



# Article Heat Transfer in Flow Past Two Cylinders in Tandem and Enhancement with a Slit

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**Abstract:** This study aimed to determine the flow structures and heat transfer for flow past a tandem cylinder array and the effect of a slit on the enhancement of heat transfer. Different distances between cylinders and inclination angles of the slit were simulated to determine the effects on the flow pattern and heat transfer. Overall, the Nusselt number of the array is increased by 6–15% with applying a slit on a cylinder. However, in some special conditions, the slit induces two kinds of flow pattern transforms which are Suppression and Revival. The suppression mode inhibits the vortex shedding and reduces the heat transfer. In contrast, the revival mode triggers the vortex shedding and increases heat transfer.

Keywords: tandem; cylinder; slit; enhanced heat transfer

#### 1. Introduction

Flow past a group of circular cylinders in different arrangements plays an important role in various engineering applications and has drawn attention from many researchers for decades. Among these arrangements, two-cylinder configurations profoundly influence the designs of the array of heat exchanger tubes and the developing categories: micro pin-fins heat sink and micro heat exchangers [1–3]. Flow over circular cylinders provides different fundamental fluid dynamic phenomena, such as boundary-layer separation, shear-layer development, and vortex dynamics. A tandem arrangement represents the most general form of a pair of cylinders in a fluid flow. However, most of the studies focus on hydrodynamics such as flow field, drag, lift and vortex shedding frequency, etc. [4–7]. For the design of the pin-fins heat sink, it is crucial to understand the mechanism between flow pattern and heat transfer. However, the studies simultaneously involve the flow structure and heat transfer are rare.

An earlier study of the heat transfer problem of two tandem cylinders was reported by Buyruk [8]. He numerically studied the effect of the space ratio on the heat transfer for two isothermal cylinders in tandem with the flow at Re = 400 and Pr = 0.7. His results show that the upstream cylinder is not necessarily affected if the space ratio  $L/D \ge 3$ . However, as L/D < 3, the Nusselt number on the rear surface of the upstream cylinder decreases as the space ratio decreases due to the flow blockage by the downstream cylinder. He also simulated the tube bank, which is composed of three cylinders in a staggered arrangement or four cylinders in a two-line arrangement for flow at Re = 80, 120, and 200. A larger Reynolds number causes the impact point to move upstream of the downstream cylinder for inline arrangement. For three staggered arrangements, the local heat transfer coefficient of the downstream cylinder front region is larger than the upstream cylinder front region due to the effect of flow acceleration.

Harimi and Saghafian [9] used a finite volume method with an overset grid to study the flow and heat transfer for two and three equal size cylinders in a tandem array. The simulations were performed for the Prandtl numbers of 0.7 and 7 at the Reynolds numbers of 100 and 200 for flow past the isothermal cylinders. The spacing ratio L/D ranged



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/). from 2 to 10. Two correlations were found for the calculation of mean Nusselt number for upstream cylinder and downstream cylinder, respectively, in terms of space ratio and Reynolds number and Prandtl number.

Mahir and Altac [10] used ANSYS FLUENT software (Ansys Inc., Canonsburg, PA, USA) to investigate the flow past two cylinders in a tandem array with the same surface temperature at Reynolds numbers Re = 100 and 200. This study determines the variation in the flow structure and the Nusselt number on the surface for cylinders that are separated by different distances (L/D = 2-10). The results show that the maximum Nusselt number in the upstream cylinder occurs at the stagnation point. Along the surface from the stagnation point to the separation point, the Nusselt number decreases from a maximum to a minimum because the thickness of the boundary layer gradually grows. Behind the separation point, the Nusselt number recovers slightly because recirculation behind the cylinder enhances the heat transfer. This phenomenon is similar to heat transfer for a single cylinder. When  $L/D \leq 3$ , most of the fluid that separates from the upstream cylinder reattaches to the downstream cylinder. This is known as shear layer reattachment. The shear flow detaches from the surface of the downstream cylinder again after reattachment. Only a small amount of fluid flows into the region between the upstream and downstream cylinders, so the Nusselt number (Nu) at the front stagnation point of the downstream cylinder is low. The rear stagnation point for the downstream cylinder still immerses the wake of the flow that detaches from the upstream cylinder, so the velocity and mass of fluid at this point are almost zero, which results in a low local Nusselt number.

The maximum value for *Nu* occurs at about 80° from the front stagnation point along the upper or the lower surface of the downstream cylinder because the boundary layer is thinnest at these points. Therefore, the distribution of the heat transfer coefficient for the downstream cylinder is different from that for the upstream cylinder. When  $L/D \ge 4$ , both cylinders exhibit vortex shedding. The distribution of the Nusselt number for the upstream cylinder is similar to that for a single cylinder. However, for the downstream cylinder, the maximum value for the Nusselt number occurs at the front stagnation point due to the impingement of the shedding vortex from the upstream cylinder. The minimum values still occur at the separation points. The points at which the maximum and minimum values occur move periodically, depending on the motion of vortex shedding of the upstream cylinder.

Zhou and Xi [11] solved the heat transfer problems for staggered cylindrical arrays by using a finite difference method with the SIMPLE (Patankar and Spalding, 1972) scheme. The study determined the effects of flow interaction from fluid past an array of staggered cylinders. The results show that the distance between the staggered cylinders is the key factor that affects heat transfer. When the space between rows is small (about 2–3 times the cylindrical diameter), there is a blockage effect. When the space between the upstream and downstream cylinders is increased. This blocking effect and the instability of the flow enhance the heat transfer, so the Nusselt number for the downstream cylinder is always greater than that for the upstream cylinder. The study shows that the blocking effect, the flow instability effect between the cylinders, the vortex shedding oscillation, the flow acceleration effect between the cylinders, the effect of recovery of velocity, and the temperature affect heat transfer for a cylindrical array.

For the enhancement of heat transfer in a cylinder, Tsutsui and Igarashi [12] employed a small rod in front of a cylinder to study the optimal enhanced heat transfer for the cylinder. The Reynolds number for these experiments ranged from  $1.5 \times 10^4$  to  $6.2 \times 10^4$ . The diameter ratio between the cylinder and the rod, d/D, ranged from 0.025 to 0.3 and the distance ratio between the circular cylinder and the rod, L/D, varied between 1.0 and 3.0. Two flow patterns were found. For flow pattern A, the fluid flowed past the rod with vortex shedding. For flow pattern B, flow separated from the rod and reattached to the cylinder. The results showed that the heat transfer of the front surface of cylinder is increased for both flow patterns. However, pattern B enhanced the heat transfer more effectively than pattern A for all Reynolds numbers. The optimal heat transfer enhancement was found to be at L/D = 1.25 and d/D = 0.25. The resulting overall Nusselt number value at  $6.2 \times 10^4$  is over 40% higher than that of the single cylinder.

Ma [13] numerically study the slit effect on the enhancement of heat transfer for flow past a cylindrical tube bank, for which each cylindrical tube contains a horizontal slit. The results show that the total heat rate is increased by 25–53% for flows at Re = 700 to 6000, with Pr = 7.

Al-Damook et al. [14] experimentally and numerically studied the heat transfer enhancement of a heat sink with circular holes on cylindrical pin fins at Re = 3250 to 6580. In this study, a heat transfer coefficient is based on the projected area in the flow direction was proposed. The results show that the average Nusselt numbers for this kind of perforated fin based on the total wetted area or the projected area both are greater than those for a solid fin. Wu et al. [15] conducted the simulations of airflow at 3500 < Re < 6500 through cylindrical pin fins with the circular perforations with constant heat flux. The results show that with the optimal perforation diameter, 1 mm and perforation spaces 2.75 mm, the circular perforated pin fins have an 8% greater average Nusselt number than solid pins. Their results also reconfirmed the points of Al-Damook et al. [14].

Hsu [16] applied a slit on a cylinder to study the feasibility of the enhancement of heat transfer. Different inclination angles of the slit ranging from  $0^{\circ}$  to  $90^{\circ}$  were simulated to study the effect of the slit on heat transfer of a cylinder. The results show the slit can increase the frequency of vortex shedding and 12.6% of the Nusselt number for flow at Re = 500. The maximum increase in heat transfer rate exists as the inclination angle of the slit is close to a critical angle that divides the flow pattern into an injection mode and a blowing/suction mode [17].

The rapid development of new high-performance chips drives the demand for effective cooling methods for high-power chips or electronic devices. Micro pin-fins heat sinks due to their high heat transfer effectiveness, compact size, are one of the unique devices for cooling microelectronic chips.

John et al. [1] studied the overall performance of two different micro pin-fin heat sinks, one with square-shaped pin-fins and the other with circular pin-fins as flow at Re = 50-500. The effects of pitch distance in axial and transverse directions, aspect ratio of the pin-fin, hydraulic diameters of the pin-fin on thermal resistance and pressure drop were investigated in this study. The results show that as Re < 300, the performance of heat sinks with circular pin-fins is better than the heat sinks with square pin-fins and vice versa at Re > 300.

Seyf and Feizbakhshi [2] numerically investigated flow and heat transfer behavior in the micro pin-fin heat sinks where DI-water (deionized water) with nanoparticles was used as the coolant fluid. The flow past the staggered cylindrical pin-fin array at Re < 100. Shafeie et al. [3] conducted a numerical study of laminar forced convection in heat sinks with micro pin-fin structure as flow at Re = 200. The cylindrical pin fins in different staggered or oblique array which were employed in the microchannel or micro heat sink. The result shows the microchannel with fabricated pin-fins performed slightly better than the micro heat sink with pin-fins at the same pumping powers.

The Reynolds number of the flow regime in the micro-pin-fin heat sinks or micro heat exchangers usually is small. Hence, it is difficult with an experimental approach to obtain the detailed flow structure or local heat transfer around pins. In this research, the spectral element method is used to determine the distance ratio effect on the flow structure and heat transfer of a two-cylinder tandem array with the flow at Re = 100 to 200. Secondly, the slit at different inclinations is applied to a cylinder of the array to study the effectiveness of enhancement for heat transfer.

## 2. Numerical Method

A direct numerical technique, the spectral element method, is used to simulate flow past a tandem array cylinders system by solving the incompressible Navier–Stokes equations as follows:

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u}$$
(1)

$$\nabla \cdot \vec{u} = 0 \tag{2}$$

where  $\overline{u}$  is the velocity vector, p is the pressure,  $\rho$  is the density and  $\nu$  is the kinematic viscosity. A time splitting scheme [18] is used to treat the nonlinear convective terms explicitly while the pressure and velocity diffusion terms may be solved implicitly as Equations (3)–(5).

$$\frac{\vec{u} - \vec{u}^n}{\Delta t} = \sum_{r=0}^2 \beta_r \left( -\vec{u} \cdot \nabla \vec{u} \right)^{n-r}$$
(3)

$$\frac{\hat{\vec{u}} - \hat{\vec{u}}}{\Delta t} = -\frac{1}{\rho} \nabla p^{n+1} \tag{4}$$

$$\frac{\vec{u}^{n+1} - \hat{\vec{u}}}{\Delta t} = \nu \cdot \nabla^2 \vec{u}^{n+1}$$
(5)

where the intermediate time step values for velocity,  $\hat{u}$  and  $\hat{u}$ , and  $\wedge$  and  $\hat{\wedge}$ , are between the n<sub>th</sub> and n + 1<sub>st</sub> time steps. The nonlinear convective term is treated explicitly by using a third order Adam-Bashforth scheme with coefficients,  $\beta_r$  as Equation (3).

The pressure term was treated separately as an elliptical problem by taking the divergence on both sides of Equation (4). Due to the divergence free condition, the surface term is zero. Hence,

$$\nabla^2 p^{n+1} = \frac{\rho}{\Delta t} \nabla \cdot \overset{\widehat{}}{u} \tag{6}$$

Let

$$\phi_h = \overline{u}^{n+1} \tag{7}$$

$$\lambda^2 = \frac{1}{\nu \cdot \Delta t} \tag{8}$$

$$g = -\frac{u}{v \cdot \Delta t} \tag{9}$$

Hence, the diffusion terms are modeled as general Helmholtz form as Equation (10). The pressure term can be treated as the same manner.

ŝ

$$(\nabla^2 - \lambda^2)\phi = g \text{ on } \Omega \tag{10}$$

The variational form of Equation (10) is discretized by the spectral element method. The solution domain is broken up into macro elements as shown in Figure 1, u and p are substituted by discrete approximations as follows

$$\phi_h^k = \sum_{p=0}^N \sum_{q=0}^M \phi_{pq}^k h_p(r) h_q(s)$$
(11)

where the *h* functions are the Lagrangian interpolants as Equtaion (12).  $L_N$  is a Legendre polynomial.  $\xi_i$  are the collocation points of Lagrangian interpolation.

$$h_p(r) = -\frac{1}{N(N+1)L_N(\xi_i)} \frac{(1-r^2)L'_N(r)}{r-\xi_i}$$
(12)



**Figure 1.** The closer view of the elements and grids with order of the Legendre polynomial, N = 11, of a two-cylinder tandem array with space ratio, L/D = 3, whose upstream cylinder has a slit at  $\alpha = 30^{\circ}$  and whose downstream cylinder is a regular one.

Appling Gauss–Lobatto quadrature on the integrals of variational form and summing contributions from adjacent elements, the global matrix equation is obtained as follows:

$$\left(A - \lambda^2 B\right)\phi = Bg \tag{13}$$

where *A* is the discrete Laplacian operator and *B* is the mass matrix. The matrix equation is solved using a preconditioned conjugate gradient.

The energy equation is

$$\rho C_p \left( \frac{dT}{dt} + \vec{u} \cdot \nabla T \right) = k \nabla \cdot \nabla T + q \tag{14}$$

where the density,  $\rho$ , is assumed to be a constant. A time-splitting scheme was also used to separate diffusion terms from convective terms. The convective terms were treated explicitly and the diffusion terms were solved implicitly as follows:

$$\frac{\hat{T} - T^n}{\Delta t} = \frac{1}{\rho C_p} \sum_{r=0}^2 \beta_r \left( -\vec{u} \cdot \nabla T \right)^{n-r}$$
(15)

$$\frac{T^{n+1} - \hat{T}}{\Delta t} - \frac{q}{\rho C_p} = \frac{k}{\rho C_p} \cdot \nabla^2 T^{n+1}$$
(16)

The diffusion term of the energy equation could also be treated as a Helmholtz problem. The further discretized process is similar to those in the momentum equation shown above.

The following simulations were implemented by using an in-house code written in Fortran and two desktop computers which were equipped with Intel Core i5-4570 CPU @ 3.20 GHz.

# 2.1. Nusselt Number

The local and mean Nusselt numbers on the surface of the cylinder were calculated using Equations (17)–(20), as shown below:

$$-k\frac{\partial T}{\partial n} = h_{\theta}(T_{w} - T_{\infty}) \tag{17}$$

$$Nu_{\theta} = \frac{h_{\theta}D}{k} \tag{18}$$

$$Nu_{\theta} = -\frac{\partial T}{\partial n} \frac{D}{(T_w - T_{\infty})}$$
(19)

$$\overline{Nu} = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} Nu_{\theta} d\theta \tag{20}$$

## 2.2. Boundary Conditions for Heat Transfer

The computational domain extends to a distance of 20 diameter of the cylinder (D) from inlet to the upstream cylinder center and a distance of 50 D downstream is shown in Figure 2a. The distances from the upper and lower boundaries to the center of the cylinder are both 20 D. The Dirichlet boundary conditions ( $u = U_0$ , v = 0,  $T = T_\infty$ ) which are uniform flow with constant temperature  $T_\infty = 20$  °C were used for the inflow and the upper and lower boundaries. The Neumann outflow condition was used on the downstream boundary. Upon the surfaces of the cylinder and slit were imposed the boundary conditions (u = 0, v = 0,  $T = T_s$ ,  $T_s = 100$  °C), as shown in Figure 2b, where  $\alpha$  is the angle of inclination of slit; S is the width of slit.



Figure 2. The computational domain and boundary conditions: (a) domain and (b) model of cylinder with a slit.

## 3. Validation

### 3.1. Grid Independence

The macro elements as shown in Figure 1 are called spectral elements. Each element contains  $N_x \times N_y$  grids.  $N_x$  or  $N_y$  is the order of the Legendre polynomial which composes the shape function or the Lagrangian interpolant in each dimension. For simplicity, the orders of the shape function in both dimensions are kept the same. Hence, if a two-dimensional domain contains k spectral elements, the total number of grids in the domain can be simply estimated as  $kN^2$ .

For testing the grid independence, the order of the Legendre polynomial (*N*) was serially increased with a specific number of elements, k = 324, to determine the convergence of the Nusselt number on the surface of a regular two-cylinder tandem array with a distance ratio L/D = 5 in a flow of Re = 200. Figure 3 shows that the distribution of Nusselt numbers converges when  $N \ge 11$ . Hence, the following simulation for flow past a tandem array at Re = 100 to 200 used the order of the Legendre polynomial, N = 11, and the number of elements, k = 308 to 368, dependent on the distance between cylinders, with the same computational domain as in Figure 2a.

₽ 10



120 150 180 210 240 270 300 330 360 240 270 300 330 360 θ θ (b) (a)

**Figure 3.** Convergence test with different orders of the Legendre polynomial (N), the Nusselt number on the surface of a regular two-cylinder tandem array with distance ratio, L/D = 5, Re = 200: (**a**) upstream cylinder; (**b**) downstream cylinder.

### 3.2. Validation

The present results for two regular cylinders in tandem at different distances were compared with those of several previous studies. Figure 4 shows that the drag coefficients in the present results agree with most of the references. Some deviation may have been caused by the size of the computational domain, as in the investigation by Posdziech and Grundmann [19], whose study indicates that the hydrodynamic force coefficients of flow past a single cylinder are strongly dependent on the size of the computational domain, especially the inflow length and lengths of upper and lower boundary. The suggested size of domain for the two-cylinder array was not found in the previous studies. Hence, the validation was conducted by comparing it with the limited existing references [4,9,10,20–23], in which the domain sizes are divergent. Among those studies, few involve heat transfer. Hence, the domain size chosen in this study was extended to be larger than that in most of the references as the capacity of our computing facility can do.



**Figure 4.** The drag coefficients for a two-cylinder tandem array with different distance ratios,  $Cd_1$  is the drag coefficient for upstream cylinder,  $Cd_2$  is the drag coefficient for downstream cylinder: (**a**) Re = 100; (**b**) Re = 200.

Tables 1 and 2 show the drag, lift coefficients, and Nusselt numbers for each cylinder in a tandem array with L/D = 2 to 10 and flow of Re = 100 and Re = 200, respectively. The corresponding domain sizes of references are listed in Table 3. The domain size and solution of Han [23] are the closest to the present study. However, there are no data available for L/D > 3. Hence, the comparison of the present results and the references show that the present numerical method and domain size provide a sufficiently accurate solution for the study of the problems of flow and heat transfer for flows at Re = 100 and 200 with Pr = 0.7.

Authors	L/D	Cd <sub>1</sub>	Cl <sub>1</sub>	Cd <sub>2</sub>	Cl <sub>2</sub>	St	Nu <sub>1</sub>	Nu <sub>2</sub>
Present	2	1.160	0	-0.091	0	0	4.892	2.145
Sharman et al. [4]	2	1.169		-0.090		0.121		
Mussa et al. [21]	2	1.178		-0.077		0.123		
Harimi and Saghafian [9]	2	1.173		-0.065			4.688	2.029
Mahir and Altac [10]	2	1.225	±0.0075	$\pm 0.00012$	$\pm 0.0258$		4.74	2.03
Present	3	1.130	0	-0.049	0	0	4.952	2.369
Sharman et al.	3	1.111		-0.027		0.109		
Koda and Lien [20]	3	1.171		0.002		0.106		
Mussa et al.	3	1.171		0.074		0.114		
Harimi and Saghafian	3	1.160		0.0018		0.1111	4.782	2.299
Mahir and Altac	3	1.205	0	-0.048	$0.0014\pm0.004$		4.804	2.293
Present	5	$1.303\pm0.015$	$\pm 0.409$	$0.746\pm0.150$	$\pm 1.519$	0.1563	5.461	4.202
Koda and Lien	5	1.316		0.793		0.149		
Harimi and Saghafian	5	1.311		0.805		0.1568	5.028	3.264
Mahir and Altac	5	$1.369\pm0.013$	$\pm 0.437$	$0.874 \pm 0.165$	$\pm 1.617$	0.161	$5.180 \pm 0.005$	$4.28\pm0.040$
Present	7	$1.285\pm0.011$	$\pm 0.333$	$0.589\pm0.066$	$\pm 1.184$	0.1546	5.270	4.039
Harimi and Saghafian	7	1.317		0.701		0.1593	5.036	3.29
Mahir and Altac	7	$1.355\pm0.014$	$\pm 0.363$	$0.682\pm0.07$	$\pm 1.309$	0.166	$5.159 \pm 0.003$	$4.21\pm0.26$
Present	10	$1.322\pm0.010$	$\pm 0.346$	$0.611\pm0.073$	$\pm 0.983$	0.1624	5.296	3.964
Sharman et al.	10	1.321		0.629		0.161		
Mussa et al	10	1.329		0.66		0.161		
Harimi and Saghafian	10	1.337		0.741		0.1639	5.058	3.493
Mahir and Altac	10	$1.383\pm0.011$	±0.359	$0.731\pm0.087$	$\pm 1.14$	0.172	$5.174 \pm 0.003$	$4.27\pm0.18$

**Table 1.** Hydrodynamic forces coefficient and Nusselt number of a two-cylinder tandem array for flow at Re = 100.

Note: Cd: drag coefficient, Cl: lift coefficient, St: Strouhal number, Nu: Nusselt number; 1 indicates an upstream cylinder and 2 indicates a downstream cylinder.

Authors	L/D	$Cd_1$	Cl <sub>1</sub>	Cd <sub>2</sub>	Cl <sub>2</sub>	St	Nu <sub>1</sub>	Nu <sub>2</sub>
Present	2	$1.051\pm0.009$	$\pm 0.039$	$-0.197 \pm 0.006$	$\pm 0.147$	0.1304	6.618	2.97
Mahir and Altac [10]	2	$1.06\pm0.0004$	$\pm 0.034$	$-0.21 \pm 0.0036$	$\pm 0.17$		6.460	$2.88\pm0.03$
Harimi and Saghafian [9]	2	1.038		-0.175			6.359	2.82
Meneghini and Saltara [22]	2	1.03		-0.17		0.130		
Han et al. [23]	2	1.041		-0.199		0.132		
Present	3	$1.020\pm0.011$	$\pm 0.029$	$-0.114 \pm 0.014$	$\pm 0.306$	0.1244	6.733	3.481
Mahir and Altac	3	$1.051\pm0.025$	±0.029	$-0.56 \pm 0.012$	±0.269	0.130	$6.56\pm0.001$	$3.54\pm0.11$
Harimi and Saghafian	3	1.016		-0.085			6.482	3.388
Koda and Lien [20]	3	1.043	0.023	-0.129	0.212			
Meneghini and Saltara	3	1.0		-0.08		0.125		
Han et al.	3	1.005		-0.119		0.127		
Present	4	$1.278\pm0.053$	$\pm 0.768$	$0.502\pm0.4$	$\pm 1.775$	0.1809	7.41	5.46
Harimi and Saghafian	4	1.291		0.648			6.981	4.234
Mahir and Altac	4	$1.34\pm0.056$	$\pm 0.805$	$0.558\pm0.22$	±1.99	0.181	$7.44\pm0.046$	$6.15\pm0.51$
Koda and Lien	4	1.287		0.442				
Meneghini and Saltara	4	1.18		0.38		0.174		
Present	5	$1.259\pm0.051$	$\pm 0.682$	$0.388\pm0.156$	$\pm 1.444$	0.1801	7.455	5.728
Mahir and Altac	5	$1.327\pm0.055$	$\pm 0.731$	$0.455\pm0.16$	$\pm 1.569$	0.186	$7.43\pm0.034$	$5.96 \pm 0.47$
Harimi and Saghafian	5	1.291		0.583			7.027	4.427
Koda and Lien	5	1.295	$\pm 0.489$	0.459	$\pm 1.111$			
Present	7	$1.298 \pm 0.05$	$\pm 0.691$	$0.367\pm0.166$	$\pm 1.325$	0.1894	7.510	5.394
Mahir and Altac	7	$1.356\pm0.049$	$\pm 0.742$	$0.442\pm0.15$	±1.328	0.194	$7.45\pm0.029$	$5.83\pm0.32$
Harimi and Saghafian	7	1.30		0.468			7.066	4.454
Present	10	$1.329\pm0.051$	$\pm 0.685$	$0.366\pm0.168$	$\pm 1.307$	0.1947	7.556	5.009
Mahir and Altac	10	$1.359\pm0.054$	$\pm 0.70$	$0.524\pm0.153$	$\pm 1.287$	0.191	$7.46\pm0.0\overline{27}$	$5.86\pm0.26$
Harimi and Saghafian	10	1.334		0.485			7.102	4.62

Table 2. Hydrodynamic forces coefficient and Nusselt number of a two-cylinder tandem array for flow at Re = 200.

**Table 3.** Dimensions of computational domain for simulation of flow past a two-cylinder tandem array.

Authors	L <sub>front</sub> /D	L <sub>rear</sub> /D	L <sub>up</sub> /D	L <sub>down</sub> /D
Sharmen et al. [4]	12.5	20	25	25
Mussa et al. [21]	13.5	25.5	23.5	23.5
Harimi and Saghafian [9]	15	25	12	12
Mahir and Altac [10]	8.5	20	10	10
Koda and Lien [20]	10	15	20	20
Han et al. [23]	20	30	20	20
Meneghini and Saltara [22]	10.5	10.5	10.5	25
Present	20	50	20	20

Note: distance from the inlet, outlet, upper boundary and lower boundary of the domain to the center of the object are L<sub>front</sub>, L<sub>rear</sub>, L<sub>up</sub>, L<sub>down</sub>, respectively; D: diameter of cylinder.

# 4. Results and Discussion

According to previous studies [24–26], the flow of the two-cylinder tandem array was classified into three main flow patterns: (a) the extended body, (b) the reattachment, and (c) the co-shedding. For a small distance ratio (L/D = 1 to 1.2), the two cylinders

behave as a single bluff body or "extended body." The separated shear layers from the upstream cylinder enclose around the downstream cylinder without any reattachment onto its surface before rolling up alternately into Karman vortices behind the downstream cylinder. That is, the downstream cylinder locates inside the vortex formation region of the upstream cylinder. In this case, very little fluid flow into the gap between cylinders. The Karman vortices are more elongated compared to a single cylinder.

The reattachment pattern exists when the distance ratio is 2 < L/D < 5. The shear layers from the upstream cylinder could reattach to the downstream cylinder. This behavior principally involves the reattachment of the shear layers from the upstream cylinder and the formation and shedding of eddies in the gap region between the two cylinders. The gap eddies could be symmetric or asymmetric. Karman vortex shedding occurs only from the downstream cylinder.

The co-shedding pattern exists when the pitch ratio is  $L/D \ge 5$ . The downstream cylinder is sufficiently far away so that Karman vortex shedding could occur from the upstream cylinder as well as the downstream one. The downstream cylinder is located outside the vortex formation region of the upstream cylinder and experiences the periodic impingement of shed Karman vortices from the upstream cylinder.

# 4.1. Flow Patterns and Heat Transfer of Tandem Regular Cylinders 4.1.1. Re = 100

When the flow is at Re = 100, the distance ratio between cylinders, L/D, is 2; the flow separates from the upstream cylinder and reattaches on the shoulders of the downstream cylinders, as shown in Figure 5a where a pair of symmetric vortex attaches on the rear surface of each cylinder. This is one of the reattachment flow patterns. For the upstream cylinder, the distribution of the surface pressure is similar to that of a single cylinder, as shown in Figure 6a. However, the Nusselt number around the surface is slightly different from the single cylinder, as shown in Figure 6b. Though the maximum of Nu still exists on the stagnation point, unlike a single cylinder, there is no dune-shape recovery of the Nusselt number between  $\theta = 135^{\circ}$  and 225°. This is because the vortex shedding is suppressed by the downstream cylinder within a short distance. Usually, the location of the minimum Nusselt number is almost the same as that of flow separation. Hence, the separation points for the upstream cylinder could be estimated at  $\theta = 135^{\circ}$  and  $\theta = 225^{\circ}$ , respectively. For the downstream cylinder, the maximum Nusselt numbers exists on the points of reattachment at about  $\theta = 75^{\circ}$  and  $\theta = 285^{\circ}$ , where the pressure is also the highest. The nose of the downstream cylinder has a minimum Nusselt number because only a very small amount of cooler fluid flows into the gap between cylinders. In the rear surface of the downstream cylinder between  $\theta = 150^{\circ}$  and  $\theta = 210^{\circ}$ , the Nusselt number has no recovery because the whole downstream cylinder still immerses in the wake of the upstream cylinder. The flow separates again from the downstream cylinder at  $\theta = 150^{\circ}$  and  $\theta = 210^{\circ}$ , as in the estimation from Figures 5a and 6b.

The case of L/D = 3 is similar to that of L/D = 2, with a stable reattachment pattern. In summary, this reattachment flow pattern has a stagnation point and two separation points on the upstream cylinder, and two reattached points and two separation points on the downstream cylinder. The thermal boundary layer around the base point of the upstream cylinder and the stagnation point of the downstream cylinder are thick, as shown in Figure 5b. This indicates that the heat in the region between the cylinders is not easily dissipated. As shown in Figure 6b, the Nusselt numbers on the rear surface of the upstream cylinder and the front surface of the downstream cylinder are lower. Larger heat transfer occurs on the stagnation point of the upstream cylinder and the reattached points of the downstream cylinder. The short distance between cylinders causes a shielding effect on the downstream cylinder and hinders cooler fluid flowing around the downstream cylinder. That is why the heat transfer of the downstream cylinder is much lower than that of the upstream cylinder.



**Figure 5.** Flow patterns of Re = 100, streamlines (left) and temperature contours (right): (**a**,**b**) L/D = 2, reattachment; (**c**,**d**) L/D = 5, co-shedding; (**e**,**f**) L/D = 7, co-shedding.



**Figure 6.** Re = 100, L/D = 2, pressure and Nusselt number distribution around the cylinders: (**a**) surface pressure coefficient; (**b**) Nusselt number.

At  $L/D \ge 5$ , both cylinders have vortex shedding, as shown in Figure 5c–f. The distribution of the surface pressure and Nusselt number of co-shedding mode are very different from the reattachment mode, especially for the downstream cylinder, as shown in Figure 7. The pressure on the stagnation point of the downstream cylinder is larger than that of the reattachment mode, which means more fluid impact on the front surface, as a comparison of Figures 6a and 7a shows. The periodic vortex shedding from the upstream cylinder introduced more cooler fluid from the vicinity of the array into the gap between cylinders and results in larger Nusselt numbers for both cylinders, as a comparison of Figures 6b and 7b shows. The ramping-up of the Nusselt number of the upstream cylinder moves periodically around  $\theta = 180 \pm 15^{\circ}$  on the rear surface, as shown in Figure 8e. Because the wake of the upstream cylinder oscillates, the stagnation point of

the downstream cylinder periodically sways around the nose with  $\theta = \pm 67^{\circ}$ , as shown in Figure 8f. Due to the moving of the stagnation point, the separation point of downstream cylinder also moves periodically at  $\theta = 180 \pm 40^{\circ}$ , as shown in Figure 8a-f. For L/D = 7 or 10, the flow has a similar behavior to that shown in Figure 5e,f and Figure 9.



**Figure 7.** Re = 100, L/D = 5, time mean pressure and Nusselt number distribution around the cylinders by taking the average of five periods of the cycle: (**a**) surface pressure coefficient; (**b**) Nusselt number.



Figure 8. Cont.



**Figure 8.** Transient temperature and Nusselt number distribution of the co-shedding pattern, Re = 100, L/D = 5: (a) 1/4 T; (b) 2/4T; (c) 3/4T; (d) T; (e) Nusselt number of the upstream cylinder; (f) Nusselt number of the downstream cylinder.



Figure 9. Transient Nusselt number distribution of the co-shedding pattern, Re = 100, L/D = 7: (a) 1/4T; (b) 2/4T; 3/4T; T.

## 4.1.2. Re = 200

Unlike Re = 100, as Re = 200, L/D = 3, there is a pair of vortices with alternately unequal sizes behind the upstream cylinder, as shown in Figure 10a. Flow separates from the upper and lower surfaces of the upstream cylinder and reattaches at the unsymmetrical locations on the downstream cylinder. This results in a pair of the unequally sized vortices between the cylinders. The alternately unequal size of vortices causes imbalanced pressure on the downstream cylinder and results in vortex shedding, as shown in Figure 10a,b. This is called unstable reattachment. When L/D  $\geq$  5, each cylinder has vortex shedding to form a co-shedding pattern. However, the vortex shedding from the downstream cylinder is more irregular than that of Re = 100, as shown in Figure 10c–f. For L/D = 3, the Nusselt number distribution on the upstream cylinder is similar to that of a single cylinder, as shown in Figure 11a. However, for the downstream cylinder, the locations of larger Nusselt number which also represents the reattached points, sway between  $\theta = 67^{\circ}$  and  $73^{\circ}$  for the upper surface and  $287^{\circ}$  to  $293^{\circ}$  for the lower surface, as shown in Figure 11b. Two prominent pairs of reattached points exist. One pair is at  $\theta = 67^{\circ}$  and  $287^{\circ}$  on the upper and lower surfaces, respectively. The other pair locates at  $\theta = 73^{\circ}$  and  $293^{\circ}$ .



**Figure 10.** Flow patterns of Re=200, streamlines (**left**) and temperature contours (**right**): (**a**,**b**) L/D = 3, unstable reattachment; (**c**,**d**) L/D = 5, co-shedding; (**e**,**f**) L/D = 7, co-shedding.

When  $L/D \ge 5$ , the distributions of the Nusselt number for both cylinders has a larger magnitude and amplitude of oscillation than those of Re = 100, as shown in Figure 11c-f. For L/D = 5, the stronger recovery of Nusselt number of the upstream cylinder moves periodically around  $\theta = 180 \pm 15^{\circ}$  on the rear surface, as shown in Figure 11c. The stagnation point of the downstream cylinder periodically sways around the nose with  $\theta = \pm 72^{\circ}$  and the pair of separation points of the downstream cylinder also move periodically between  $\theta = 180 \pm 41^{\circ}$  and  $\theta = 180 \pm 47^{\circ}$ , as shown in Figure 11d. The phenomenon is more obvious when L/D = 7, as shown in Figure 9e,f.

The Characteristics of Tandem Array

The pressure difference between stagnation and the base point of a single cylinder is larger than that of the upstream cylinder in a tandem array, as shown in Figure 12a. The pressure difference becomes smaller when the distance ratio is shorter due to the stronger suppression effect by the downstream cylinder. This implies that the drag of the upstream cylinder in a tandem array is smaller than that of the single cylinder. A shorter distance reduces the drag. For the downstream cylinder, as  $L/D \ge 5$ , the trend of pressure distribution resembles that of the single cylinder, whose maximum is located on the stagnation point and whose minimum is located around the separation points. However, when  $L/D \leq 3$ , the maximum pressure is located on the reattached points and the minimum is located on the stagnation point, as shown in Figure 12b. In terms of the Nusselt number, though the distribution of the Nusselt number of the upstream cylinder resembles that of the single cylinder at any distance ratio, the recovery on the rear surface is stronger in the longer distance ratio due to the more significant periodic vortex shedding, as shown in Figure 12c. The distribution of the Nusselt number for the downstream cylinder converts from a "W" shape for the co-shedding flow pattern to an "M" shape for the reattachment flow pattern, as shown in Figure 12d.



**Figure 11.** Transient Nusselt number distribution of the co-shedding pattern, Re = 200: (**a**,**b**) L/D = 3; (**c**,**d**) L/D = 5; (**e**,**f**) L/D = 7.

Effect of Distance

When L/D > 3, the time mean drag and lift amplitude increases significantly; meanwhile, the flow pattern changes from the reattachment mode to the co-shedding mode, as shown in Figure 13a,b. The Nusselt number of the co-shedding mode is also larger than that of the reattachment mode, as shown in Figure 13c,d. The reason is that, in the co-shedding mode, more of the cooler fluid is flapped into the region between cylinders and more heat is dissipated, along with vortex shedding from the downstream cylinder than in reattachment mode. Drag, the amplitude of the lift coefficient, and Nusselt number reach their maximum values at L/D = 5, which is close to the critical distance. When the distance ratio became less than the critical distance ratio, the flow enters reattachment mode. When the distance ratio is larger than the critical distance ratio, the flow is in co-shedding mode.



**Figure 12.** Time mean pressure and Nusselt number distributions of two cylinders in the different distance ratios, Re = 100: (a) surface pressure distribution on the upstream cylinder; (b) surface pressure distribution on the downstream cylinder; (c) Nusselt number distribution on the upstream cylinder; (d) Nusselt number distribution on the downstream cylinder.

Overall, the drag of the downstream cylinder s smaller than that of the upstream cylinder due to the shielding effect of the upstream cylinder, as shown in Figure 13a. As L/D increases, the drag of the upstream cylinder is increased toward the magnitude of the single cylinder; the drag of the downstream cylinder is increased but is still less than that of the upstream cylinder. Unlike the drag, the amplitude of oscillated lift of the downstream cylinder is larger than that of the upstream cylinder, as shown in Figure 13b. The lift oscillation of upstream cylinder is constrained by the downstream cylinder due to the blockage effect when  $L/D \leq 3$ . When L/D becomes larger than five, the amplitude of oscillated lift of the upstream cylinder gradually resembles that of the single cylinder, but the amplitude of lift of the downstream cylinder is still larger than that of a single cylinder or the upstream cylinder. The reason is that the vortex shedding from the upstream cylinder induces and provides constructive interference to the downstream cylinder, as shown in Figure 14.



**Figure 13.** Effect of distance ratio on the time mean hydrodynamic forces and Nusselt number at Re = 100 and Re = 200: (a) drag; (b) amplitude of lift; (c) Nusselt number of each cylinder; (d) Nusselt number of the array system.



Figure 14. The history of drag and lift coefficients for Re = 200, L/D = 7: (a) drag coefficient; (b) lift coefficient.

# 4.2. Flow Patterns and Heat Transfer of Tandem Slotted Cylinders

A slit with slit ratio S/D of 0.1 was applied on the upstream cylinder of a tandem array with different distances between cylinders to study the flow pattern and effect of a

slit on heat transfer. The inclination of the slit varied from  $0^{\circ}$  to  $90^{\circ}$ . The distance between cylinders ranged from L/D = 3 to L/D = 7.4.2.1. Flow patterns.

When the slit was applied on the tandem array of cylinders, three major flow patterns were seen. The first was stable reattachment. Each cylinder was attached with a pair of symmetric vortices on its rear surface, e.g., in the case of Re = 100, L/D = 3, with a horizontal slit on the upstream cylinder, as shown in Figure 15a. The second type was unstable reattachment with a pair of unsymmetrical vortices and vortex shedding behind the upstream cylinder and downstream cylinder, respectively, e.g., the case of Re = 200, L/D = 3, with a horizontal slit, as shown in Figure 15b. The third type was the co-shedding pattern, e.g., in the case of Re = 100, L/D = 5, with an inclined slit on the upstream cylinder, as shown in Figure 15.



**Figure 15.** Three major flow patterns of the slotted tandem array: streamline (**left**); temperature contours (**right**). (**a**,**b**) Stable reattachment, Re = 100, L/D = 3,  $\alpha = 0^{\circ}$ ; (**c**,**d**) unstable reattachment, Re = 200, L/D = 3,  $\alpha = 0^{\circ}$ ; (**e**,**f**) co-shedding, Re = 100, L/D = 5,  $\alpha = 45^{\circ}$ .

4.2.1. Flow Pattern Transform

Three cases of flow pattern transform occurred when a slit was employed on the array. The flow pattern transforms can be classified into two kinds as follows.

(i) Suppression

As in the previous discussion, the flow pattern of L/D = 5, Re = 100 is known as co-shedding, as shown in Figure 16a,b. However, after the upstream cylinder is slotted horizontally, the vortex shedding disappears from both cylinders, as shown in Figure 16c,d. Instead, each cylinder is attached with a pair of symmetric vortices. Hence, the flow pattern is changed from co-shedding mode to reattachment mode. The periodically oscillated lift coefficients for two regular cylinders, as shown in Figure 16e, are suppressed to be zero without oscillation when a horizontal slit is used, as shown in Figure 16f. The local Nusselt number near the inlet of the upstream cylinder is induced by a slit to have a higher value. However, it is limited to the local areas around the inlet. For most locations, the Nusselt number is very close to that of a regular upstream cylinder. Nevertheless, the Nusselt number of the downstream cylinder decreases significantly compared with the regular



downstream cylinder, as shown in Figure 16h. Hence, the horizontal slit decreases the overall Nusselt number of the tandem array.

**Figure 16.** Re = 100, L/D = 5 with slit  $\alpha = 0^{\circ}$ . (**a**–**c**) Streamlines, temperature contours, and lift history of the regular cylinder array; (**d**–**f**) streamlines, temperature contours, and lift history of the cylinder array with the slit; (**g**) time mean Nusselt number distribution of the upstream cylinder in regular and slotted array; (**h**) time mean Nusselt number distribution of the downstream cylinder in regular and slotted array.

Under the same conditions as above, the suppression phenomenon also occurs when the upstream cylinder is slotted vertically. The flow pattern is also changed from the coshedding mode, shown in Figure 16a, to the reattachment mode, as shown in Figure 17a,b. Although the local Nusselt number around the inlet and outlet of the slit is increased; most of the surfaces have almost the same value as a regular upstream cylinder, as shown in Figure 17c. However, the Nusselt number of the downstream cylinder is reduced significantly after the flow pattern transform, as shown in Figure 17d. Hence, the system Nusselt number of the array is also decreased by a vertical slit.



**Figure 17.** Re = 100, L/D = 5 with slit  $\alpha$  = 90° and (**a**) streamlines, (**b**) temperature contours, (**c**) time mean Nusselt number distribution of the upstream cylinder in regular and slotted array, and (**d**) time mean Nusselt number distribution of the downstream cylinder in regular and slotted array.

In summary, as L/D = 5, Re = 100, the horizontal slit or vertical silt can suppress the oscillation of flow in the array. The suppression mode reduces the Nusselt numbers of both cylinders.

(ii) Revival

The original flow pattern of L/D = 3 at Re = 100 is the reattachment mode, as shown in Figure 18a,c. However, the flow pattern is changed if an inclined slit with  $\alpha = 45^{\circ}$  is employed on the upstream cylinder. A pair of unsymmetrical vortices forms alternately behind the rear surface of the upstream cylinder instead of the symmetric vortices. Meanwhile, periodic vortex shedding appears behind the downstream cylinder, as shown in Figure 18b,d. This flow pattern is called the unstable reattachment mode because the flow separates from the upstream cylinder and alternately reattaches to two unsymmetrical locations of the downstream cylinder. The non-oscillated lift coefficients of two regular cylinders in the array are both revived by the slit to be periodically oscillated lift coefficients, as shown in Figure 18e,f. The Nusselt number of both cylinders increase slightly, as shown in Figure 18g,h. The inclined slit destroys the balanced pressure between the upper and lower surfaces of each cylinder and results in unsymmetrical vortices and vortex shedding. Nevertheless, this unstable flow is favorable to the heat transfer for the array.



**Figure 18.** Re = 100, L/D = 3 with slit  $\alpha$  = 45°. (**a**–**c**) Streamlines, temperature contours, and lift history of the regular cylinder array; (**d**–**f**) streamlines, temperature contours, and lift history of the cylinder array with the slit; (**g**) time mean Nusselt number distribution of the upstream cylinder in regular and slotted array; (**h**) time mean Nusselt number distribution of the downstream cylinder in regular and slotted array.

4.2.2. Effect of the Inclination of a Slit

When L/D = 5, Re = 100, a slit at  $\alpha = 0^{\circ}$  or 90° suppresses the co-shedding flow pattern and causes the system to default to the reattachment flow pattern. Therefore, the heat transfer of both cylinders is reduced, as shown in Figure 19a–c. When L/D = 3, a slit at any inclination increases the Nusselt number. When  $\alpha = 45^{\circ}$ , the Nusselt number of the array is increased more obviously because the flow pattern is changed by the slit from stable reattachment mode to unstable reattachment mode. For Re = 200, there was no change in flow pattern as a slit was applied on the cylinder with any angle of inclination. For example, when L/D = 3, the flow pattern is maintained as unstable reattachment with a single vortex shedding. At L/D = 5 and 7, the flow patterns remain in co-shedding mode as the slit is applied. Although the inclined slit did not change the original flow pattern of the array for flow at Re = 200, the Nusselt number of the array still increased by about 6–9% by using the slit, as shown in Figure 20.



**Figure 19.** The time mean Nusselt number as a slit with different angles is employed on the upstream cylinder of the tandem array at Re = 100: (a) upstream cylinder; (b) downstream cylinder; (c) array system.

#### 4.2.3. Effect of Distance Ratio

From the analyses of slit inclination, we can see that the best Nusselt number for the array exists at around  $\alpha = 45^{\circ}$ . Therefore, a slit at  $\alpha = 45^{\circ}$  was chosen to employ on the upstream cylinder under different distance ratios to study the effect of distance on the heat transfer of the array. Figure 21 shows that the Nusselt number is increased by about 6–15% by using a slit when L/D = 2–10. The increase in the Nusselt number is larger when there is an array with a shorter distance between cylinders at a lower Reynolds number, as shown in Figure 21a. When L/D = 3, Re = 100, the increase in Nusselt number is the greatest at 14.8%, compared to the array of regular cylinders.



**Figure 20.** The time mean Nusselt number as a slit with different angles is employed on the upstream cylinder of the tandem array at Re = 200: (a) upstream cylinder; (b) downstream cylinder; (c) array system.



**Figure 21.** The time mean Nusselt number of the array as a slit at  $\alpha = 45^{\circ}$  is employed on the upstream cylinder of the tandem array with different cylinder distance: (a) Re = 100; (b) Re = 200.

In summary, two kinds of flow pattern transformation exist at Re = 100. Hence, flow pattern transformation occurs only at a flow of Re = 100. The revival type of flow transform is favorable to heat transfer. However, the suppression type of flow transform is disadvantageous to heat transfer. Hence, a slit has to be employed carefully. A slit did not change the flow type at Re = 200. However, the slit increases the Nusselt number for flow at Re = 200 at any distance ratio and any angle of slit inclination. Overall, a slit is more sensitive for flow at Re = 100 in terms of changing the flow pattern than Re = 200.

## 5. Conclusions

Different distances between the cylinders result in three flow patterns in a tandem array: reattachment, unstable reattachment, or co-shedding. There is a critical distance ratio between L/D = 3 and 5 for flow at Re = 100 and 200. When the distance ratio is less than the critical distance, the flow is in reattachment mode. When the distance ratio is larger than the critical distance ratio, the flow is in co-shedding mode. The drag, lift, and heat transfer of the co-shedding mode are larger than those of the reattachment mode. If the slit is employed on the upstream cylinder, two kinds of flow pattern transform were found: Suppression and Revival. For the suppression mode, the existing vortex shedding disappears from both cylinders after a slit is applied. The suppression reduces the Nusselt numbers of both cylinders because flow oscillation is inhibited. For revival mode, the restful flow around both cylinders is perturbed to generate vortex shedding after applying a slit. As a result, the Nusselt numbers for both cylinders are increased slightly. Therefore, a slit can suppress or trigger vortex shedding, which is favorable to the heat transfer of the array. Hence, a slit has to be employed properly. If the co-shedding flow pattern exists in the regular cylinder array, the slit is not advisable to use for the enhancement of heat transfer. If the reattachment mode exists in the regular cylinder array, the inclined slit may increase the heat transfer. Overall, a slit employed in the upstream cylinder can improve the heat transfer of array by 6-15%. Among those, the optimal slit inclination for enhancement of heat transfer is at  $\alpha = 45^{\circ}$  to  $60^{\circ}$ .

This work provides the details of flow structure and the corresponding Nusselt number of a two-cylinder array. The results may be useful for the design of a pin-fins array or heat exchanger. Especially, thanks to the scope of the simulations at the flow of low Reynolds numbers, the results and conclusions in this study are suitable references for the design of micro pin-fins or micro heat exchangers.

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