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Vehicle Dynamic Control with 4WS, ESC and TVD under Constraint on Front Slip Angles

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Abstract: To enhance vehicle maneuverability and stability, a controller with 4-wheel steering (4WS), electronic stability control (ESC) and a torque vectoring device (TVD) under constraint on the front slip angles is designed in this research. In the controller, the control allocation method is adopted to generate yaw moment via 4WS, ESC and TVD. If the front steering angle is added for generating yaw moment, the steering performance of the vehicle can be further deteriorated. This is because the magnitude of the lateral tire forces are limited and the required yaw moment is insufficient. Constraint is imposed on the magnitude of the front slip angles in order to prevent the lateral tire forces from saturating. The driving simulation is performed by considering the limit of the front slip angle proposed in this study. Compared to the case that uses the existing 4WS, the results of this study are derived from the actuator combination that enhances performance while maintaining stability.

Keywords: vehicle dynamics; 4-wheel steer; in-wheel driving system; slip angle; tire force saturation; active front steering; rear wheel steering; electronic stability control; torque vectoring device

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1. Introduction

Electronic stability control (ESC) was introduced in the early 1990's. The ESC system was developed to enhance vehicle stability using the distribution of brake forces [1]. Therefore, ESC was imprinted to improve driving stability. Accordingly, in the 2010's, ESC was essentially installed in vehicles. In vehicle dynamics, stability is determined by the sideslip angle [2]. Thus, brake forces are distributed to maintain sideslip angle within a certain range. However, when using the brake forces, driving performance can be reduced. To improve both vehicle stability and drivability, a torque vectoring device (TVD) was developed [3]. Historically, TVDs were derived from the torque split system. In the late 1980's, Porsche developed a PSK (Porsche-Steuer-Kupplung) torque split system to maximize traction [4]. Using this system, the torque split-ratio between the front and the rear varied from 4:6 to 2:8, according to weight transfer. This system is known as the origin of TVD. For the latest car models, this can be realized using a TVD drivetrain [5]. The aim of the TVD is to optimize torque distribution at each wheel, depending on driving condition. In addition, the TVD enhances agility and stability [6].

However, using ESC and TVD can consume significant amounts of energy [7], so using 4-wheel steering (4WS) is appropriate from an energy-efficiency point of view. 4WS itself means that the front and rear wheels can be steerable. In low-speed driving situations, the rear steer is positioned opposite to the front steer. This can reduce the turning radius. At high-speed, the rear steer is positioned in the same direction as the front for stability [8]. Over the past 30 years, the maximum value for rear wheel steer has been limited to five degrees [9]. This is due to vehicle stability as well as kinematic limitations. In the latest rear wheel steering system of the Mercedes-Benz, the maximum value is 10 degrees, so that the new large-sized sedan handles similarly to a compact car [10].

In the view of vehicle dynamic control, only rear wheel steering (RWS) is regarded as an actuator in 4WS, because the front wheels are steered by a driver [9]. As an actuator for vehicle stability control, an active front steering (AFS) was developed in the middle of the 2000's. An AFS is a device that can steer front wheels, regardless of the driver's steering input. An RWS is a device that can steer rear wheels. Differently from the AFS, RWS steers wheels not by a driver but by a control logic. The disadvantage of AFS are that it makes the front lateral tire forces limited. The corrective steering angles of AFS are added to those intended by the human driver. It can increase the slip angles at the front wheels. As a result, the front lateral tire forces are easily saturated.

The integration of the systems—ESC, 4WS and TVD, are called the vehicle stability control (VSC). Generally, VSC is designed in order to enhance the maneuverability and lateral stability of a vehicle [11]. Maneuverability refers to the driving of a vehicle to follow the driver's intention, which is expressed as a reference yaw-rate calculated from the steering wheel angle. Therefore, stability controllers enhance maneuverability by reducing the clearance between the actual yaw-rate and driver's intent. Lateral stability means that the sideslip angle of the vehicle is within a certain range. This is because when the slip angle exceeds a certain range, it diverges, which reduces stability. Therefore, for vehicle stability, the chassis must be controlled so that the sideslip angle converges into a certain area. In general, the sideslip is limited to within ± 3 degrees for stability [11].

In most cases, vehicle dynamic controllers consist of upper and lower level controllers [12]. The upper one determines the target motion of the vehicle body. Specifically, the total force on each wheel and target yaw moment on the vehicle body are determined, using the 2-DOF vehicle dynamic model. The lower one calculates each tire force to satisfy the target motion. In particular, tire force distribution can generate additional yaw moment [12]. To implement tire force distribution, several methods have been adopted. In various studies, the LQ (Linear Quadratic) optimal has been used [13]. In addition, sliding mode control and fuzzy control have been considered [14,15].

To implement tire force distribution, several actuators of vehicle chassis have been used. Among them, ESC is the essential system [16]. When necessary, the ESC distributes the braking pressure delivered to each wheel differently. This can generate additional yaw moment using each longitudinal tire force. The TVD is also a system to generate yaw moment using longitudinal tire force distribution [17]. Within the limits of the driving torque capability, TVD produces different tire forces for each wheel. In contrast, AFS and RWS control lateral tire forces [18,19]. In this paper, 4WS means the combination of AFS and RWS. By distributing the lateral tire force as needed, the yaw moment is controlled. This is a more direct method of control, from a vehicle dynamics point of view. Thus, both longitudinal and lateral tire forces can be controlled using a combination of ESC, TVD, AFS and RWS. The idea was named integrated chassis control (ICC) [20]. In addition, the combination of those four actuators is considered in this paper.

In adopting ICC, it is important to know how to distribute the control input to the various actuators. In other words, methods to satisfy the target vehicle motion using these actuators are the most important issue. This is in order to find the problem of deriving an optimal control input to satisfy target values, such as stability and maneuverability. Recently, steer-by-wire systems have been commercialized. Accordingly, various studies have been conducted to control the combination of ESC, TVD, AFS and RWS. In order to control the yaw, a study on integrated chassis control, applied the optimal control method, was conducted [21]. Another optimization-based method for yaw moment distribution is the weighted pseudo-inverse based control allocation (WPCA) [20,22–24]. The WPCA can be used to solve problems for various inputs, such as driving, braking, and steering. In this paper, in order to control the yaw moment, the optimal actuator inputs are derived by using the variable weights of WPCA [20,22–24].

If WPCA is applied to control yaw moment, smaller weights of AFS make ICC use AFS only. AFS, in contrast to ESC, maintains driving speed and does not compromise ride comfort [13]. However, the lateral tire force saturates beyond the critical point of the slip

angle. Therefore, the limit for generating yaw moments with AFS is low [22]. For that reason, the yaw moment must be controlled, taking into account the limits of the lateral tire force controlled by the AFS.

Studies on this issue have recently been conducted. Jing considered the effect of lateral tire force saturation for 4-wheel driving (4WD) [25]. Another study dealt with the lateral tire force saturation situation of front wheels based on WPCA [22,23]. In these studies, the front lateral tire forces were constrained. However, the lateral tire force is the function of the slip angle. So, it is necessary to constrain the front slip angles for the purpose of limiting the front lateral tire forces. For this reason, Yim proposed 4-wheel independent steering system (4WIS), with consideration of the constraint on front slip angles [25]. However, these studies are not about AFS and RWS, but about 4WIS, which has not been implemented in real vehicles. However, 4WS is implemented in real vehicles [10]. Therefore, 4WS, under the constraint of the front slip angle, is considered in this paper. On the other hand, Li proposed an MPC-based lateral controller that takes into account the limitation of lateral tire force [26]. This study is concerned with the lateral control to converge to the stability domains of the sideslip angle and yaw-rate when lateral tire force is limited. However, this study assumes a situation where the driving force and braking force are zero. This means that the integrated chassis control is excluded.

This study presents which of the combinations of ESC, TVD, and 4WS improve dynamics, while maintaining stability. To do so, the yaw moment must be controlled, considering the constraint on front slip angles. The yaw moment control is implemented by determining the longitudinal and transverse tire forces of each wheel. Four longitudinal tire forces and four lateral tire forces are determined by control allocation. The key is to consider the situation in which the front lateral tire force is limited to a maximum value. In this limited situation, the yaw moment cannot be generated. To compensate for this shortfall, RWS, ESC and TVD are used. The performance of the proposed method was verified through the simulation of the double-lane change situation. Through simulation, it was confirmed that this controller distributes the control input taking the slip angle constraint into account. In addition, this simulation has confirmed which combination of ESC, TVD and 4WS most satisfies maneuverability and stability.

This paper is organized as follows: In Section 2, the yaw moment controller is designed in order to stabilize the lateral and yaw motions of the vehicle. The control allocation method, needed to determine the steering angles, traction torque and brake pressure of 4WS, TVD and ESC, is proposed in Section 2. In Section 3, a method needed to handle the constraint on the front slip angle is proposed. To validate the proposed method, simulation is conducted and simulation results are analyzed in Section 4. In Section 5, the conclusion of this research is drawn.

2. Design of Vehicle Dynamic Controller

The vehicle dynamic controller was designed to decide inputs of 4WS, ESC and TVD. It consists of the yaw moment controller, control allocation and determination of the steering angles.

2.1. Design of the Yaw Moment Controller

The yaw moment controller calculates the yaw moment required for turning. The driver's turning intention can be replaced with the desired yaw-rate γ_d . In addition, the actual yaw-rate γ is determined in the moment, due to the lateral tire force. In this case, the longitudinal tire force is assumed to be zero. In other words, the longitudinal velocity v_x is assumed to be constant [12]. The difference between the desired yaw-rate and the actual yaw-rate is defined as the yaw-rate error. The yaw moment controller calculates the required yaw moment ΔM_c to converge the yaw-rate error to zero.

The difference between the left and right steering angles of the Ackerman steering is very small. Therefore, it is assumed that the left and right steering angles are the same. Next, the turning situation can be expressed as a bicycle model, as shown in Figure 1.

The angle between the vehicle's speed vector and the heading direction is defined as the sideslip angle β . The distance from the center of gravity to the front wheel is defined as l_f , and the distance to the rear wheel is defined as l_r . The front wheel steering angle of 4WS is defined as δ_f and the rear wheel steering angle is defined as δ_r . The moment of inertia about the z-axis is called I_z . The bicycle model in Figure 1 can be expressed by the differential equation of Equation (1).

$$mv_x(\dot{\beta} + \gamma) = F_{yf} \cos \delta_f + F_{yr} \cos \delta_r \quad (1)$$

$$I_z \dot{\gamma} = l_f F_{yf} \cos \delta_f - l_r F_{yr} \cos \delta_r + \Delta M_c$$

$$\begin{cases} \alpha_f = \frac{v_y + l_f \gamma}{v_x} - \delta_f = \beta + \frac{l_f \gamma}{v_x} - \delta_f \\ \alpha_r = \frac{v_y - l_r \gamma}{v_x} - \delta_r = \beta - \frac{l_r \gamma}{v_x} - \delta_r \end{cases} \quad (2)$$

$$F_{yf} = f(\alpha_f), \quad F_{yr} = f(\alpha_r) \quad (3)$$

Figure 1. 2-DOF bicycle model.

The tire slip angles of the front α_f and the rear α_r are defined as the angle between the direction of wheel and the steering angle, as given in Equation (2). This formula assumes that $\tan^{-1}(\theta) \approx \theta$. In Equation (1), the relationship between the lateral tire forces, F_{yf} and F_{yr} , and slip angles, α_f and α_r , is assumed to be linear. However, their linear relationship will be limited to the maximum value in the next section, as shown in Equation (3) [12]. The desired yaw-rate, γ_d , can be calculated by the driver's steering input δ_f , cornering stiffness, C_f and C_r , l_f , l_r , v_x and m .

Controlling a vehicle in vehicle dynamics can be viewed from two perspectives. Maneuverability refers to the ability to steer as intended by the driver. This issue is addressed by reducing the yaw-rate error. Stability is achieved by controlling the sideslip angle to not exceed a certain range. To satisfy these two, the sliding mode control method was adopted. The sliding mode surface is expressed as Equation (4). In Equation (4), η is a weighting factor of the sideslip angle. In order for this surface to converge, the condition of Equation (5) must be satisfied [14].

$$s = (\gamma - \gamma_d) + \eta \cdot \beta \quad (4)$$

$$\dot{s} = -Ks \quad (K > 0) \quad (5)$$

From (1), (4) and (5), ΔM_c can be obtained as Equation (6).

$$\begin{aligned} \Delta M_c = I_z \cdot \dot{\gamma}_d + I_z \cdot \eta \cdot \left(\frac{F_{yf} \cos \delta_f + F_{yr} \cos \delta_r}{mv_x} - \gamma \right) \\ - l_f F_{yf} \cos \delta_f + l_r F_{yr} \cos \delta_r - I_z \cdot K \cdot (\gamma - \gamma_d + \eta \cdot \beta) \end{aligned} \quad (6)$$

2.2. Control Allocation

ΔM_c can be realized by the tire forces of each wheel. Tire forces on each wheel are generated by ESC, TVD and 4WS. That is, four braking forces are generated by the ESC, four tractive forces are generated by the TVD and four lateral tire forces are generated by the 4WS. This is called yaw moment distribution. In this paper, a weighted pseudo-inverse based control allocation (WPCA) is adopted to distribute the control yaw moment into the

tire forces generated by ESC, TVD and 4WS [20]. WPCA is a quadratic programming with equality constraints used for control allocation in VSC. For the sake of intuitive description, WCPA is called control allocation in this paper.

Figure 2 shows tire force distribution of each wheel according to ΔM_c , when ΔM_c is positive [20]. In Figure 2, F_{x1} to F_{x4} are longitudinal tire forces. F_{y1} to F_{y4} stand for lateral tire forces. The AFS angle is added to the steering angle handled by the driver to generate F_{y1} and F_{y2} . RWS directly affects the generation of F_{y3} and F_{y4} . Additional lateral tire forces on wheel one and two, generated by AFS, will be called ΔF_{yf} in this section. In the same way, RWS generates ΔF_{yr} , while ESC and TVD generates from ΔF_{x1} to ΔF_{x4} . These tire forces should be determined to generate ΔM_c . Control allocation is used to determine these eight tire forces.

Figure 2. Coordinate system corresponding to tire forces and control yaw moment.

As previously explained, yaw moment control, ΔM_c , is generated by the additional tire forces using the actuators. Equation (7) shows the geometric relation between the tire forces and ΔM_c , given in Figure 2. In Equation (7), the elements of the vector $\mathbf{H}$ are given in Equation (8). As shown in Figure 2, the elements of vector $\mathbf{H}$ are determined by the steering angle of each wheel and the dimension of the vehicle. Vector $\mathbf{H}$ is given in Equation (9).

$$\underbrace{\begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} \Delta F_{yf} \\ \Delta F_{yr} \\ \Delta F_{x1} \\ \Delta F_{x2} \\ \Delta F_{x3} \\ \Delta F_{x4} \end{bmatrix}}_{\mathbf{q}} = \Delta M_c \quad (7)$$

$$\begin{cases} h_1 = 2l_f \cos \delta_f, & h_2 = -2l_r \cos \delta_r \\ h_3 = -l_f \sin \delta_f + \frac{t_f}{2} \cos \delta_f, & h_4 = -l_f \sin \delta_f - \frac{t_f}{2} \cos \delta_f \\ h_5 = l_r \sin \delta_r + \frac{t_r}{2} \cos \delta_r, & h_6 = l_r \sin \delta_r - \frac{t_r}{2} \cos \delta_r \end{cases} \quad (8)$$

The objective cost function, J , of control allocation is defined as follows:

$$\begin{aligned} J &= \frac{\rho_1 \Delta F_{yf}^2 + \rho_3 \Delta F_{x1}^2}{(\mu F_{z1})^2} + \frac{\rho_1 \Delta F_{yf}^2 + \rho_4 \Delta F_{x2}^2}{(\mu F_{z2})^2} + \frac{\rho_2 \Delta F_{yr}^2 + \rho_5 \Delta F_{x3}^2}{(\mu F_{z3})^2} + \frac{\rho_2 \Delta F_{yr}^2 + \rho_6 \Delta F_{x4}^2}{(\mu F_{z4})^2} \\ &= \mathbf{q}^T \mathbf{W} \mathbf{q} \\ \mathbf{W} &= \text{diag} \left[\frac{1}{\xi_1^2} + \frac{1}{\xi_2^2}, \frac{1}{\xi_3^2} + \frac{1}{\xi_4^2}, \frac{1}{\xi_1^2}, \frac{1}{\xi_2^2}, \frac{1}{\xi_3^2}, \frac{1}{\xi_4^2} \right] \boldsymbol{\rho}, \quad \xi_i = \mu F_{zi} \quad i = 1, 2, 3, 4 \\ \boldsymbol{\rho} &= \text{diag} [\rho_1 \quad \rho_2 \quad \rho_3 \quad \rho_4 \quad \rho_5 \quad \rho_6] \end{aligned} \quad (9)$$

In Equation (9), the radii of friction circles, μF_{zi} , must be estimated. For this problem, several estimation methods exist. Assuming that the road surface is flat, it can generally be estimated using longitudinal and lateral accelerations [21]. For rough or slippery road surfaces, it can be estimated using a slip-ratio [27]. As the driving situation on a flat road

surface is assumed, the first method is cited in this paper. In Equation (9), ρ is the weight vector for each element. Thus, ρ_i serves to change the combination of each actuator [20,22]. In addition, slip control of ESC or TVD can be performed in control allocation. Another purpose of using ρ_i is to tune vehicle dynamics. For example, by inducing excessive slip of the rear tire, the vehicle can increase agility at the cost of lower stability. More details will be described thereafter.

This quadratic programming optimization problem can be solved using the Lagrange multiplier. By using the Lagrange multiplier, this problem is solved as Equation (10). From $\mathbf{x}_{opt}$ in Equation (10), each tire force can be obtained. Tire forces should be converted into wheel torques, braking pressures and steer angles. The steering angles will be treated in Section 2.3.

$$\mathbf{x}_{opt} = \mathbf{W}^{-1} \mathbf{H}^T (\mathbf{H} \mathbf{W}^{-1} \mathbf{H}^T)^{-1} \Delta M_c \quad (10)$$

For various combinations of ESC, TVD and 4WS, a particular constraint according to direction of yaw moment should be decided. For instance, when only ESC is available and the counter clock-wise yaw moment (CCWM) is needed, braking force ΔF_{x1} and ΔF_{x3} must be activated. This example will be called ‘ESC CCWM’ in the next paragraph.

In the explanation of Equation (9), it was described that ρ means the degree of activation of each tire force. For the ESC CCWM case, ΔF_{x1} and ΔF_{x3} must be activated and the rest must not be activated. For this purpose, ρ_3 and ρ_5 must be very small values, e.g., 0.001. Accordingly, ρ_1, ρ_2, ρ_4 and ρ_6 should be relatively high values, e.g., 1, as shown in Equation (11). By this principle, various combinations of actuators are possible using very small values ε_1 and ε_2 . Equations (11)–(14) explain this principle [11,20,23,24].

ESC (electronic stability control—four independent brakes) activation

$$\begin{aligned} \rho &= \text{diag} [1 \quad 1 \quad \varepsilon_1 \quad 1 \quad \varepsilon_2 \quad 1] \quad \text{if } \Delta M_c > 0 \\ \rho &= \text{diag} [1 \quad 1 \quad 1 \quad \varepsilon_1 \quad 1 \quad \varepsilon_2] \quad \text{if } \Delta M_c < 0 \end{aligned} \quad (11)$$

AFS (active front steering) activation

$$\begin{aligned} \rho &= \text{diag} [\varepsilon_1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \quad \text{if } \Delta M_c > 0 \\ \rho &= \text{diag} [\varepsilon_1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \quad \text{if } \Delta M_c < 0 \end{aligned} \quad (12)$$

RWS (rear wheel steering) activation

$$\begin{aligned} \rho &= \text{diag} [1 \quad \varepsilon_1 \quad 1 \quad 1 \quad 1 \quad 1] \quad \text{if } \Delta M_c > 0 \\ \rho &= \text{diag} [1 \quad \varepsilon_1 \quad 1 \quad 1 \quad 1 \quad 1] \quad \text{if } \Delta M_c < 0 \end{aligned} \quad (13)$$

RWS and TVD (torque vectoring device) activation

$$\begin{aligned} \rho &= \text{diag} [1 \quad \varepsilon_1 \quad 1 \quad \varepsilon_2 \quad 1 \quad \varepsilon_3] \quad \text{if } \Delta M_c > 0 \\ \rho &= \text{diag} [1 \quad \varepsilon_1 \quad \varepsilon_2 \quad 1 \quad \varepsilon_3 \quad 1] \quad \text{if } \Delta M_c < 0 \end{aligned} \quad (14)$$

where $\varepsilon_1, \varepsilon_2$ and ε_3 activate or deactivate RWS, front and rear traction or braking, respectively. When one of these weights increases, the corresponding actuator will be used less. For example, when the ε_1 rises, the usage of AFS decreases.

The tire force of each wheel determined in control allocation should be transmitted as a control input to each actuator. From the determined longitudinal tire force, the brake pressure is determined as in Equation (14). The torque input delivered to the motor inverter is defined as the longitudinal tire force multiplied by the tire radius. At this time, the

anti-lock braking system (ABS) and traction control system (TCS), related to the slip-ratio, utilize existing commercialized logic. Therefore, this content is omitted from this paper.

$$P_{Bi} = \frac{r_w}{K_B} \Delta F_{xi} \quad (i = 1, 2, 3, 4) \quad (15)$$

2.3. Determination of Steer Angles

Control allocation determines tire force of each wheel. As wheel torque and brake pressure were calculated separately, the steering angles had to be determined as well. For AFS, as the steering angle by the driver's operation is already determined, the additional steering angle $\Delta\delta_f$ is determined by the controller. For RWS, the controller determines the angle, δ_r . These angles can be achieved from the definition of tire slip angle. By substituting the steer angles of AFS and RWS into Equation (2), Equation (16) is obtained. From Equation (16), steer angles of AFS and RWS can be obtained, as in Equation (17). $\Delta\delta_f$ and δ_r can be derived using linearized lateral tire force formulation, as in Equation (18). In these formulations, the sideslip angle β must be estimated. In previous studies, various sideslip angle estimation methods exist [28]. Among these, the Kalman filter-based method was adopted.

$$\alpha_i = \begin{cases} \beta + \frac{l_f \gamma}{v_x} - (\delta_i + \Delta\delta_i), & i = 1, 2 \\ \beta - \frac{l_r \gamma}{v_x} - \delta_i, & i = 3, 4 \end{cases} \quad (16)$$

$$\begin{cases} \Delta\delta_i = -\alpha_i + \beta + \frac{l_f \gamma}{v_x} - \delta_i, & i = 1, 2 \\ \delta_i = -\alpha_i + \beta - \frac{l_r \gamma}{v_x}, & i = 3, 4 \end{cases} \quad (17)$$

$$\begin{cases} \Delta\delta_i = -\frac{F_{yi}}{\sigma C_i} + \beta + \frac{l_f \gamma}{v_x} - \delta_i, & i = 1, 2 \\ \delta_i = -\frac{F_{yi}}{\sigma C_i} + \beta - \frac{l_r \gamma}{v_x}, & i = 3, 4 \end{cases} \quad (18)$$

3. Yaw Moment Compensation under Constraint on Front Wheels

As explained in the previous chapter, when only AFS is used in chassis control, there is no braking force, so there is less deceleration. However, AFS encounters the limit of lateral tire force when excessive steering is applied. Therefore, the theoretical optimal solution by control allocation may not derive the optimal performance in practice. To solve this problem, it is necessary to consider the characteristics of the lateral tire force. Figure 3 shows the lateral tire force characteristics with respect to slip angle and the coefficient of friction [22,23]. The lateral tire force arrives at its maximum if the slip angle is α_m . So, if AFS is utilized when the slip angle is larger than α_m , its effect can be reduced. In this situation, AFS should be limited, and the role of RWS, ESC, TVD, etc. should be increased.

Figure 3. Lateral tire force with respect to slip angle.

The maximum of the lateral tire force is calculated with Equation (19). In Equation (19), $q_1(F_z, \mu)$ is the maximum of the lateral tire force. It can be obtained statically from experimental data using F_z and μ . $q_2(F_x, F_z, \mu)$ is the maximum value, obtained from the definition

of the friction circle, $(\mu F_{zi})^2$. As shown in Equation (19), $F_{y,max}$, the maximum of the lateral tire force is determined as the smaller of q_1 and q_2 .

$$F_{y,max} = \min\{q_1(F_z, \mu), q_2(F_x, F_z, \mu)\} \quad (19)$$

$$q_2(F_x, F_z, \mu) = \sqrt{(\mu F_z)^2 - F_x^2} \quad (20)$$

The front slip angle should not exceed α_m , such that ΔF_{yf} generated by AFS should not exceed $F_{y,max}$. If ΔF_{yf} exceeds $F_{y,max}$, ΔF_{yf} must be $F_{y,max}$. As a result, the objective function Equation (7) is changed to Equation (21) in WPCA, because ΔF_{yf} becomes the constant, $F_{y,max}$. Moreover, the set of variable weights should be set as Equation (22), such that AFS should not be used. Equation (22) represents the variable weights corresponding to RWS, ESC and TVD, as given from Equation (11) to Equation (14). The solution of the optimization problem with the objective function Equation (21), the weight vector (22), and the equality constraint Equation (7), is easily obtained with Equation (10). This problem is called constrained WPCA (C-WPCA) [23,24].

$$\Delta M_n = \Delta M_c - h_1 \cdot \Delta F_{yf} \quad (21)$$

$$\rho = \text{diag} [1 \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet] \quad (22)$$

4. Simulations and Validation

Simulations were performed to verify the proposed method. The SW package used is CarSim and includes a vehicle model, a driver model, and a driving environment [18]. To confirm both maneuverability and stability, the elk test (ISO 3888-2) was performed. The vehicle model is a sub-compact SUV model, listed in CarSim, as shown in Table 1. The driver model can predict 0.75 s ahead, meaning an inexperienced driver [29]. The driver model drives the vehicle at 80 kph. The coefficient of friction between the tire and the road is 0.6. Therefore, the simulation scenario is a driving condition that tends to lose stability. The maximum front and rear steer angles are bounded to 10 degrees and 5 degrees, respectively. The brake actuators were also modeled as first-order systems, with 0.05 s. The proposed controller and actuator delay were implemented in MATLAB Simulink. The controller transmits each braking pressure, driving torque and steering angle to the vehicle model of CarSim. The criterion for satisfactory maneuverability is 4.58 deg/s of yaw-rate error, and this criterion follows FMVSS 126. In addition, the criterion for satisfaction of stability is 3 degrees of the sideslip angle [30].

Table 1. Parameters of a sub-compact SUV model, listed in CarSim.

m_s	1146 kg	l_f	0.880 m
I_z	1302.1 kg·m ²	l_r	1.32 m
C_f	35,900 N/rad	$K_{B,front}$	149 N·m/MPa
C_r	49,800 N/rad	$K_{B,rear}$	69 N·m/MPa
v_x	80 km/h	r_w	0.398 m

Two types of simulation were performed. The first simulation was performed to compare the unconstrained and constrained cases when using only 4WS for yaw moment distribution. In this case, only RWS was activated to compensate for the yaw moment. The second simulation was performed to compare several cases of the RWS–ESC–TVD combination, when the front lateral tire forces are $F_{y,max}$.

Figures 4–6 show the results of the first simulation. In these figures, the legends 4WS and 4WSC represent the simulation results of 4WS with WPCA and C-WPCA, respectively. In Figure 5, the legends, FL, FR, RL and RR, represent the front left, front right, rear left and rear right wheels, respectively. As shown in Figure 4, there are small differences between the unconstrained and constrained cases in using 4WS. In other words, the control

allocation complements the performance by compensating the required yaw moment using RWS. The notable difference between the unconstrained and constrained cases is the yaw-rate error, as shown in Figure 4a. More specifically, the maximum yaw-rate errors of the unconstrained and constrained cases are 1.7 deg/s and 2.3 deg/s, respectively. These are satisfactory in terms of the maneuverability. Moreover, there is a slight difference between the sideslip angles of 4WS and 4WSC, as shown in Figure 4b. This means that the lateral stability was maintained under the constraint of the front slip angles with 4WS. As shown in Figure 4c, the driver's steering wheel angles of 4WS and 4WSC were nearly identical to each other. This means that a driver cannot recognize the condition of constraint on the front slip angles.

Figure 5 shows the slip angles of each wheel for each case. As shown in Figure 5b, the front slip angles are effectively constrained by the proposed method. Figure 6 shows the lateral tire forces with respect to the slip angles of each wheel for each case. In addition, the lateral tire forces were not saturated due to the constraint on the front slip angles, as shown in Figure 6b. On the lateral tire force characteristic curve with respect to slip angles, it is observed that most of the result values exist in the linear section, with the proposed method. As shown in Figures 4–6, it is verified that the proposed control method is able to limit the magnitude of the front slip angles without a significant deterioration of the control performance.

Figure 4. Simulation results for unconstrained (4WS) and constrained (4WSC) cases.

Figure 5. Slip angles for unconstrained and constrained cases.

Figure 6. Lateral tire forces with respect to slip angles for unconstrained and constrained cases.

The second simulation was performed to compare the several cases of RWS–ESC–TVD combinations under the limitation on the front slip angles. Regarding the actuator configuration, four cases—CASE1, CASE2, CASE3 and CASE4 were considered in simulation. CASE1, CASE2, CASE3 and CASE4 mean the actuator combinations: RWS, RWS and TVD;

RWS and ESC; and RWS, ESC, and TVD, respectively. Through these simulations, it can be confirmed which combination had the best performance.

Figures 7–9 show the results for each case. Table 2 shows the maximum yaw-rate errors, sideslip angles and the minimum longitudinal velocity for each case. In Table 2, 4WS represents the unconstrained case with 4WS only. In Figure 8, the title of the vertical axis and 4WS angle, means the angle generated at the front and rear wheels. For front wheels, it is a corrective steering angle added to the driver's wheel. For rear wheels, it is just a steering angle. As shown in Figure 9, the front slip angles were constrained within the given value for all cases. These results are similar to those given in Figure 6. As shown in Figure 7, CASE3 and CASE4 show poor performance compared to CASE1 and CASE2 under the constraint of the front slip angles. Both CASE3 and CASE4 have ESC. In other words, the yaw-rate error, the sideslip angle and the driver's steering wheel angle were increased, while the longitudinal velocity was reduced, if ESC was used to generate yaw moment. This fact can be also found in Table 2. The reduction of the longitudinal velocity means the loss of the energy for driving. So, it is recommended that ESC should not be used for yaw moment generation under the constraint on the front slip angles.

Figure 7. Simulation results for each case.

(a) Corrective steering angles for each case

(b) Traction torques for each case

(c) Applied brake pressures for each case

Figure 8. Control inputs for each case.

Figure 9. Lateral tire forces with respect to slip angles for each case.

Table 2. Comparison of results for each case.

Cases	Max $ \dot{\gamma} - \dot{\gamma}_d $ (deg/s)	Max $ \beta $ (deg)	Min v_x (km/h)
4WS	1.7	1.1	65.8
CASE1	2.3	1.2	66.0
CASE2	2.5	1.2	69.4
CASE3	2.9	2.0	57.2
CASE4	2.8	1.8	59.5

As shown in Figure 7 and Table 2, the TVD had little effects on the control performance of RWS under the constraint of the front slip angles, except for the longitudinal velocity. As shown in Figure 7c, the longitudinal velocity was increased by the TVD. This is a notable feature of the TVD, as compared to ESC. As shown in Figure 8, the corrective steering angles of RWS were reduced by the use of the other actuators, i.e., ESC and TVD. This is the natural consequence of multiple actuator cooperation, as presented in previous studies [11,20,22–24].

From the first simulation, the proposed method, i.e., WPCA with 4WS under the constraint on the front slip angles, shows nearly identical performance to that without any constraints in terms of the yaw-rate error, the sideslip angle and the driver's steering wheel angle. From the second simulation, it is recommended that the TVD has the benefit in increasing the vehicle speed without performance deterioration, and ESC should not be used due to performance deterioration under the constraint on the front slip angles.

5. Conclusions

The stability control system with 4WS deteriorates control performance due to the saturation of the front lateral tire force. In this paper, a control allocation method was proposed to generate the required yaw moment for vehicle stability under the condition that the front lateral tire forces are saturated. For this purpose, the constrained WPCA (C-WPCA) was adopted. This method can effectively constrain the front slip angle by tuning the weight vector of WPCA when it is 4WS. To compensate for the loss of the required yaw moment, RWS was applied. To observe the effects of the ESC and TVD compensating for the required yaw moment, the combinations of ESC, TVD and 4WS were considered. To verify the proposed method, simulations for various actuator combinations were performed.

For 4WS, with the constraint on the front slip angles, the vehicle stability was maintained at a similar level, compared to that without the constraint. For several actuator combinations of ESC, TVD and 4WS, with the constraint on the front slip angles, it was satisfied in terms of the vehicle stability. Among those, yaw moment compensation using RWS and TVD showed the least amount of performance degradation. ESC can be combined with 4WS to slightly improve the vehicle stability. However, this combination significantly deteriorates driving performance. Therefore, it can be said that the combination of 4WS and TVD, which is excellent in terms of performance while maintaining stability, is the best one under the constraint on the front slip angle. It is better to add an ESC to the combination of 4WS and TVD only in cases where it is obvious that it is outside the stability.

As described in Sections 2 and 3, WPCA is easy to solve because it requires only algebraic calculation when finding the optimum solution. Moreover, the lateral tire force and the slip angle can be estimated with an observer or estimator. As mentioned in Section 1, ESC is mandatory in several countries, and 4WS and TVD have been gradually implemented in real vehicles. So, the proposed method can be easily implemented in real vehicles with 4WS, ESC and TVD.

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