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Modeling Wind Speed Based on Fractional Ornstein-Uhlenbeck Process

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Abstract: The primary task of the design and feasibility study for the use of wind power plants is to predict changes in wind speeds at the site of power system installation. The stochastic nature of the wind and spatio-temporal variability explains the high complexity of this problem, associated with finding the best mathematical modeling which satisfies the best solution for this problem. In the known discrete models based on Markov chains, the autoregressive-moving average does not allow variance in the time step, which does not allow their use for simulation of operating modes of wind turbines and wind energy systems. The article proposes and tests a SDE-based model for generating synthetic wind speed data using the stochastic differential equation of the fractional Ornstein-Uhlenbeck process with periodic function of long-run mean. The model allows generating wind speed trajectories with a given autocorrelation, required statistical distribution and provides the incorporation of daily and seasonal variations. Compared to the standard Ornstein-Uhlenbeck process driven by ordinary Brownian motion, the fractional model used in this study allows one to generate synthetic wind speed trajectories which autocorrelation function decays according to a power law that more closely matches the hourly autocorrelation of actual data. In order to demonstrate the capabilities of this model, a number of simulations were carried out using model parameters estimated from actual observation data of wind speed collected at 518 weather stations located throughout Russia.

Keywords: wind energy; wind speed model; stochastic differential equations; fractional Brownian motion; time-series modeling

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1. Introduction

Currently, most of the budget of the Russian Federation is provided by revenues from the sale of oil and gas, which determines the leading role of the oil and gas industry in the social-economic development of the country. The development of the oil and gas sector requires updates and the development for new fields of trunk oil and new gas pipelines which are mainly located in areas remote from the central electric grid. Thus, reliable and efficient power supply utilizing technological facilities in decentralized regions is an important and urgent task of the oil and gas industry, which determines high requirements for the reliability of power supply systems. The well-known standard designs of power supply systems do not always meet the established reliability requirements, which requires the use of new technical solutions. One of such solutions is the use of wind power plants in power supply systems.

The positive experience of using offshore wind power plants for the power supply of offshore oil fields is given in [1]. A thorough assessment of the prospects for the use of wind turbines for the power supply of the gas industry facilities was obtained in [2,3]. The results of the research carried out prove that the use of wind turbines in the power supply systems of technological facilities in the oil and gas industry provides a decrease in overall energy consumption and the cost of extracted geo resources, as well as increases environmental and energy security in areas of decentralized energy supply.

To solve the problems of feasibility and design of wind energy systems, it is necessary to have reliable forecasting of wind speeds at short and long time intervals is required. The stochastic nature of the wind and its great variability in time determine the high complexity of this problem. The solution of this problem requires methods of mathematical modeling.

Wind speed can be described by stochastic processes, the characteristics of which are:

- Probability Density Function (PDF) describing the statistical distribution of wind speeds that depend on the specific site and used to estimate potential energy production [4].
- Autocorrelation Function (ACF) that shows the strength of the relationship between two successive values of the same time series [5]. Its accurate modeling is necessary to improve the reliability of forecasting wind speed fluctuations over short time intervals.
- Systematic daily and seasonal cycles [6] which significantly affect the performance of wind turbines [7] and whose modeling is necessary to predict electricity generation [8]; analyze the aerodynamic interaction between wind turbines in wind farms [9]; and coordinate the operation modes of hybrid RES-based power supply systems.

The initial data for modeling power systems based on wind turbines are the time series of meteorological observations accumulated over a long period of time at meteorological stations. In the task of modeling and evaluating the performance of wind farms, data are required at least with an hourly interval between observations, while most meteorological stations in the territory of the Russian Federation observations taken 8 or 4 times per day with a time interval between observations equal to three and six hours, respectively. This problem necessitates the development of models that allow synthesizing a time series with the required time resolution while maintaining statistical characteristics (autocorrelation, statistical distribution, and non-stationary cyclic components). This work is devoted to the study of this type of model based on a stochastic differential equation. The remainder of this paper is organized in the following manner. Section 2 provides an analysis of the sources devoted to the research topic. Section 3 elaborates on our methods and algorithms. Section 4 describes the estimation of model parameters of research. The model validation and the obtained results are discussed in Section 5. In conclusion, the research findings are recapitulated.

2. Literature Review

Wind speed models based on AR/MA, Markov chain models and other discrete models [10–12] do not allow for the variation of simulation time step simulation. This limitation does not allow the use of these models for the simulation of wind power systems with a high degree of time sampling, which is a necessary condition for selecting the optimal configuration of power-generating equipment and analyzing steady-state and transient processes in RES-based electric power systems.

In article [13] an overview of predictive models is provided and a short description of a hybrid approach is introduced based on a neural network with long-term short-term memory; a method of hierarchical evolutionary decomposition using an improved optimization algorithm for tuning hyper-parameters. In article [14] an approach to increase the level of accurate forecasting of wind energy is proposed based on the use of met heuristic evolutionary algorithms (GA, PSO) to adjust the parameters of a neural network (in this case, GA is used as a meta-optimization algorithm in order to find the optimal PSO parameters). In [15], a hybrid forecasting technique is proposed, the principle of which is to decompose time series data and perform forecasting using a recurrent neural network and error correction methods. In paper [16] the use of a long-term short-term memory (LSTM) network for modeling and forecasting energy consumption is considered. In general, the results presented in the listed works show a significant correlation with actual data and a significant predictive ability of such models in comparison with traditional forecasting methods.

Models based on stochastic differential equations (SDE) can also be used to model wind speed, the construction method of which is described in articles [17–19]. Given that the stochastic process described by the SDE is continuous, the model based on it allows one to change the simulation step in time, depending on the purpose of the study. Moreover, the model based on the SDE does not require a training procedure and significant recalculation of parameters when changing the temporal resolution.

In [17,18], models based on the SDE of the standard Ornstein-Uhlenbeck process are proposed and tested, which allow synthesizing wind speed trajectories with an exponential decay of the autocorrelation function and a given statistical distribution obtained by means of memoryless transformation. However, as the authors note, the exponential form of the correlation between the levels of the wind speed time series is a special case and rarely agrees with the actual autocorrelation characteristic over long time intervals, which can be more adequately described by a power function. Moreover, the model does not allow for the possibility of reproducing the non-stationary components of the time series (daily and seasonal fluctuations), which is necessary when assessing the productivity of wind power plants. This drawback is eliminated in [19], which describes a stochastic model based on SDE, which allows for the generation of a time series of wind speed with a daily cyclic component, but the possibility of modeling the seasonal component is not provided.

The purpose of this work is to develop a wind speed model which can be used for the simulation of wind energy system operating modes at different time intervals and one that is able to capture certain wind speed time series characteristics include autocorrelation, probability distribution, and provide the incorporation of non-stationary daily and seasonal components. The basis for the construction of such a model was the method of modeling the wind speed based on the SDE of the standard Ornstein-Uhlenbeck process, described in [17,18]. Taking into account that the autocorrelation of real processes occurring in nature rarely correspond to an exponential function, in this work we investigated the possibility of using a model based on the fractional Ornstein-Uhlenbeck process, which allows for the generation of synthetic wind speed data with the property of long-range dependence and autocorrelation, a function that decreases according to a power law.

3. Methods and Algorithms

The algorithm for the generation of synthetic wind speed by SDE consists of three stages: model calibration, which consists in estimating the parameters of the model using data of real long-term observations, numerical simulation of SDE, and transforming the distribution of the obtained auto correlated sequence to given distribution corresponding to the data of real observations. The flowchart of the simulation algorithm is shown in Figure 1.

To simulate the stochastic component of the wind speed, the Ornstein-Uhlenbeck process driven by fractional Brownian motion is used. A stochastic differential equation of this process can be written in the following form [20]:

$$dX_t = \theta \cdot (\mu - X_t) \cdot dt + \sigma \cdot dW_t^H, \tag{1}$$

where θ—mean-reversion rate (θ > 0), μ—long-run mean, σ—diffusion parameter, W_t^H—fractional Brownian motion with Hurst exponent $H \in (0, 1)$.

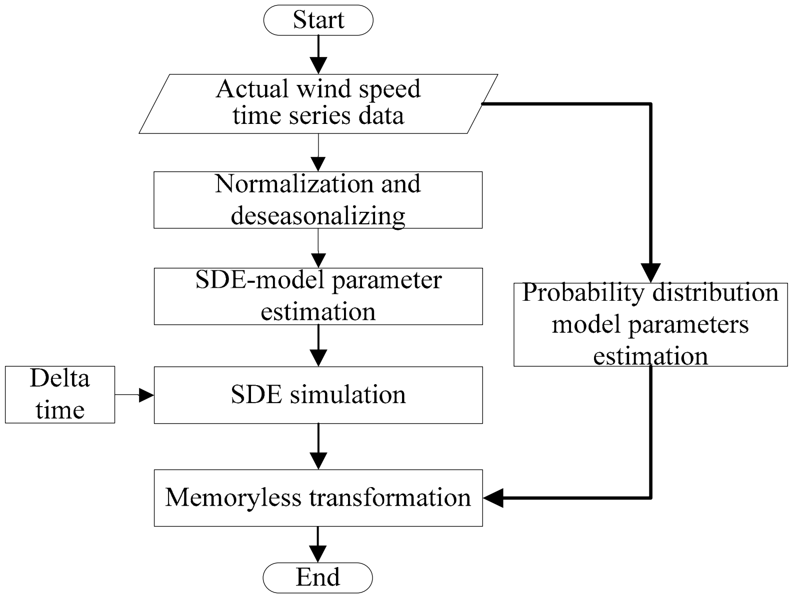

Figure 1. Wind speed simulation flowchart.

The fractional Brownian motion (fBm) is centered on the Gaussian process with continuous sample paths and the covariance function is given by [21]:

$$E\left(W_t^H, W_s^H\right) = \frac{1}{2}\left(|s|^{2H} + |t|^{2H} - |t-s|^{2H}\right) \tag{2}$$

For $H = 1/2$, the fBm is reduced to a standard Brownian motion where increments are independent (Wiener process). In this case SDE (1) describes the stationary Gauss-Markov process with exponentially decreasing autocorrelation. For $H > 1/2$, the process has the property of a long-range dependence and autocorrelation function that decays along a power-law.

The original form of SDE (1) describes wind speed as a stationary process with given autocorrelation where decay characteristics depends on the Hurst exponent parameter and mean-reverting rate parameter. To simulate the wind speed variations during the day, the periodic time dependent function (3) used as a long-run mean term in Equation (1):

$$y(t) = \alpha \cdot \cos\left(\frac{2\pi\left(t - t_{peak}\right)}{24}\right) \tag{3}$$

where α—daily wind speed amplitude parameter ($0 \leq \alpha \leq 1$), t—simulation time, and t_{peak}—hour of daytime peak wind speed ($0 \leq t_{peak} \leq 24$).

Taking into account this component, the initial Equation (1) is transformed into the following form [22]:

$$dX_t = \left(\frac{dy(t)}{dt} + \theta \cdot [y(t) - X_t]\right) \cdot dt + \sigma \cdot dW_t^H \tag{4}$$

The numerical simulation of the stochastic process described by Equation (3) is performed using the Euler-Maruyama scheme, suitable for discretization of the fractional Ornstein-Uhlenbeck process with long memory ($H \geq 1/2$) [23]:

$$\hat{X}_{n+1} = \hat{X}_n + \left[dy(t_n) + \ \hat{\theta}\ \cdot \left(y(t_n) - \hat{X}_n\right)\right] \cdot \Delta t + \hat{\sigma} \cdot \left(W_{n+1}^H - W_n^H\right), \tag{5}$$

where Δt—delta time, t—simulated time, W_n^H—fractional Gaussian noise with $H > 1/2$.

In the simulation process, the circulant embedding method [24] is used to generate fractional Gaussian noise.

Since the realizations of the stochastic process have a normal distribution with zero mean and unity variance, it is necessary to transform them into a distribution form corresponding to the real observation data. In this study, the two-parameter Weibull distribution is chosen to approximate statistical distribution of actual wind speed data.

Memoryless transformation method [17] is used to obtain a sequence with a given distribution:

$$V = F^{-1}[\Phi(X)] \tag{6}$$

where $\Phi(\cdot)$—cumulative distribution function of Normal distribution, F^{-1}—inverse cumulative distribution function of given distribution.

In order to simulate seasonal variations each value of the simulated trajectory $X(t)$ is sequentially transformed into a Weibull distribution variable $V(t)$ with parameters (c,k) defined for each month (i):

$$V(t) = c_i(t) \cdot [-\ln(1 - \Phi[X(t)])]^{1/k_i(t)}, \tag{7}$$

where $V(t)$—simulated wind speed sequence with given distribution; c_i, k_i—scale and shape parameters of Weibull distribution estimated separately for each month of observed data ($i = 1, 2 \ldots 12$).

4. Model Parameters Estimation

In the process of model calibration, an assessment of the following parameters is required: the Hurst exponent, diffusion and mean reversion rate parameters, daily mean amplitude, and hour of maximum of the wind speed, as well as the probability distribution model parameters. At the first stage, the parameters of the distribution model are estimated, for which the sample of initial data is divided by month into 12 groups and estimates of the Weibull distribution parameters are determined for each group. Before evaluating the rest of the model parameters, a series of sample data should be normalized so that the sample mean and variance are equivalent to the mean and variance of the stochastic process. In addition, the seasonal monthly component should be removed from the time series data.

4.1. Hurst Exponent Estimator

The aggregated variance method was used to estimate the Hurst parameter [25]. The original time series data is divided into blocks (k) of size (m) and for each block the average values are calculated as:

$$X^{(m)}(k) = \frac{1}{m} \cdot \sum_{i=(k-1)m+1}^{km} X(i) \quad k = 1, 2 \ldots N/m \tag{8}$$

where N—sample size, m—size of aggregated time-series block, k—the sequence number of the aggregated block.

Since the variance of fractional Gaussian noise is $Var X^{(m)} = \sigma_0 m^{\beta}$ as $m \to \infty$, where $\beta = 2H - 2 < 0$, then its estimate is the sample variance $\overline{Var} X^{(m)}$, which should be calculated for multiple values (m) as follows:

$$\overline{Var} X^{(m)} = \frac{1}{N/m} \sum_{k=1}^{N/m} \left(X^{(m)}(k)\right)^2 - \left(\frac{1}{N/m} \sum_{k=1}^{N/m} X^{(m)}(k)\right)^2 \tag{9}$$

The size of blocks (m) are chosen so that the points $\log(m)$ on the plot $\log\left(\overline{Var} X^{(m)}\right)$ versus $\log(m)$ are equidistant from each other, i.e., $m_{i+1}/m_i = C$, where C is a constant depending on the sample size and the desired number of points. Then the dependence can be approximated by a straight line, the slope of which is equal to (β) and the estimate of the Hurst exponent is defined as $\hat{H} = \frac{\beta+2}{2}$.

4.2. Diffusion Parameter Estimator

The diffusion parameter (σ) of SDE can be estimated by quadratic variations of observed time-series data using the following formula [26]:

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{N-1}\left(X_{(i+1)} - X_i\right)^2}{(N-1)\cdot \Delta t^{2\hat{H}}} \tag{10}$$

where X_i, $X_{(i+1)}$—time series values at the i-time step and next time step, Δt—time step of observations, $\hat{H}$—estimated Hurst exponent of time series data, N—sample size.

4.3. Mean-Reversion Rate Estimator

Since the wind speed can be considered as a locally stationary process with a long-term dependence ($H > 1/2$ and $\theta > 0$), the mean-reversion rate parameter of the fractional Ornstein-Uhlenbeck process can be evaluated using the ergodic type estimator [27]:

$$\hat{\theta} = \left[\frac{1}{\hat{\sigma}^2 \hat{H}\Gamma(2\hat{H})N} \cdot \sum_{i=1}^{N} X_i^2\right]^{-\frac{1}{2\hat{H}}} \tag{11}$$

where X_i—time-series data, Γ—gamma-function, $\hat{\sigma}$—estimated diffusion parameter, by Equation (10), $\hat{H}$—Hurst exponent estimate, N—sample size.

4.4. Estimation Parameters of Periodic Function of Daily Pattern

The evaluation of the parameters of the amplitude and time of the maximum daily average wind speed is carried out by fitting the curve of periodic Equation (3) to the points of the actual daily profile, the values of which for each observation period are calculated according to the following formula:

$$DP(\tau) = \frac{1}{N/n_\tau} \cdot \sum_{i=0}^{(N/n_\tau)-1} X_{(i\cdot n_\tau)+\tau}, \ \tau = 1, 2 \ldots n, \tag{12}$$

where N—sample size, n_τ—total number of observations, τ—index of observation time point.

5. Model Validation and Results

Validation of the model consists of assessing the ability of the model to generate a synthetic time series of wind speed, the statistical characteristics of which, including autocorrelation, probability distribution, daily and seasonal variability are consistent with the corresponding characteristics of the actual wind speed data. For the analysis, this study used a large set of time series of wind speed data collected at 518 weather stations throughout Russia provided by All-Russian Institute of Hydrometeorological Information [28]. The time series contains data from eight-term observations recorded at standard synoptic times with a time interval equal to three hours. The geographic locations of all weather stations are shown in Figure 2.

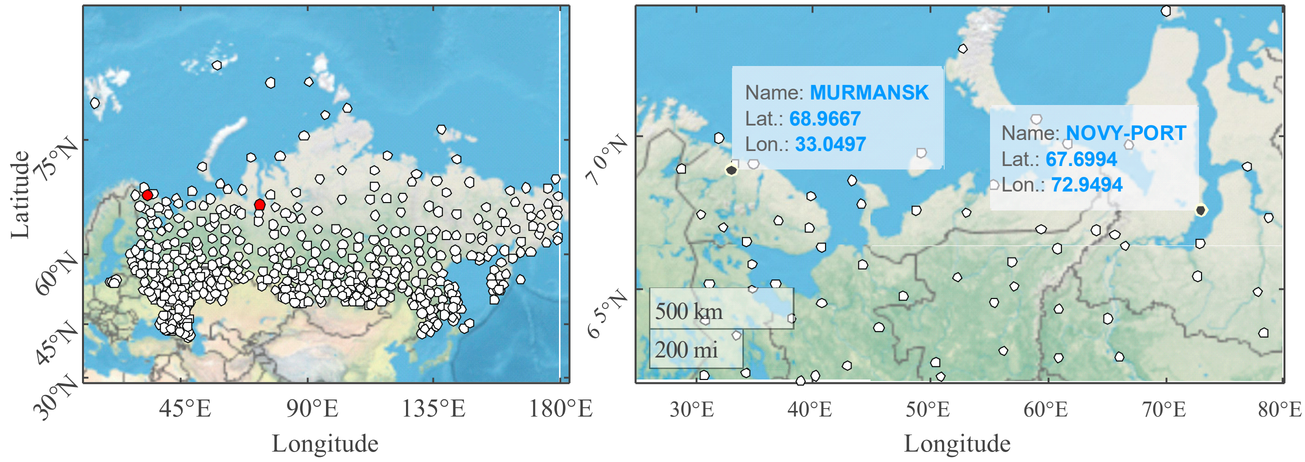

Figure 2. Map of weather stations locations.

For the separate data of all meteorological stations, the model parameters were estimated and a simulation was performed. The distribution of which is shown in the Figure 3.

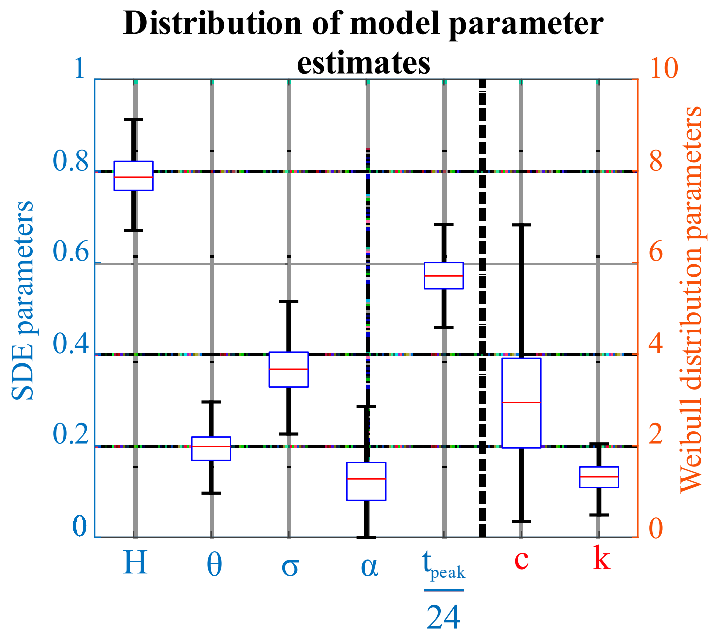

Figure 3. Boxplot of estimated model parameters.

Most of the original time series used have a time range from 1966 to 2019 (~91.5%), the minimum interval is 1966–1994. Due to the many missing values, changes in the location of weather stations and unreliable data recorded in the early years, only data from the last 20 years of observations were used in the study.

To demonstrate the capabilities of this model and analyze the results, the simulated wind speed data of two meteorological stations are presented, where wind power facilities are currently commissioned (Murmansk) or there is a potential for their integration into the power supply systems of oil and gas facilities (Novy Port). Information on weather stations and data sampling is given in Table 1.

Table 1. Weather stations data.

#	WMO ID	Weather Station	Lat., dd	Lon., dd	Altitude, m	Time Range			Wind Speed Statistic, m/s			
						Start	End	Δt, h	Min	Max	Mean	std
1	22113	Murmansk	68.97	33.05	57	1 January 1999	1 January 2019	3	0	21	4.58	2.56
2	23242	Novy Port	67.70	72.95	12				0	28	5.50	3.13

The obtained estimates of the model parameters for the sample data of the selected meteorological stations (Table 1) are calculated by the method described in Section 4 of this article are shown in Figure 4.

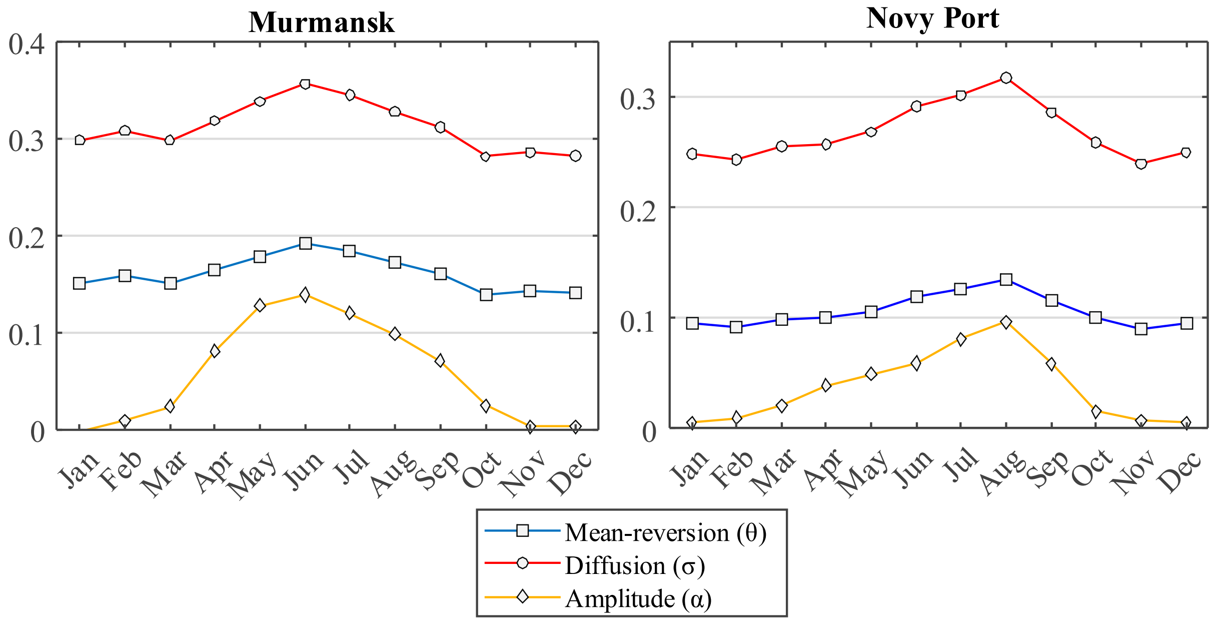

Figure 4. Estimates of SDE-model parameters for each month.

The estimates of the parameters of the Weibull distribution are shown in Figure 5.

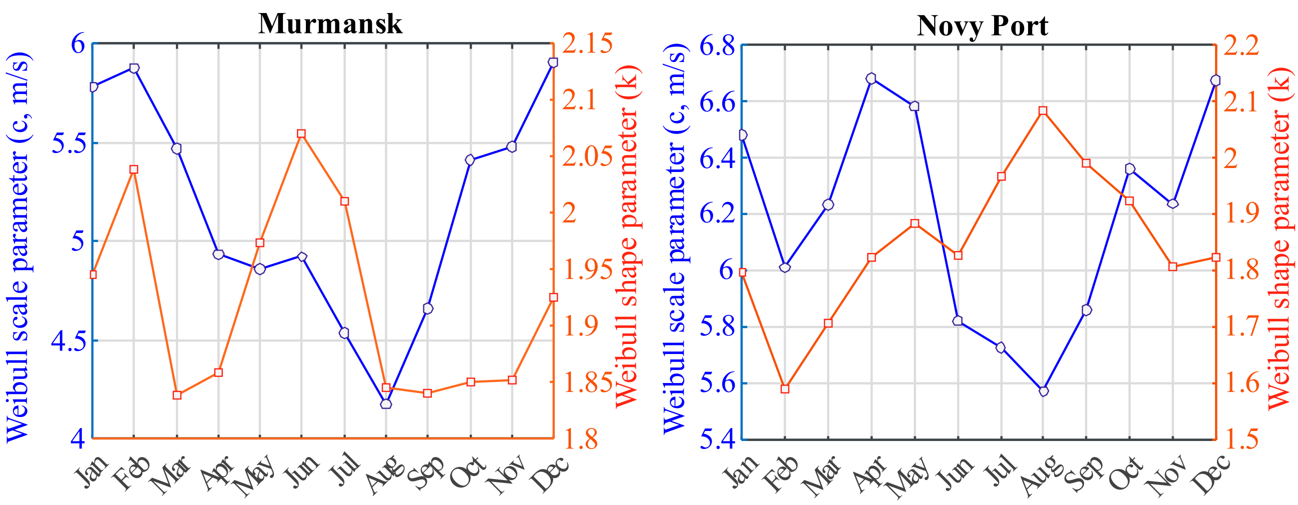

Figure 5. Month-wise Weibull distribution parameter estimates.

The wind speed data was simulated on a time interval equal to 20 years, with a sampling step equal to the initial interval between observations ($\Delta t = 3$ h) and with an hourly interval (Δt = 1 h). For the simulated time series of wind speed, the hourly autocorrelation, autocorrelation of daily average values during the year, and daily profiles and histograms of probability distribution were compared with same characteristics of actual data.

The determination coefficient [29] used as comparison criteria is:

$$R^2 = 1 - \frac{\sum (y_i - f_i)^2}{\sum (y_i - \overline{y}_i)^2} \tag{13}$$

where y_i—actual wind speed data characteristic, f_i—simulated wind speed characteristic.

Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are used to compare the models [30,31]:

$$\text{AIC} = -2\ln(\hat{L}) + 2k \tag{14}$$

$$\text{BIC} = -2\ln(\hat{L}) + \ln(n)k \tag{15}$$

where $\hat{L}$—maximum value of likelihood function of model; k—number of estimated model parameters; n—sample size.

To obtain statistically significant estimates in the testing process, in each case, 30 independent trajectories of the stochastic process were generated.

Actual and synthesized hourly and daily average time series data are shown in Figure 6.

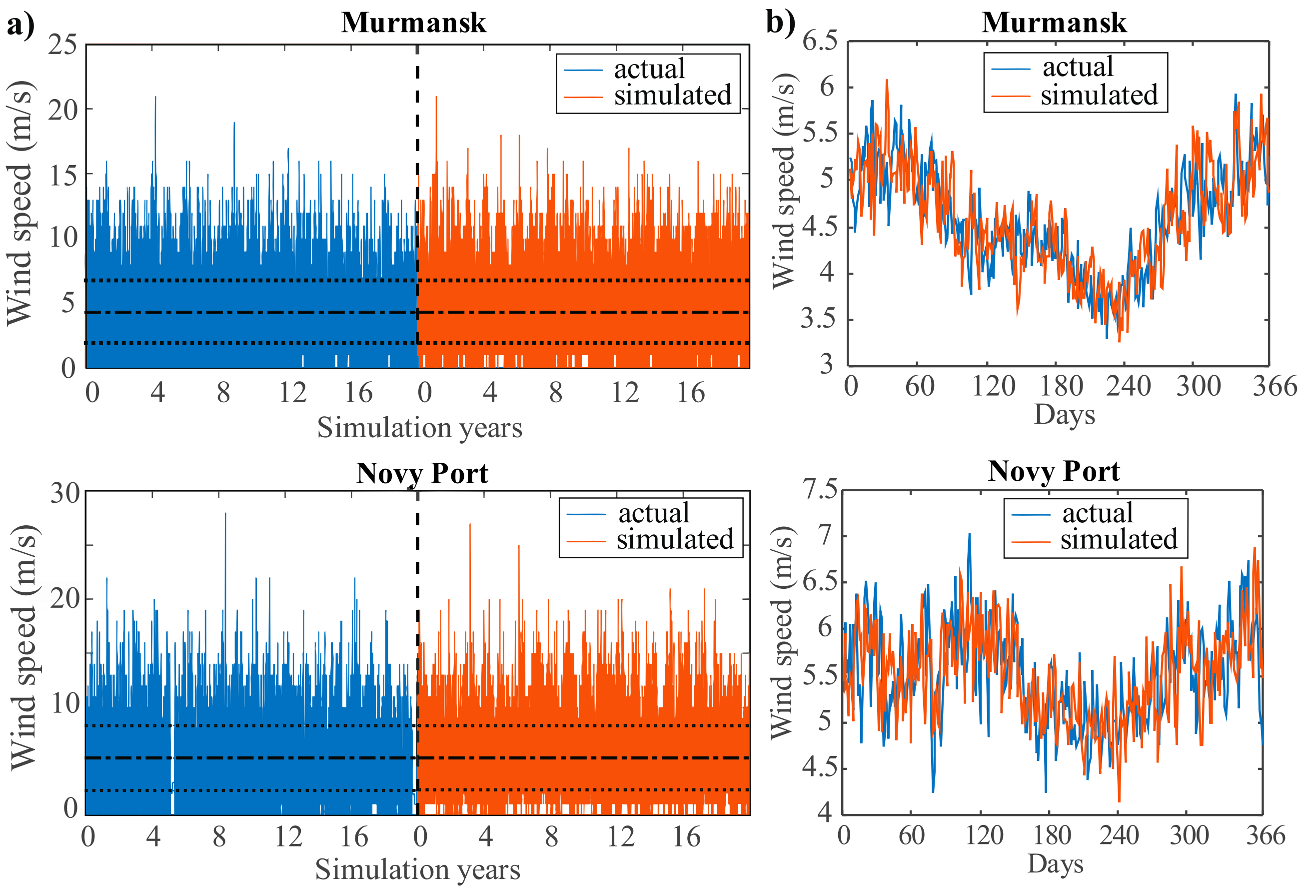

Figure 6. Actual and simulated wind speed time-series: (**a**)—three-hour averaged, (**b**)—daily averaged profile.

Figure 7 shows autocorrelograms of actual and simulated time series calculated within a 120 h range. It can be seen that when modeling the wind speed by SDE process without long-term dependence ($H = 1/2$), the ACF of the simulated wind speed is consistent only on first time lag, and then decreases to almost zero. At the same time, the model of the fractional process with long-term memory ($H > 1/2$) has a power-law decaying ACF which is more consistent to ACF of empirical observation data, which is confirmed by a high coefficient of $R^2 > 90\%$.

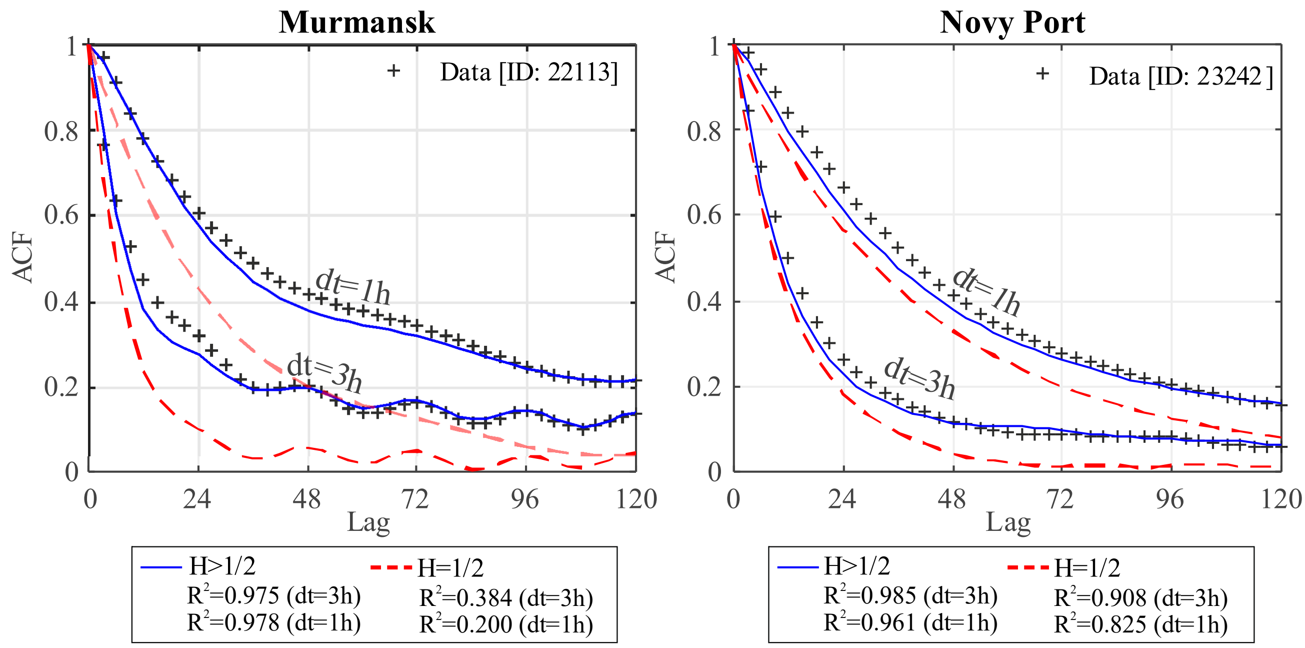

Figure 7. Autocorrelation of observed and simulated wind speed data.

Figure 8 shows the daily profile of averaged wind speeds for each observation period during the time of day, which are approximated by the periodic analytical function $y(t)$ (3). It can be seen that the daily profiles calculated for simulated data exactly follows the shape of the curve of the analytic function and closely match the actual data which is measured by coefficient of determination ($R^2 > 0.9$).

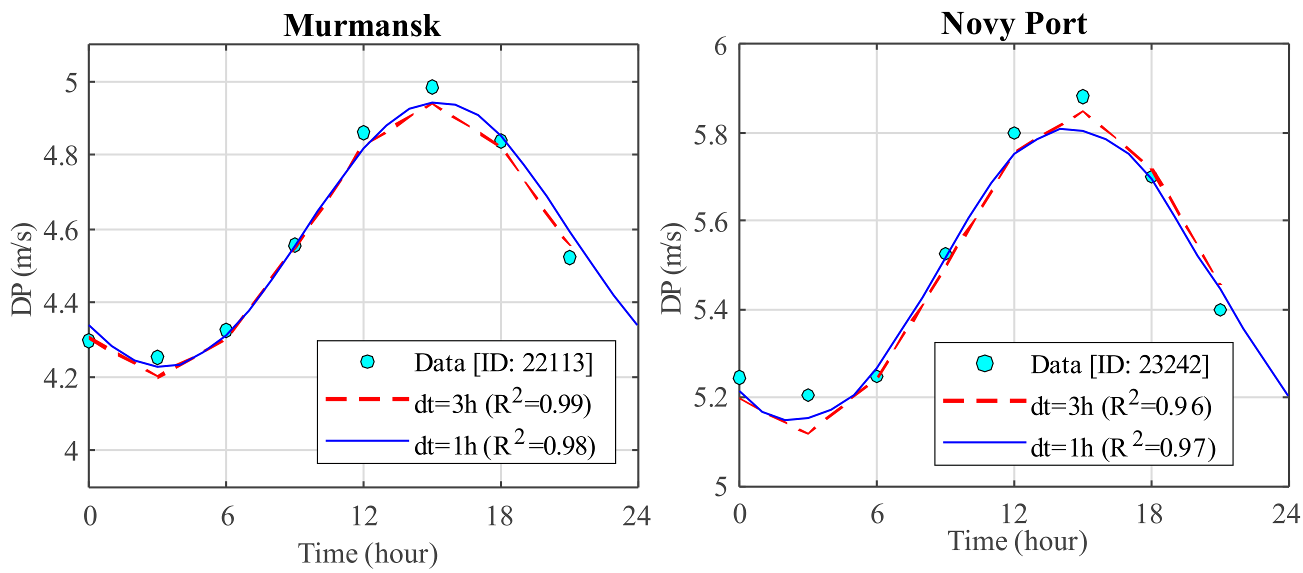

Figure 8. Daily wind speed profiles.

The autocorrelograms shown in Figure 9 demonstrate the correlation between the day average wind speeds across the year. The obtained characteristics show a clearly pronounced seasonal component, changing along a periodic trajectory. The simulation results demonstrate the ability of the model to reproduce this pattern with an acceptable degree of accuracy (R^2 > 90%).

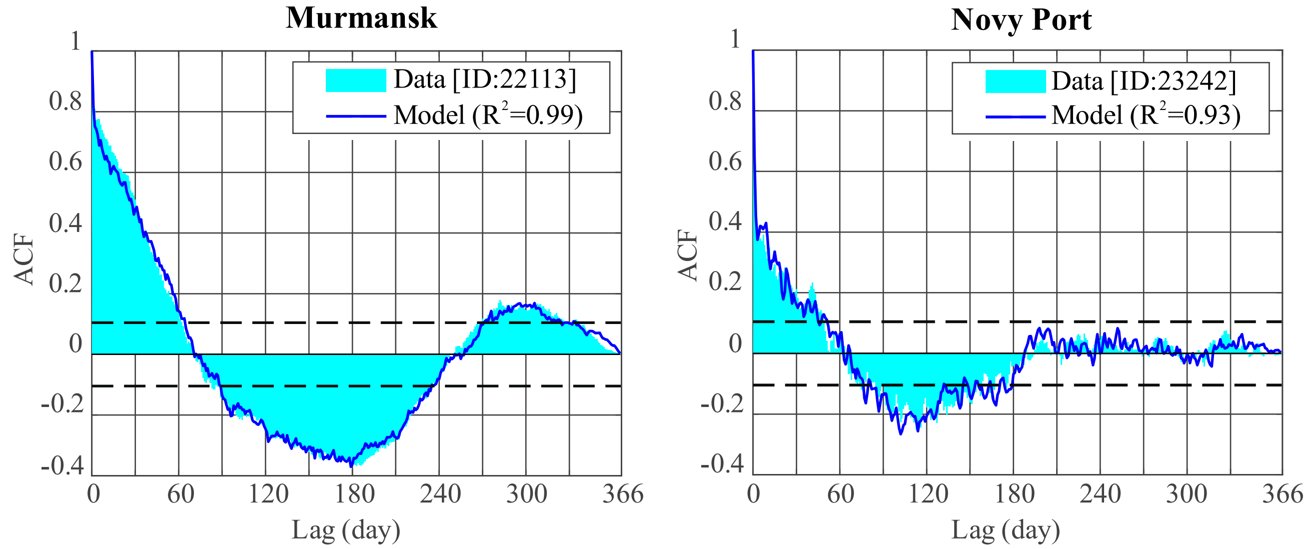

Figure 9. Daily average wind speed autocorrelation.

The histograms of the real distribution of wind speed approximated by the Weibull distribution are shown in Figure 10 and are compared with the distribution of the simulated data. It can be seen that the probability distribution of the simulated data, as well as the analytical Weibull PDF, are quite accurately fit to the histograms of the actual distribution.

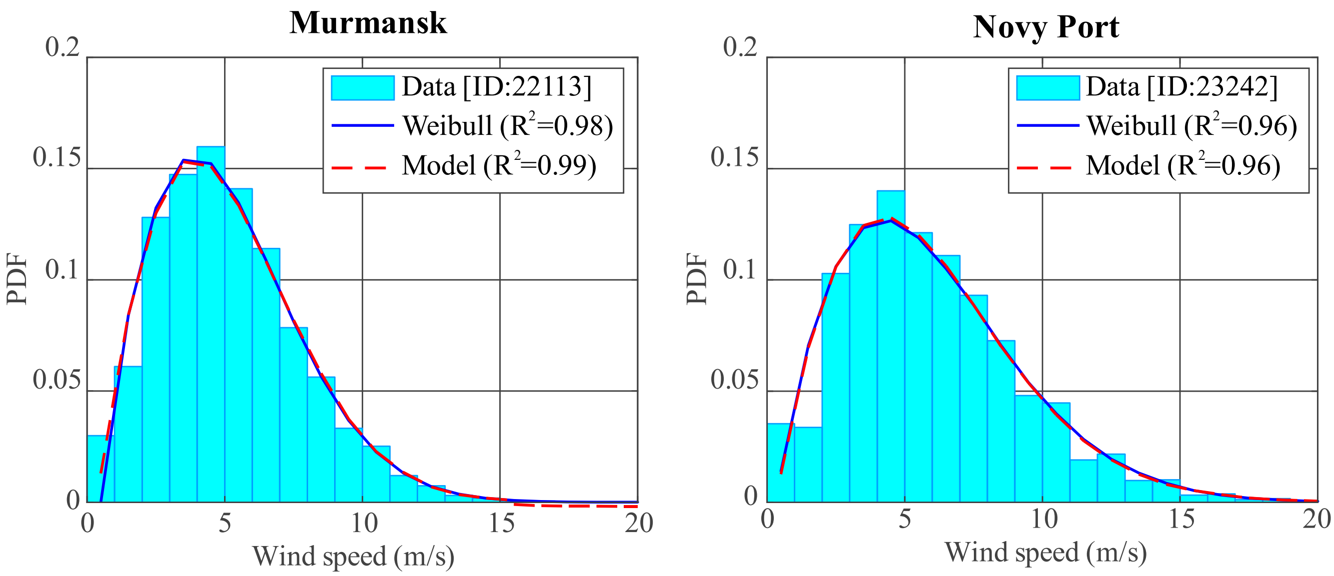

Figure 10. Probability distribution of real and simulated wind speed.

A comparison of statistical point estimates of a real data sample and synthesized time series of wind speed using the standard and fractional models are shown in Table 2.

Table 2. Descriptive statistics of real and simulated wind speed data.

Weather Station	Observed Data				Standard Model (H = 1/2)				Fractional Model (H > 1/2)			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
						Δt = 3						
1	0	21	4.58	2.56	0.01	20.25	4.58	2.55	0.01	20.67	4.58	2.55
2	0	28	5.50	3.13	0.01	25.53	5.50	3.13	0.01	26.11	5.50	3.13
						Δt = 1						
1	0	21	4.58	2.56	0.01	21.06	4.58	2.55	0	20.75	4.58	2.55
2	0	28	5.50	3.13	0.01	26.81	5.50	3.13	0	26.36	5.50	3.13

Figure 11 shows the relationship between Hurst exponent estimates calculated for the time series of actual observations and synthetic wind speed data generated for each dataset included in the analysis. The data obtained demonstrate the main difference between the standard and fractional model, which is the ability to simulate the long-term correlation dependence inherent in a time series.

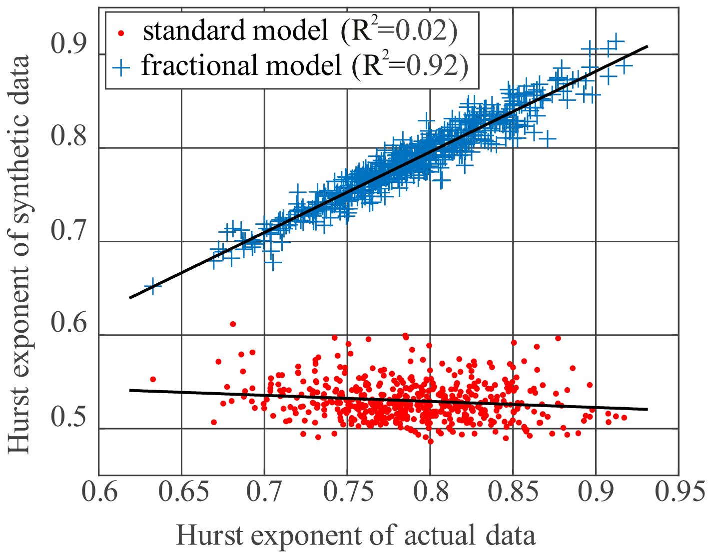

Figure 11. Hurst exponent of simulated and actual wind speed data.

The results of evaluating the accuracy of simulation carried out according to the above-described methodology for each of the 518 time series of data used in the study are summarized in Table 3.

Table 3. Model comparison results.

Criterion	Standard Model (H = 1/2)				Fractional Model (H > 1/2)			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
$\Delta t = 3$								
R^2 (ACF)	0	0.9780	0.4675	0.2679	0.3928	0.9931	0.9327	0.0632
R^2 (DP)	0.0167	0.9947	0.9027	0.1263	0.0506	0.9962	0.9201	0.0994
R^2 (Daily ACF)	0.5894	0.9997	0.9784	0.0437	0.7121	0.9996	0.9848	0.0310
R^2 (PDF)	0.6445	0.9954	0.9432	0.0444	0.6372	0.1171	0.9432	0.0446
AIC(1×10^{-3})	36.727	520.165	238.049	83.621	23.451	519.740	236.995	84.217
BIC(1×10^{-3})	38.27	521.708	239.588	83.624	25.016	521.304	238.555	84.220
R^2 (H)	0.0210				0.9215			
$\Delta t = 1$								
R^2 (ACF)	0	0.9899	0.3137	0.2732	0	0.9967	0.9110	0.1000
R^2 (DP)	0.0641	0.9899	0.8858	0.1193	0.0641	0.9933	0.9187	0.0959
R^2 (Daily ACF)	0.6782	0.9997	0.9827	0.0374	0.7506	0.9995	0.9843	0.0290
R^2 (PDF)	0.6371	0.9956	0.9436	0.0446	0.6804	0.9957	0.9436	0.0438
AIC(1×10^{-3})	36.730	1549.40	696.113	255.202	14.964	1550.40	689.560	257.190
BIC(1×10^{-3})	38.432	1551.10	697.810	255.205	16.689	1552.10	691.281	257.193
R^2 (H)	0.0151				0.9056			

6. Conclusions

The article presents a wind speed SDE-model based on a fractional Ornstein-Uhlenbeck process with a periodic long-run mean to capture the diurnal cycle and incorporate seasonal variations, and thus perform more accurate modeling operating modes of wind energy based power systems. The SDE-based model describes a time-continuous stochastic process and allows synthesizing wind speed data at different time intervals and varying the sampling step, which makes it possible to use this model to simulate both steady-state and transient operating modes of power systems based on wind power plants.

The main difference of fractional model from the standard Ornstein-Uhlenbeck process is the ability to simulate wind speed data with long-range dependence property and autocorrelation that decreases according to power law, which is closely matched to hourly autocorrelation of actual observational data. This is confirmed by the result of comparing the hourly autocorrelation of the synthesized data with the actual ACF. The deviation from which was less than 10% ($R^2 > 0.9$) on average out of 518 cases.

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Nomenclature

SDE	Stochastic differential equation
PDF	Probability density function
ACF	Autocorrelation function
RES	Renewable energy source
AR	Autoregressive model
MA	Moving-average model
ARMA	Autoregressive moving-average model
GA	Genetic algorithm
PSO	Particle swarm optimization
fBm	Fractional Brownian motion
DP	Daily profile
AIC	Akaike information criterion
BIC	Bayesian information criterion
θ	Mean-reversion rate
μ	Long-run mean
σ	Diffusion
W_t^H	Fractional Brownian motion (с параметром Херста $H \neq 1/2$)
α	daily wind speed amplitude parameter
t_{peak}	hour of daytime peak wind speed
Δt	delta time
Φ	cumulative distribution function of Normal distribution
F^{-1}	inverse cumulative distribution function

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