Article

Mathematical Programming and Solution Approaches for Transportation Optimisation in Supply Network

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Abstract: The problem of transport is a special type of mathematical programming designed to search for the optimal distribution network, taking into account the set of suppliers and the set of recipients. This article proposes an innovative approach to solving the transportation problem and devises source codes in GNU Octave (version 3.4.3) to avoid the necessity of carrying out enormous calculations in traditional methods and to minimize transportation costs, fuel consumption, and CO₂ emission. The paper presents a numerical example of a solution to the transportation problem using: the northwest corner, the least cost in the matrix, the row minimum, and Vogel's Approximation Methods (VAM). The joint use of mathematical programming and optimization was applicable to real conditions. The transport was carried out with medium load trucks. Both suppliers and recipients of materials were located geographically within the territory of the Republic of Poland. The presented model was supported by a numerical example with interpretation and visualization of the obtained results. The implementation of the proposed solution enables the user to develop an optimal transport plan for individually defined criteria.

Keywords: transportation optimization; CO₂ emissions; mathematical modeling; programming; code verification

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1. Introduction

It is particularly important to be able to optimize road transport operations and to use economic theories to solve transportation problems. In literature, problems related to the reduction of computational complexity of the transportation algorithms using, e.g., the C++ programming language or the Matlab environment, were repeatedly addressed. However, it must be acknowledged that they require the user to have a thorough knowledge of the functions available and be proficient in their use. A GNU Octave also allows to use programming techniques, generate new user functions, or perform complex numerical calculations. So far, determining the optimal solution of the transportation problem using programming in GNU Octave was not studied. Therefore, the aim of this article was to devise source codes in free GNU Octave (version 3.4.3) environment to minimize transportation costs, fuel consumption, and CO₂ emission.

1.1. Transportation of Materials in Supply Network

From a mathematical point of view, transportation problems have a wide range of applications in logistic systems [1]. Service network design problems arise wherever there is a need to determine cost minimizing routes and schedules, given the constraints on resource availability and level of service [2]. In [3], an agent-based simulation model studying the evolution of a coalition over time taking into account various trust-related issues was developed. Providing efficient distribution systems for services became a challenging issue for logistics companies. Mixed delivery approach, which combines attended home delivery

(AHD) and shared delivery locations (SDL) usage in an innovative way, was proposed in [4]. In real-life situations, the traditional transportation problems deal with issues such as transportation costs, selection of delivery routes, selection of production places, or reduction of carbon emissions [5–8]. Indeed, it is necessary to plan the transportation of materials within the supply network rationally and optimally [9]. In the case of a balanced transportation problem, the balance between supply and demand is assumed. Due to the practical nature of the problem and the effectiveness of the methodology, the transportation algorithm is often used [10–12]. The paper [13] is concerned with the design of efficient exact and heuristic algorithms for addressing a bilevel network pricing problem, where demand is a nonlinear function of travel cost. Metaheuristic approach for solving transportation problem with fixed costs associated to the routes was presented in [14]. In distribution networks, rational transport management also requires organizational efforts related to the dynamics and type of tasks [15–17]. The costs of transport tasks should be optimized using methods and tools used not only in commercial solutions [18–20]. The literature often addresses cases aimed at developing the most effective method of solving transportation problems according to time or cost, while using the available IT tools [21,22]. In the paper [23] one of the first basic transportation problem was formalized, which was then further developed in [24], and then expanded even more in [25] by using the simplex method. There are a number of methods leading to a basic feasible solution to the transportation problem. The most common methods used to support the decision making process are: the northwest corner method, the row minimum method, the least cost in the matrix, and VAM method (Vogel's Approximation Method).

1.2. Modifications of Existing Methods

An innovative method of solving the transport problem based on the northwest corner method is described in the paper [26]. The authors, by deliberately modifying the cost matrix before applying the northwest corner method, proved that the proposed solution positively influences the effectiveness of the method by reducing the number of iterations performed. In [27], an analysis was made of the number of steps necessary to determine the optimal transport problem using the northwest corner method, the minimum cost method, and the VAM method. The subject of the research were transport cost matrices with dimensions of 5×5 , 10×10 , 20×20 , 40×40 , 80×80 , 160×160 , 320×320 , while the values of supply and demand were generated in a random way from a set of numbers from 5 to 50,000. On the basis of the obtained results, it was found that the number of iterations required to obtain an optimal solution when using the northwest corner method in the case of matrices with dimensions of 320×320 is as much as six-times higher than when using the minimum cost method or the VAM method. In [28], an alternative method of solving the balanced transport problem, so-called TCM (Tuncay Can Approximation Method), based on the geometric average of transportation costs, was used. A novel method called KSAM (Karagül-Sahin Approximation Method) used to find an initial solution to the transportation problem was proposed in [29]. The solutions obtained by KSAM were as good as the solutions obtained with the VAM method and as fast as the northwest corner method. In [30] the MOMC method (Maximum Supply with Minimum Cost) was used and the results were compared with three classical methods, i.e., the northwest corner method, the minimum cost method and the VAM method. The MOMC method provides a solution that minimizes the objective function and offers a computational advantage in the form of faster data processing and reduced memory usage. The article [31] analyses the VAM method and proposes how to improve it by using the total alternative cost matrix and alternative allocation costs. Both parametric and nonparametric statistical tests available in the MINITAB-15 statistical package were used in the calculations. In [32], the limitations resulting from the use of the Vogel approximation were discussed and an improved LD-VAM algorithm (Logical Development of Vogel's Approximation Method) providing a lower objective function value than the VAM method was developed. The paper [33]

proposes an ATM (Allocation Table Method) method leading to a basic feasible solution with a lower transportation cost than the solution obtained with traditional algorithms.

1.3. Reduction of Computational Complexity Using the C++ Programming Language

In article [34], a new heuristic method called TOC (Total Opportunity-Cost Method) was proposed to determine the basic feasible solution to the transportation problem, and the results obtained were compared with the classical VAM method. In the publication [35], several variants of the VAM method were analyzed taking into account the concept of Total Opportunity-Cost Method (TOC). Based on the calculations carried out, using the programming language Turbo C++, it was found that the classic VAM method combined with the Total Opportunity-Cost method, the so-called VAM-TOC, results in an optimal or close to optimal solution. Implementation of C++ programming language is also described in [36]. Using the JHM method (Juman & Hoque Method), 18 transportation problems were solved, 11 of which were taken from the literature, and 7 were generated at random. The calculations show that the JHM method in as many as 16 problems considered resulted in a minimum transport cost being determined. In [37], the transportation problem was solved using the northwest corner, minimal cost, the row minimum method, the least cost in the matrix, and the VAM methods. The calculations were carried out in two ways, i.e., by using traditional methodology and object-oriented programming. Due to the need to perform a number of iterations leading to the solution of the problem in question, the C++ programming language was used.

1.4. Practical Application of the Matlab Environment in Transportation Problems

In the literature, there are many solutions to the transportation problem that use the Matlab software. In the paper [38], the problem of transport of chemical substances for a pharmaceutical company using the northwest corner method was analyzed. The Matlab software was also used in [39]. Proposed solution based on the Vogel's Approximation and Modified Distribution Method would ease the computation of different problems, especially when the problem has a larger cost matrix. In [40], the validity of using the Matlab package was proven not only due to the high efficiency of the algorithm used, but also due to the reduction of time necessary to perform the calculations. In addition, the implementation of the proposed procedure makes it possible to solve the traveling salesman problem. The descriptions of algorithms based on the northwest corner method, the minimum cost method, the VAM method, and the MODI method are included in [41]. The proposed approach emphasizes the superiority of computer programming methods over classical analytical methods.

1.5. Sustainable Transport

Considerable attention is paid to minimizing transport costs with strictly defined supply and demand volumes. However, it should be remembered that during the transport process, the values of both supply and demand can vary over time. The transportation costs to develop a new supplier–retailer inventory model, under the condition that the supplier and the retailer have adopted the two-level trade credit policy, were discussed in article [42]. In [43], a model for taking stock costs into account was defined to determine the lower and upper limit of total transportation costs in a situation of changing supply and demand. The calculations were done using the Matlab software (version 7.10). The significance of the impact of demand variability on the minimization of the supply chain costs is discussed in [44]. The work focuses on optimizing the vehicle speed not only to reduce the transportation costs, but also to reduce carbon emissions while taking into account the variability of demand. The Matlab software was also used for the calculations. The paper [45] presented an analysis of the variability of exhaust emissions from conventional, hybrid, and electric vehicles. The possibility of reducing risks in the supply chain and of using the compound Poisson process was proposed as a possible further research direction. A similar subject was addressed in the paper [46], which explained that rational adjustment

of vehicle speeds can offer not only financial but also environmental benefits by reducing carbon dioxide emissions.

1.6. Multicriteria Decision Making Processes

The literature also often addresses issues related to the multicriteria decision making process. In the paper [47] the optimal transport and logistics solution was determined, concerning the distribution of home appliance goods from China to warehouses located in Poland, while ensuring the most favorable conditions for both the manufacturing company and the customers. An integral part of the logistic system is the subsystem of movement and transportation, the aim of which is to ensure the timely and efficient movement of personnel, equipment and supplies in crisis situations. Logistics and transportation simulations that can be used to provide insights into potential outcomes of proposed military deployment plans were presented in [48]. The system of transporting military cargo by means of road transport in peacetime is characterized by high stability. During the period of exercises or combat operations, the high pace of operations requires the commanders to make an effort to monitor and continuously modify the supply routes.

Comparison of research contributions between our paper and related articles was presented in Table 1. The use of our code in GNU Octave allows for effective management of vehicles and for determining optimal distribution plan within supply chain. Moreover, an easy adaptation of the source code could be implemented in other linear programming problems.

Table 1. Comparison of transportation problems in this paper and related articles.

Problem Chosen from	Cost	CO ₂	Time	C++	Matlab	GNU Octave
Our paper	✓	✓	–	–	–	✓
[1]	–	–	✓	–	–	–
[6]	✓	✓	–	–	–	–
[14]	✓	–	–	–	–	–
[15]	–	–	✓	–	–	–
[18]	✓	–	–	–	–	–
[26]	✓	–	–	–	–	–
[27]	✓	–	–	–	–	–
[29]	✓	–	–	–	–	–
[30]	✓	–	–	–	–	–
[32]	✓	–	–	–	–	–
[33]	✓	–	–	–	–	–
[35]	✓	–	–	✓	–	–
[36]	✓	–	–	✓	–	–
[37]	✓	–	–	✓	–	–
[38]	✓	–	–	–	✓	–
[39]	✓	–	–	–	✓	–
[40]	✓	–	–	–	✓	–
[41]	✓	–	–	–	✓	–
[44]	✓	✓	–	–	–	–
[46]	✓	✓	–	–	–	–
[49]	✓	–	–	–	–	–
[50]	✓	✓	–	✓	–	–

2. Theoretical Backgrounds for Mathematical Model Development

A transportation problem where total supply is equal to aggregated demand is called a balanced transportation problem. Otherwise, it is an unbalanced transportation problem. Every unbalanced transportation problem can be converted to a balanced transportation problem by adding an artificial supplier or recipient [51,52]. The needs of each recipient as well as the resources of each supplier are known. The distribution of the product should be planned so that transportation costs are minimal [49,53]. The notations used to formulate this problem are presented in Table 2.

Table 2. List of variables.

Notations	Details
$F_{obj}(X, C)$	The objective function whose arguments are cost matrix and basic feasible solution,
$F_{deg}(X)$	The degeneration function whose arguments are base elements,
X	The matrix of the feasible solution to the transportation problem,
x_{ij}	Number of units to be transported from the i -th supplier to the j -th recipient,
C	The transportation cost matrix,
C_{NW}	The total transportation cost for the northwest corner method,
C_{MKW}	The total transportation cost for the row minimum method,
C_{MK}	The total transportation cost for the least cost in the matrix method,
C_{VAM}	The total transportation cost for the Vogel's approximation method,
c_{ij}	The transportation cost from the i -th supplier to the j -th recipient,
m	Total number of supply nodes, number of suppliers,
n	Total number of demand nodes, number of recipients,
a_i	The resource of the i -th supplier, $a_i > 0, i = 1, \dots, m$,
a_{NW}	The new value of supply for the northwest corner method,
a_{MKW}	The new value of supply for the row minimum method,
a_{MK}	The new value of supply for the least cost in the matrix method,
a_{VAM}	The new value of supply for the Vogel's approximation method,
b_j	The demand of the j -th recipient, $b_j > 0, j = 1, \dots, n$,
b_{NW}	The new value of demand for the northwest corner method,
b_{MKW}	The new value of demand for the row minimum method,
b_{MK}	The new value of demand for the least cost in the matrix method,
b_{VAM}	The new value of demand for the Vogel's approximation method,
r_i	The difference between the lowest and second lowest cost $c_{ij} > 0$ in each row in C ,
s_j	The difference between the lowest and second lowest cost $c_{ij} > 0$ in each column in C .

The transportation problem can be stated mathematically as a linear programming problem. The objective function described in the formula in Equation (1) minimizes the total cost of transportation between suppliers and recipients:

$$F_{obj}(X, C) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}. \quad (1)$$

Subject to Equations (2) and (3):

$$\sum_{j=1}^n x_{ij} = a_i, \quad (2)$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad (3)$$

where $x_{ij} \geq 0, i = 1, \dots, m, j = 1, \dots, n$.

If total demand is equal to aggregated supply then the relationship in Equation (4) can be noted as:

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j. \quad (4)$$

The feasible solution to the transportation problem is the matrix $X = [x_{ij}]$ that meets the conditions (2) and (3), while the optimal solution is a feasible solution that minimizes the objective function (1). The matrix $X = [x_{ij}]$ is referred to as the basic feasible solution to the transportation problem relative to base set B if:

$$(i, j) \notin B \Rightarrow x_{ij} = 0. \quad (5)$$

The variables $\{x_{ij} | (i, j) \in B\}$ and $\{x_{ij} | (i, j) \notin B\}$ are called base and nonbase variables, respectively, in relation to set B . The next steps of the transportation algorithm are shown below:

1. Determine the base set B and basic feasible solution $X_B = [x_{ij}^B]$,

2. Determine the zero matrix $C_B = [c_{ij}^B]$ equivalent to the cost matrix $C = [c_{ij}]$ in relation to the base set B ,
3. For one of the unknowns, take any value $u_1, u_2, \dots, u_m$ and $v_1, v_2, \dots, v_n$:
 - (a) other unknowns are to be determined by solving the equation system $c_{ij} + u_i + v_j = 0$ for $(i, j) \in B$,
 - (b) determine $c_{ij}^B = c_{ij} + u_i + v_j$ where $i = 1, \dots, m, j = 1, \dots, n$.
4. Check if the matrix C_B is non-negative. If $C_B \geq 0$ then X_B is the optimal solution. Otherwise, go to step 4.
5. Determine a new base set B_1 and the corresponding basic feasible solution $X_{B_1} = [x_{ij}^{B_1}]$:
 - (a) designate a node (k, l) meeting the condition $c_{kl}^B = \min_{(i,j)} c_{ij}^B$,
 - (b) determine the cycle Γ contained in the set $B \cup \{(k, l)\}$. This cycle should be divided into a positive and a negative semicyclical Γ_1 and Γ_2 , where Γ_1 is in the (k, l) ,
 - (c) designate a node (p, q) semi-cyclical Γ_2 meeting the condition $x_{pq}^B = \min_{(i,j) \in \Gamma_2} x_{ij}^B$,
 - (d) assume $B_1 = B \cup \{(k, l)\} - \{(p, q)\}$,
 - (e) determine $X_{B_1} = [x_{ij}^{B_1}]$ from the formula:

$$x_{ij}^{B_1} = \begin{cases} x_{ij}^B + x_{pq}^B & \text{for } (i, j) \in \Gamma_1 \\ x_{ij}^B - x_{pq}^B & \text{for } (i, j) \in \Gamma_2 \\ x_{ij}^B & \text{for } (i, j) \notin \Gamma \end{cases}$$

6. Go to step 2.

The transportation algorithm is presented in the form of a flowchart in Figure 1.

Figure 1. Flowchart of transportation problem.

3. Methods

3.1. The Northwest Corner Method

The northwest corner method, otherwise known as the upper-left corner method, provides a feasible solution to the considered transportation problem. It does not take into account the transport cost matrix, which may result in high cost of the solution. It consists of subsequent assignment of appropriate values to variables, each time for routes located in the upper left-hand corner of the transport table [54]. It is necessary to come up with a plan for the transport of products from $i = 1, \dots, m$ suppliers to $j = 1, \dots, n$ recipients in such a way as to ensure minimum transportation costs, with no more than a_i of the product at each point of delivery and no less than b_j of the product at each point of receipt. Transportation takes place along the planned arcs connecting the delivery vertices with the collection vertices (the route from the i -th supplier to the j -th recipient), forming a directed transport network in which the unit transportation costs along each arc are calculated c_{ij} . The solution technique by the northwest corner method is presented as Algorithm 1.

Algorithm 1 Pseudocode for the Northwest Corner Method

Input: m, n, c_{ij}, a_i, b_j
Output: $C_{NW}, F_{zdeg}(X)$
 $a_{NW} \leftarrow a_i$
 $b_{NW} \leftarrow b_j$
while $i \leq m; j \leq n$
 The first non-zero element x_{ij} is located in the first cell in the northwest corner in matrix X
calculate: the minimum value among supply or demand for the first cell in the northwest corner $X = \min(a_{NW}, b_{NW})$
 the new value of supply $a_{NW} = a_{NW} - X$
 the new value of demand $b_{NW} = b_{NW} - X$
 if the new value $a_{NW} = 0$ **then** the remaining cells in this column should be filled with 0
 endif
 if the new value $b_{NW} = 0$ **then** the remaining cells in this row should be filled with 0
 endif
 the next non-zero elements x_{ij} are located in the next cells in the northwest corner in matrix X
endwhile
calculate: total transportation cost for the northwest corner method
 $C_{NW} = F_{obj}(X, C)$ and the number of basic variables $F_{zdeg}(X)$
check: degeneration of the solution
 if $F_{zdeg}(X) = m + n - 1$ **then** solution is degenerated
 else solution is non-degenerated
 endif

Six suppliers located in towns: Zlocieniec, Wesola, Gizycko, Rzeszow, Krakow and Brzeg have 24, 31, 19, 49, 40, 37 loading units, respectively. Eight recipients located geographically in the territory of the Republic of Poland presented in Figure 2 placed an order for 18, 29, 15, 34, 26, 21, 36, 21 loading units respectively. The transportation model was represented by the network shown in Figure 3. The arcs represent the routes linking the destinations and sources.

Figure 2. Location of suppliers and recipients in territory of Poland.

Figure 3. Network structure of transportation problem.

Taking into account dynamic and necessity to act in crisis situations, analysis of the transportation costs was based on military guidelines and the decision of Chief of the Inspectorate for Armed Forces Support from 19 February 2020. The transport is to be carried out with medium load trucks for which the following unit cost factors were adopted:

- amortisation €1.20,

- maintenance €0.97,
- engine fuels and lubricants €0.44.

The total cost index per exploitation unit is €2.62. Data for numerical problem are presented in Table 3.

Table 3. Transportation table for numerical example.

	P1	P2	P3	P4	P5	P6	Demand
S1	736.0	1400.0	1647.0	1504.0	1063.0	545.0	18
S2	1011.0	674.0	197.0	1556.0	1478.0	1400.0	29
S3	1349.0	791.0	156.0	1608.0	1530.0	1660.0	15
S4	1271.0	267.0	882.0	596.0	648.0	908.0	34
S5	1193.0	337.0	1037.0	674.0	622.0	700.0	26
S6	545.0	648.0	895.0	1530.0	1115.0	674.0	21
S7	1712.0	726.0	1219.0	156.0	493.0	1193.0	36
S8	1764.0	506.0	415.0	1089.0	1245.0	1375.0	21
Supply	24	31	19	49	40	37	

At the beginning a table should be prepared with the dimension of m —columns (number of suppliers) and n —lines (number of recipients) taking into account the demand and supply constraints:

$$\begin{aligned}
 x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} &= 18, \\
 x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} &= 29, \\
 x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} &= 15, \\
 x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} &= 34, \\
 x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} &= 26, \\
 x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} &= 21, \\
 x_{71} + x_{72} + x_{73} + x_{74} + x_{75} + x_{76} &= 36, \\
 x_{81} + x_{82} + x_{83} + x_{84} + x_{85} + x_{86} &= 21, \\
 x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81} &= 24, \\
 x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} + x_{72} + x_{82} &= 31, \\
 x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} + x_{83} &= 19, \\
 x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} &= 49, \\
 x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} + x_{75} + x_{85} &= 40, \\
 x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} + x_{76} + x_{86} &= 37.
 \end{aligned}$$

To solve the transportation problem using the GNU Octave software (Version 3.4.3/John W. Eaton, Madison, WI, USA), it is necessary to input the following values: the number of suppliers, the number of recipients and the transportation costs from the i -th supplier to the j -th recipient. In addition, it is necessary to determine demand and supply. Using the northwest corner method, table completion should start with the first cell in the left corner, which corresponds to specific supply and demand values. In the next step, the lower value from among them should be selected and entered in the field corresponding to the first cell of the left corner, and then both supply and demand should be reduced by the value entered. For the first cell, supply assumes a value of 24, while demand is 18. A smaller value is 18, so in the next step, the first cell was supplemented with 18, after which it was subtracted from both supply and demand. At the same time, the software checks which of the demand or supply values is equal zero. If the demand takes a zero value, then the remaining cells in the row under consideration should be completed with zeros. In case the supply would be zeroed, the remaining cells of the column would have to be

filled with zeros. Following the same procedure in the next steps, the feasible solution presented in Table 4 was obtained. All nonzero elements are called base elements, while zero elements are called nonbase elements. The solution is degenerated when the number of base elements is $m + n - 1$ meaning $8 + 6 - 1 = 13$. Obtaining an undegenerated solution will make it impossible to check the optimality of the solution using the potential method.

Table 4. Results of subsequent iterations and feasible solution for northwest corner method.

step 13						
18	0	0	0	0	0	0
6	23	0	0	0	0	0
0	8	7	0	0	0	0
0	0	12	22	0	0	0
0	0	0	26	0	0	0
0	0	0	1	20	0	0
0	0	0	0	20	16	16
0	0	0	0	0	21	21

Transportation cost was computed by using the objective function described in Equation (1). The calculations resulted in a degenerated solution for which the total cost of transport was €165,109.0. Computational source code written in Notepad++ and generated in GNU Octave for finding the basic feasible solution using Northwest Corner Method is given in Appendix A. Lines from 1 to 112 are common to each method. The command window also displays information about the value of the objective function, the number of the base elements, and the degeneration of the received solution.

3.2. The Row Minimum Method

The row minimum method consists of selecting the elements of the C-cost matrix, for which the cost c_{ij} in the first row is minimal. The indicated c_{ij} element corresponds to the value x_{ij} , from which the construction of the base matrix $X = [x_{ij}]$ starts. Then, the arcs corresponding to the zero elements of the transformed cost matrix are selected. To determine the initial feasible solution, it is necessary to supplement the X matrix with elements corresponding to arcs with the lowest unit transportation costs in the subsequent rows. Completing the results table using the row minimum method consists of comparing transportation costs and the corresponding values of supply and demand starting from the first row. The solution technique by the row minimum method is presented as Algorithm 2.

The lowest cost in the first row is 545.0, with the demand of 18 and supply of 37. In the next step both supply and demand were reduced by 18, resulting in a zero value for demand, as a consequence the remaining cells of the first row were supplemented with zeroes. Following the same procedure in the next steps, the feasible solution presented in Table 5 was obtained.

Table 5. Results of subsequent iterations and feasible solution for row minimum method.

step 13						
0	0	0	0	0	18	18
0	10	19	0	0	0	0
0	15	0	0	0	0	0
0	6	0	28	0	0	0
0	0	0	0	26	0	0
21	0	0	0	0	0	0
0	0	0	21	14	1	1
3	0	0	0	0	18	18

The value of the objective function determined using the row minimum method was lower than the value obtained using the northwest corner method. The calculations resulted in a degenerated solution for which the total cost of transport was equal to €119,478.0. The source code written in Notepad++ and generated in GNU Octave for finding the basic feasible solution using the row minimum method is given in Appendix B.

Algorithm 2 Pseudocode for the Row Minimum Method

Input: m, n, c_{ij}, a_i, b_j
Output: $C_{MKW}, F_{zdeg}(X)$
 $a_{MKW} \leftarrow a_i$
 $b_{MKW} \leftarrow b_j$
while $i \leq m; j \leq n$
Find the element in the first row of the C matrix for which c_{ij} is minimal
Indicate element c_{ij} which corresponds to the first non-zero element x_{ij}
calculate: the minimum value among supply or demand for the x_{ij} in the first row of the C matrix
 $X = \min(a_{MKW}, b_{MKW})$
the new value of supply $a_{MKW} = a_{MKW} - X$
the new value of demand $b_{MKW} = b_{MKW} - X$
if the new value $a_{MKW} = 0$ **then** the remaining cells in this column should be completed with 0
endif
if the new value $b_{MKW} = 0$ **then** the remaining cells in this row should be completed with 0
endif
the next non-zero elements x_{ij} (corresponding to the minimal value of costs in the next rows of the C matrix) are located in the matrix X
endwhile
calculate: total transportation cost for the row minimum method
 $C_{MKW} = F_{obj}(X, C)$ and the number of basic variables $F_{zdeg}(X)$
check: degeneration of the solution
if $F_{zdeg}(X) = m + n - 1$ **then** solution is degenerated
else solution is non-degenerated
endif

3.3. The Least Cost in the Matrix Method

This method consists in supplementing the table with routes with the lowest unit costs following the order of the non-decreasing sequence of values for unit costs of transport. The cost matrix should be transformed in such a way that there is at least one 0 value in each column and row. The minimum element in the row (column) under consideration should be subtracted from the elements in each row (column). Then, from the zero elements of the transformed cost matrix, the one for which the cost c_{ij} is the lowest should be selected. The c_{ij} element corresponds to the x_{ij} arc from which the base matrix $X = [x_{ij}]$ creation should be started. Then, the arcs corresponding to the zero elements of the transformed cost matrix are selected. To determine the basic feasible solution, it is necessary to supplement the X matrix with additional elements corresponding to arches with the lowest unit transportation costs. The solution technique by the least cost in the matrix method is presented as Algorithm 3.

The lowest cost is 156.0 with a demand of 15 and a supply of 19. In the next step, both supply and demand were reduced by 15, resulting in a zero value for demand, as a consequence of which the remaining cells of row three were also supplemented with zeroes. The result of the described scheme is presented in Table 6.

The calculations resulted in a degenerated solution for which the total cost of transport using the least cost in the matrix method amount to €114,888.0. The source code written in Notepad++ and generated in GNU Octave for finding the basic feasible solution using the least cost in the matrix method is given in Appendix C.

Table 6. Results of subsequent iterations and feasible solution for least cost in matrix method.

step 13						
	0	0	0	0	0	18
	3	0	4	0	3	19
	0	0	15	0	0	0
	0	31	0	3	0	0
	0	0	0	0	26	0
	21	0	0	0	0	0
	0	0	0	36	0	0
	0	0	0	10	11	0

Algorithm 3 Pseudocode for the Least Cost in the Matrix Method

```

Input:  $m, n, c_{ij}, a_i, b_j$ 
Output:  $C_{MK}, F_{zdeg}(X)$ 
 $a_{MK} \leftarrow a_i$ 
 $b_{MK} \leftarrow b_j$ 
while  $i \leq m; j \leq n$ 
  Find the element of the C matrix for which  $c_{ij}$  is minimal
  Indicate element  $c_{ij}$  which corresponds to the first non-zero element  $x_{ij}$ 
  calculate: the minimum value among supply or demand for the  $x_{ij}$  in the C matrix
                $X = \min(a_{MK}, b_{MK})$ 
               the new value of supply  $a_{MK} = a_{MK} - X$ 
               the new value of demand  $b_{MK} = b_{MK} - X$ 
               if the new value  $a_{MK} = 0$  then the remaining cells in this column
               should be completed with 0
               endif
               if the new value  $b_{MK} = 0$  then the remaining cells in this row should
               be completed with 0
               endif
               the next non-zero elements  $x_{ij}$  (corresponding to the minimal
               value of costs in the next rows/columns of the C matrix) are
               located in the matrix X
endwhile
calculate: total transportation cost for the least cost in the matrix method
                $C_{MK} = F_{obj}(X, C)$  and the number of basic variables  $F_{zdeg}(X)$ 
check: degeneration of the solution
               if  $F_{zdeg}(X) = m + n - 1$  then solution is degenerated
               else solution is non-degenerated
               endif

```

3.4. The Vogel's Approximation Method

The VAM method takes into account the transportation cost matrix, thus making it possible to find a low-cost solution. The application of GNU Octave software to determine the optimal solution with VAM method is to calculate the ratios, i.e., the difference r_i and s_j between the lowest and second lowest cost options in each row and in each column, respectively. In the next step it is necessary to indicate the cell with the highest difference values $r_k = \max_i \{r_i\}$ and $s_l = \max_j \{s_j\}$. When $r_k < s_l$ the highest indicator corresponds to a column and then the lowest cost in the column under consideration should be indicated. If the highest difference would correspond to the row $r_k > s_l$ the lowest cost in the given row should be indicated. The solution technique by the Vogel's approximation method is presented as Algorithm 4.

Algorithm 4 Pseudocode for the Vogel's Approximation Method

Input: m, n, c_{ij}, a_i, b_j
Output: $C_{VAM}, F_{zdeg}(X)$
 $a_{VAM} \leftarrow a_i$
 $b_{VAM} \leftarrow b_j$
while $i \leq m; j \leq n$
calculate: Ratios $r_k = \max_i \{r_i\}$ and $s_l = \max_j \{s_j\}$ between the lowest and second lowest cost c_{ij} in each row and in each column
if $r_k < s_l$ **then** the lowest cost c_{ij} in the column under consideration should be indicated
else
the lowest cost c_{ij} in the row under consideration should be indicated
endif
Indicate the minimum element c_{ij} (in the column/row under consideration) which corresponds to the first non-zero element x_{ij}
calculate: the minimum value among supply or demand for the x_{ij} in the C matrix
 $X = \min(a_{VAM}, b_{VAM})$
the new value of supply $a_{VAM} = a_{VAM} - X$
the new value of demand $b_{VAM} = b_{VAM} - X$
if the new value $a_{VAM} = 0$ **then** the remaining cells in this column should be completed with 0
endif
if the new value $b_{VAM} = 0$ **then** the remaining cells in this row should be completed with 0
endif
The next nonzero elements x_{ij} (corresponding to the minimal value of costs in the subsequent rows/columns of the C matrix for which the maximum value of r_k or s_l was obtained) are located in the matrix X
endwhile
calculate: total transportation cost for the Vogel's approximation method
 $C_{VAM} = F_{obj}(X, C)$ and the number of basic variables $F_{zdeg}(X)$
check: degeneration of the solution
if $F_{zdeg}(X) = m + n - 1$ **then** solution is degenerated
else solution is non-degenerated
endif

The results obtained on the basis of the described scheme are shown in Table 7.

Table 7. Results of subsequent iterations and feasible solution for Vogel's Approximation Method (VAM).

step 13						
0	0	0	0	0	0	18
15	10	4	0	0	0	0
0	0	15	0	0	0	0
0	0	0	13	21	0	0
0	0	0	0	19	7	0
9	0	0	0	0	12	0
0	0	0	36	0	0	0
0	21	0	0	0	0	0

Total transportation costs with the VAM method amount to €102,152.0. The source code written in Notepad++ and generated in GNU Octave for finding the basic feasible solution using the VAM method is given in Appendix D. Algorithm in Appendix D was extended and in consequence allows for comparison of the number of iterations, the value of objective function and the degeneration of received solution, as presented in Table 8.

Table 8. Comparison of results obtained by using GNU Octave.

Final Comparison of Methods				
Method	NW	MKW	MK	VAM
The number of iterations	14	14	14	14
Total transportation cost (€)	165,109.0	119,478.0	114,888.0	102,152.0
Solution is degenerated	YES	YES	YES	YES

Abbreviation:
 NW—The Northwest Corner Method
 MK—The Least Cost in the Matrix Method
 MKW—The Row Minimum Method
 VAM—The Vogel's Approximation Method

When comparing the initial values of the basic solutions to the transportation problem in question, it should be noted that, depending on the method used, different objective function values were obtained. The calculations show that the highest transportation cost was estimated using the northwest corner method, while the least cost in the matrix method led to a lower value of transportation cost than the commonly used row minimum method. The lowest value of the objective function was obtained using the VAM method. In each of the presented methods, the necessary condition for degenerating a feasible solution was met. The lowest cost solution to the transportation problem within the supply network is presented in Figure 4.

Figure 4. Graphical interpretation of lowest cost solution.

The solution of transportation problem obtained by used methods and deviation from the lowest cost solution are summarized in Table 9. Solutions obtained by NW, MKW, and MK methods were not optimal, and therefore the potentials method was used.

Table 9. Objective function and deviation from lowest cost solution.

Method	Value (€)	Deviation from the Lowest Cost Solution (%)
NW	165,109.0	61.6
MKW	119,478.0	16.9
MK	114,888.0	12.4
VAM	102,152.0	0.0

4. Results

4.1. Optimization of the Basic Feasible Solution

Based on the input data set and the basic feasible solution obtained by the northwest corner method, the results table was first prepared so that the cells corresponding to supply and demand values remained empty. The transportation costs in the base cells were supplemented. It is assumed that the value of the potential is $Y_1 = 0$. The cost corresponding to this potential should be found and then the potential X_1 being the difference between the cost and Y_1 potential should be calculated. In the task, we obtained the following values $X_1 = 736 - Y_1 = 736$. The next cost in the column that corresponds to the X_1 should be found and the Y_2 should be determined. The Y_2 potential corresponding to the determined cost should be calculated as the difference between cost 1011 and potential X_1 . The procedure was repeated to determine the remaining potentials. The remaining cells should be filled in with the sums of potentials $X_t + Y_u$ where $t = 1, \dots, n$ and $u = 1, \dots, m$ keeping in mind that n is the number of recipients and m is the number of suppliers [55]. The results of the applied conversions are shown in Table 10.

Table 10. Summary of indirect costs and costs resulting from task content.

Indirect Costs						Transportation Costs						
X_1	X_2	X_3	X_4	X_5	X_6							
736	399	−236	−521	−936	−236							
736	399	−236	−521	−936	−236	$Y_1 = 0$	736	1400	1647	1504	1063	545
1011	674	39	−246	−661	39	$Y_2 = 275$	1011	674	197	1556	1478	1400
1128	791	156	−130	−545	156	$Y_3 = 392$	1349	791	156	1608	1530	1660
1854	1517	882	596	182	882	$Y_4 = 1118$	1271	267	882	596	648	908
1932	1595	960	674	259	960	$Y_5 = 1196$	1193	337	1037	674	622	700
2788	2451	1815	1530	1115	1815	$Y_6 = 2051$	545	648	895	1530	1115	674
2165	1828	1193	908	493	1193	$Y_7 = 1429$	1712	726	1219	156	493	1193
2347	2010	1375	1089	674	1375	$Y_8 = 1611$	1764	506	415	1089	1245	1375

In the next step, the values of optimization indicators should be determined, understood as the difference between indirect costs and costs resulting from the numerical data. They are listed in Table 11.

Table 11. Values of optimization indicators for solution obtained using northwest corner method.

$X_1 = 736$	$X_2 = 399$	$X_3 = -236$	$X_4 = -521$	$X_5 = -936$	$X_6 = -236$	
0	−1001	−1883	−2025	−2000	−781	$Y_1 = 0$
0	0	−158	−1802	−2140	−1362	$Y_2 = 275$
−220	0	0	−1738	−2075	−1504	$Y_3 = 392$
584	1250	0	0	−467	−26	$Y_4 = 1118$
739	1258	−78	0	−363	259	$Y_5 = 1196$
2243	1802	921	0	0	1141	$Y_6 = 2051$
454	1102	−26	752	0	0	$Y_7 = 1429$
584	1504	960	0	−571	0	$Y_8 = 1611$

There are positive numbers among the indicators, which means that the solution obtained is not optimal. The cycle design leads to an feasible solution at a lower cost. The first element of the positive cycle corresponds to the maximum optimization indicator. In a row containing an element of a positive cycle, the component that will have its equivalent in the column should be indicated. The procedure should be repeated until the cycle is closed. The lowest value from the components of the negative cycle must then be indicated and subtracted from all components of the negative cycle and added to all components of the positive cycle. The created cycle and the new feasible solution are shown in Table 12.

Table 12. Cycle design and new feasible solution.

Cycle						New Feasible Solution					
18	0	0	0	0	0	18	0	0	0	0	0
6(−)	23(+)	0	0	0	0	5	24	0	0	0	0
0	8(−)	7(+)	0	0	0	0	7	8	0	0	0
0	0	12(−)	22(+)	0	0	0	0	11	23	0	0
0	0	0	26	0	0	0	0	0	26	0	0
0(+)	0	0	1(−)	20	0	1	0	0	0	20	0
0	0	0	0	20	16	0	0	0	0	20	16
0	0	0	0	0	21	0	0	0	0	0	21

The cost of the current solution is €162,883.0, so it is slightly lower than the one obtained in the basic feasible solution, which means that the obtained solution is better. By repeating the above procedure 12 times, a new feasible solution was obtained. The feasible solution and the final values of the optimization indicators for the northwest corner method are presented in Table 13.

Table 13. New solution and optimization indicators.

New Feasible Solution						Optimization Indicators					
0	0	0	0	0	18	−1463	−6018	−9310	−4956	−2714	0
15	10	4	0	0	0	0	0	0	−2478	−1888	−1180
0	0	15	0	0	0	−1723	−720	0	−2903	−2313	−2549
0	0	0	13	21	0	−3068	−35	−5003	0	0	−826
0	0	0	0	19	7	−2832	−472	−5829	−472	0	0
9	0	0	0	0	12	0	−2006	−5298	−4484	−2360	0
0	0	0	36	0	0	−7080	−4130	−8543	0	−1298	−4130
0	21	0	0	0	0	−4189	0	−1758	−3009	−3717	−4307

All values of the optimization indicators are negative, so the received solution of €102,152.0 is optimal. The final transportation costs and the result of the potential method leading to an optimal solution are presented in Table 14.

Table 14. Comparison of methods and costs of solutions (€) obtained.

Method	NW	MKW	MK	VAM
Basic Feasible Solution	165,109.0	119,478.0	114,888.0	102,152.0
1st improvement of the solution	162,866.0	118,060.0	113,931.0	
2nd improvement of the solution	150,356.0	116,218.0	108,991.0	
3rd improvement of the solution	146,232.0	113,397.0	107,213.0	
4th improvement of the solution	138,616.0	106,665.0	105,503.0	
5th improvement of the solution	131,311.0	102,675.0	102,201.0	
6th improvement of the solution	124,333.0	102,201.0	102,152.0	
7th improvement of the solution	120,769.0	102,152.0		
8th improvement of the solution	118,600.0			
9th improvement of the solution	115,072.0			
10th improvement of the solution	112,349.0			
11th improvement of the solution	108,137.0			
12th improvement of the solution	102,152.0			

As can be seen from Table 14, the presented scheme of optimization of the obtained optimal solution was repeated:

- twelve times for the northwest corner method,
- seven times for the row minimum method,
- six times for the least cost in the matrix method,
- once for the VAM method.

Values of the transportation costs after subsequent steps of improvement are illustrated in Figures 5 and 6.

Figure 5. Next improvements of solutions.

Figure 6. Deviation from optimal solution.

4.2. Fuel Consumption and CO₂ Emissions

The evolution of the transport system has a significant impact on the socioeconomic development of the modern world. The concept of sustainable transport takes into account not just economic and social criteria, but also environmental ones. Despite the significant role of the transport system in the development of the economy, transport has a negative impact on the quality of life and health of people, as well as on the environment due to its significant contribution to greenhouse gas emissions. International cooperation cannot be limited only to the expansion of the transport network (although this is an important aspect), but must also take environmental preservation into account. Sustainable transport development and environmental preservation are linked to the formation of a green transport system. Under the European Green Deal Communication [56], the European Union (EU) member states have committed to reducing greenhouse gas emissions by at least 55% by 2030, compared to 1990 levels. As part of the development of the market for low- and zero-emission vehicles, the European Commission has adopted the following targets for reducing CO₂ emissions from newly manufactured passenger cars and delivery vehicles: a 55% reduction in CO₂ emissions from passenger cars and a 50% reduction in CO₂ emissions from delivery vehicles by 2030; zero CO₂ emissions from new passenger cars by 2035. For new trucks, the target is to reduce CO₂ emissions by an average of 15% from 2025 and 30% from 2030 when compared to 2019 levels. CO₂ emissions from trucks, buses and coaches currently account for 6% of total EU CO₂ emissions and 27% of total road transport CO₂ emissions [57]. It is therefore necessary to adopt an environmentally friendly transport policy and to create tools to support decision-making processes, depending on the criteria adopted. Table 15 illustrates the total distance traveled expressed in (km), depending on the successive solutions obtained by the potential method.

Table 15. Distance traveled (km) depending on solution obtained.

Method	NW	MKW	MK	VAM
Basic Feasible Solution	11,226.0	9218.0	9307.0	7030.0
1st solution	10,241.0	9036.0	8503.0	
2nd solution	9930.0	7778.0	7777.0	
3rd solution	9787.0	7778.0	7194.0	
4th solution	9102.0	6559.0	7090.0	
5th solution	8454.0	6714.0	6597.0	
6th solution	8312.0	6597.0	7030.0	
7th solution	8442.0	7030.0		
8th solution	8717.0			
9th solution	7498.0			
10th solution	7446.0			
11th solution	6875.0			
12th solution	7030.0			

The calculations show that the basic feasible solution determined by the northwest corner method required the longest route of 11,226.0 (km). The optimal solution in terms of transport cost determined by the northwest corner method was not optimal in terms of total distance traveled. In the case of the least cost in the matrix method, the shortest distance required to complete the transport task equal to 6597.0 (km) was obtained as the result of the fifth iteration of the potential method. Of all the methods considered, the shortest route equal to 6559.0 (km) corresponded to the fourth iteration of the potentials method applied to the basic feasible solution determined by the row minimum method. A graphical interpretation of the results obtained is shown in the Figure 7.

Figure 7. Distance traveled (km) depending on different solutions obtained using potentials method.

The values of fuel consumption and CO₂ emissions depending on the distance traveled are presented in Table 16. The Vehicle Energy Consumption Calculation Tool (VECTO version 3.3.9.2175) was used to calculate them [58].

Implementation of the transport plan in accordance with the basic solution determined by the northwest corner method is associated with the highest CO₂ emissions of 4939.4 (kg). From an environmental point of view, the optimal solution offering the lowest CO₂ emission of 2886.0 (kg) was obtained using the row minimum method as a result of applying the potentials method four times. The resulting solution leads to a reduction in total CO₂ emissions by as much as 2053.5 (kg). The results of the performed calculations are shown in Figure 8.

Figure 8. CO₂ emissions (kg) depending on solution obtained.

Table 16. Total fuel consumption and total CO₂ emissions as a function of distance traveled.

Method	Total Fuel Consumption (L)				Total CO ₂ Emission (kg)			
	NW	MKW	MK	VAM	NW	MKW	MK	VAM
Basic feasible solution	3143.3	2581.0	2606.6	1968.4	4939.4	4055.9	4095.1	3093.2
1st solution	2867.5	2530.1	2380.8		4506.0	3975.8	3741.3	
2nd solution	2780.4	2177.8	2177.6		4369.2	3422.3	3421.9	
3rd solution	2740.4	2177.8	2014.3		4306.3	3422.3	3165.4	
4th solution	2548.6	1836.5	1985.2		4004.9	2886.0	3119.6	
5th solution	2367.1	1879.9	1847.2		3719.8	2954.2	2902.7	
6th solution	2327.4	1847.2	1968.4		3657.3	2902.7	3093.2	
7th solution	2363.8	1968.4			3714.5	3093.2		
8th solution	2440.8				3835.5			
9th solution	2099.4				3299.1			
10th solution	2084.9				3276.2			
11th solution	1925.0				3025.0			
12th solution	1968.4				3093.2			

5. Discussion and Conclusions

The essential element of the logistic system is the subsystem of movement and transportation, whose main task is to ensure the timely movement of personnel and goods. The constant development of the automotive industry, the introduction of vehicles with increasing payloads and the ability to cross a variety of terrain, and the expansion of the road network indicate the growing importance of road transport. The dynamic change of the situation causes the distances of the supply routes to constantly change, which requires organizational effort. It is particularly important to be able to optimize the operations of road transport, especially when the time to make decisions is limited. There are many studies available in the literature concerning the optimization of the transportation problem depending on the established objective function, while taking into account the existing constraints.

The solutions used are aimed not only at determining the best method of solving the transportation problem, but also at developing an algorithm that leads to a reduction of calculation complexity by reducing the time needed for completing the calculations. The literature repeatedly addressed the possibilities of using the C++ programming language or

the Matlab environment to increase the efficiency of the transportation algorithm, whereas division of source codes in GNU Octave was not yet research subject. More and more tools are now available for using programming techniques or performing complex numerical calculations, including a free GNU Octave environment. Determining the optimal solution of the transportation problem using the GNU Octave software was not yet considered in any publication. Therefore, the purpose of this article was to present an innovative approach to solving the transportation problem aimed at minimizing the transportation costs, fuel consumption, and CO₂ emissions using programming in the GNU Octave.

Proposed solution using the GNU Octave has great practical and theoretical importance. The programming in GNU Octave saves a lot of time from complex and iterative calculations. The solutions used are aimed not only at determining the best method of solving the transportation problem, but also at decision support for individually defined criteria in other linear programming problems. The presented model was supported by a numerical example together with an interpretation of the results obtained. The paper focuses on determining the optimal method of product distribution within an assumed supply network using: the northwest corner, the least cost in the matrix, the row minimum and the Vogel Approximation Method (VAM) methods to calculate the minimal value of the objective function. A distribution network consisting of six suppliers and eight recipients of materials was considered. The transport was carried out with medium load trucks for which the cost ratio per unit of operation was €2.62.

On the basis of the calculations carried out, it was found that determining the basic feasible solution for each of the presented methods required 14 iterations. When comparing the initial basic solutions of the problem in question, the values of the objective function varied according to the method used. The analysis shows that the highest cost of transport was obtained using the northwest corner method, which is the simplest to calculate but the least accurate. In turn, the least cost in the matrix method provided a lower value of the objective function than the commonly used row minimum method. The lowest transportation cost was obtained by means of the Vogel approximation. For each of the methods used, the necessary condition for the degeneration of the calculated feasible solution was met, which made it possible to apply the potentials method. Determining the optimal solution based on the northwest corner method required as much as twelve repetitions of the scheme for calculating the optimization indicators. The negative values of the optimization indicators were obtained in the case of the row minimum method after seven repetitions of the scheme. The application of the least cost in the matrix method required six repetitions of the potential method.

The evolution of the transport system has a significant impact on the socioeconomic development of the world. However, it should be emphasized that despite economic growth, transport has a negative impact on the environment through its significant contribution to greenhouse gas emissions. Therefore, it is necessary to implement a proecological transport policy and to develop tools to support rational decision-making depending on the criterion adopted. We propose one such tool, provided in the form of attached source codes, only requiring the user to enter numerical data. Taking into account the ecological aspect, it was observed that the implementation of the transport task in accordance with the basic feasible solution obtained from the northwest corner method results in a total of 4939.4 (kg) of CO₂ being emitted into the atmosphere, while the application of the fourth iteration of the potentials method in relation to the row minimum method reduces CO₂ emissions to 2886.0 (kg), which amounts to 58.0%. The optimal solution obtained in this way reduces the total CO₂ emissions by as much as 2053.3 (kg).

The implementation of the proposed solution shortens the calculation algorithm and allows the user to determine the optimal distribution plan for material resources for individually determined criteria. The advantage of the software used is that the GNU Octave command window displays the individual steps leading to the basic feasible solution for each of the methods presented. In addition, an attempt by the user to enter negative numbers generates a message that the entered values must be changed. Moreover,

the source code for each of the methods checks whether the distribution network under consideration is a so-called balanced transportation problem. The disadvantage of the software used is that if the user inputs the data incorrectly, it is necessary to reinput all data again.

The use of IT tools allows effective management of the vehicle fleet and provides the possibility of organizing the workload of drivers to perform initial calculations of the costs at the planning stage of the transport operations. The increasing computing power of computers provides the ability to solve increasingly complex decision-making problems. Therefore, the right direction of further research will be to extend the function of the program with an algorithm based on the potentials method and to use the Gnuplot graphic tool to visualize the results in the form of a graph of the optimal distribution network.

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Appendix A

```

1  printf('SOLUTION OBTAINED BY THE NORTHWEST CORNER METHOD')
2  % OBJECTIVE FUNCTION
3  function cost=cost(T,c)
4  cost = 0;
5  m = columns(T);
6  n = rows(T);
7  for i=1:m
8  for k=1:n
9  cost = cost+T(k,i)*c(k,i); %sum of the unit costs of transport and volume of cargo from the suppliers
    to the recipients
10 endfor
11 endfor
12 endfunction
13 % DEGENERATION OF THE SOLUTION
14 function degeneration=degeneration(T)
15 degeneration=0;
16 m = columns(T);
17 n = rows(T);
18 for i=1:m
19 for k=1:n
20 if T(k,i)>0
21 degeneration=degeneration+1;
22 endif
23 endfor
24 endfor
25 endfunction
26 pt=input('Go to next step 0 (NO), 1 (YES)')
27 if pt==1
28 % NUMBER OF SUPPLIERS AND RECIPIENTS
29 m=input('Input the number of suppliers, m = ');
30 while m<=0 printf('number of suppliers must be > 0\n')
31 m=input('Input again the number of suppliers, m= ');
32 endwhile

```

```

33 n=input('Input the number of recipients , n = ');
34 while n<=0 printf('number of recipients must be > 0\n');
35 n=input('Input again the number of recipients , n= ');
36 endwhile
37 for k=1:n
38     for i=1:m
39         T(k,i)=-1;
40     endfor
41 endfor
42 % TRANSPORTATION MATRIX C BETWEEN SUPPLIERS AND RECIPIENTS
43 printf('Input the transportation matrix (c) between %g suppliers and %g recipients\n',m,n')
44 for k=1:n
45     for i=1:m
46         printf('Input transportation cost from supplier P%g to recipient S%g, c(%g,%g) = ',i,k,k,i)
47         c(k,i)=input('');
48         while c(k,i)<=0
49             printf('Transportation cost must be > 0, c(%g,%g)= ', k,i)
50             c(k,i)=input('');
51         endwhile
52     endfor
53 endfor
54 % SUPPLY AND DEMAND
55 for i=1:m printf('Input supply of P%g supplier , (%g) = ',i,i')
56     supply(1,i)=input('');
57     while supply(1,i)<=0 printf('Supply must be > 0, supply(%g) = ',i')
58         supply(1,i)=input('');
59     endwhile
60 endfor
61 for k=1:n printf('Input demand of S%g recipient , (%g) = ',k,k')
62     demand(k,1)=input('');
63     while demand(k,1)<=0 printf('Demand must be > 0, demand(%g) = ',k')
64         demand(k,1)=input('');
65     endwhile
66 endfor
67 % CHECKING IF TOTAL DEMAND IS EQUAL TO AGGREGATED SUPPLY
68 while or(sum(demand) < sum(supply), sum(demand)>sum(supply))
69     printf('Total demand must be equal to aggregated supply\n')
70     tn=input('Would you like to change supply? 0 (YES) or 1 (NO) ')
71     if tn==0
72         for i=1:m printf('Input supply of P%g supplier , supply(%g) = ',i,i')
73             supply(1,i)=input('');
74             while supply(1,i)<=0 printf('Supply must be > 0, supply(%g) = ',i')
75                 supply(1,i)=input('');
76             endwhile
77         endfor
78     else
79         if tn==1
80             for k=1:n printf('Input demand of S%g recipient , demand(%g) = ',k,k')
81                 demand(k,1)=input('');
82                 while demand(k,1)<=0 printf('Demand must be > 0, demand(%g) = ',k')
83                     demand(k,1)=input('');
84                 endwhile
85             endfor
86         else
87             tn=input('Would you like to change supply? 0 (YES) or 1 (NO) ')
88         end
89     endif
90 endwhile
91 % TRANSPORTATION COSTS MATRIX
92 printf('\n')
93 for i=1:m
94     if i<=m printf(' | %g\t',supply(1,i))
95     else
96         printf(' | ')
97     endif
98 endfor
99 printf('\n')
100 for i=1:m
101     if i<=m printf(' | P%g\t',i)

```

```

102 else
103     printf(' | ')
104 endif
105 endfor
106 printf('\n')
107 for k=1:n printf(' ')
108 for i=1:m printf(' | %g\t',c(k,i))
109 endfor
110 printf(' | S%g | %g\t\n',k,demand(k,1))
111 endfor
112 printf('\n')
113
114 TNW = T;
115 printf('Table\n ')
116 for i=1:m
117 if i<=m
118 printf(' | %g\t',supply(1,i))
119 else
120 printf(' ')
121 endif
122 endfor
123 printf('\n\n')
124 for k=1:n
125 printf(' ')
126 for i=1:m
127 if TNW(k,i)<0
128 printf(' | x\t',TNW(k,i))
129 else
130 printf(' | %g\t',TNW(k,i))
131 endif
132 endfor
133 printf(' | %g\t\n',demand(k,1))
134 endfor
135 printf('\n')
136 printf('THE NORTHWEST CORNER MEIHOD \n')
137 supplyNW = supply; demandNW=demand;
138 stepNW = 1; i=1;k=1;l=0; w=1;
139 input(' ');
140 while i<=m
141 while k<=n
142 printf('step %g\n',stepNW)
143 TNW(k,i)=min(supplyNW(i),demandNW(k));
144 supplyNW(i)=supplyNW(i)-TNW(k,i);
145 demandNW(k)=demandNW(k)-TNW(k,i);
146 stepNW = stepNW+1;
147 if and(demandNW(k)==0,k<=n)
148 for j=i+1:m
149 if TNW(k,j)==-1
150 TNW(k,j)=0;
151 endif
152 endfor
153 endif
154 k=k+1;
155 if and(supplyNW(i)==0,i<=m)
156 w=k;
157 k=n+1;
158 else
159 printf(' ')
160 for o=1:n
161 if o<=m
162 printf(' | %g\t',supplyNW(1,o))
163 else
164 printf(' ')
165 endif
166 endfor
167 printf('\n\n')
168 for o=1:n
169 printf(' ')
170 for u=1:m

```

```

171 if TNW(o,u)<0
172 printf(' | x\t',TNW(o,u))
173 else
174 printf(' | %g\t',TNW(o,u))
175 endif
176 endfor
177 printf(' | %g\t\n',demandNW(o,1))
178 endfor
179 printf('\n')
180 input(' ');
181 endif
182 endwhile
183 for j=w:n
184 if TNW(j,i)==-1
185 TNW(j,i)=0;
186 endif
187 endfor
188 printf(' ')
189 for o=1:m
190 if o<=m
191 printf(' | %g\t',supplyNW(1,o))
192 else
193 printf(' ')
194 endif
195 endfor
196 printf('\n\n')
197 for o=1:n
198 printf(' ')
199 for u=1:m
200 if TNW(o,u)<0
201 printf(' | x\t',TNW(o,u))
202 else
203 printf(' | %g\t',TNW(o,u))
204 endif
205 endfor
206 printf(' | %g\t\n',demandNW(o,1))
207 endfor
208 printf('\n')
209 if min(min(TNW))
210 input(' ');
211 endif
212 i=i+1;
213 l=1;
214 for k=1:n
215 if demandNW(k)==0
216 l=l+1;
217 end
218 endfor
219 k=l;
220 endwhile
221 % RESULTS
222 costNW = cost(TNW,c);
223 printf('Total transportation costs for the northwest corner method amount to %g\n',costNW)
224 printf('The number of basic variables - %g \n',degeneration(TNW))
225 if degeneration(TNW)==n+m-1
226 printf('Solution obtained by the northwest corner method is degenerated\n')
227 degenerationTNW = 'YES';
228 else
229 printf('Solution obtained by the northwest corner method is non-degenerated, %g<%g\n',degeneration
(TNW),n+m-1)
230 degenerationTNW = 'NO';
231 endif
232 printf('\n')

```

Appendix B

```

1 % Attention: copy lines from 1 to 112 from the Appendix A
2 TMKW = T;
3 printf( 'Table\n ' )
4 for i=1:m
5     if i <=m
6         printf( ' | %g\t ', supply(1,i) )
7     else
8         printf( ' ' )
9     endif
10 endfor
11 printf( '\n\n' )
12 for k=1:n
13     printf( ' ' )
14     for i=1:m
15         if TMKW(k,i) < 0
16             printf( ' | x\t ', TMKW(k,i) )
17         else
18             printf( ' | %g\t ', TMKW(k,i) )
19         endif
20     endfor
21     printf( ' | %g\t\n ', demand(k,1) )
22 endfor
23 printf( '\n' )
24 printf( 'THE ROW MINIMUM METHOD\n' )
25 dMKW = c; TMKW = T; supplyMKW = supply; demandMKW = demand;
26 cmin=0; cmax=0; imin=0; kmin=0; stepMKW=1;
27 cmax=max(max(dMKW));
28 input( ' ' );
29 while min(min(TMKW)) < 0
30     printf( 'step %g\n ', stepMKW )
31     stepMKW = stepMKW + 1;
32     krows=1;
33     while krows <= n
34         if min(TMKW(krows,:)) < 0
35             k = krows;
36             krows=n;
37         endif
38         krows=krows+1;
39     endwhile
40     i=1;
41     cmin=min(dMKW(k,:));
42     while or(k <= n, k==1)
43         while or(i <= m, i==1)
44             if i <= m
45                 if dMKW(k,i) == cmin
46                     imin = i;
47                     kmin = k;
48                     i=m+1;
49                     k=n+1;
50                 else
51                     i=i+1;
52                 endif
53             endif
54         endwhile
55         if k <= n
56             k=k+1;
57             i=1;
58         endif
59     endwhile
60     cMKWmin(kmin, imin)=cmin;
61     TMKW(kmin, imin)=min(supplyMKW(1, imin), demandMKW(kmin, 1));
62     supplyMKW(imin)=supplyMKW(imin)-TMKW(kmin, imin);
63     demandMKW(kmin)=demandMKW(kmin)-TMKW(kmin, imin);
64     if demandMKW(kmin)==0
65         for i=1:m
66             if TMKW(kmin, i)==-1
67                 TMKW(kmin, i)=0;

```

```

68     endif
69     dMKW(kmin,i)=cmax+1;
70     endfor
71     endif
72     if supplyMKW(imin)==0
73     for k=1:n
74     if TMKW(k,imin)==-1
75     TMKW(k,imin)=0;
76     endif
77     dMKW(k,imin)=cmax+1;
78     endfor
79     endif
80     printf(' ')
81     for o=1:m
82     if o<=m
83     printf(' | %g\t',supplyMKW(1,o))
84     else
85     printf(' ')
86     endif
87     endfor
88     printf('\n\n')
89     for o=1:n
90     printf(' ')
91     for u=1:m
92     if TMKW(o,u)<0
93     printf(' | x\t',TMKW(o,u))
94     else
95     printf(' | %g\t',TMKW(o,u))
96     endif
97     endfor
98     printf(' | %g\t\n',demandMKW(o,1))
99     endfor
100    printf('\n')
101    if min(min(TMKW))
102    input(' ');
103    endif
104    endwhile
105    % RESULTS
106    costMKW = cost(TMKW,c);
107    printf('Total transportation cost for the row minimum method amount to %g\n',costMKW)
108    printf('The number of basic variables - %g \n',degeneration(TMKW))
109    if degeneration(TMKW)==n+m-1
110    printf('Solution obtained by the row minimum method is degenerated \n')
111    degenerationTMKW = 'YES';
112    else
113    printf('Solution obtained by the row minimum method is non-degenerated, %g<%g\n',degeneration(TMKW),
        n+m-1)
114    degenerationTMKW = 'NO';
115    endif
116    printf('\n')

```

Appendix C

```

1    % Attention: copy lines from 1 to 112 from the Appendix A
2    TMK = T;
3    printf('Table\n ')
4    for i=1:m
5    if i<=m
6    printf(' | %g\t',supply(1,i))
7    else
8    printf(' ')
9    endif
10   endfor
11   printf('\n\n')
12   for k=1:n
13   printf(' ')
14   for i=1:m
15   if TMK(k,i)<0

```



```

16     printf( ' | x\t ',TMK(k,i))
17     else
18     printf( ' | %g\t ',TMK(k,i))
19     endif
20     endfor
21     printf( ' |      %g\t\n ',demand(k,1))
22     endfor
23     printf( '\n')
24     printf( 'THE LEAST COST IN MATRIX\n')
25     cMK = c; TMK = T; supplyMK = supply; demandMK = demand;
26     cmin=0; cmax=0; imin=0; kmin=0; stepMK=1;
27     cmax=max(max(cMK));
28     input( ' ');
29     while min(min(TMK))<0
30     printf( 'step %g\n',stepMK)
31     stepMK = stepMK + 1;
32     cmin=min(min(cMK));
33     k=1;
34     i=1;
35     while or(k <= n,k==1)
36     while or(i <= m,i==1)
37     if i<=m
38     if cMK(k,i) == cmin
39     imin = i;
40     kmin = k;
41     i=m+1;
42     k=n+1;
43     else
44     i=i+1;
45     endif
46     endif
47     endwhile
48     if k<=n
49     k=k+1;
50     i=1;
51     endif
52     endwhile
53     cMKmin(kmin,imin)=cmin;
54     TMK(kmin,imin)=min(supplyMK(1,imin),demandMK(kmin,1));
55     supplyMK(imin)=supplyMK(imin)-TMK(kmin,imin);
56     demandMK(kmin)=demandMK(kmin)-TMK(kmin,imin);
57     if demandMK(kmin)==0
58     for i=1:m
59     if TMK(kmin,i)==-1
60     TMK(kmin,i)=0;
61     endif
62     cMK(kmin,i)=cmax+1;
63     endfor
64     endif
65     if supplyMK(imin)==0
66     for k=1:n
67     if TMK(k,imin)==-1
68     TMK(k,imin)=0;
69     endif
70     cMK(k,imin)=cmax+1;
71     endfor
72     endif
73     printf( ' ')
74     for o=1:m
75     if o<=m
76     printf( ' | %g\t ',supplyMK(1,o))
77     else
78     printf( ' ')
79     endif
80     endfor
81     printf( '\n\n')
82     for o=1:n
83     printf( ' ')
84     for u=1:m

```

```

85     if TMK(o,u)<0
86         printf(' | x\t',TMK(o,u))
87     else
88         printf(' | %g\t',TMK(o,u))
89     endif
90 endfor
91 printf(' |      %g\t\n',demandMK(o,1))
92 endfor
93 printf('\n')
94 if min(min(TMK))
95     input(' ');
96 endif
97 endwhile
98 % RESULTS
99 costMK = cost(TMK,c);
100 printf('Total transportation cost for the least cost in matrix amount to %g\n',costMK)
101 printf('The number of basic variables - %g \n',degeneration(TMK))
102 if degeneration(TMK)==n+m-1
103     printf('Solution obtained by the least cost in matrix is degenerated \n')
104     degenerationTMK = 'YES';
105 else
106     printf('Solution obtained by the least cost in matrix is non-degenerated , %g<%g\n',degeneration(TMK) ,
107           n+m-1)
108     degenerationTMK = 'NO';
109 endif
110 printf('\n')

```

Appendix D

```

1 % Attention: copy lines from 1 to 112 from the Appendix A
2 TVAM = T;
3 printf('Table\n ')
4 for i=1:m
5     if i<=n
6         printf(' | %g\t',supply(1,i))
7     else
8         printf(' ')
9     endif
10 endfor
11 printf('\n\n')
12 for k=1:n
13     printf(' ')
14     for i=1:m
15         if TVAM(k,i)<0
16             printf(' | x\t',TVAM(k,i))
17         else
18             printf(' | %g\t',TVAM(k,i))
19         endif
20     endfor
21     printf(' |      %g\t\n',demand(k,1))
22 endfor
23 printf('\n')
24 printf('VOGEL"S APPROXIMATION METHOD\n')
25 cVAM = c; TVAM = T; supplyVAM = supply; demandVAM = demand;
26 cmin=0; cmax=0; imin=0; kmin=0; stepVAM=1;
27 cmax=max(max(cVAM));
28 input(' ');
29 while min(min(TVAM))<0
30     printf('step %g\n',stepVAM)
31     stepVAM = stepVAM + 1;
32     for i=1:m
33         cVAMsort = sort(cVAM(:,i));
34         cVAMc(1,i) = cVAMsort(2)-cVAMsort(1);
35         cVAMcmin(1,i)=cVAMsort(1);
36     endfor
37     for k=1:n
38         cVAMsort = sort(cVAM(k,:));
39         cVAMr(k,1) = cVAMsort(2)-cVAMsort(1);

```

```

40 cVAMrmin(k,1)=cVAMsort(1);
41 endfor
42 cVAMmax=max(max(cVAMc),max(cVAMr));
43 for i=1:m
44     if cVAMc(1,i)==cVAMmax
45         else
46         cVAMcmin(1,i)=cmax+1;
47     endif
48 endfor
49 for k=1:n
50     if cVAMr(k,1)==cVAMmax
51         else
52         cVAMrmin(k,1)=cmax+1;
53     endif
54 endfor
55 cVAMmin = min(min(cVAMcmin),min(cVAMrmin));
56 kmin = 0; k=1; i=1; imin=0;
57 while i<=m
58     if cVAMcmin(1,i) == cVAMmin
59         imin = i; i=m+1;
60     endif
61     i=i+1;
62 endwhile
63 if imin == 0
64     while k<=n
65         if cVAMrmin(k,1)==cVAMmin
66             kmin = k; k=n+1;
67         endif
68         k=k+1;
69     endwhile
70     endif
71     i=1;k=1;
72     if kmin >0
73         while i<=m
74             if cVAM(kmin,i)==cVAMmin
75                 imin = i; i=m+1;
76             endif
77             i=i+1;
78         endwhile
79         else
80             if imin>0
81                 while k<=n
82                     if cVAM(k,imin)==cVAMmin
83                         kmin = k; k=n+1;
84                     endif
85                     k=k+1;
86                 endwhile
87             endif
88             endif
89             TVAM(kmin,imin)=min(supplyVAM(1,imin),demandVAM(kmin,1));
90             supplyVAM(imin)=supplyVAM(imin)-TVAM(kmin,imin);
91             demandVAM(kmin)=demandVAM(kmin)-TVAM(kmin,imin);
92             if supplyVAM(imin)==0
93                 for k=1:n
94                     if TVAM(k,imin)==-1
95                         TVAM(k,imin) = 0;
96                     endif
97                     cVAM(k,imin)=cmax*2;
98                 endfor
99             endif
100             if demandVAM(kmin)==0
101                 for i=1:m
102                     if TVAM(kmin,i)==-1
103                         TVAM(kmin,i) = 0;
104                     endif
105                     cVAM(kmin,i)=cmax*2;
106                 endfor
107             endif
108             printf(' ')

```

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