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Comparative Study of an EKF-Based Parameter Estimation and a Nonlinear Optimization-Based Estimation on PMSM System Identification

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Abstract: In this study, two different parameter estimation algorithms are studied and compared. Iterated EKF and a nonlinear optimization algorithm based on on-line search methods are implemented to estimate parameters of a given permanent magnet synchronous motor whose dynamics are assumed to be known and nonlinear. In addition to parameters, initial conditions of the dynamical system are also considered to be unknown, and that comprises one of the differences of those two algorithms. The implementation of those algorithms for the problem and adaptations of the methods are detailed for some other variations of the problem that are reported in the literature. As for the computational aspect of the study, a convexity study is conducted to obtain the spherical neighborhood of the unknown terms around their correct values in the space. To obtain such a range is important to determine convexity properties of the optimization problem given in the estimation problem. In this study, an EKF-based parameter estimation algorithm and an optimization-based method are designed for a given nonlinear dynamical system. The design steps are detailed, and the efficacies and shortcomings of both algorithms are discussed regarding the numerical simulations.

Keywords: EKF; nonlinear optimization; parameter estimation; state estimation; system identification

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1. Introduction

The question of system identification when it comes to nonlinear dynamical systems becomes more challenging due to the immense implications that nonlinearities cause. Although it is a standard practice to determine whether a given LTI system is observable, this process is not standard and there is no one unique algorithm for a given nonlinear system [1]. There are some linear algebra-based techniques that are available in the literature to conduct that process; however, those methods have certain limitations depending on the nonlinearity. In the literature, there are nonlinear observer types that are quite popular and have the ability to deal with the modelling uncertainties, some of which can be listed as SMO (sliding mode observer) [2], HGO (high gain observer) [3], EKF (extended Kalman filter), and variations thereof [4,5]. For mild nonlinearities, EKF seems to be the prominent choice due to its straightforward design and working principle. EKF is often used to estimate the states that are not measurable, and there is an available system model based on which the estimator is designed.

Although EKF may result in sufficient accuracy in many real-world applications, there are some cases where the assumptions of EKF may not hold. It is important to address under which conditions EKF might fail. There are two main starting points that cause the Kalman filter to work efficiently: the normal distributed process and sensor noise terms and the invariance of a random variable with normal distribution under liner operations as in linear time invariant systems. The assumption that process noise with a normal distribution may not hold in practice but as the central limit theorem suggests that it can be reached for certain circumstances as time progresses [6]. Another factor, the invariance property, allows the Kalman filter to update the state estimation error pdf at each iteration rather than storing the complete pdf, as in the case of a particle filter. Especially for linear systems or systems that can be represented as linear for a stated operating point, it is known that Kalman filter operates satisfactorily. For nonlinear systems, however, since a stochastic signal with normal distribution operated by a nonlinear term results in another stochastic signal with a distribution different than the input signal, the notion that only mean and variance terms can be stored and updated fails. There are some methods that focus on the higher-order moments of the stochastic signal; however, these cases do not cover a wide range of applications. EKF tries to linearize the nonlinear system model at each time step using its own state estimates. For highly nonlinear and discontinuity possessing systems this technique may diverge. However, especially when sampled at a sufficiently high frequencies and for smooth nonlinearities, EKF generally converges. A variation of EKF focuses on the mean and variance computation of the output signal obtained by a stochastic input signal with normal distribution passing through a nonlinear operation [7]. UKF employs some heuristic techniques to compute central moments of the output signal rather than linearizing the nonlinear system. This technique is especially popular in SLAM applications. Another parameter estimation method that is situated in the same class as EKF is a particle filter, which, like EKF, uses Bayesian estimation but focuses on the problem with more general settings. PF tries to reconstruct the stochastic pdfs at each time step. For the cases where it is evident that the stochastic signals that are present in the problem are non-Gaussian and/or multi-modal, this technique outperforms its counterparts at the expense of increased computational load. There are techniques that employ PF as a main parameter estimator tool for certain system identification problems, but the computational aspect is the only drawback [8].

Another significant usage of the observers is for parameter estimation. In cases where there are enough measured signals and some parameters are required to be estimated for monitoring or control purposes, EKF also can be used by extending the state vectors by the parameters to be estimated. For a small set of unknown parameters, that technique works sufficiently accurately. However, for the cases where the number of parameters is much greater than the number of states, there can be some numerical issues raise during the estimation process. To circumvent that problem, it is a common practice to design two separate EKFs, one for the state estimation and other for the parameter estimation, and the information exchange between the two estimators is regulated by another regulation mechanism [9]. In doing so, the numerical problems are mitigated to an extent. The same type of method is also used for the parameter estimation of systems where the number of parameters is greater than the state dimension and there are enough data points to work with. In industrial applications, those requirements often arise for system monitoring and fault-detection problems [10].

As for the plant that was tested as the test problem, PMSM is considered to be the main counterpart of induction motors that are widely utilized in a wide range of industrial applications. The generator version is prominent not only for consumption, but also due to the increased wind energy harvesting systems. Owing to its power-to-volume ratio, it is used in applications where the weight of the total system is limited, such as in EV applications and surveillance tools [11].

In this study, a line search-based nonlinear optimization technique is implemented, and one of the most important developments in recent years in the control and estimation literature is the implementation of many optimization techniques in controller and estimator design processes. Starting with the LMI techniques that have been used in robust linear controller design, some other convex optimization techniques have become prominent in the field of control [12]. Due to the increased computational power, many Lyapunov function construction problems for stability analysis in nonlinear systems can be reduced to linear programming using some heuristic methods [13,14]. In this study, a line search-based nonlinear optimization method is discussed. Furthermore, whether the problem is convex is also discussed. This question is important due to the fact that the presence of convexity for such a problem may result in a possible on-line nonlinear observer construction.

Another significant subject where optimization algorithms are widely used is model predictive control, which basically solves the optimal trajectory and optimal input signal problem for a limited horizon. For linear cases, since there is a structure to be exploited that results in less computational load, MPC is especially preferred. However, depending on the availability of computation power and the importance of the application, MPC is also deployed in many nonlinear systems [15,16]. Using MPC-based methods, it is also possible to address some specific controllability and observability problems for LTI or nonlinear systems. For example, it is possible to compute a linear static controller so that the given hard input signal amplitude constraints are satisfied, and the stability of the closed loop is guaranteed. Although recent compared to the linear quadratic controller-type control design techniques, the literature on this subject is quite rich.

In this study, another method that is used to compare against the EKF-based method and the line-search nonlinear optimization method is genetic algorithms. GAs are utilized in parameter estimation and many optimization problems, especially for the problems where the decision variable number is too high and/or the optimization problem cannot be written in closed form and only a cost function can be generated for a given decision variable. GAs are used to estimate the parameters of PMSM and the results of these three algorithms are analyzed [17]. The importance of the research can be stated as the comparison of three separate classes of parameter estimation methods: the EKF, a sequential MMSE-based method; a convex optimization-based identification algorithm; and a heuristic parameter estimation method GA. In electromechanical systems drive and in power electronics literature, comparisons of these type of separate classes of parameter estimation algorithms are not prevalent. Estimating the parameters by using the algorithms detailed in the paper can make it possible to obtain an adaptive control scheme that has the potential to mitigate the performance degradation due to parameter variation, and by monitoring the parameters can contribute to designing a fault-detection strategy that can be desired in a critical operation.

An EKF-based iterative parameter estimation algorithm and line search-based nonlinear optimization algorithm are given and the efficacies of the estimation methods are examined through a numerical simulation. The plant dynamics are given in Section 2. The parameter estimation techniques, EKF-based search algorithm, and line-search algorithm are detailed in Section 3. Validation of the performance simulation problem and the corresponding results are given in Section 4. Finally, in Section 5 the results are discussed.

2. Nonlinear Dynamical Model of PMSM

In this study, to test the parameter accuracy of the estimation algorithm that is to be presented in the next section, a PMSM model was given whose model possesses non-severe nonlinearities that are suitable for designing an EKF. There are four states, two of which are available for measuring, and five related system parameters to be estimated [18]. The lower and upper bounds of the parameters are considered to be known and the process and measurement noises have normal distributions. The plant model is given as

$$\frac{d}{dt}x_1 = \left(-\frac{P_1}{P_2}\right)x_1 + \left(\frac{P_3}{P_2}\right)x_3\sin(x_4) + \left(\frac{1}{P_2}\right)u_1 \tag{1}$$

$$\frac{d}{dt}x_2 = \left(-\frac{P_1}{P_2}\right)x_2 + \left(-\frac{P_3}{P_2}\right)x_3\cos(x_4) + \left(\frac{1}{P_2}\right)u_2 \tag{2}$$

$$\frac{d}{dt}x_3 = \left(-\frac{3P_3}{2P_4}\right)x_1\sin(x_4) + \left(\frac{3P_3}{2P_4}\right)x_2\cos(x_4) + \left(-\frac{P_5}{P_4}\right)x_3 \tag{3}$$

$$\frac{d}{dt}x_4 = x_3 \tag{4}$$

The relevant parameters to be estimated and their respective definitions and units are given in Table 1.

Table 1. The plant parameters and definitions.

Parameter	Unit	Definition
P_1	Ω	Winding resistance
P_2	H	Winding inductance
P_3	-	Magnetic flux constant of the motor
P_4	kg m^2	Moment of inertia of the rotor and the load
P_5	-	Viscous friction constant of the rotor

The system states are given in Table 2 with their respective units.

Table 2. The plant states and definitions.

Parameter	Unit	Definition
x_1	A	Direct-axis stator winding current
x_2	A	Quadrature-axis stator winding current
x_3	rad/s	Rotor mechanical rotation speed
x_4	rad	Rotor mechanical position

At this stage, to prevent the numerical problems that can arise due to division operation, a given change of parameters is introduced in Equations (5)–(9).

$$p_1 = -\frac{P_1}{P_2} \tag{5}$$

$$p_2 = \frac{P_3}{P_2} \tag{6}$$

$$p_3 = \frac{1}{P_2} \tag{7}$$

$$p_4 = \left(\frac{-3P_3}{2P_4}\right) \tag{8}$$

$$p_5 = \frac{-P_5}{P_4} \tag{9}$$

As a result of this change of variables, the nonlinear system model based on which the estimation algorithms are designed is given in Equations (10)–(13).

$$\frac{d}{dt}x_1 = p_1x_1 + p_2x_3\sin(x_4) + p_3u_1 + w_1 \tag{10}$$

$$\frac{d}{dt}x_2 = p_1x_2 - p_2x_3\cos(x_4) + p_3u_2 + w_2 \tag{11}$$

$$\frac{d}{dt}x_3 = p_4x_1\sin(x_4) - p_4x_2\cos(x_4) + p_5x_3 + w_3 \tag{12}$$

$$\frac{d}{dt}x_4 = x_3 + w_4 \tag{13}$$

With the introduced change of variables, the estimation algorithm was designed based on this model. However, the first two states were assumed to be measured as given in,

$$y_1 = x_1 + v_1 \tag{14}$$

$$y_2 = x_2 + v_2 \tag{15}$$

where the process noise and sensor noise are given by the w and v terms, respectively. Their statistics are given in the simulation section.

3. Parameter Estimation Algorithms

3.1. EKF-Based Parameter Estimation Algorithm

In this section, an EKF-based parameter estimation algorithm is presented for system identification purposes. As stated in the previous section, there are five parameters and the bounds of them are known. Two of the four states are measured, and the rest of the state signals are to be estimated, which results in seven variable estimations, five of which can be considered constant signals. Instead of extending the state vector by stacking the parameters and designing an observer for a nine-dimensional system with two measured states, which would result in numerical difficulties, five separate EKFs were designed. In each EKF, whose design steps are to be presented, four states and one parameter are considered while fixing the rest of the parameters. This type of iterative parameter estimation has a considerable computational complexity; however, compared to the one with larger dimensions and numerical stability issues, there may be some cases where the parameter estimation algorithm or a version thereof is desired. The overall scheme of the method is presented in the block diagram given in Figure 1.

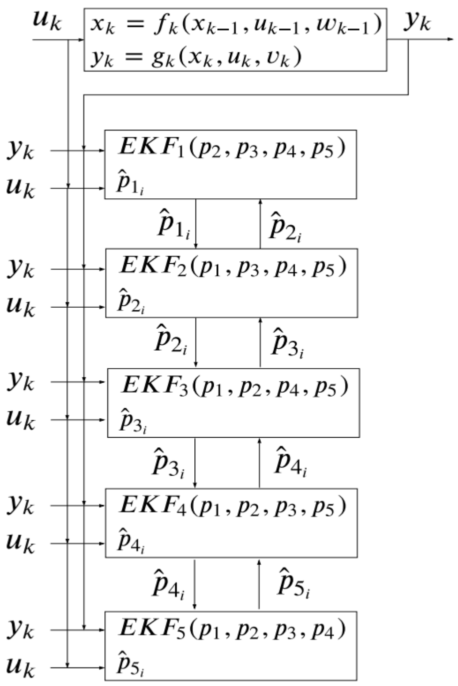

Figure 1. Block diagram and illustration of the EKF-based parameter estimation algorithm.

3.1.1. General EKF Design for Parameter Estimation

For the parameter estimation, the state vector is expanded by the parameters and a new dynamic artificial system is defined. The system is represented as

$$x_k = f_k(x_{k-1}, u_{k-1}, w_{k-1}) \tag{16}$$

$$y_k = g_k(x_k, u_k, v_k) \tag{17}$$

For the given system, four Jacobian matrices are defined as

$$A_k = \frac{\partial f_k(x_{k-1}, u_{k-1}, w_{k-1})}{\partial x_{k-1}}, B_{w_k} = \frac{\partial f_k(x_{k-1}, u_{k-1}, w_{k-1})}{\partial w_{k-1}} \tag{18}$$

$$C_k = \frac{\partial g_k(x_k, u_k, v_k)}{\partial x_k}, D_{v_k} = \frac{\partial g_k(x_k, u_k, v_k)}{\partial v_k} \tag{19}$$

The EKF steps are given below:

$$\hat{x}_{k|k-1} = f_k\left(\hat{x}_{k-1|k-1}, u_{k-1}\right) \tag{20}$$

$$\hat{y}_{k|k-1} = g_k\left(\hat{x}_{k-1|k-1}, u_{k-1}\right) \tag{21}$$

$$A_k = \left[\frac{\partial f_k(x_{k-1}, u_{k-1}, w_{k-1})}{\partial x_{k-1}}\right]\Bigg|_{\substack{x_{k-1} = \hat{x}_{k-1|k-1} \\ u_{k-1} = u_{k-1} \\ w_{k-1} = 0}} \tag{22}$$

$$B_{w_k} = \left[\frac{\partial f_k(x_{k-1}, u_{k-1}, w_{k-1})}{\partial w_{k-1}}\right]\Bigg|_{\substack{x_{k-1} = \hat{x}_{k-1|k-1} \\ u_{k-1} = u_{k-1} \\ w_{k-1} = 0}} \tag{23}$$

$$\Sigma_{\widetilde{x}\widetilde{x}_{k|k-1}} = A_k \Sigma_{\widetilde{x}\widetilde{x}_{k-1|k-1}} A_k^T + B_{w_k} Q_k B_{w_k}^T \tag{24}$$

$$C_k = \left[\frac{\partial g_k(x_k, u_k, v_k)}{\partial x_k}\right]\Bigg|_{\substack{x_k = \hat{x}_{k|k-1} \\ u_k = u_k \\ v_k = 0}} \tag{25}$$

$$D_{v_k} = \left[\frac{\partial g_k(x_k, u_k, v_k)}{\partial v_k}\right]\Bigg|_{\substack{x_k = \hat{x}_{k|k-1} \\ u_k = u_k \\ v_k = 0}} \tag{26}$$

$$K_k = \Sigma_{\widetilde{x}\widetilde{x}_{k|k-1}} C_k^T \left[C_k \Sigma_{\widetilde{x}\widetilde{x}_{k|k-1}} C_k^T + D_{v_k} R_k D_{v_k}^T\right] \tag{27}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left[y_k - \hat{y}_{k|k-1}\right] \tag{28}$$

$$\hat{y}_{k|k} = g_k\left(\hat{x}_{k|k}, u_k\right) \tag{29}$$

$$\Sigma_{\widetilde{x}\widetilde{x}_{k|k}} = [I - K_k C_k] \Sigma_{\widetilde{x}\widetilde{x}_{k|k-1}} \tag{30}$$

The related parameters are given in the Table 3 below.

Table 3. EKF-related terms and definitions thereof.

Parameter	Definition	Parameter	Definition
$\hat{x}_{k\|k-1}$	Predicted state	$\Sigma_{\widetilde{x}\widetilde{x}_{k\|k-1}}$	A priori state error covariance matrix
$\hat{y}_{k\|k-1}$	Predicted output	$\Sigma_{\widetilde{x}\widetilde{x}_{k\|k}}$	A posteriori state error covariance matrix
Q_k	Process noise covariance matrix	$\hat{x}_{k\|k}$	Estimated state
R_k	Measurement noise covariance matrix	$\hat{y}_{k\|k}$	Estimated output
$A_k, B_{w_k}, C_k, D_{v_k}$	Jacobian matrices Linearized around the current operation point	K_k	Kalman gain

3.1.2. EKF Design for p_1

The system on which the EKF is based is given as

$$\frac{d}{dt} x_1 = x_5 x_1 + p_2 x_3 \sin(x_4) + p_3 u_1 + w_1 \tag{31}$$

$$\frac{d}{dt} x_2 = x_5 x_2 - p_2 x_3 \cos(x_4) + p_3 u_2 + w_2 \tag{32}$$

$$\frac{d}{dt}x_3 = p_4 x_1 \sin(x_4) - p_4 x_2 \cos(x_4) + p_5 x_3 + w_3 \tag{33}$$

$$\frac{d}{dt}x_4 = x_3 + w_4 \tag{34}$$

$$\frac{d}{dt}x_5 = w_5 \tag{35}$$

This model is obtained by renaming p_1 as x_5 and additionally defining the x_5 dynamics. The term w_5 is introduced to make it possible for the observer to change the estimation value. The four prominent Jacobian matrices are given as

$$A_k = \begin{bmatrix} x_5 & 0 & p_2\sin(x_4) & p_2 x_3 \cos(x_4) & x_1 \\ 0 & x_5 & -p_2\cos(x_4) & p_2 x_3 \sin(x_4) & x_2 \\ p_4\sin(x_4) & -p_4\cos(x_4) & p_5 & p_4 x_1\cos(x_4) + p_4 x_2 \sin(x_4) & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} T_s + I \tag{36}$$

$$B_{w_k} = IT_s, C_k = \begin{bmatrix} I_2 & 0_{2x3} \end{bmatrix}, D_{v_k} = I_2 \tag{37}$$

The p_1 term is to be estimated in addition to the four state signals while four parameters are fixed and assumed to be known for this subsystem.

3.1.3. EKF Design for p_2

The system on which the EKF is based is given as

$$\frac{d}{dt}x_1 = p_1 x_1 + x_5 x_3 \sin(x_4) + p_3 u_1 + w_1 \tag{38}$$

$$\frac{d}{dt}x_2 = p_1 x_2 - x_5 x_3 \cos(x_4) + p_3 u_2 + w_2 \tag{39}$$

$$\frac{d}{dt}x_3 = p_4 x_1 \sin(x_4) - p_4 x_2 \cos(x_4) + p_5 x_3 + w_3 \tag{40}$$

$$\frac{d}{dt}x_4 = x_3 + w_4 \tag{41}$$

$$\frac{d}{dt}x_5 = w_5 \tag{42}$$

This model is obtained by renaming p_2 as x_5 and additionally defining the x_5 dynamics. The term w_5 is introduced to make it possible for the observer to change the estimation value. The Jacobian matrix related to the state vector is given as

$$A_k = \begin{bmatrix} p_1 & 0 & x_5\sin(x_4) & x_3 x_5 \cos(x_4) & x_3\sin(x_4) \\ 0 & p_1 & -x_5\cos(x_4) & x_3 x_5 \sin(x_4) & -x_3\cos(x_4) \\ p_4\sin(x_4) & -p_4\cos(x_4) & p_5 & p_4 x_1\cos(x_4) + p_4 x_2 \sin(x_4) & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} T_s + I \tag{43}$$

The p_2 term is to be estimated in addition to the four state signals while four parameters are fixed and assumed to be known for this subsystem.

3.1.4. EKF Design for p_3

The system on which the EKF is based is given as

$$\frac{d}{dt}x_1 = p_1 x_1 + p_2 x_3 \sin(x_4) + x_5 u_1 + w_1 \tag{44}$$

$$\frac{d}{dt}x_2 = p_1 x_2 + p_2 x_3 \cos(x_4) + x_5 u_2 + w_2 \tag{45}$$

$$\frac{d}{dt}x_3 = p_4 x_1 \sin(x_4) - p_4 x_2 \cos(x_4) + p_5 x_3 + w_3 \tag{46}$$

$$\frac{d}{dt}x_4 = x_3 + w_4 \tag{47}$$

$$\frac{d}{dt}x_5 = w_5 \tag{48}$$

This model is obtained by renaming p_3 as x_5 and additionally defining the x_5 dynamics. The term w_5 is introduced to make it possible for the observer to change the estimation value. The Jacobian matrix related to the state vector is given as

$$A_k = \begin{bmatrix} p_1 & 0 & p_2 \sin(x_4) & p_2 x_3 \cos(x_4) & u_1 \\ 0 & p_1 & -p_2 \cos(x_4) & p_2 x_3 \sin(x_4) & u_2 \\ p_4 \sin(x_4) & -p_4 \cos(x_4) & p_5 & p_4 x_1 \cos(x_4) + p_4 x_2 \sin(x_4) & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} T_s + I \tag{49}$$

3.1.5. EKF Design for p_4

The system on which the EKF is based is given as

$$\frac{d}{dt}x_1 = p_1 x_1 + p_2 x_3 \sin(x_4) + p_3 u_1 + w_1 \tag{50}$$

$$\frac{d}{dt}x_2 = p_1 x_2 - p_2 x_3 \cos(x_4) + p_3 u_2 + w_2 \tag{51}$$

$$\frac{d}{dt}x_3 = x_5 x_1 \sin(x_4) - x_5 x_2 \cos(x_4) + p_5 x_3 + w_3 \tag{52}$$

$$\frac{d}{dt}x_4 = x_3 + w_4 \tag{53}$$

$$\frac{d}{dt}x_5 = w_5 \tag{54}$$

This model is obtained by renaming p_4 as x_5 and additionally defining the x_5 dynamics. The term w_5 is introduced to make it possible for the observer to change the estimation value. The Jacobian matrix related to the state vector is given as

$$A_k = \begin{bmatrix} p_1 & 0 & p_2 \sin(x_4) & p_2 x_3 \cos(x_4) & 0 \\ 0 & p_1 & -p_2 \cos(x_4) & p_2 x_3 \sin(x_4) & 0 \\ x_5 \sin(x_4) & -x_5 \cos(x_4) & p_5 & x_5 x_1 \cos(x_4) + x_5 x_2 \sin(x_4) & x_1 \sin(x_4) - x_2 \cos(x_4) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} T_s + I \tag{55}$$

The p_4 term is to be estimated in addition to the four state signals while four parameters are fixed and assumed to be known for this subsystem.

3.1.6. EKF Design for p_5

The system on which the EKF is based is given as

$$\frac{d}{dt}x_1 = p_1 x_1 + p_2 x_3 \sin(x_4) + p_3 u_1 + w_1 \tag{56}$$

$$\frac{d}{dt}x_2 = p_1 x_2 - p_2 x_3 \cos(x_4) + p_3 u_2 + w_2 \tag{57}$$

$$\frac{d}{dt}x_3 = p_4 x_1 \sin(x_4) - p_4 x_2 \cos(x_4) + x_5 x_3 + w_3 \tag{58}$$

$$\frac{d}{dt}x_4 = x_3 + w_4 \tag{59}$$

$$\frac{d}{dt}x_5 = w_5 \tag{60}$$

This model is obtained by renaming p_5 as x_5 and additionally defining the x_5 dynamics. The term w_5 is introduced to make it possible for the observer to change the estimation value. The Jacobian matrix related to the state vector is given as

$$A_k = \begin{bmatrix} p_1 & 0 & p_2\sin(x_4) & p_2x_3\cos(x_4) & 0 \\ 0 & p_1 & -p_2\cos(x_4) & p_2x_3\sin(x_4) & 0 \\ p_4\sin(x_4) & -p_4\cos(x_4) & x_5 & p_4x_1\cos(x_4)+p_4x_2\sin(x_4) & x_3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} T_s + I \tag{61}$$

The p_5 term is to be estimated in addition to the four state signals while four parameters are fixed and assumed to be known for this subsystem.

3.2. Line Search-Based Nonlinear Optimization Method

For parameter estimation, a line search-based nonlinear optimization is used. First, the problem is stated such that input signal history and output signal history are available and the terms that are desired to be computed are initial conditions of the state variables and parameters of the nonlinear PMSM dynamical system. The optimization algorithm focuses not only the parameter computation but also on the initial conditions of the state variables. This strategy may result in a higher dimensional parameter estimation problem, but as is the case in constrained optimization, manipulating the problem in such a way that the dimension is increased may prove useful [19]. Since the relation between input and output signals is affected not only by the internal parameters but also by the initial values of the state variables, and for the cases where at least some bound is available to the algorithm, it is reasonable to expand the decision variable dimension. In the original problem it is stated that the state space dimension is four, and the number of parameters to be estimated is five. Therefore, the problem is expressed as the optimization of those nine numerical values. The model of the nonlinear dynamics is available and it is assumed that the output signal is available, and for a given value of decision variables, the cost of the optimization problem can be constructed as a mean square error of the real and the estimated one. Therefore, the optimization problem can be stated as

$$\begin{array}{cc} \underset{\substack{\hat{p}_1,\ldots,\hat{p}_5 \\ \hat{x}_1(0),\hat{x}_2(0),\hat{x}_3(0),\hat{x}_4(0)}}{\text{min.}} & \left\{ \sum_{k=0}^{N} \left[(x_1(k)-\hat{x}_1(k))^2 + (x_2(k)-\hat{x}_2(k))^2 \right] \right\} \\ \underset{\forall k=0,\ldots,N}{\text{s.t.}} & \left\{ \begin{array}{c} \hat{x}_1(k+1)-\hat{x}_1(k) = T_s[\hat{p}_1\hat{x}_1(k)+\hat{p}_2\hat{x}_3(k)\sin(\hat{x}_4(k))+\hat{p}_3u_1(k)] \\ \hat{x}_2(k+1)-\hat{x}_2(k) = T_s[\hat{p}_1\hat{x}_2(k)-\hat{p}_2\hat{x}_3(k)\cos(\hat{x}_4(k))+\hat{p}_3u_2(k)] \\ \hat{x}_3(k+1)-\hat{x}_3(k) = T_s[\hat{p}_4\hat{x}_1(k)\sin(\hat{x}_4(k))-\hat{p}_4\hat{x}_2(k)\cos(\hat{x}_4(k))+\hat{p}_5\hat{x}_3(k)] \\ \hat{x}_4(k+1)-\hat{x}_4(k) = T_s[\hat{x}_3(k)] \end{array} \right\} \end{array} \tag{62}$$

However, for computational considerations the problem can be dissected into smaller sequential optimization parameters and the problem can be stated as follows: First, random values are assigned at the each of the decision variables subject to their box constraints. In the optimization stage, only $\hat{p}_1$ is optimized while keeping other decision variables fixed. This problem of optimizing over a single variable can be stated as a line search, as the parameter takes values between the stated range. There are golden-section and Fibonacci-search algorithms to reduce the computational burden of the problem. When the optimization ends, the next decision variable, i.e., $\hat{p}_2$, is optimized while keeping other decision variables fixed using the updated value of $\hat{p}_1$ determined in the previous step. The process continues until convergence. It should be noted that for severe multi-model functions this method faces some local-minima issues [20]. However, considering this algorithm only computes the state trajectories and the cost function value, at multiple times in each step instead of performing a matrix inversion, it is computationally more favorable to the iterated EKF parameter estimation technique stated in the previous section.

3.3. Genetic Algorithm Parameter Estimation

Genetic algorithms are a class of global optimization tools influenced by evolution [21]. It searches the parameter space to find a global minimum for a given evaluation function with use of mutation and crossover followed by the selection of a population of parameters. Even though the iterative nature of GAs guarantees no worse performance in the subsequent generations, depending on the non-linearity of the evaluation function with respect to the parameters, GAs can get stuck on local minimums. GAs can also be sensitive to the values of mutation and crossover rate as well as the choice of a selection technique [22]. Despite this, GAs are suitable candidates for non-convex optimization-related problems and for problems where the typical assumptions of the optimization problem are not valid, nor where the conventional methods are infeasible to implement.

A good choice for the evaluation function or the cost function of the GA can again be as follows:

$$J = \sum_{k=0}^{N} \Big[(x_1(k) - \hat{x}_1(k))^2 + (x_2(k) - \hat{x}_2(k))^2 \Big] \tag{63}$$

This is due to our assumption of process and sensor noise being zero-mean Gaussian. One benefit of this type of parameter estimation is the ease of extendibility to find a more statistically reliable answer with the inclusion of simulation results with different sets of initial conditions. Then, the cost function can be altered as

$$\sum_{i=0}^{M} \sum_{k=0}^{N} \Big[(x_{i,1}(k) - \hat{x}_{i,1}(k))^2 + (x_{i,2}(k) - \hat{x}_{i,2}(k))^2 \Big] \tag{64}$$

where M denotes the number of simulations with different sets of initial conditions. Despite this, GAs are suitable candidates for non-convex optimization-related problems and for problems where the typical assumptions of the optimization problem are not valid or the conventional methods are infeasible to implement.

4. Numerical Simulations

In this section, the numerical simulations that were conducted are discussed. Table 4 below states the important values that were used in the simulation.

Table 4. Simulation settings.

Terms	Values
Duration	5 sec
T_s	0.01 sec
$x(0)$	$[0.1, 0.1, 0.1, 0.1]$
p_{lower}	$[-5, 1, 1, -5, -5]$
p_{upper}	$[-0.1, 20, 20, -0.1, -0.1]$
$\sigma^2_{pro-noise}$	$10^{-4} I_4$
$\sigma^2_{sen-noise}$	$10^{-4} I_2$
$u_1(t)$	$1\sin(2\pi[1]t)$
$u_2(t)$	$1\sin\big(2\pi[1]t + \frac{\pi}{2}\big)$
$[p_1, p_2, p_3, p_4, p_5]$	$[-1, 10, 10, -1.5, -1]$

4.1. Simulation for EKF-Based Parameter Estimation Algorithm

For the given simulation-related terms, the EKF-based parameter estimation algorithm was simulated, and the efficacy of the algorithm is presented in Figure 2. In the simulation, to represent the fact that the initial point of the estimation is independent of the convergence properties of the estimation algorithm, the different initial point of estimation was employed and simulated for the same nonlinear model of the PMSM. It is noted that the parameter estimates converged to the real values under the disturbing effects of the process and measurement noise presence.

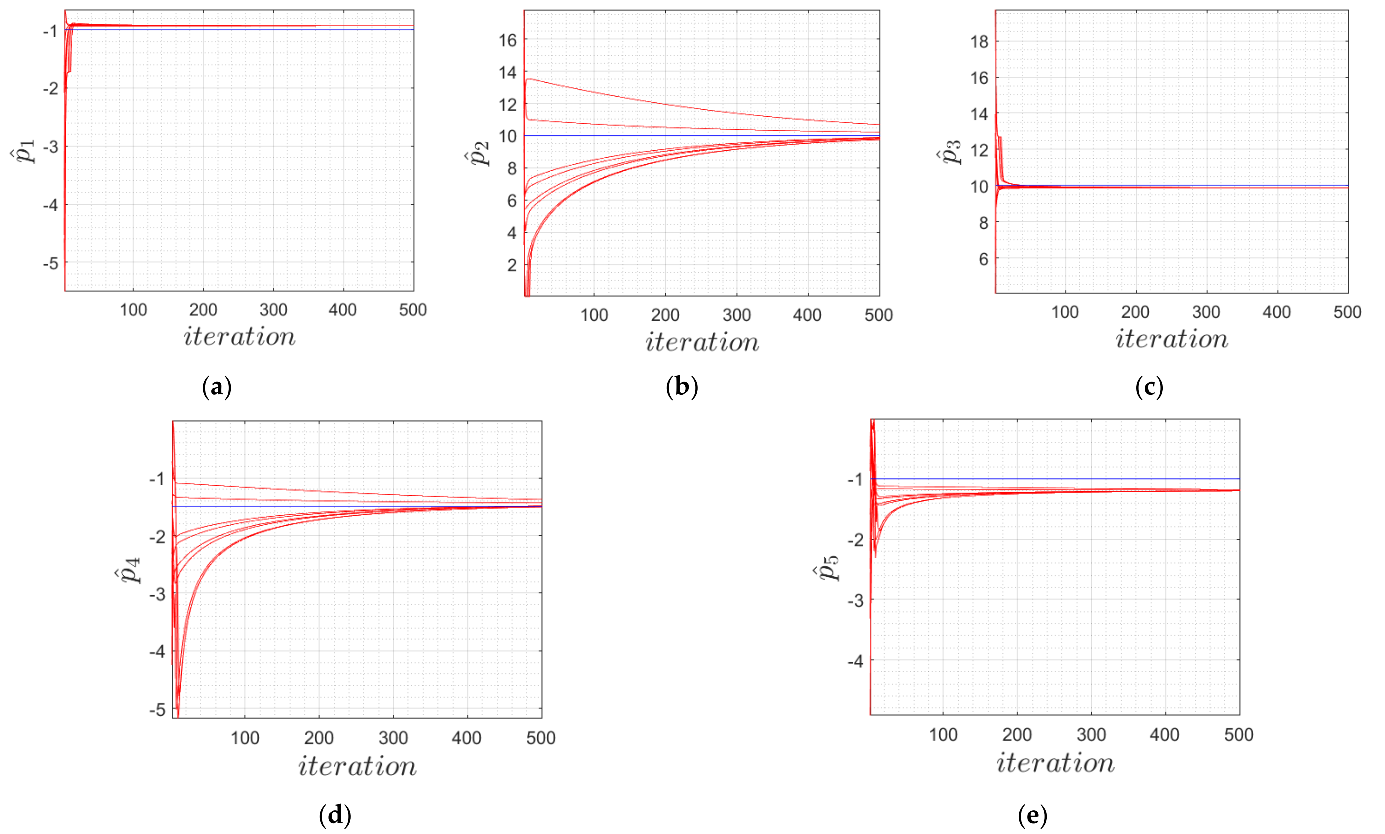

Figure 2. Convergence of the estimated parameters using EKF-based parameter estimation. (**a**) $\hat{p}_1$ convergence, (**b**) $\hat{p}_2$ convergence, (**c**) $\hat{p}_3$ convergence, (**d**) $\hat{p}_4$ convergence, (**e**) $\hat{p}_5$ convergence.

4.2. Simulation of Line Search-Based Nonlinear Optimization Algorithm

For the given simulation-related terms, a line search-based algorithm was simulated and the efficacy of the algorithm is presented in Figure 3. For the demonstration of the consistency of the algorithm, four different initial estimates were given and the convergence of the algorithm was observed.

4.3. Simulation for GA Optimization

To compare the performance of the two previously presented estimators, another optimization GA was used to estimate both the initial conditions of the state variables and the parameters. The performance is illustrated in Figure 4. It can be seen that there was a dependence on the initial estimate values of the parameter estimates. Since the GA focused on minimizing the objective function using the parameters but not in a sequential manner, as was the case in the optimization algorithm previously stated, this type of local minimum convergence can be seen. It can be also noted that there were some procedures to prevent the GA from converging to a local minimum to an extent, but since this algorithm is only included for comparative purposes, these details are not presented here.

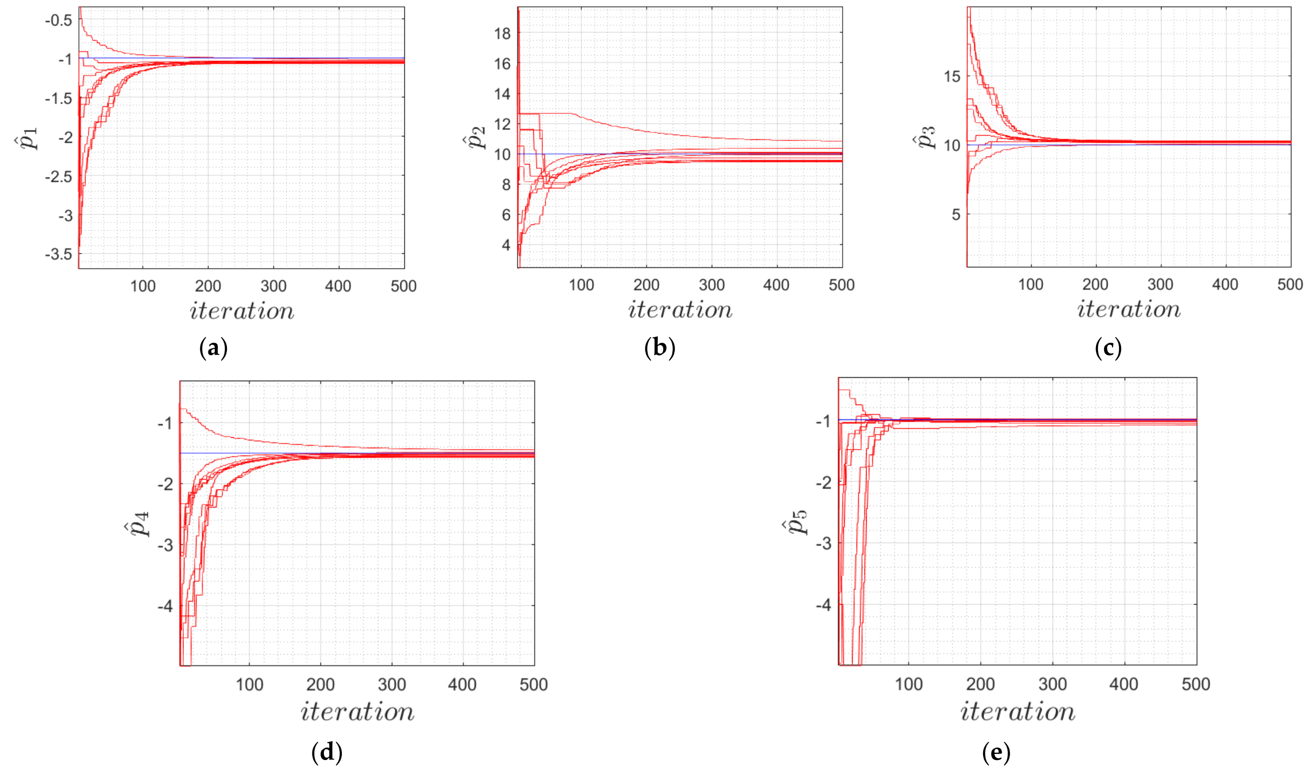

Figure 3. Convergence of the estimated parameters using line search-based nonlinear optimization algorithm. (**a**) $\hat{p}_1$ convergence, (**b**) $\hat{p}_2$ convergence, (**c**) $\hat{p}_3$ convergence, (**d**) $\hat{p}_4$ convergence, (**e**) $\hat{p}_5$ convergence.

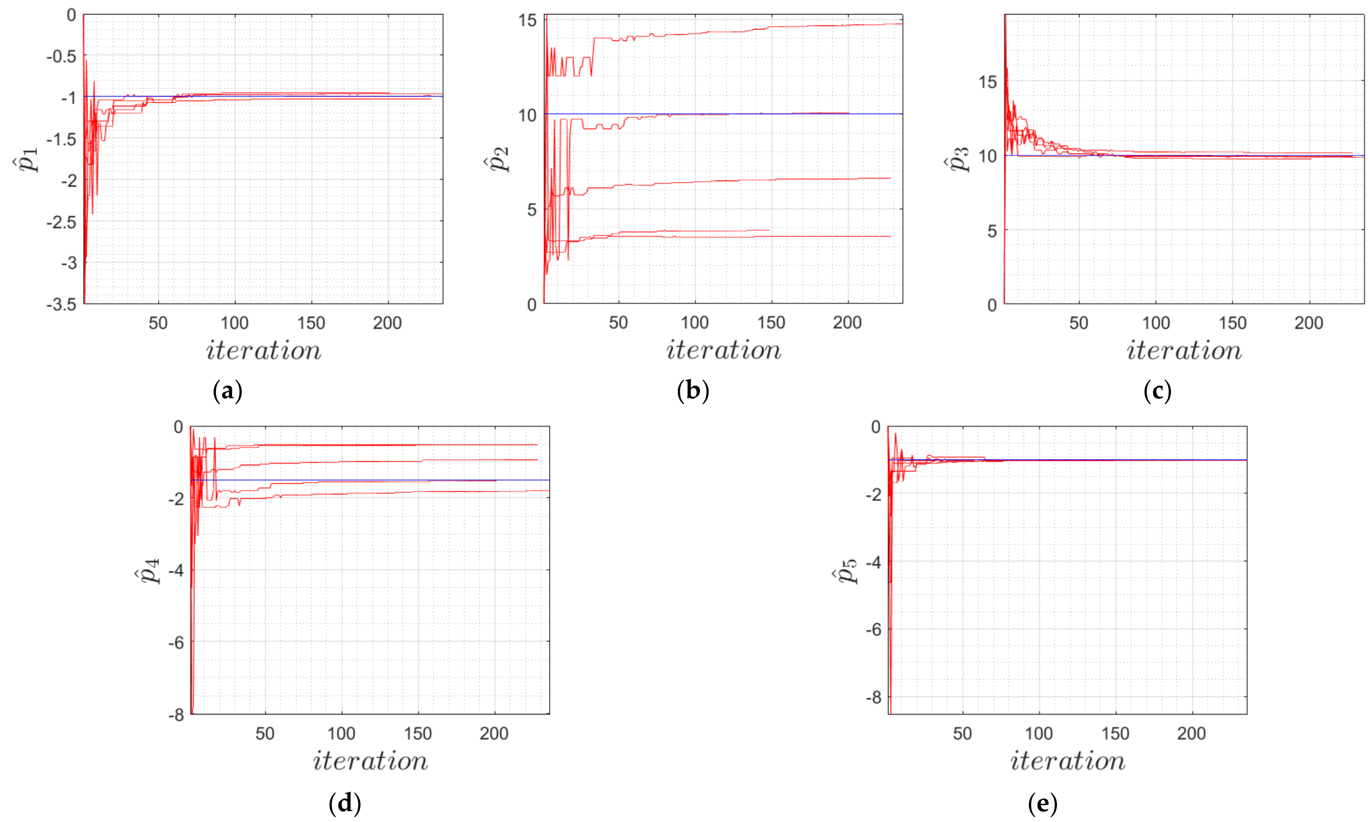

Figure 4. Convergence of the estimated parameters using GA optimization. (**a**) $\hat{p}_1$ convergence, (**b**) $\hat{p}_2$ convergence, (**c**) $\hat{p}_3$ onvergence, (**d**) $\hat{p}_4$ convergence, (**e**) $\hat{p}_5$ convergence.

5. Conclusions

In this study, PMSM, whose nonlinear dynamical model is given, was studied. The parameters of the system were estimated using the stated iterated EKF-based parameter estimation algorithm and the line search-based method. For both of the cases, it was seen that the convergence of the estimates was independent of the initial estimate points given to the estimation algorithms, and for none of the starting points was a divergence observed, which indicates that the algorithms performed reliably. Additionally, for comparison purposes, GAs were used to find the parameters and the initial values of the state variables. The numerical simulations were conducted in MATLAB. For the computational aspects of the study, the matrix inversion operation that is present in the EKF-based method cause an additional computational burden. Although, as the simulation results indicate, the performances were similar, the line search-based nonlinear optimization method outperformed in terms of time complexity. It is important to point out as final conclusions that these types of parameter estimation analyses are significant in designing a parameter estimator that is desired to be run on-line due to the fact that these analysis techniques or variations thereof can conclude which parameters can be estimated or which parameters are easier to estimate. For parallel parameter estimation operations or fault-detection mechanisms, these questions are fundamental.

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