu_{i1}))}{\cos(2 \cdot \gamma_1) - \cos(2 \cdot (\gamma_1 + \mu_{i1}))} + \tan(\phi_{i1}) \quad (34)$$

$$\Delta y_9^{dc} = -\cos(\gamma_1) + \cos(\gamma_1 + \mu_{i1}) + \frac{2 \cdot R_{i1} \cdot I_{i1}}{k_i \cdot a_{i1} \cdot V_{i1}} \quad (35)$$

$$\Delta y_{10}^{dc} = P_{r1}^{dcsch} - P_{r1}^{dc} \quad (36)$$

$$\Delta y_{11}^{dc} = -Vd_{r1} + 0.97 \cdot (k_{r1} \cdot a_{r1} \cdot V_{r1} \cdot \cos(\alpha_{r1}^{min}) - R_{r1} \cdot I_{r1}) \quad (37)$$

$$\Delta y_{10}^{dc} = Id_{r2}^{dcsch} - Id_{r2}^{dc} \quad (38)$$

$$\Delta y_{11}^{dc} = -Vd_{r2} + 0.97 \cdot (k_{r2} \cdot a_{r2} \cdot V_{r2} \cdot \cos(\alpha_{r2}^{min}) - R_{r2} \cdot I_{r2}) \quad (39)$$

$$\Delta y_{12}^{dc} = \cos \gamma_{i1}^{sch} - \cos \gamma_{i1} \quad (40)$$

$$\Delta y_{13}^{dc} = Vd_{i1}^{dcsch} - Id_{i1}^{dc} \quad (41)$$

$$\Delta y_{14}^{dc} = I_{r1} - \frac{(Vd_{r1} - Vd_{r2})}{R_{dc}} - \frac{2 \cdot (Vd_{r1} - Vd_x)}{R_{dc}} \quad (42)$$

$$\Delta y_{15}^{dc} = I_{r2} - \frac{(Vd_{r2} - Vd_{r1})}{R_{dc}} - \frac{(Vd_{r2} - Vd_{i1})}{R_{dc}} \quad (43)$$

$$\Delta y_{16}^{dc} = I_{i1} - \frac{(Vd_{i1} - Vd_{r2})}{R_{dc}} - \frac{2 \cdot (Vd_{i1} - Vd_x)}{R_{dc}} \quad (44)$$

$$\Delta y_{17}^{dc} = I_x - \frac{2 \cdot (Vd_x - Vd_{r1})}{R_{dc}} - \frac{2 \cdot (Vd_x - Vd_{i1})}{R_{dc}} \quad (45)$$

The DC variables of the problem are included in the following order: ϕ_{r1} , I_{r1} , μ_{r1} , ϕ_{r2} , I_{r2} , μ_{r2} , ϕ_{i1} , I_{i1} , μ_{i1} , a_{r1} , α_{r1} , a_{r2} , α_{r2} , a_{i1} , γ_{i1} , Vd_{r1} , Vd_{r2} , Vd_{i1} , and Vd_x .

The solution of the general linear system of Equation (16) provides the values for the variables in the steady state of the AC/DC system.

4. Results

The proposed methodology is verified through the study of two test systems. The first one is a modified version of the IEEE 14-bus system, and the second one is a 4-bus test system considering three HVDC systems with different DC networks.

4.1. Modified IEEE 14-Bus

The first study is an AC-DC system with three terminals and two HVDC lines, shown in [15], based on the well-known IEEE 14-bus. The system topology is shown in Figure 3. The base power is 100 MVA.

Figure 3. IEEE 14-bus test system with a radial three-converter HVDC multiterminal [15].

The parameters of the AC system are shown in Tables 4–8.

Table 4. Capacitor shunt compensation (MVar).

AC Bus No.	9	2	4	5
Rated reactive power	15.0	20.0	20.0	20.0

Table 5. Reactance and tap of transformers (%).

From Bus No.	To Bus No.	Ratio	X_t
4	7	0.978	20.912
4	9	0.969	55.618
5	6	0.932	25.202

Table 6. Generator Parameters.

Bus No.	V_g (p.u.)	Q_{max} (MVar)	Q_{min} (MVar)	P_g (MW)
1	1.060	150.00	−150.00	232.38
2	1.045	50.00	−40.00	40.00
3	1.010	40.00	0.00	0.00
6	1.070	40.00	−6.00	0.00
8	1.090	40.00	−6.00	0.00

V_g = generator terminal voltage; Q_{max} = maximum reactive power generation; Q_{min} = minimum reactive power generation; P_g = output active power generation.

Table 7. System load configuration.

Load Bus No.	$P_l(p.u.)$	$Q_l(p.u.)$
2	0.217	0.127
3	0.942	0.190
4	0.478	−0.039
5	0.076	0.016
6	0.112	0.075
9	0.295	0.166
10	0.090	0.058
11	0.035	0.018
12	0.061	0.016
13	0.135	0.058
14	0.149	0.050

P_l = active load power; Q_l = reactive load power.

Table 8. Parameters of transmission lines and transformers.

From Bus No.	To Bus No.	R (%)	X (%)	y (MVar)
1	2	1.938	5.917	5.28
1	5	5.403	22.304	4.92
2	3	4.699	19.797	4.38
3	4	6.701	17.103	1.28
4	5	1.335	4.211	0.00
6	11	9.498	19.890	0.00
6	12	12.291	25.581	0.00
6	13	6.615	13.027	0.00
7	8	0.000	17.615	0.00
7	9	0.000	11.001	0.00
9	14	12.711	27.038	0.00
9	10	03.181	08.450	0.00
10	11	8.205	19.207	0.00
12	13	22.092	19.988	0.00
13	14	17.093	34.802	0.00

R = series resistance; X = series reactance; y = total shunt admittance.

The parameters of the HVDC system and the control modes in this study are shown in Tables 9 and 10, respectively.

Table 9. The HVDC system configuration.

Converter No.	AC Bus No.	$a_c(p.u.)$	Type	$X_c(p.u.)$	HVDC Line	$R_d(p.u.)$
1	5	0.950	Rect.	0.083	1–2	0.02
2	2	0.975	Rect.	0.125	2–3	0.02
3	4	1.000	Inv.	0.125		

Table 10. Control modes configuration for Cases 1 and 2.

Converter No.	Case 1			Case 2		
	1	2	3	1	2	3
Control mode	CP	CC	CEA	CIA	CP	CEA
Scheduled value	0.47	0.47	17	14	0.56	17

Using the proposed methodology, it is possible to solve the problem in both operating conditions. The results for Cases 1 and 2 are shown in Tables 11–14. It is important to point

out that the proposed methodology converged in eight iterations in both cases. The adopted solution tolerances were 10^{-6} p.u. for the AC system and 10^{-9} p.u. for the DC system. All results were validated with the reference [15].

Table 11. AC bus voltage result.

Bus No.	Case 1		Case 2	
	Magnitude (p.u.)	Ang (°)	Magnitude (p.u.)	Ang (°)
1	1.0600	0.000	1.0600	0.000
2	1.0450	−4.084	1.0450	−4.077
3	1.0100	−13.550	1.0100	−13.561
4	0.9940	−12.189	0.9938	−12.213
5	0.9940	−12.132	0.9937	−12.162
6	1.0700	−17.380	1.0700	−17.409
7	1.0493	−15.546	1.0492	−15.571
8	1.0900	−15.546	1.0900	−15.571
9	1.0425	−17.267	1.0425	−17.293
10	1.0400	−17.570	1.0399	−17.597
11	1.0514	−17.595	1.0514	−17.623
12	1.0541	−18.188	1.0541	−18.217
13	1.0485	−18.197	1.0485	−18.226
14	1.0270	−18.687	1.0270	−18.715

Table 12. Active and reactive generation result.

Bus No.	Case 1		Case 2	
	P (MW)	Q (MVar)	P (MW)	Q (MVar)
1	233.1400	0.0028	233.1800	0.0044
2	39.3671	41.7683	39.3652	41.3397
3	0.0000	38.8600	0.0000	38.9700
4	0.0000	0.0787	0.0000	0.0005
5	0.0000	0.1516	0.0000	0.0388
6	0.0000	30.3600	0.0000	30.5100
8	0.0000	25.2000	0.0000	25.2600

Table 13. HVDC system result for Case 1.

Parameter	Converter No.—Control Mode		
	1—CP	2—CC	3—CEA
V_{d_c} (p.u.)	1.2069	1.1991	1.1819
ϕ_c (°)	18.7800	29.2480	27.8680
I_c (p.u.)	0.3894	0.4700	−0.8594
δ_c (°)	14.0600	24.1800	17.0000
μ_c (°)	8.7690	9.6480	19.5290
a_c (p.u.)	0.9500	0.9750	1.0000
P_c^{dc} (MW)	46.9967	56.3571	−101.5734
Q_c^{dc} (MVar)	15.9810	31.5594	53.7087

The comparison of Tables 10 and 13 shows that the proposed methodology was effective in controlling the scheduled values in Case 1 considering the CP, CC, and CEA modes, respectively.

Regarding Case 2, the comparison of Tables 10 and 14 also shows that the proposed methodology was effective in controlling the scheduled values considering the CIA, CP, and CEA modes, respectively.

MTDC systems. The proposed methodology uses the simultaneous strategy for the AC-DC power flow solution. This choice is related mainly to the following factors. First, because of the work proposed in [8], in which an analysis for the simultaneous solution of AC-DC power flow for steady-state studies is developed, for a DC system of only two terminals, based on the DC transmission link of Itaipu, which has served as the premise for the studies proposed in this paper. Second, the full Newton methodology is a well-established tool in the general literature on the subject, and, depending on the system feature, the choice of a sequential strategy could result in a delay in the AC convergence process, so the simultaneous strategy might be more interesting in these scenarios. Third, for the purposes of the study, the chosen method is best suited to the needs of this work. Finally, another decisive factor in this choice is related to the fact that there are few studies nowadays about the solution of Multi-Terminals HVDC systems using the simultaneous strategy.

5. Conclusions

This paper addresses the main assumptions involved in the modeling of Multi-Terminal HVDC systems in the AC-DC power flow problem for steady-state studies. The proposed methodology is based on a full Newton formulation of the AC-DC power flow problem for a multi-terminal HVDC system with a generic methodology for representing the DC network in steady-state studies.

The results show that the proposed methodology is valid and effective in power transmission studies via Multi-Terminal HVDC systems. It can be stated that the methodology and the proposed models constitute valuable tools in solving current problems of AC-DC power flow, as a result of the increase in electricity demand, the interconnection of systems operating in different frequencies, and a more intensive use of Multi-Terminal HVDC transmission structures in electric power systems.

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Abbreviations

The following abbreviations are used in this manuscript:

HVDC	High Voltage Direct Current
MTDC	Multi-Terminal HVDC
VSC	Voltage Source Converter
CSC	Current Source Converter
DC	Direct Current
AC	Alternating Current

The following notation are used in this manuscript:

$\hat{V}_c$	Primary phasor voltage of the converter transformer
$\hat{E}_c$	Secondary phasor voltage of the converter transformer
a_c	Converter transformer turns ratio
X_c	Converter commutating reactance per bridge/phase
R_c	Converter equivalent commutating resistance
ϕ_c	Converter power factor angle
μ_c	Commutating converter angle
δ_c	Converter firing angle
α	Rectifier ignition angle
γ	Inverter extinction angle
V_{dc}	Converter direct voltage per pole
I_c	Converter direct current per pole
R_{dc}	DC resistance
P_c^{dc}	Injection of active power on the AC interface bus associated with the converter
Q_c^{dc}	Injection of reactive power on the AC interface bus associated with the converter
n_b	Number of series-connected bridges in a terminal

superscripts:

+	for positive pole quantities
−	for negative pole quantities
c	for converter
r	for rectifier
i	for inverter
dc	for direct current
max	for maximum values
min	for minimum values
sch	for scheduled values

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