

Article

A Hybrid Optimization Algorithm for Solving of the Unit Commitment Problem Considering Uncertainty of the Load Demand

Aml Sayed¹, Mohamed Ebeed² , Ziad M. Ali^{3,4}, Adel Bedair Abdel-Rahman^{5,6}, Mahrous Ahmed⁷ , Shady H. E. Abdel Aleem⁸ , Adel El-Shahat^{9,*}  and Mahmoud Rihan¹

- ¹ Department of Electrical Engineering, Faculty of Engineering, South Valley University, Qena 83523, Egypt; amsayed19@eng.svu.edu.eg (A.S.); mahmoudrihan@eng.svu.edu.eg (M.R.)
- ² Department of Electrical Engineering, Faculty of Engineering, Sohag University, Sohag 82524, Egypt; mebeed@eng.sohag.edu.eg
- ³ Electrical Engineering Department, College of Engineering, Prince Sattam Bin Abdulaziz University, Wadi Addawaser 11991, Saudi Arabia; dr.ziad.elhalwany@aswu.edu.eg
- ⁴ Electrical Engineering Department, Aswan Faculty of Engineering, Aswan University, Aswan 81542, Egypt
- ⁵ Electronics and Communications Engineering Department, Egypt-Japan University of Science and Technology, Alexandria 21934, Egypt; adel.bedair@ejust.edu.eg
- ⁶ Electronics and Communications Engineering Department, Faculty of Engineering, South Valley University, Qena 83523, Egypt
- ⁷ Department of Electrical, College of Engineering, Taif University, Taif 21944, Saudi Arabia; m.elsamman@tu.edu.sa
- ⁸ Department of Electrical Engineering, Valley High Institute of Engineering and Technology, Science Valley Academy, Qalyubia 44971, Egypt; engyshady@ieee.org
- ⁹ Energy Technology Program, School of Engineering Technology, Purdue University, West Lafayette, IN 47907, USA
- * Correspondence: Asayedah@purdue.edu



Citation: Sayed, A.; Ebeed, M.; Ali, Z.M.; Abdel-Rahman, A.B.; Ahmed, M.; Abdel Aleem, S.H.E.; El-Shahat, A.; Rihan, M. A Hybrid Optimization Algorithm for Solving of the Unit Commitment Problem Considering Uncertainty of the Load Demand. *Energies* **2021**, *14*, 8014. <https://doi.org/10.3390/en14238014>

Academic Editor: Ali Nabavi

Received: 1 November 2021

Accepted: 24 November 2021

Published: 30 November 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

Abstract: Unit commitment problem (UCP) is classified as a mixed-integer, large combinatorial, high-dimensional and nonlinear optimization problem. This paper suggests solving the UCP under deterministic and stochastic load demand using a hybrid technique that includes the modified particle swarm optimization (MPSO) along with equilibrium optimizer (EO), termed as MPSO-EO. The proposed approach is tested firstly on 15 benchmark test functions, and then it is implemented to solve the UCP under two test systems. The results are basically compared to that of standard EO and previously applied optimization techniques in solving the UCP. In test system 1, the load demand is deterministic. The proposed technique is in the best three solutions for the 10-unit system with cost savings of 309.95 USD over standard EO and for the 20-unit system it shows the best results over all algorithms in comparison with cost savings of 1951.5 USD over standard EO. In test system 2, the load demand is considered stochastic, and only the 10-unit system is studied. The proposed technique outperforms the standard EO with cost savings of 40.93 USD. The simulation results demonstrate that MPSO-EO has fairly good performance for solving the UCP with significant total operating cost savings compared to standard EO compared with other reported techniques.

Keywords: unit commitment; optimization; equilibrium optimizer; particle swarm optimization; uncertainty

1. Introduction

1.1. Unit Commitment

UCP is a very complicated optimization problem in electrical power system operation that involves both binary and continuous variables and considers a large set of constraints, including unit and system constraints, which complicates the problem further. It is classified as a short-term problem as it is usually considered for 24 consecutive hours,

comprising one day. The UCP aims to figure out the best on/off status for generating units at each power station and determine individual power outputs of the scheduled generation units to minimize total operating costs while meeting system load demand at each time interval [1]. Stochastic unit commitment (SUC) refers to the uncertainty in the UCP, which can appear on both the load side and/or generation side. Here, the uncertainty in the load side only is considered, so instead of assuming that the load demand is constant, it is considered vary over the day during each hour, known as load uncertainty, which obliges the units' power to track the load to keep balance operation in the power system [2]. The planning of generating units in the power system should be done so that there is an adequate generation reserve to avert failures and incidental conditions. There are several constraints in the UCP, including system and physical constraints, and the problem should be solved to satisfy all constraints over the study period [3]. A literature review states the various efforts introduced in solving UCP.

1.2. Literature Review

Both deterministic and meta-heuristic techniques inspired by nature are employed to solve the UCP [4]. Deterministic techniques including Lagrangian relaxation (LR) [5–7], priority list [8,9], mixed-integer programming (MIP) [10] and dynamic programming (DP) [11,12] belong to numerical optimization techniques, which are considered the classical methods and have the advantages of simplicity and fast convergence, but mostly suffer from poor solution quality and premature convergence. Recently, meta-heuristic techniques have been widely applied to several optimization problems, and this subsection of the research highlights some outstanding scientific efforts in solving the UCP using various meta-heuristic algorithms. In [13,14], the solution was obtained by genetic algorithms (GAs). The authors of [12] used the varying quality function approach and added specific operators to avoid using standard operators (crossover and mutation) to solve the UCP. In [13], a modified GA algorithm was used, where a matrix representation is used to encode the problem and a specific operator is applied to improve computational time and solution quality. In [15,16], the solution methodology was based on the particle swarm optimizer (PSO). The authors of [14] tended to use more information about particles to control the mutation process and apply new strategies for choosing parameters to enhance the solution of the UCP. In [15], the authors deal with binary variables of the UCP as integers, with each integer expressing the unit's continuous on/off status to reduce the number of decision variables and thus get over the defects of stochastic algorithms. In [17], the gravitational search algorithm (GSA) was used for the UCP. Reference [18] presented a binary version of fish migration optimization (BFMO) and an advanced version of binary fish migration optimization (ABFMO) to solve the UCP. In [19], a binary real-coded firefly algorithm (BRCFFA) was applied to the UCP in such a manner that the binary coded FF produced the generators' operating states through the tanh function, and the real-coded FF produced the output powers of committed generators. The authors applied simulated annealing (SA) in solving the UCP [20] by dividing the main problem into two subproblems. A combinatorial problem and a nonlinear programming problem are the two subproblems. The SA algorithm was used to solve the combinatorial problem, and a quadratic programming technique was used to address the nonlinear programming problem. The authors of [21] developed a novel adaptive binary salp swarm algorithm to solve the UCP as a mixed integer optimization problem considering the ramp rate limits. Later, hybrid approaches evolved for solving the UCP more efficiently. The authors of [22] employed evolutionary programming (EP) coupled with the tabu search method to meet the requirements of the UCP. In [23], the authors suggested an effective hybrid approach that combines PSO and grey wolf (GWO) to combine the strengths of both algorithms. The hybridization is made so that the updating process was made firstly by PSO then by GWO. In [24], a hybridization between Lagrangian relaxation, evolutionary programming and quadratic programming was introduced to solve the UCP through two coordination procedures. A combination between PSO and BPSO was proposed in [25],

where BPSO deals with binary variables and PSO deals with real variables to solve a mixed heat and power unit commitment. Additionally, a hybrid genetic algorithm and differential evolution were implemented [26,27]. Despite the fact that there is no optimizer that can be effective enough for all optimization problems, each optimizer has its own strengths and weaknesses, so the hybridization process between two optimizers seeks to avoid weak points of the optimizers and get the most out of them.

1.3. Contributions

This paper's main contribution can be summarized in three points:

1. Solving the UCP under deterministic and probabilistic states. In a stochastic case, the uncertainty in the load side is considered.
2. An efficient hybrid approach between modified particle swarm optimization and equilibrium optimizer (MPSO-EO) is proposed for solving the UCP.
3. Validation the performance of the MPSO-EO through standard benchmark functions.
4. A comparison between the proposed algorithm and well-known techniques such as EO, PSO, GWO and SCA for the solution of the UCP.

The remainder of the paper is laid out as follows: Section 2 involves mathematical problem formulation with constraints and load uncertainty modelling. Section 3 provides an overview of applied algorithms and presents the proposed hybrid methodology for solving the UCP. Section 4 illustrates the effectiveness of proposed technique through applying benchmark test functions and different test systems of the UCP. Finally, in Section 5, a conclusion is provided.

2. Problem Formulation

This section involves the UCP's objective function, problem constraints and modelling for load uncertainty.

2.1. Objective Function

The UCP's objective function is to reduce the system total operating cost by estimating optimal schedule and power output for the available generation units while satisfying several constraints. Fuel cost (power production costs), start-up and shutdown costs make up the power generation's total operating cost function. Mathematical representation for the objective function of the UCP is represented by Equation (1) as follows:

$$\text{Min } F_T = \sum_{t=1}^T \sum_{i=1}^N U_i^t \times FC_i^t + SC_i^t + SD_i^t \quad (1)$$

2.1.1. Fuel Cost

A quadratic equation can be used to express the cost of fuel and is represented by Equation (2):

$$FC_i^t = a_i + b_i \times P_i(t) + c_i \times P_i^2(t) \quad (2)$$

where a_i , b_i and c_i represent the fuel cost coefficients for i th generating unit.

2.1.2. Start-Up Cost

It is the incurred cost at the starting of a generating unit. Thermal units must be "warmed up" before they can be brought online. The warming up process costs money and thus affects the total operating cost. The cost of re-starting a unit is determined by how long it has been off. Different units have different start-up costs and the cost of starting up unit i can be calculated as in Equation (3):

$$SC_i^t = \begin{cases} SC_{i_{hot}} \rightarrow MDT_i \leq T_{OFF_i}^t \leq MDT_i + T_{cold_i} \\ SC_{i_{cold}} \rightarrow T_{OFF_i}^t > MDT_i + T_{cold_i} \end{cases} \quad (3)$$

2.1.3. Shutdown Cost

The cost of shutting down all units is the same, but it is not considered in this study.

2.2. Constraints

2.2.1. Thermal Units Constraints

(a) Generation power limits

Output power limits from the thermal units is given in Equation (4):

$$P_{i_{min}} \leq P_i(t) \leq P_{i_{max}} \quad (4)$$

(b) Minimum up/down time constraints

• Minimum up time constraint

Once a unit is running, it may not be turned off instantly and this constraint is expressed in Equation (5):

$$T_{ON_i} \geq MUT_i \quad (5)$$

• Minimum down time constraint

A unit cannot be restarted instantly after it has been turned off, and this constraint is expressed in Equation (6):

$$T_{OFF_i} \geq MDT_i \quad (6)$$

(c) Spinning reserve

The system should have additional capacity to face sudden accidents, such as sudden load increase or generator outage known as spinning reserve, and is represented by Equation (7):

$$\sum_{i=1}^N U_i(t) \times P_{i_{max}} \geq SR^t + P^t_L \quad (7)$$

2.2.2. System Constraints

(a) Power balance constraint

$$\sum_{i=1}^N U_i(t) \times P_i(t) = P^t_L \quad (8)$$

2.3. Load Uncertainty Model

The modelling of load demand uncertainty in a power system is made using probability density functions [28], which can be represented using Equation (9).

$$PDF_{LD}(S^t_{LD}) = \frac{1}{\sqrt{2\pi\sigma^t_{LD}}} \exp\left[-\frac{(S^t_{LD} - \mu^t_{LD})^2}{\sigma^t_{LD}^2}\right] \quad (9)$$

where PDF_{LD} represents the load demand probability density function and S^t_{LD} is the load demand apparent power at time t .

The proposed work depends on Monte Carlo simulation (MCS) and scenario-based reduction techniques to deal with load demand uncertainty.

3. Optimization Algorithm

3.1. Particle Swarm Optimization (PSO)

In 1995, Eberhart and Kennedy introduced (developed) a population-based optimization algorithm as a substitute for GAs, known as 'particle swarm [29] optimization' (PSO) [30]. PSO was motivated by creatures' social behavior like flocks of birds, schools of fishes, etc. PSO depends on the fact of seeking for finding the optimal solution in a

multidimensional search area. The strength of PSO comes from the social interactions between individuals as they search the space collaboratively to obtain the best solution globally. In PSO, the swarm is referred to as a population and each individual is referred to as a particle. Each particle stands for a candidate solution for the solved optimization problem and has two associated vectors defined as position and velocity vectors. At each iteration, each particle tracks two values: (1) the particle's best previous position known as the personal best (P_{best}) and (2) the best position ever found between all particles in the population known as the global best (G_{best}).

Let X and V be the i th particle's position and velocity vectors in a search space, respectively. Then, in each iteration, the velocity and position of each particle are updated based on the two tracked values. They are represented mathematically by Equations (10) and (11):

$$V^{it+1}_i = \omega * V^{it}_i + c_1 * Rand_1 * (P_{best} - X^{it}_i) + c_2 * Rand_2 * (G_{best} - X^{it}_i) \quad (10)$$

$$X^{it+1}_i = X^{it}_i + V^{it+1}_i \quad (11)$$

where ω is called inertia weight; $Rand_1$ and $Rand_2$ are random vectors in range of $[0, 1]$; c_1 and c_2 are called acceleration coefficients and have values between 0 and 2.5; X^{it}_i and V^{it}_i are the i th particle's position and velocity vectors at iteration it , respectively; and X^{it+1}_i and V^{it+1}_i are the i th particle's position and velocity vectors at iteration $it + 1$, respectively.

The appropriate selection of inertia weight ω is important as it affects the exploration properties of PSO. It is given in Equation (12):

$$\omega = \omega_{mini} + \frac{\omega_{maxi} - \omega_{mini}}{it_{max}} * (it_{max} - it) \quad (12)$$

where ω_{mini} and ω_{maxi} are the inertia weight's minimum and maximum values and are generally taken as 0.4 and 0.9, respectively. it denotes the current iteration while it_{max} denotes the maximum number of iterations.

PSO has gained wide popularity due to its simplification of application and the ease in adjusting its few parameters. Its flexibility in adjustment makes it a preferred choice in the hybridization process with most modern algorithms to enhance the solution of several optimization problems such as those in [31], where the authors employed PSO with the firefly algorithm to solve the issues of the multi-objective optimal power flow. Additionally, in [32], a hybrid algorithm of PSO and grey wolf optimizer (GWO) was developed to solve the problem of optimal power flow under uncertainty of solar and wind power. A modified version of PSO is employed with EO to solve the UCP under deterministic and stochastic load demand.

3.2. Equilibrium Optimizer (EO)

Equilibrium optimizer (EO) was introduced by Faramarzi depending on the physical basis to process the continuous optimization problems [33]. The performance of the grey wolf optimizer (GWO) and the solution of the mass balance equation on a control volume were the inspirations for EO. EO tries to find the state of equilibrium that implements the mass balance between the entered, generated and output mass of a control volume. The inspiring mass balance equation is given in Equation (13):

$$V \frac{dc}{dt} = QC_{eq} - QC + G \quad (13)$$

where V represents the control volume, C_{eq} gives the concentration of the equilibrium state, Q denotes the flow rate, C denotes the concentration and G represents the mass generation rate. After that, by solving Equation (13) for the concentration (C) as a function of time (t), it is possible to either find the concentration that exists in the control volume as the turnover rate is known or to determine the average turnover rate when the generation rate and other conditions are known. In the EO algorithm, each particle refers to a candidate solution

and its concentration refers to the position of this particle and both are acting as a search agent. Each search agent randomly updates its position based on best solutions found so far, called equilibrium candidates, to reach the state of equilibrium (optimal solution). The approach for updating the particles' (search agents') positions in EO algorithms can be summarized as follows.

3.2.1. Initialization

EO, like other meta-heuristic optimizers, uses the initial population to establish the particles' initial positions randomly in the search space, according to the equation given in (14):

$$\vec{C}_i^{initial} = lb + Rand_i(ub - lb) \quad (14)$$

where $i = 1, 2, \dots, N$ and N represents the population size; lb and ub are the control variables' lower and upper limits, respectively; and $Rand_i$ is random vector in range of $[0, 1]$. Then the fitness function for the initial particles is calculated.

3.2.2. Equilibrium Candidates and Equilibrium Pool

The particles are sorted depending on their corresponding positions and the four particles with the best positions are estimated and their average is calculated, as shown in Equation (15), to create a fifth particle whose position is equal to the calculated average value.

$$\vec{C}_{eq(ave)} = \frac{\vec{C}_{eq1} + \vec{C}_{eq2} + \vec{C}_{eq3} + \vec{C}_{eq4}}{4} \quad (15)$$

These five particles are called equilibrium candidates. They form a vector called equilibrium pool, which represented by Equation (16):

$$\vec{C}_{eq(pool)} = \left\{ \vec{C}_{eq1}, \vec{C}_{eq2}, \vec{C}_{eq3}, \vec{C}_{eq4}, \vec{C}_{eq(ave)} \right\} \quad (16)$$

where $\vec{C}_{eq(pool)}$ represents the equilibrium pool; $\vec{C}_{eq1}, \vec{C}_{eq2}, \vec{C}_{eq3}$ and \vec{C}_{eq4} are the four individual particles with the best positions found so far; $\vec{C}_{eq(ave)}$ denotes the average of the best four particles. The equilibrium pool particles are updated at each iteration.

3.2.3. Exponential Term and Concentrations Update

During the update of the particle positions (concentrations) throughout the iterations, the exponential term (F) is essential in the EO algorithm to balance exploration and exploitation. Mathematically, it is represented by (17):

$$\vec{F} = a_1 \text{sign}(\vec{r} - 0.5) \left[e^{-\lambda t} - 1 \right] \quad (17)$$

where

$$t = \left(1 - \frac{it}{it_{max}} \right)^{\left(a_2 \frac{it}{it_{max}} \right)} \quad (18)$$

where λ is called control volume and is random vector in range of $[0, 1]$; r is uniform random vector in range of $[0, 1]$; and a_1, a_2 are constants and their values are 2 and 1, respectively. They are used to adjust the exponential value; it represents the current iteration and it_{max} represents the maximum number of iterations. It is worth mentioning that a_1 affects the exploration ability of the algorithm while a_2 affects the exploitation (sign $(r - 0.5)$ controls the exploitation and the exploration direction).

3.2.4. Generation Rate and Concentrations Update

The second important term in EO approach for updating the particles' positions (concentrations) during optimization process is called the generation rate (G). The generation

rate controls the exploitation process and is given mathematically as a function of time in Equation (19):

$$\vec{G} = \vec{G}_0 e^{-\vec{k}(t-t_0)} \quad (19)$$

where G_0 indicates the initial value and k represents a decay constant. For having a more controllable search pattern and to control the number of random variables, EO assumes that $\vec{k} = \vec{\lambda}$. Then, the final generation rate expression is represented by Equation (20) as follows:

$$\vec{G} = \vec{G}_0 e^{-\vec{\lambda}(t-t_0)} = \vec{G}_0 * F \quad (20)$$

where

$$\vec{G}_0 = \overrightarrow{GCP} \left(\vec{C}_{eq} - \vec{\lambda} \vec{C} \right) \quad (21)$$

and

$$\overrightarrow{GCP} = \begin{cases} 0.5 r_1 & r_2 \geq GP \\ 0 & r_2 < GP \end{cases} \quad (22)$$

where r_1 and r_2 are random vectors in range of $[0, 1]$ and \overrightarrow{GCP} is the control parameter of the generation rate (G).

The final updating equation for EO depending on the previous approach is given in Equation (23):

$$\vec{C} = \vec{C}_{eq} + \left(\vec{C} - \vec{C}_{eq} \right) \cdot \vec{F} + \frac{\vec{G}}{\vec{\lambda} V} \left(1 - \vec{F} \right) \quad (23)$$

3.2.5. Memory Saving for Particles

The mechanism of memory saving in EO resembles the concept of P_{best} in PSO. The addition of a memory-saving mechanism helps each particle to keep in track with its best positions so far in the search space. The fitness value of each individual particle in the current iteration is compared to its fitness value from the previous iteration in this step of the algorithm, and the fittest value is preserved. Although this technique aids exploitation capability, it may also increase the chance of falling into local minima.

3.3. The Proposed Hybrid Methodology

This paper uses a hybrid strategy to tackle the unit commitment optimization problem, which combines modified particle swarm optimization (PSO) with the equilibrium optimizer (EO). Although EO solves various optimization problems effectively, it depends on the five particles in the equilibrium pool known as equilibrium candidates in the updating process. These particles suffer from the shortcomings of reduced population variety and trapping into the local optimum. PSO also has several drawbacks, such as stagnation and the particles' proclivity to become idle after a certain number of iterations, resulting in the lack of local and global search capabilities. To overcome the defects of standard versions of EO and PSO, this paper proposes a hybridization combined between the two optimizers so that EO's population diversity increases and the ability of PSO to escape from the local minima increases, but the convergence rate of the hybrid algorithm slows down. As a result, the performance of the optimizers is enhanced, and it is ensured to get the best possible optimal solution for the UCP that avoids local stagnation; thus, over the schedule period, the total operating cost is reduced.

The modified PSO refers to depend on time-varying acceleration coefficient (c_1 and c_2). Proper choice of the coefficients c_1 and c_2 can affect the speed and efficiency of the algorithm, resulting in faster convergence of the algorithm and avoidance for the local optima.

It has been noticed that the best performance of PSO is when c_1 is changing in a descending manner while c_2 is changing in ascending manner, and generally this change is between 0 and 2.5 over the full course of iterations. There are various time-varying

updating strategies to determine c_1 and c_2 [34]. The formula used in this paper is given in Equations (24) and (25) as follows:

$$c_1 = -2 * \frac{it^3}{it_{max}^3} + 2 \tag{24}$$

$$c_2 = 2 * \frac{it^3}{it_{max}^3} \tag{25}$$

In which c_1 and c_2 change between 0 and 2.

The positions of particles are initially updated using the modified PSO (MPSO) algorithm and then further updated using the EO method in the proposed hybrid modified PSO–EO algorithm. Figure 1 depicts the process for the suggested hybrid approach.

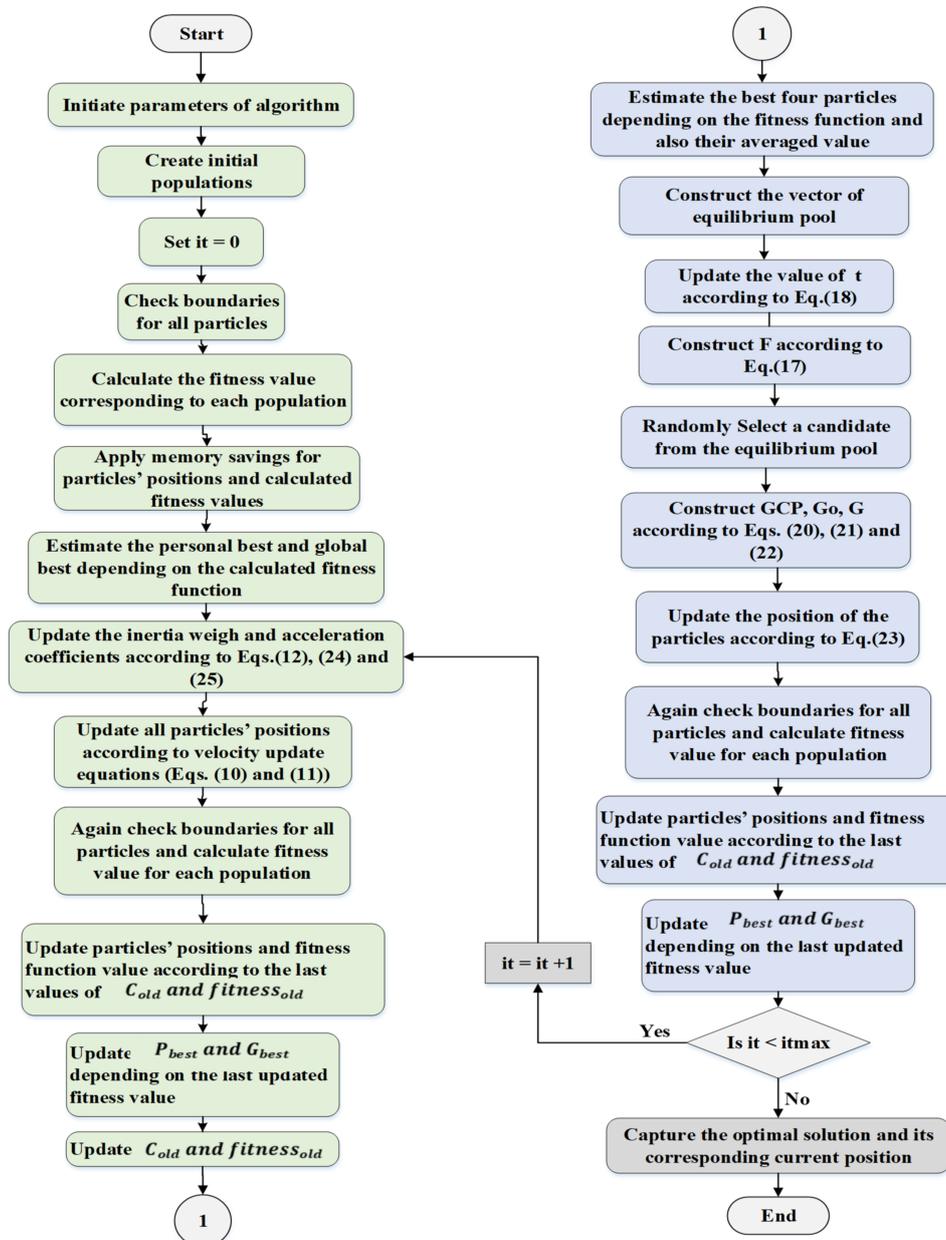


Figure 1. Flow chart of proposed MPSO-EO.

4. Results and Discussion

4.1. First: Application on Benchmark Test Functions

To characterize the performance of the proposed hybrid MZPSO–EO algorithm, a group of 15 familiar benchmark test functions are used. Then, the results are compared along with four of the famous meta-heuristic algorithms such as the equilibrium optimizer (EO), grey wolf optimizer (GWO), particle swarm optimizer (PSO) and sine cosine algorithm (SCA).

4.1.1. Benchmark Test Functions

Generally, the benchmark test functions are 29 functions and are categorized into four sections which are unimodal, multimodal with no local minima, multimodal with many local minima and composite functions. The first section includes seven test functions (F₁–F₇). The second section includes six test functions (F₈–F₁₃). The third section involves 10 test functions (F₁₄–F₂₃). The last section consists of six composite test functions (F₂₄–F₂₉). All of these functions represent minimization problems.

4.1.2. Benchmark Test Functions Comparison

The hybrid approach's performance is tested by comparing with the mentioned algorithms in Section 4.1 based on the unimodal and multimodal functions (15 test functions are selected). The unimodal functions have a single optimum solution and are used to evaluate the ability of meta-heuristic algorithms to be exploited. The multimodal functions have many optimal solutions and are used to test the exploration ability of the examined meta-heuristic algorithms. The maximum number of iterations and population size are set to 800 and 50 for functions (F₁–F₇), 300 and 30 for functions (F₈–F₁₃) and 100 and 20 for functions (F₁₄–F₁₅), respectively. To deal with stochastic nature of these algorithms, 25 trial runs were performed for each benchmark function. The best-of-run solution, the worst-of-run solution, the standard deviation and the average solution of all runs are all reported in Table 1. The convergence characteristics' curves comparison between the proposed technique and previously mentioned techniques is given in Figure 2. As shown from the results in Table 1, MPSO–EO achieved the best performance for unimodal functions (F₁–F₇), except for function (F₆), in which it achieved the second-best performance after the original EO. These results show the superiority of the proposed technique and indicate that the applied hybridization improved the exploitation ability of the original EO. For multimodal functions, MPSO–EO succeeded in achieving the best performance for function F₈, which is considered the most complicated function among all other functions in this section. For F₉ and F₁₁, both MPSO–EO and standard EO reached the global optimal, but MPSO–EO outperformed in the mean value and standard deviation. For F₁₄, all algorithms except for SCA reached the global optimal, but MPSO–EO outperformed in the mean value and occupied the second position after standard EO for the standard deviation value. For functions F₁₀, F₁₂, F₁₃ and F₁₅, MPSO–EO performed the best among all proposed algorithms. The discussed ranking is based on the best-of-run value.

Table 1. Comparison for benchmark test functions results.

Function No.		MPSO–EO	EO	PSO	GWO	SCA
F1	Best	9.0495×10^{-106}	1.1579×10^{-83}	0.1215795	1.6997×10^{-58}	9.3414×10^{-6}
	Worst	3.1214×10^{-100}	4.2725×10^{-79}	1.531071	6.1888×10^{-55}	3.765
	Mean	1.573×10^{-101}	4.3309×10^{-80}	0.4307582	7.1958×10^{-56}	0.1748
	Std	5.7719×10^{-101}	9.83229×10^{-80}	0.2737527	1.3523×10^{-55}	0.6923
F2	Best	3.6375×10^{-56}	6.6036×10^{-46}	0.7375	1.3512×10^{-32}	2.9791×10^{-6}
	Worst	3.4786×10^{-54}	1.1718×10^{-43}	41.2712	1.009×10^{-30}	0.00096
	Mean	4.2934×10^{-55}	2.4282×10^{-44}	4.8417	2.2264×10^{-31}	0.00019
	Std	7.2358×10^{-55}	2.9957×10^{-44}	7.4781	2.389×10^{-31}	0.0002

Table 1. Cont.

Function No.		MPSO-EO	EO	PSO	GWO	SCA
F3	Best	3.558×10^{-38}	1.1168×10^{-27}	33.78574	1.7032×10^{-20}	17.18137
	Worst	1.7073×10^{-26}	5.8834×10^{-20}	112.7994	8.5859×10^{-14}	7716.624
	Mean	5.7827×10^{-28}	2.9493×10^{-21}	65.95589	4.1645×10^{-15}	2545.905
	Std	3.1155×10^{-27}	1.0939×10^{-20}	21.51473	1.5601×10^{-14}	1954.96
F4	Best	8.9129×10^{-23}	1.0421×10^{-22}	1.0515	1.8572×10^{-15}	4.7894
	Worst	3.0663×10^{-16}	1.1162×10^{-19}	1.8308	6.9359×10^{-13}	46.8004
	Mean	2.9137×10^{-17}	1.3109×10^{-20}	1.4866	8.9864×10^{-14}	18.5352
	Std	6.3935×10^{-17}	2.8049×10^{-20}	0.2206	1.333×10^{-13}	10.9167
F5	Best	23.1873	23.8427	139.5195	24.9101	28.3771
	Worst	24.2688	24.5458	1110.277	27.9375	10,848.22
	Mean	23.6952	24.1897	342.3329	26.40021	689.1504
	Std	0.29534	0.1921	211.188	0.7468	1999.06
F6	Best	1.3896×10^{-11}	2.0002×10^{-13}	0.0669	1.2974×10^{-5}	3.7721
	Worst	7.1921×10^{-9}	6.1705×10^{-10}	0.8661	1.0023	5.5307
	Mean	8.3805×10^{-10}	3.686×10^{-11}	0.3458	0.4026	4.4089
	Std	1.3708×10^{-9}	1.1153×10^{-10}	0.2197	0.2869	0.4089
F7	Best	7.6647×10^{-5}	0.0001	0.2622	0.0002	0.0029
	Worst	0.0005	0.00097	21.6872	0.0019	0.1073
	Mean	0.0003	0.00046	5.2364	0.0008	0.0285
	Std	0.0001	0.00019	5.8439	0.0004	0.0254
F8	Best	-9865.201	-9719.074	-7770.439	-7315.422	-4246.805
	Worst	-7414.904	-6908.452	-2747.769	-3016.587	-3119.04
	Mean	-8589.561	-8503.638	-5479.792	-5846.141	-3616.961
	Std	712.506	719.2974	1361.11	1328.698	283.2907
F9	Best	0	0	163.1299	5.5707×10^{-12}	21.3267
	Worst	5.6843×10^{-14}	2.2737×10^{-13}	307.7645	20.5118	180.1624
	Mean	4.5474×10^{-15}	2.9559×10^{-14}	243.6789	6.4631	70.2827
	Std	1.5739×10^{-14}	5.2201×10^{-14}	38.1449	4.8576	33.9405
F10	Best	2.2204×10^{-14}	2.4603×10^{-13}	2.4881	3.2538×10^{-9}	1.3701
	Worst	5.7732×10^{-14}	3.2623×10^{-12}	4.237	2.7796×10^{-8}	20.4224
	Mean	3.4852×10^{-14}	1.0424×10^{-12}	3.4253	1.1751×10^{-8}	15.9039
	Std	7.7484×10^{-15}	7.9499×10^{-13}	0.4637	6.4636×10^{-9}	7.5859
F11	Best	0	0	0.1689	6.3283×10^{-15}	0.9503
	Worst	0.0197	0.0246	0.6164	0.0296	9.5659
	Mean	0.0008	0.0042	0.3849	0.0087	2.1191
	Std	0.0039	0.0082	0.0847	0.0115	1.8833
F12	Best	2.7601×10^{-6}	5.3834×10^{-6}	0.0383	0.0201	3.3341
	Worst	0.1039	0.1037	1.6209	0.1148	668,487
	Mean	0.0044	0.0042	.3266	0.0547	344,523.6
	Std	0.02076	0.02073	0.3562	0.0273	134,615
F13	Best	2.1984×10^{-5}	0.0002	0.7789	0.4995	3.8972
	Worst	0.4761	0.4005	2.9736	1.2256	1.83×10^7
	Mean	0.1437	0.081	1.5124	0.7896	134,352
	Std	0.15062	0.102	0.5359	0.1984	369,891
F14	Best	0.998	0.998	0.998	0.998	0.998
	Worst	4.9505	2.9821	11.7187	15.5038	10.7631
	Mean	1.3945	1.5539	5.0092	5.8148	3.6726
	Std	0.9033	0.8142	3.1811	4.648	3.3029
F15	Best	0.00031	0.00037	0.0008	0.0005	0.0006
	Worst	0.0204	0.02036	0.0203	0.0203	0.0025
	Mean	0.0013	0.0015	0.0068	0.0069	0.0015
	Std	0.0039	0.0039	0.0087	0.0094	0.0006

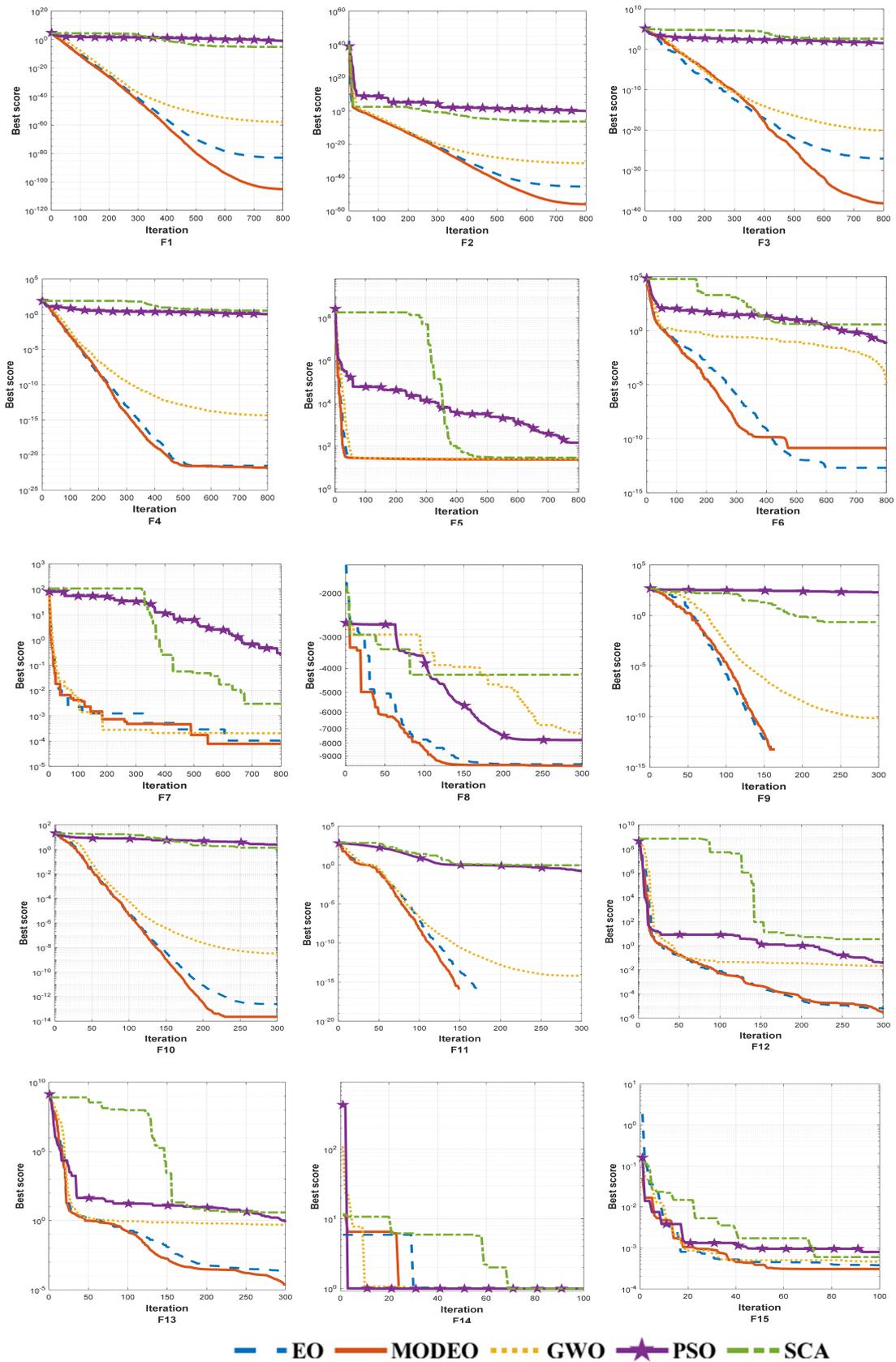


Figure 2. Comparison of convergence curves of the five algorithms for benchmark functions (F₁-F₁₅).

4.2. Second: Application on the UCP

The proposed hybrid technique MPSO-EO was further implemented to solve the UCP under different test systems with a variety of dimensions. The simulation studies were implemented in the MATLAB 2020a environment on a PC with an Intel Core i7 processor, 8 GB RAM and Microsoft Windows operating system. The suggested technique's simulation results under two test systems are presented and discussed. After that, a comparison of developed approaches with previously applied methods in solving the UCP is shown to verify MPSO-EO efficiency in solving the UCP.

Population size estimation: simulation studies and results observations of solution quality and execution time were used to estimate the ideal population size for carrying out numerical experiments of researched test systems.

Thirty trial runs were made for the under-study test systems to ensure the robustness of the proposed algorithm.

4.2.1. Performance of MPSO-EO for Test System 1

This test system considers deterministic load and ignores the variability of the load. The forecast load data over a 24-h horizon are given in Table 2 [23]. The reserve values are taken as 10% of the load and are given in Table 2 [23]. The solution quality is tested on 10-unit system and 20-unit system. The input data for the 10-unit system are given in Table 3 [35] and it is taken as a standard system. In Table 3, the term “initial state” refers to the unit's initial state at the start of the scheduling period. The (+) symbol indicates that the unit is turned on, while the (−) symbol indicates that it is turned off. Input data from a typical 10-unit system is duplicated for the 20-unit system. The population size for optimal results is presented in Table 4. The optimal commitment and generation schedules for a 10-unit system utilizing MPSO-EO are produced in Tables 5 and 6, respectively. The graphical representation for the performance of generating units is given in Figure 3. For a 20-unit system, Table 7 shows the commitment schedules and Figure 4 gives the graphical representation for the performance of generating units. Table 8 shows the total fuel cost, the start-up cost and the total operating cost for the two studied dimensions in test system 1 using MPSO-EO.

Table 2. Load and reserve data (test system 1) [23].

Time	1	2	3	4	5	6	7	8	9	10	11	12
Load demand	700	750	850	950	1000	1100	1150	1200	1300	1400	1450	1500
Reserve values	70	75	85	95	100	110	115	120	130	140	145	150
Time	13	14	15	16	17	18	19	20	21	22	23	24
Load demand	1400	1300	1200	1050	1000	1100	1200	1400	1300	1100	900	800
Reserve values	140	130	120	105	100	110	120	140	130	110	90	80

Table 3. The 10-unit system input data [35].

Unit	a (\$/h)	b (\$/MWh)	c (\$/MW ² h)	$P_{i_{max}}$ (MW)	$P_{i_{min}}$ (MW)	$SC_{i_{hot}}$ (\$)	$SC_{i_{cold}}$ (\$)	MUT_i (h)	MDT_i (h)	T_{cold_i} (h)	Initial State (h)
Un 1	1000	16.19	0.00048	455	150	4500	9000	8	8	5	8
Un 2	970	17.26	0.00031	455	150	5000	10,000	8	8	5	8
Un 3	700	16.6	0.002	130	20	550	1100	5	5	4	−5
Un 4	680	16.5	0.00211	130	20	560	1120	5	5	4	−5
Un 5	450	19.7	0.00398	162	25	900	1800	6	6	4	−6
Un 6	370	22.26	0.00712	80	20	170	340	3	3	2	−3
Un 7	480	27.74	0.00079	85	25	260	520	3	3	2	−3
Un 8	660	25.92	0.00413	55	10	30	60	1	1	0	−1
Un 9	665	27.27	0.00222	55	10	30	60	1	1	0	−1
Un 10	670	27.79	0.00173	55	10	30	60	1	1	0	−1

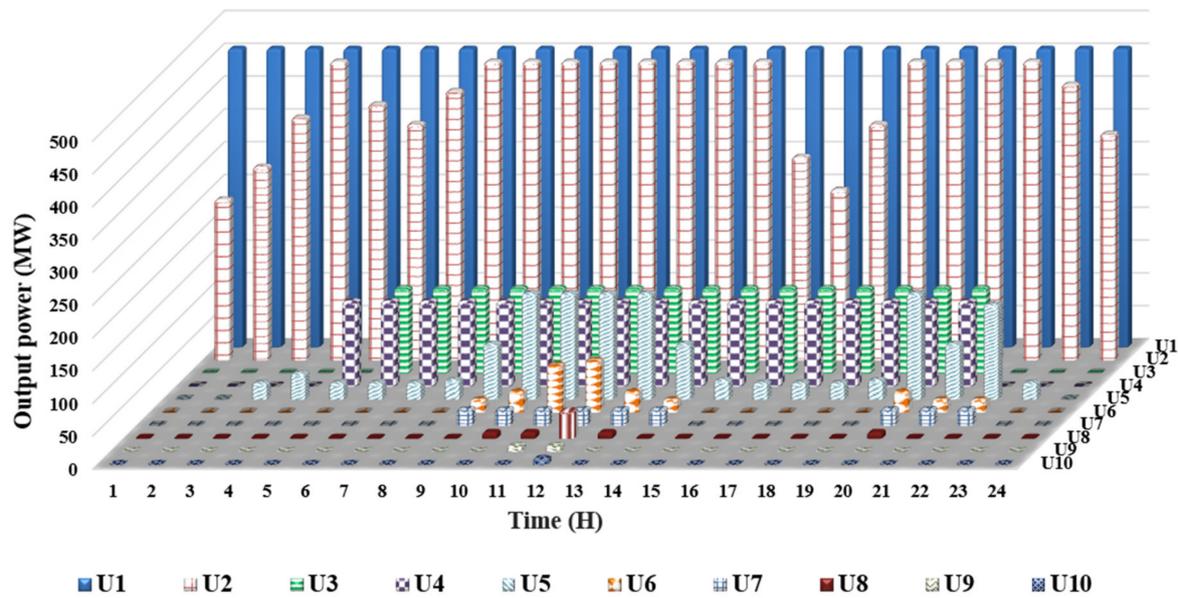


Figure 3. Performance of 10-generating units over 24 h (test system 1).

Table 7. Optimal scheduled operation for the 20-unit system over 24 h (test system 1).

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Un 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Un 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Un 3	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Un 4	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Un 5	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
Un 6	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0
Un 7	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
Un 8	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	0	0	0
Un 9	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
Un 10	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Un 11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Un 12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Un 13	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Un 14	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Un 15	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Un 16	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0
Un 17	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
Un 18	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0
Un 19	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
Un 20	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0

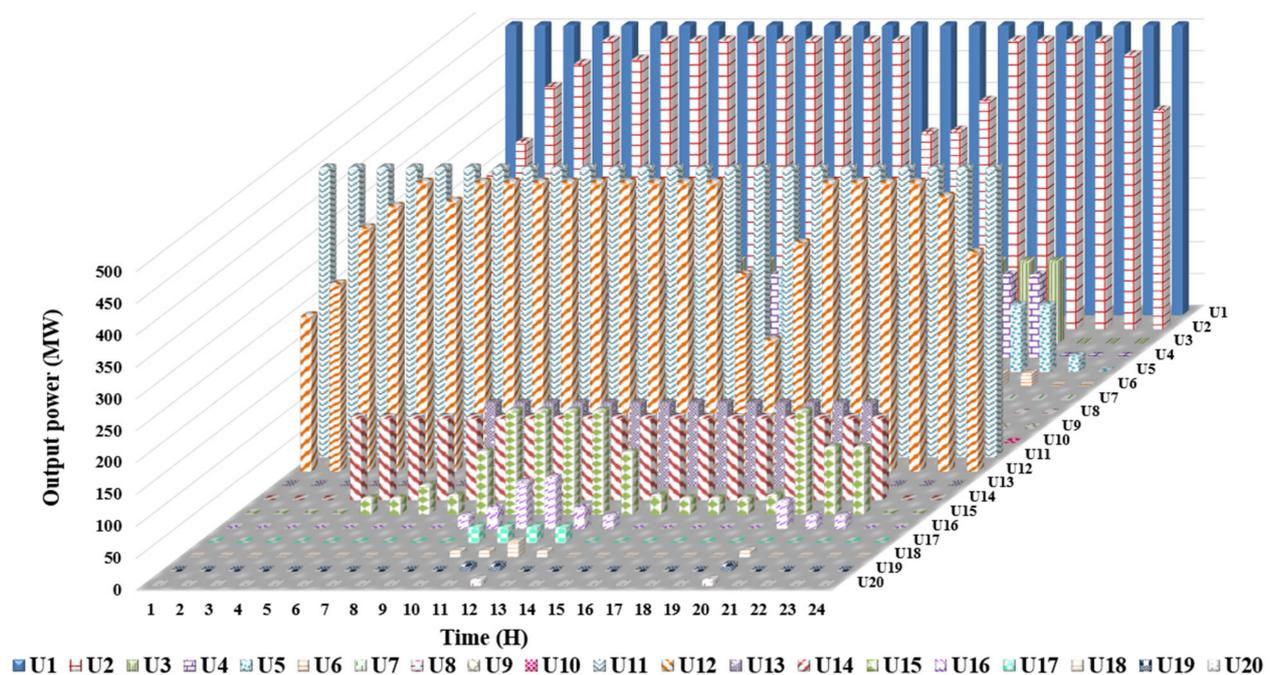


Figure 4. Performance of 10-generating units over 24 h (test system 1).

Table 8. Fuel cost, start-up cost and total operating cost obtained by MPSO-EO (test system 1).

Scale	Fuel Cost	Start-Up Cost	Total Operating Cost
10 unit	559,887.0172	4090	563,977.0172
20 unit	1,114,911.5105	8400	1,123,311.5105

The solution quality of the 10- and 20-unit systems using MPSO-EO is compared with the basic EO algorithm and other existing algorithms applied for solving the UCP (Table 9) to prove the priority of MPSO-EO in solving the UCP. The analysis of numerical results in Table 9 can be summarized as follows: MPSO-EO improves the solution quality and achieves better performance over standard EO, LR [13], EP [36], SA [37], MA [38], ICGA [39], GRASP [40], PSO-GWO [23], DPLR [6], ABFMO [18] and BFMO [18] for both the 10- and 20-unit systems; on the other hand, MPSO-EO gives the same operating cost as IQEA [41] and IBPSO [42] for the 10-unit system, but it outperforms both of them in the 20-unit system results. QEA [43], BGWO1 [44] and hGADE/cur1 [27] outperform MPSO-EO with slightly better results (less operating cost) for the 10-unit system, but the last overcomes all of them in the 20-unit system with significant cost savings. The analysis of simulation results proves the superiority and effectiveness of the MPSO-EO in solving the UCP and supports the presented modification. The execution time comparison is not considered as it differs by the differences in operating system and processor speed. The convergence curves of MPSO-EO for the 10- and 20-unit systems are introduced in Figure 5.

Table 9. Comparison of MPSO-EO with other algorithms (test system 1).

Approach	10 Unit			20 Unit		
	Worst	Average	Best	Worst	Average	Best
MPSO-EO	568,400.16	564,795.331	563,977.017	1,127,752.147	1,124,356.478	1,123,311.510
EO	577,281.91	568,893.790	564,286.949	1,140,682.515	1,131,797.256	1,125,263.048
LR [13]	565,825	565,825	565,825	1,130,660	1,130,660	1,130,660
EP [36]	566,231	565,352	564,551	1,129,793	1,127,257	1,125,494
SA [37]	566,260	565,988	565,828	1,129,112	1,127,955	1,126,251
MA [38]	567,022	566,787	566,686	1,128,403	1,128,213	1,128,192
ICGA [39]	566,404	566,404	566,404	—	—	1,127,244
GRASP [40]	565,825	565,825	565,825	—	—	—
PSO-GWO [23]	—	—	565,210.2	—	—	—
DPLR [6]	564,049	564,049	564,049	—	—	1,128,098
IQEA [41]	563,977	563,977	563,977	1,124,504	1,124,320	1,123,890
IBPSO [42]	565,312	564,155	563,977	1,125,216	1,125,448	1,125,730
QEA [43]	564,672	563,969	563,938	1,125,715	1,124,689	1,123,607
BGWO1 [44]	565,518.14	564,378.58	563,976.64	1,127,393.2	1,126,126.3	1,125,546.4
hGADE/cur1 [27]	564,350	564,088	563,959	1,125,076	1,124,502	1,123,426
ABFMO [18]	—	565,136	—	—	1,131,551	—
BFMO [18]	—	564,864	—	—	1,131,958	—

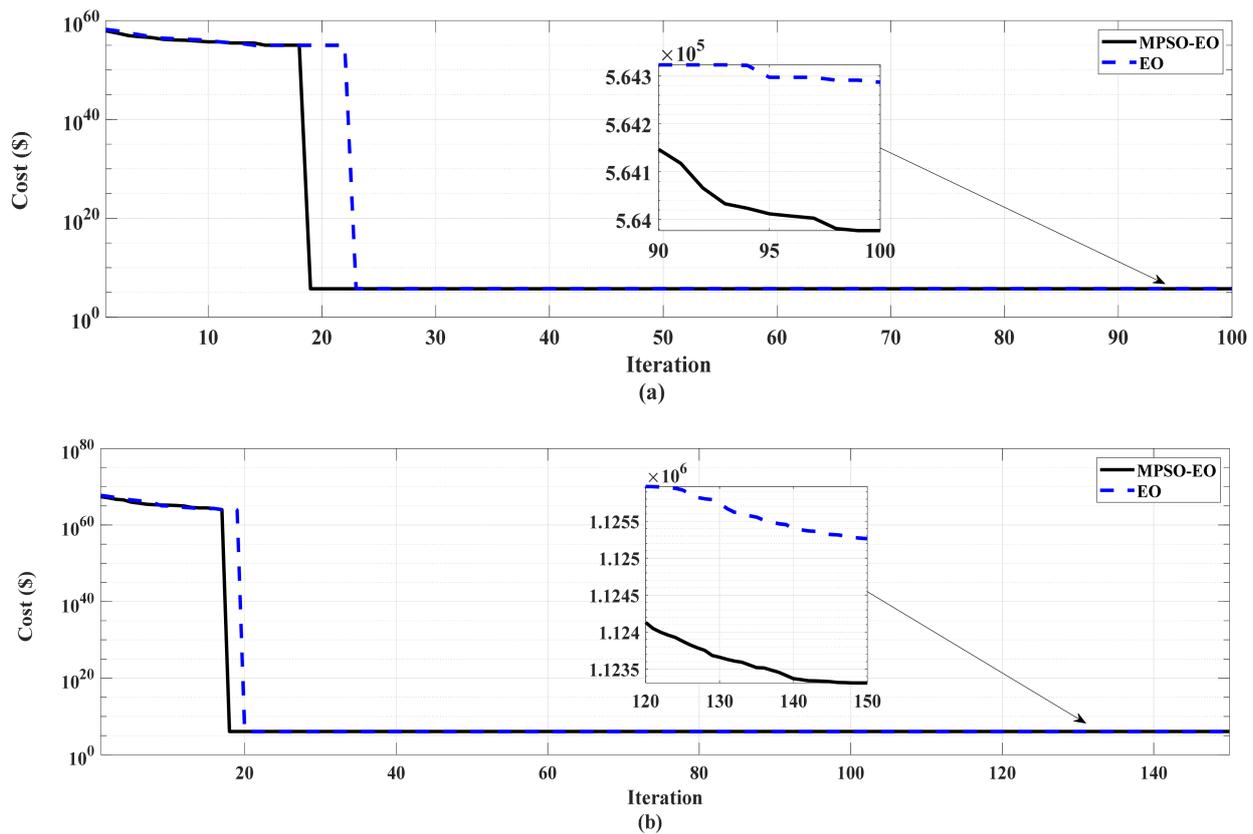


Figure 5. Comparison of convergence characteristics between MPSO-EO and EO (test system 1): (a) 10-unit system and (b) 20-unit system.

4.2.2. Performance of MPSO-EO for Test System 2

This test system considers the variability of the load in each hour, and the load is varied with a predefined standard deviation. Only the 10-unit system case is considered under random load, which is distributed over two values known as the mean and standard deviations (μ and σ) [45]. Table 10 involves typical values for μ and σ of the load [45–48].

These values are given for each hour and are used to estimate the load value as a univariate function. Tables 11 and 12 provide the optimum commitment and generation schedules, respectively. The graphical representation for the performance of generating units is given in Figure 6. In Table 13, total fuel cost, start-up cost and total operating cost using MPSO-EO are presented. Table 14 introduces a comparison for the total operation cost between MPSO-EO and standard EO. Figure 7 represents the convergence curve of MPSO-EO. The statistics show that the developed hybridization is more effective than the standard method in solving the UCP.

Table 10. Typical values for μ and σ (test system 2) [45].

Time	1	2	3	4	5	6	7	8	9	10	11	12
Mean deviation (μ)	1035.71	832.06	778.66	827.79	723.28	876.95	870.79	810.08	899.87	850.46	957.60	713.67
Standard deviation (σ)	9.448	9.627	10.960	11.435	8.367	9.364	10.076	10.131	9.928	12.044	10.465	10.123
Time	13	14	15	16	17	18	19	20	21	22	23	24
Mean deviation (μ)	890.86	816.91	1099.55	825.49	943.54	788.79	894.74	697.60	859.55	901.18	941.85	850.42
Standard deviation (σ)	9.668	10.432	9.505	10.651	8.501	9.229	10.588	8.637	9.783	11.136	9.694	9.475

Table 11. Optimal scheduled operation for the 10-unit system over 24 h (test system 2).

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Un 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Un 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Un 3	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
Un 4	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
Un 5	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
Un 6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Un 7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Un 8	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Un 9	1	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
Un 10	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0

Table 12. Optimal output power for the 10-unit system over 24 h (test system 2).

Hour	Un 1	Un 2	Un 3	Un 4	Un 5	Un 6	Un 7	Un 8	Un 9	Un 10
1	455	455	0	0	106	0	0	10	10	0
2	455	150	92	110	25	0	0	0	0	0
3	455	150	65	85	25	0	0	0	0	0
4	455	150	89	108	25	0	0	0	0	0
5	455	150	36	58	25	0	0	0	0	0
6	455	150	116	130	25	0	0	0	0	0
7	455	390	0	0	25	0	0	0	0	0
8	455	330	0	0	25	0	0	0	0	0
9	455	425	0	0	0	0	0	0	10	10
10	455	384	0	0	0	0	0	0	0	10
11	455	455	0	0	0	0	0	45	10	10
12	455	249	0	0	0	0	0	0	0	10
13	455	416	0	0	0	0	0	0	10	10
14	455	361	0	0	0	0	0	0	0	0
15	455	358	130	130	25	0	0	0	0	0
16	455	150	88	108	25	0	0	0	0	0
17	455	204	130	130	25	0	0	0	0	0
18	455	150	69	90	25	0	0	0	0	0
19	455	156	130	130	25	0	0	0	0	0
20	455	150	23	45	25	0	0	0	0	0
21	455	150	106	124	25	0	0	0	0	0
22	455	162	130	130	25	0	0	0	0	0
23	455	202	130	130	25	0	0	0	0	0
24	455	150	101	119	25	0	0	0	0	0

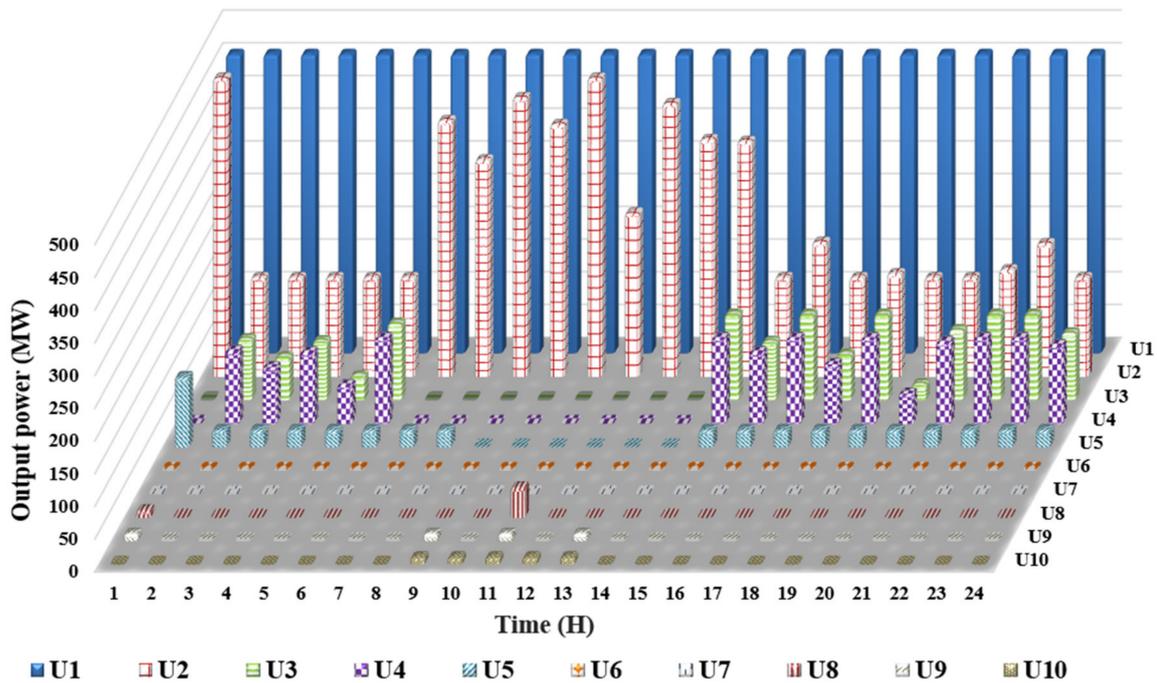


Figure 6. Performance of 10-generating units over 24 h (test system 2).

Table 13. Fuel cost, start-up cost and total operating cost obtained by MPSO-EO (test system 2).

	Fuel Cost	Start-Up Cost	Total Operating Cost
10 unit	433,369.9353	4320	437,689.9353

Table 14. Comparison between MPSO-EO and standard EO (test system 2).

Approach	10 Unit		
	Worst	Average	Best
MPSO-EO	440,761.2383	438,129.907	437,689.9353
EO	448,188.7534	441,034.2414	437,730.8655

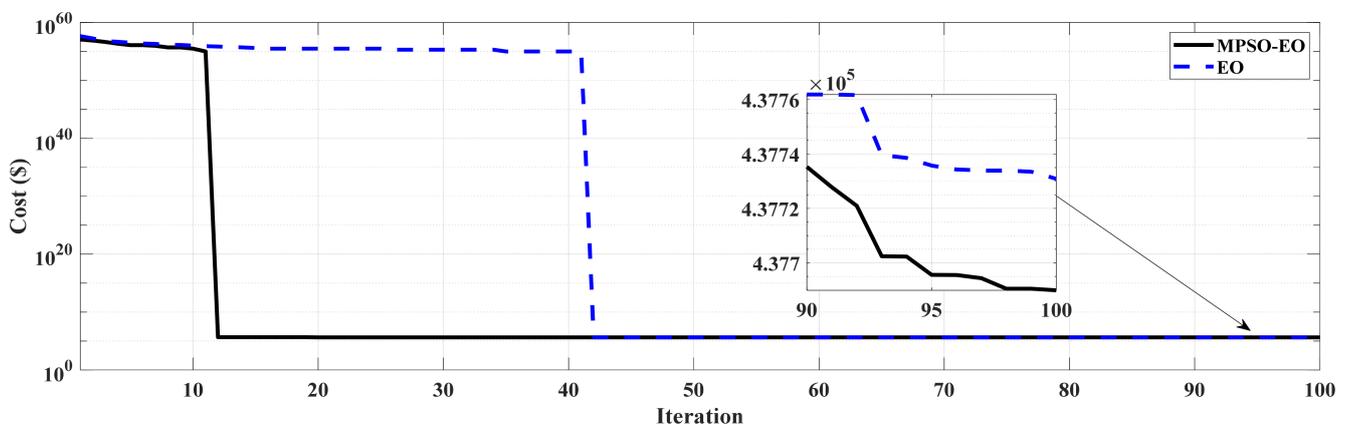


Figure 7. Comparison of convergence characteristics between MPSO-EO and EO for the 10-unit system (test system 2).

5. Conclusions

This paper proposes a novel EO algorithm to solve the single-area UCP through hybridization between standard EO and the modified version of PSO. The proposed algorithm MPSO-EO is simple to implement as it depends on improving the update process of particles' positions to improve the population diversity. The problem is solved under two test systems: deterministic and stochastic systems. The robustness and effectiveness of MPSO-EO are tested with different dimensions. The results are promising and show the advantage of the proposed modification over the standard EO. In the case of deterministic load, MPSO-EO provides cost savings over standard EO and over most other algorithms for the 10-unit system and offers the best cost savings among all algorithms when compared, including standard EO for the 20-unit system. In case of stochastic load, a 10-unit system is studied, and MPSO-EO outperforms standard EO with less operating cost. The main limitation with the proposed algorithm is that the computational time is high to some extent but, on the other hand, it gives highly effective performance and auspicious results.

The future work will be extended to include the following:

1. Solving the UCP with the integration of renewable energy sources and energy storage systems;
2. Solving the stochastic UCP by considering uncertainty in both load and generation sides to have a more reliable solution.

Author Contributions: Conceptualization, M.E.; methodology, Z.M.A.; software, A.B.A.-R.; validation, M.A.; formal analysis, S.H.E.A.A.; investigation, Z.M.A.; resources, A.S.; data curation, M.E.; writing—original draft preparation, A.S. and A.E.-S.; writing—review and editing, M.E. and M.R.; visualization, M.R., A.B.A.-R. and A.E.-S.; supervision, M.E. and S.H.E.A.A.; project administration, Z.M.A. and M.A.; funding acquisition, M.A., Z.M.A., S.H.E.A.A. and A.E.-S. All authors have read and agreed to the published version of the manuscript.

Funding: This work is funded by the Taif University Researchers Supporting Project Number (TURSP-2020/146), Taif University, Taif, Saudi Arabia.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to their large size.

Acknowledgments: The authors would like to acknowledge the financial support received from Taif University Researchers Supporting Project Number (TURSP-2020/146), Taif University, Taif, Saudi Arabia.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

t	Index of time horizon for a set of T
i	Index of thermal generating units for a set of N
$FC_i(t)$	Fuel cost function of thermal unit i at time t
$SC_i(t)$	Start-up cost function of thermal unit i at time t
$SD_i(t)$	Shutdown cost function of thermal unit i at time t
$SC_{i_{hot}}$	Hot start-up cost of thermal unit i
$SC_{i_{cold}}$	Cold start-up cost of thermal unit i
MUT_i	Minimum up time of thermal unit i
MDT_i	Minimum down time of thermal unit i
T_{OFF_i}	Time period that unit i was continuously off
T_{ON_i}	Time period that unit i was continuously on

T_{cold_i}	Time period for cooling down of unit i
$P_{i_{min}}$	Minimum generation limit of thermal unit i
$P_{i_{max}}$	Maximum generation limit of thermal unit i
P_L^t	Load demand of the system at time t
SR^t	Spinning reserve requirements of the system at time t
σ_{LD}^t	Standard deviation of the load demand at time t
μ_{LD}^t	Mean deviation of the load demand at time t
F_T	Total operating cost (objective function)
$P_i(t)$	Output power of thermal unit i at time t
$U_i(t)$	On/off state of the thermal unit i at time t

References

- Bhardwaj, A.; Tung, N.S.; Shukla, V.K.; Kamboj, V.K. The important impacts of unit commitment constraints in power system planning. *Int. J. Emerg. Trends Eng. Dev.* **2012**, *5*, 301–306.
- Burns, R. Optimization of priority lists for a unit commitment program. In Proceedings of the IEEE Power Engineering Society Summer Meeting, San Francisco, CA, USA, 20–25 July 1975.
- Zhu, J. *Optimization of Power System Operation*; John Wiley & Sons: New York, NY, USA, 2015.
- Ananth, D.; Vineela, K. A review of different optimisation techniques for solving single and multi-objective optimisation problem in power system and mostly unit commitment problem. *Int. J. Ambient. Energy* **2021**, *42*, 1676–1698. [[CrossRef](#)]
- Virmani, S.; Adrian, E.C.; Imhof, K.; Mukherjee, S. Implementation of a Lagrangian relaxation based unit commitment problem. *IEEE Trans. Power Syst.* **1989**, *4*, 1373–1380. [[CrossRef](#)]
- Ongsakul, W.; Petcharaks, N. Unit commitment by enhanced adaptive Lagrangian relaxation. *IEEE Trans. Power Syst.* **2004**, *19*, 620–628. [[CrossRef](#)]
- Bakirtzis, A.; Zoumas, C. Lambda of Lagrangian relaxation solution to unit commitment problem. *IEE Proc.-Gener. Transm. Distrib.* **2000**, *147*, 131–136. [[CrossRef](#)]
- Senjyu, T.; Shimabukuro, K.; Uezato, K.; Funabashi, T. A fast technique for unit commitment problem by extended priority list. *IEEE Trans. Power Syst.* **2003**, *18*, 882–888. [[CrossRef](#)]
- Tingfang, Y.; Ting, T. Methodological Priority List for Unit Commitment Problem. In Proceedings of the 2008 International Conference on Computer Science and Software Engineering, Washington, DC, USA, 12–14 December 2008; IEEE: Manhattan, NY, USA, 2008; pp. 176–179.
- Muckstadt, J.A.; Wilson, R.C. An application of mixed-integer programming duality to scheduling thermal generating systems. *IEEE Trans. Power Appar. Syst.* **1968**, *PAS-87*, 1968–1978. [[CrossRef](#)]
- Su, C.-C.; Hsu, Y.-Y. Fuzzy dynamic programming: An application to unit commitment. *IEEE Trans. Power Syst.* **1991**, *6*, 1231–1237.
- Patra, S.; Goswami, S.; Goswami, B. Fuzzy and simulated annealing based dynamic programming for the unit commitment problem. *Expert Syst. Appl.* **2009**, *36*, 5081–5086. [[CrossRef](#)]
- Kazarlis, S.A.; Bakirtzis, A.; Petridis, V. A genetic algorithm solution to the unit commitment problem. *IEEE Trans. Power Syst.* **1996**, *11*, 83–92. [[CrossRef](#)]
- Swarup, K.; Yamashiro, S. Unit commitment solution methodology using genetic algorithm. *IEEE Trans. Power Syst.* **2002**, *17*, 87–91. [[CrossRef](#)]
- Zhao, B.; Guo, C.; Bai, B.; Cao, Y. An improved particle swarm optimization algorithm for unit commitment. *Int. J. Electr. Power Energy Syst.* **2006**, *28*, 482–490. [[CrossRef](#)]
- Pappala, V.S.; Erlich, I. A new approach for solving the unit commitment problem by adaptive particle swarm optimization. In Proceedings of the 2008 IEEE Power and Energy Society General Meeting–Conversion and Delivery of Electrical Energy in the 21st Century, Pittsburgh, PA, USA, 20–24 July 2008; IEEE: Manhattan, NY, USA, 2008; pp. 1–6.
- Roy, P.K. Solution of unit commitment problem using gravitational search algorithm. *Int. J. Electr. Power Energy Syst.* **2013**, *53*, 85–94. [[CrossRef](#)]
- Pan, J.-S.; Hu, P.; Chu, S.-C. Binary fish migration optimization for solving unit commitment. *Energy* **2021**, *226*, 120329. [[CrossRef](#)]
- Chandrasekaran, K.; Simon, S.P.; Padhy, N.P. Binary real coded firefly algorithm for solving unit commitment problem. *Inf. Sci.* **2013**, *249*, 67–84. [[CrossRef](#)]
- Mantawy, A.; Abdel-Magid, Y.L.; Selim, S.Z. A simulated annealing algorithm for unit commitment. *IEEE Trans. Power Syst.* **1998**, *13*, 197–204. [[CrossRef](#)]
- Venkatesh Kumar, C.; Ramesh Babu, M. An Exhaustive Solution of Power System Unit Commitment Problem Using Enhanced Binary Salp Swarm Optimization Algorithm. *J. Electr. Eng. Technol.* **2021**, 1–19. [[CrossRef](#)]
- Rajan, C.C.A.; Mohan, M. An evolutionary programming-based tabu search method for solving the unit commitment problem. *IEEE Trans. Power Syst.* **2004**, *19*, 577–585. [[CrossRef](#)]
- Kamboj, V.K. A novel hybrid PSO–GWO approach for unit commitment problem. *Neural Comput. Appl.* **2016**, *27*, 1643–1655. [[CrossRef](#)]

24. Bavafa, M.; Monsef, H.; Navidi, N. A new hybrid approach for unit commitment using lagrangian relaxation combined with evolutionary and quadratic programming. In Proceedings of the 2009 Asia-Pacific Power and Energy Engineering Conference, Wuhan, China, 28–31 March 2009; IEEE: Manhattan, NY, USA, 2009; pp. 1–6.
25. Anand, H.; Narang, N.; Dhillon, J. Multi-objective combined heat and power unit commitment using particle swarm optimization. *Energy* **2019**, *172*, 794–807. [[CrossRef](#)]
26. Trivedi, A.; Srinivasan, D.; Biswas, S.; Reindl, T. A genetic algorithm–differential evolution based hybrid framework: Case study on unit commitment scheduling problem. *Inf. Sci.* **2016**, *354*, 275–300. [[CrossRef](#)]
27. Trivedi, A.; Srinivasan, D.; Biswas, S.; Reindl, T. Hybridizing genetic algorithm with differential evolution for solving the unit commitment scheduling problem. *Swarm Evol. Comput.* **2015**, *23*, 50–64. [[CrossRef](#)]
28. Zobaa, A.F.; Aleem, S.A. *Uncertainties in Modern Power Systems*; Academic Press: Cambridge, MA, USA, 2020.
29. Unit, S.E. *Reducing Re-Offending by Ex-Prisoners*; Social Exclusion Unit London: London, UK, 2002.
30. Kennedy, J.; Eberhart, R. Particle swarm optimization. In Proceedings of the ICNN'95-International Conference on Neural Networks, Perth, Australia, 27 November–1 December 1995; IEEE: Manhattan, NY, USA, 1995; pp. 1942–1948.
31. Khan, A.; Hizam, H.; Abdul-Wahab, N.I.; Othman, M.L. Solution of Optimal Power Flow Using Non-Dominated Sorting Multi Objective Based Hybrid Firefly and Particle Swarm Optimization Algorithm. *Energies* **2020**, *13*, 4265. [[CrossRef](#)]
32. Riaz, M.; Hanif, A.; Hussain, S.J.; Memon, M.I.; Ali, M.U.; Zafar, A. An optimization-based strategy for solving optimal power flow problems in a power system integrated with stochastic solar and wind power energy. *Appl. Sci.* **2021**, *11*, 6883. [[CrossRef](#)]
33. Faramarzi, A.; Heidarinejad, M.; Stephens, B.; Mirjalili, S. Equilibrium optimizer: A novel optimization algorithm. *Knowl.-Based Syst.* **2020**, *191*, 105190. [[CrossRef](#)]
34. Zellagui, M.; Lasmari, A.; Settoul, S.; El-Sehiemy, R.A.; El-Bayeh, C.Z.; Chenni, R. Simultaneous allocation of photovoltaic DG and DSTATCOM for techno-economic and environmental benefits in electrical distribution systems at different loading conditions using novel hybrid optimization algorithms. *Int. Trans. Electr. Energy Syst.* **2021**, *31*, e12992. [[CrossRef](#)]
35. Anita, J.M.; Raglend, I.J. Solution of unit commitment problem using shuffled frog leaping algorithm. In Proceedings of the 2012 International Conference on Computing, Electronics and Electrical Technologies (ICCEET), Nagercoil, India, 21–22 March 2012; IEEE: Manhattan, NY, USA, 2012; pp. 109–115.
36. Juste, K.; Kita, H.; Tanaka, E.; Hasegawa, J. An evolutionary programming solution to the unit commitment problem. *IEEE Trans. Power Syst.* **1999**, *14*, 1452–1459. [[CrossRef](#)]
37. Simopoulos, D.N.; Kavatza, S.D.; Vournas, C.D. Unit commitment by an enhanced simulated annealing algorithm. *IEEE Trans. Power Syst.* **2006**, *21*, 68–76. [[CrossRef](#)]
38. Saravanan, B.; Vasudevan, E.; Kothari, D. Unit commitment problem solution using invasive weed optimization algorithm. *Int. J. Electr. Power Energy Syst.* **2014**, *55*, 21–28. [[CrossRef](#)]
39. Damousis, I.G.; Bakirtzis, A.G.; Dokopoulos, P.S. A solution to the unit-commitment problem using integer-coded genetic algorithm. *IEEE Trans. Power Syst.* **2004**, *19*, 1165–1172. [[CrossRef](#)]
40. Viana, A.; de Sousa, J.P.; Matos, M. Using GRASP to solve the unit commitment problem. *Ann. Oper. Res.* **2003**, *120*, 117–132. [[CrossRef](#)]
41. Jeong, Y.-W.; Park, J.-B.; Shin, J.-R.; Lee, K.Y. A thermal unit commitment approach using an improved quantum evolutionary algorithm. *Electr. Power Compon. Syst.* **2009**, *37*, 770–786. [[CrossRef](#)]
42. Yuan, X.; Nie, H.; Su, A.; Wang, L.; Yuan, Y. An improved binary particle swarm optimization for unit commitment problem. *Expert Syst. Appl.* **2009**, *36*, 8049–8055. [[CrossRef](#)]
43. Lau, T.; Chung, C.; Wong, K.; Chung, T.; Ho, S.L. Quantum-inspired evolutionary algorithm approach for unit commitment. *IEEE Trans. Power Syst.* **2009**, *24*, 1503–1512. [[CrossRef](#)]
44. Panwar, L.K.; Reddy, S.; Verma, A.; Panigrahi, B.K.; Kumar, R. Binary grey wolf optimizer for large scale unit commitment problem. *Swarm Evol. Comput.* **2018**, *38*, 251–266. [[CrossRef](#)]
45. Saravanan, B.; Mishra, S.; Nag, D. A solution to stochastic unit commitment problem for a wind-thermal system coordination. *Front. Energy* **2014**, *8*, 192–200. [[CrossRef](#)]
46. Micev, M.; Calasan, M.; Ali, Z.M.; Hasanien, H.M.; Abdel Aleem, S.H.E. Optimal design of automatic voltage regulation controller using hybrid simulated annealing—Manta ray foraging optimization algorithm. *Ain Shams Eng. J.* **2021**, *12*, 641–657. [[CrossRef](#)]
47. Refaat, M.M.; Aleem, S.H.E.; Atia, Y.; Ali, Z.M.; El-Shahat, A.; Sayed, M.M. A Mathematical Approach to Simultaneously Plan Generation and Transmission Expansion Based on Fault Current Limiters and Reliability Constraints. *Mathematics* **2021**, *9*, 2771. [[CrossRef](#)]
48. Zobaa, A.F.; Aleem, S.H.E.A.; Abdelaziz, A.Y. *Classical and Recent Aspects of Power System Optimization*; Academic Press: Cambridge, MA, USA; Elsevier: Amsterdam, The Netherlands, 2018.