



# Article Assessment of the Impact of Per Unit Parameters Errors on Wave and Output Parameters in a Transmission Line

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**Abstract**: The assessment of the impact of per unit length parameter errors on the determination of wave parameters, currents, and voltages at the end of the line has been presented in the paper. The impact on the above-mentioned values has been indicated. This paper presents an assessment of the impact of per unit parameter errors on the determination of both wave parameters, as well as currents and voltages at the end of a transmission line, although it is mainly focused on indicating which of the per unit parameters have the strongest impact on the above-mentioned values. For this purpose, elements of incremental sensitivity have been used.

**Keywords:** circuit sensitivity; transmission line; output parameters; electrical circuit; model of a two-wire transmission

## 1. Introduction

In order to determine currents and voltages in circuits with distributed parameters transmission lines, the knowledge of the so-called wave parameters (the wave impedance and the propagation constant) is necessary. These parameters can be determined knowing the per unit parameters of the line, i.e., resistance, inductance, capacitance, and leakage given per unit of line length (usually 1 km). The determination of these quantities is often inaccurate, e.g., in an overhead line the value of capacitance and leakage depends on the weather conditions, and is usually specified in certain ranges. The study of sensitivity of wave parameters to errors in the determination of unit parameters was presented in the paper [1]. This task was performed based on the definition of the relative sensitivity of electrical circuits [2,3]. This paper presents an assessment of the impact of per unit parameter errors on the determination of both wave parameters, as well as currents and voltages at the end of a transmission line, although it is mainly focused on indicating which of the per unit parameters have the strongest impact on the above-mentioned values. For this purpose, elements of incremental sensitivity have been used [4].

## 2. Linear Circuits with Distributed Parameters

A transmission line is an electrical circuit with distributed parameters, supplied from a sinusoidal voltage source. The length of the electric circuit is comparable to the voltage wavelength  $\lambda$  [2]. The circuit is composed of elements  $R_0$ ,  $G_0$ ,  $L_0$ , and  $C_0$ , which determine the losses of active power related to heat generation and leakage, and as a result of accumulation of energy in magnetic and electric fields [5]. They are called the primary parameters of a transmission line [5]. In the case of circuits with distributed parameters in relation to the length of the line, the longitudinal parameters (resistance  $R_0$ , inductance  $L_0$ , and lateral parameters: capacitance  $C_0$ , and leak conductivity  $G_0$  [6]) are described. Their values are specified as  $\Omega/m$ , S/m, H/m, and F/m [7–9]. It is conventionally assumed that



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). if *l* is greater than 0.1  $\lambda$ , or the transmission of impulse signals with a short rise time, as is the case in high voltage cables of spark ignition engines [3,4,6,10], then such a line should be treated as a circuit with distributed parameters. This paper shall examine the properties of a homogeneous line, whose primary parameters are uniformly distributed along the line [7,9]. When analyzing an equivalent circuit of a line with distributed parameters, the following are considered:

- voltage loss on the wire resistance distributed uniformly along the line;
- voltage loss on the wire inductance distributed uniformly along the line;
- leakage current through the insulation distributed uniformly along the line (for cable line: cable insulation, for overhead line: air insulation);
- line capacitance distributed uniformly, where the plates of the capacitor are represented by two wires or a single wire and the earth.

A transmission line can be represented as a cascade connection of elementary sections of  $\Delta x$  length, in which the resistance, capacitor and coil are lumped elements. Instantaneous values of voltage u(x,t) and current i(x,t) at each point of the line are functions of two independent variables: distance and time. If the line parameters are distributed uniformly along the line, the transmission line is homogeneous. A line is linear if its parameters are not dependent on voltage or current at a given point in the line.

Figure 1 presents a model of a homogeneous two-wire transmission line with the length *l*. Terminals 1–1', called line input terminals, are connected to the e(t) voltage source with internal impedance  $Z_1$ . A receiver with impedance  $Z_2$  is connected to output terminals 2–2'.



Figure 1. Model of a two-wire transmission line.

At any point on the line distant by  $x_1$  from the beginning of the line, the voltage is  $u(x_1,t)$  and the current is  $i(x_1,t)$ , in which case for the input terminals x = 0, and for the output terminals x = l [3,10,11].

The unit resistance of a single-wire line can be determined using the formula:

$$R_0 = \frac{1000}{\gamma S} \tag{1}$$

where  $\gamma$  is the specific conductivity of the wire [m/ $\Omega$  mm<sup>2</sup>], and *S* is the cross-section of the wire [mm<sup>2</sup>].

When a two-wire line is considered, the result needs to be multiplied by two.

The per unit inductance for a single wire with a length l = 1 m is expressed by the formula:

$$L = \frac{\mu}{4\pi} \left( 0.5 + \ln\frac{a}{R} \right) \tag{2}$$

where a is the distance between the axes of the cables, *R* is the wire radius, and  $\mu$  is the electric permittivity of the environment (for air assumed as  $\mu = 4\pi \cdot 10^{-7}$  H/m).

The per unit capacitance for a two-wire line with a length l = 1 m is expressed by the formula:

$$C_0 = \frac{\pi \varepsilon_0}{\ln \frac{a}{R}} \tag{3}$$

where  $\varepsilon_0$  is the electric permittivity of the environment—air.

#### 3. Transmission Line Wave and Output Parameters

The term wave parameters of a transmission line most commonly refers to: wave impedance expressed by the Equations (6) and (7):

$$Z_c = \sqrt{\frac{R_0 + j\omega L}{G_0 = j\omega C}} \tag{4}$$

where  $R_0$  is the per unit length resistance,  $G_0$  is the per unit length inductance,  $L_0$  is the per unit length capacitance, and  $C_0$  is the per unit length leakage.

The propagation constant:

$$\gamma = \sqrt{(R_0 + j\omega L_0) \cdot (G_0 + j\omega C_0)} \tag{5}$$

Knowledge of these parameters allows for the determination of the input impedance:

$$Z_{we} = Z_C \frac{Z_2 ch\gamma l + Z_c sh\gamma l}{Z_2 sh\gamma l + Z_c ch\gamma l}$$
(6)

where  $Z_2$  is the line load impedance, and l is the line length.

Then, for a known voltage  $U_1$  at the beginning of the line, the current  $I_1$  at the beginning of the line can be determined:

$$I_1 = \frac{U_1}{Z_{we}} \tag{7}$$

and then, voltage  $U_2$  and current  $I_2$  at the end of the line:

$$\begin{cases}
U_2 = U_1 ch\gamma l - Z_c I_1 sh\gamma l \\
I_1 = -\frac{U_1}{Z_{C_1}} sh\gamma l + I_1 ch\gamma l
\end{cases}$$
(8)

The sensitivity of wave parameters as well as the output parameters of a transmission line can be traced by changing the per unit length parameters of this line [12–14].

### 4. Numerical Experiments

Numerical experiments were carried out for a 200 kHz frequency, for an overhead line with following per unit length parameters:

$$R_0 = 0.68 \ \Omega/\text{km}; G_0 = 50 \ \mu\text{S/km};$$

$$L_0 = 0.128 \text{ mH/km}; C_0 = 0.01 \ \mu\text{F/km};$$

with an impedance load  $Z_2 = 100 \Omega$ 

In order to indicate which of the per unit length parameters have the strongest impact on the values characterizing the transmission line, multiple calculations were carried out, each time substituting a different changed value of a per unit length parameter. These changes consisted in increasing and decreasing the value by 1%. The obtained results percentage changes—are presented in the form of Tables 1–11. The highest and lowest values of the modulus are highlighted by bold font in the tables. On the basis of the analysis of the above tables, it can be stated that the biggest influence on the damping factor is the change of  $R_{0}$ , and on the phase factor the change of  $C_0$ .  $C_0$  also has the greatest impact on the modulus, and the argument of both the output voltage and current [15–19].

<b>Table 1.</b> Changes to the wave impedance modul
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	$ Z_{\mathcal{C}} $	
	+1%	-1%
$\begin{matrix} R_0 \\ G_0 \\ L_0 \\ C_0 \end{matrix}$	$\begin{array}{l} 8.980621718802 \times 10^{-6} \\ -7.95516852989 \times 10^{-6} \\ 0.498747363629691 \\ -0.496273219279826 \end{array}$	$\begin{array}{l} -8.891264700064 \times 10^{-6} \\ 7.876015737812 \times 10^{-6} \\ -0.501247263019991 \\ 0.503773449510121 \end{array}$

Table 2. Changes to damping constant.

$\alpha = \mathbf{Re}\{\gamma\}$		
	+1%	-1%
R_0	0.515151228520686	-0.515151275908188
$G_0$	0.484848713525002	-0.484848750704316
$L_0$	-0.013838466672733	0.016490175659639
$C_0$	0.016313672812472	-0.013964997756260

Table 3. Changes to phase constant.

	$\beta = \operatorname{Im}\{\gamma\}$	
	+1%	-1%
$R_0$	$2.85161812570  imes 10^{-7}$	$-2.4048216284705  imes 10^{-7}$
$G_0$	$-2.27573141879  imes 10^{-7}$	$2.6715112510711  imes 10^{-7}$
L <sub>0</sub>	0.49875597170012	-0.5012560025127765
<i>C</i> <sub>0</sub>	0.49875647683926	-0.5012565178567181

Table 4. Changes to the real part of the input resistance.

	$\operatorname{Re}\{Z_{we}\}$	
	+1%	-1%
R <sub>0</sub>	0.000384588324788	-0.0003889650747472
G <sub>0</sub>	0.043391174085347	-0.0433956466098439
L <sub>0</sub>	-5.732231738707145	5.9768553112726378
<i>C</i> <sub>0</sub>	3.562167621061453	-3.2793821336200557

Table 5. Changes to the imaginary part of the input resistance.

	$\operatorname{Im}\{\mathbf{Z}_{we}\}$	
	+1%	-1%
$R_0$	-0.002777179953811	0.0027783246452566
$G_0$	0.001528422276180	-0.0015294640405447
L <sub>0</sub>	-0.041812016570611	-0.0221985063733944
$C_0$	-1.189631026642754	1.2554145000339767

	$ I_1 $	
	+1%	-1%
$R_0$	0.002747571139330	-0.0027486055903140
$G_0$	-0.000971236507435	0.0009719814772497
$L_0$	-0.031856013256254	0.0939895328519183
$C_0$	1.233753020617313	-1.2645765148497845

**Table 6.** Changes to the  $I_1$  current modulus.

**Table 7.** Changes to the argument of the current  $I_1$ .

	$arg\{I_1\}$	
	+1%	-1%
$R_0$	0.002372834589791	-0.0023694989848461
$G_0$	-0.044547012164130	0.0445533464751429
$L_0$	5.724421549430766	-5.9109167085244178
$C_0$	-2.381912054750487	1.9818184348223934

**Table 8.** Changes to the  $U_2$  voltage modulus.

	$ U_2 $	
	+1%	-1%
$R_0$	-0.009870362379686	0.0098708010242115
$G_0$	-0.010998843739695	0.0110003885878087
$L_0$	-0.053850964577535	0.0838634668968119
$C_0$	0.630798491006207	-0.6478547211964083

**Table 9.** Changes to the argument of the voltage  $U_2$ .

	$arg{U_2}$	
	+1%	-1%
R <sub>0</sub>	0.000288807602249	-0.000288916464849
$G_0$	-0.002528487115057	0.002528654755186
L <sub>0</sub>	-2.317110575876312	2.323205351184315
<i>C</i> <sub>0</sub>	-2.751776829843434	2.741194932584926

**Table 10.** Changes to the  $I_2$  current modulus.

	$ I_2 $	
	+1%	-1%
$\begin{matrix} R_0 \\ G_0 \\ L_0 \\ C_0 \end{matrix}$	-0.009870362379715 -0.010998843739739 -0.053850964577548 0.630798491006164	0.0098708010242179 0.0110003885878038 0.0838634668968006 0.6478547211963962

In order to evaluate how the values of wave and output parameters change when per unit length parameters are changed, a proprietary program has been written in the MATLAB environment. It allows for the acquisition of graphs of wave and output parameters values for a given frequency, load, input parameters, and variability of individual per unit length parameters (e.g.,  $\pm 20\%$ ) at any value of supply voltage U<sub>1</sub> and load Z<sub>2</sub>. The presented graphs were created for  $U_1 = 100$  V,  $Z_2 = 100 \Omega$ . Thus, with a change of per unit length resistance within  $\pm 20\%$ , the damping factor  $\alpha$  linearly increases from -10% to +10%, regardless of frequency (Figures 2a, 3a and 4a), while the phase factor  $\beta$  reaches a minimum around -5% and then increases, but these changes are very minor, approximately  $10^{-5\%}$ for f = 100 kHz (Figure 2b),  $10^{-6\%}$  for f = 500 kHz (Figure 3b), and  $10^{-7\%}$  for f = 1 MHz (Figure 4b). As far as the wave impedance modulus and argument are concerned, they increase almost linearly, with the exception that the modulus changes for low frequencies are around  $10^{-3}\%$  (Figure 2c) and for large ones, around  $10^{-5\%}$  (Figures 3c and 4c), whereas the argument changes are large, i.e.,  $\pm 300\%$  (Figures 2d, 3d and 4d).

 $\arg\{I_2\}$ +1% -1% $R_0$ 0.000288807602261 -0.0002889164648371 $G_0$ -0.002528487115069 0.0025286547551618  $L_0$ -2.3171105758763562.3232053511843256 -2.7517768298434552.7411949325849654  $C_0$ a) <sub>20</sub> b) ×10<sup>-5</sup> 6 10 4 alfa [%] beta [%] 0 2 0 -10 -20 -2 -20 -10 0 10 20 -20 -10 0 10 20 R0 [%] R0 [%] c) d) 10<sup>-3</sup> 400 1 abs(ZC) [%] 0.5 200 arg(ZC) [%] 0 0 -200 -0.5 -400 -10 0 10 20 -10 0 10 20 -20 -20 R0 [%] R0 [%]

**Table 11.** Changes to the argument of the current  $I_2$ .

**Figure 2.** Changes of wave parameters depending on  $R_0$  at f = 100 kHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.

Changes to the damping factor alpha when changing parameter  $L_0$  are non–linear, and vary from 1% to 0.2%, regardless of frequency (Figures 5a, 6a and 7a). The factor  $\beta$  rises almost linearly from –10% to 10%, regardless of frequency (Figures 5b, 6b and 7b). The impedance modulus rises almost linearly in the range of –10% to 10% for all tested frequencies (Figures 5c, 6c and 7c). The argument changes linearly in the range from approximately 200% to –400%, with minor changes for individual frequencies (Figures 5d, 6d and 7d).



**Figure 3.** Changes of wave parameters depending on  $R_0$  at f = 500 kHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.



**Figure 4.** Changes of wave parameters depending on  $R_0$  at f = 1 MHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.



**Figure 5.** Changes of wave parameters depending on  $L_0$  at f = 100 kHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.



**Figure 6.** Changes of wave parameters depending on  $L_0$  at f = 500 kHz; (a) damping factor  $\alpha$ , (b) phase factor  $\beta$ , (c) wave impedance modulus, and (d) wave impedance argument.

Changes to the damping factor alpha when changing parameter  $C_0$  are non–linear, and vary from 0.25% to 1.4%, regardless of frequency (Figures 8a, 9a and 10a). The factor  $\beta$  rises almost linearly from –10% to 10%, regardless of frequency (Figures 8b, 9b and 10b). The impedance modulus rises almost linearly in the range of 12% to –8% for all tested frequencies



(Figures 8c, 9c and 10c). The argument changes linearly in the range from approximately -400% to 280%, with minor changes for individual frequencies (Figures 8d, 9d and 10d).

**Figure 7.** Changes of wave parameters depending on  $L_0$  at f = 1 MHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.



**Figure 8.** Changes of wave parameters depending on  $C_0$  at f = 100 kHz; (a) damping factor  $\alpha$ , (b) phase factor  $\beta$ , (c) wave impedance modulus, and (d) wave impedance argument.

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**Figure 9.** Changes of wave parameters depending on  $C_0$  at f = 500 kHz; (a) damping factor  $\alpha$ , (b) phase factor  $\beta$ , (c) wave impedance modulus, and (d) wave impedance argument.



**Figure 10.** Changes of wave parameters depending on  $C_0$  at f = 1 MHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.

Changes to the damping factor alpha when changing the  $G_0$  parameter are non–linear, and vary from 1% to 0.2% regardless of frequency (Figures 11a, 12a and 13a). The factor  $\beta$  rises almost linearly from –10% to 10%, regardless of frequency (Figures 11b, 12b and 13b). The impedance modulus rises almost linearly in the range of –10% to 10% for all

tested frequencies (Figures 11c, 12c and 13c). The argument changes linearly in the range from approximately 40% to -200% with minor changes for individual frequencies (Figures 11d, 12d and 13d).



**Figure 11.** Changes of wave parameters depending on  $G_0$  at f = 100 kHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.



**Figure 12.** Changes of wave parameters depending on  $G_0$  at f = 500 kHz; (a) damping factor  $\alpha$ , (b) phase factor  $\beta$ , (c) wave impedance modulus, and (d) wave impedance argument.



**Figure 13.** Changes of wave parameters depending on  $G_0$  at f = 1 MHz; (**a**) damping factor  $\alpha$ , (**b**) phase factor  $\beta$ , (**c**) wave impedance modulus, and (**d**) wave impedance argument.

Changes to the  $U_2$  voltage modulus occur when the  $R_0$  parameter is changed linearly, and vary from 0.4% to -0.4%, depending on the frequency. The higher the frequency, the smaller the change (Figures 14a, 15a and 16a). The  $U_2$  argument increases almost linearly from -2% to 2%, depending on the frequency (Figures 14b, 15b and 16b). The  $I_2$  Modulus changes linearly from 0.4% to -0.4% for lower frequencies, and 0.04% to -0.04% for higher tested frequencies (Figures 14c, 15c and 16c). The  $I_2$  argument rises linearly from -2.1% to 2.1% for 500 kHz, and -0.13% to 0.13% for 1 MHz (Figures 14d, 15d and 16d).



**Figure 14.** Changes of wave parameters depending on  $R_0$  at f = 100 kHz; (**a**) U<sub>2</sub> voltage modulus, (**b**)  $U_2$  argument, (**c**)  $I_2$  modulus, (**d**)  $I_2$  argument.



**Figure 15.** Changes of wave parameters depending on  $R_0$  at f = 500 kHz; (**a**)  $U_2$  voltage modulus, (**b**)  $U_2$  argument, (**c**)  $I_2$  modulus, (**d**)  $I_2$  argument.



**Figure 16.** Changes of wave parameters depending on  $R_0$  at f = 1 MHz; (a)  $U_2$  voltage modulus, (b)  $U_2$  argument, (c)  $I_2$  modulus, (d)  $I_2$  argument.

Changes of  $U_2$  voltage modulus occur when  $L_0$  parameter is changed parabolically, and change from 7.5% to -3%, regardless of frequency. (Figures 17a, 18a and 19a). The  $U_2$  argument falls almost linearly from 50% to -50%, independently of the frequency (Figures 17b, 18b and 19b). The  $I_2$  modulus changes parabolically in the range of 7.5% to

4%, with slight differences depending on the frequency (Figures 17c, 18c and 19c). The  $I_2$  argument falls linearly in the range from 50% to -50%, independently of the frequency (Figures 17d, 18d and 19d).



**Figure 17.** Changes of wave parameters depending on  $L_0$  at f = 100 kHz; (a)  $U_2$  voltage modulus, (b)  $U_2$  argument, (c)  $I_2$  modulus, (d)  $I_2$  argument.



**Figure 18.** Changes of wave parameters depending on  $L_0$  at f = 500 kHz; (a)  $U_2$  voltage modulus, (b)  $U_2$  argument, (c)  $I_2$  modulus, (d)  $I_2$  argument.



**Figure 19.** Changes of wave parameters depending on  $L_0$  at f = 1 MHz; (a)  $U_2$  voltage modulus, (b)  $U_2$  argument, (c)  $I_2$  modulus, (d)  $I_2$  argument.

Changes of the  $U_2$  voltage modulus occur when parameter  $C_0$  is changed non–linearly, and change from -14% to 8% independently of frequency. (Figures 20a, 21a and 22a). The  $U_2$  argument falls almost linearly from 50% to -50%, independently of frequency (Figures 20b, 21b and 22b). The  $I_2$  modulus changes from -14% to 8%, independently of the frequency (Figures 20c, 21c and 22c). The  $I_2$  argument decreases linearly in the range from 50% to -50%, independently of the frequency (Figures 20c, 21c and 22c).



**Figure 20.** Changes of wave parameters depending on  $C_0$  at f = 100 kHz; (**a**)  $U_2$  voltage modulus, (**b**)  $U_2$  argument, (**c**)  $I_2$  modulus, (**d**)  $I_2$  argument.



**Figure 21.** Changes of wave parameters depending on  $L_0$  at f = 500 kHz; (**a**)  $U_2$  voltage modulus, (**b**)  $U_2$  argument, (**c**)  $I_2$  modulus, (**d**)  $I_2$  argument.



**Figure 22.** Changes of wave parameters depending on  $L_0$  at f = 1 MHz; (a)  $U_2$  voltage modulus, (b)  $U_2$  argument, (c)  $I_2$  modulus, (d)  $I_2$  argument.

Changes of the  $U_2$  voltage modulus occur when the  $G_0$  parameter is changed almost linearly, and change from 0.09% to -0.09% for 500 kHz, and 0.04% to -0.04% for 1 MHz (Figures 23a and 24a). The  $U_2$  argument decreases almost linearly from 0.02% to -0.02% at 500 kHz, and 0.01% to -0.01% at 1 MHz (Figures 23b and 24b). The  $I_2$  modulus runs almost linearly, and changes from 0.09% to -0.09% for 500 kHz, and 0.04% to -0.04% for



1 MHz (Figures 23c and 24c). The  $I_2$  argument falls almost linearly from 0.02% to -0.02% at 500 kHz, and 0.01% to -0.01% at 1 MHz (Figures 23d and 24d).

**Figure 23.** Changes of wave parameters depending on  $G_0$  at f = 500 kHz; (**a**)  $U_2$  voltage modulus, (**b**)  $U_2$  argument, (**c**)  $I_2$  modulus, (**d**)  $I_2$  argument.



**Figure 24.** Changes of wave parameters depending on  $G_0$  at f = 1 MHz; (a)  $U_2$  voltage modulus, (b)  $U_2$  argument, (c)  $I_2$  modulus, (d)  $I_2$  argument.

# 5. Conclusions

The paper presents the evaluation of the impact of per unit parameter errors on the determination of wave parameters, as well as currents and voltages at the end of the line. On the basis of numerical calculations, it was found that errors in the determination of per unit inductance have the greatest influence on the value of the part of the input resistance and on  $I_1$  current argument. However, the change of per unit resistance and leakage have the smallest effect on the value of the phase constant. It can also be seen that increasing per unit length parameters by 1% does not always result in an increase of output parameters.

A thorough analysis of the above diagrams may be used to construct optimal high voltage cables used in the ignition systems of spark–ignition engines, where the most important parameter is the  $U_2$  voltage modulus [3,4,10,20].

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