

Article

Optimization of Charge Pump Based on Piecewise Modeling of Output-Voltage Ripple

Yajun Lin ^{1,*} , Jianxin Yang ², Tin-Wai Mui ¹, Yong Zhou ¹ and Ka-Nang Leung ^{1,*}¹ Department of Electronic Engineering, The Chinese University of Hong Kong, Hong Kong; davisonmui@gmail.com (T.-W.M.); yongzhou@link.cuhk.edu.hk (Y.Z.)² School of Microelectronics, South China University of Technology, Guang Zhou 510000, China; y.jianxin@mail.scut.edu.cn

* Correspondence: linyj@link.cuhk.edu.hk (Y.L.); knleung@ee.cuhk.edu.hk (K.-N.L.)

Abstract: This work proposes a piecewise modeling of output-voltage ripple for linear charge pumps. The proposed modeling can obtain a more accurate design expression of power-conversion efficiency. The relationship between flying and output capacitance, as well as switching frequency and optimize power-conversion efficiency can be calculated. The proposed modeling is applied to three charge-pump circuits: 1-stage linear charge pump, dual-branch 1-stage linear charge pump and 4× cross-coupled charge pump. Circuit-level simulation is used to verify the accuracy of proposed modeling.

Keywords: charge pump; flying and output capacitances; output-voltage ripple; power-conversion efficiency



Citation: Lin, Y.; Yang, J.; Mui, T.-W.; Zhou, Y.; Leung, K.-N. Optimization of Charge Pump Based on Piecewise Modeling of Output-Voltage Ripple. *Energies* **2021**, *14*, 4809. <https://doi.org/10.3390/en14164809>

Academic Editor: Teuvo Suntio

Received: 4 July 2021

Accepted: 3 August 2021

Published: 6 August 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Charge pump is an important circuit in many applications such as energy harvesting, micro-sensor, flash memory and the step-up part after rectifier of wireless power transfer. There exist many variants of charge pump, for example, linear, dual-branch linear, cross-coupled, exponential, Fibonacci and Cockcroft–Walton charge pumps [1–6]. The main purposes to develop these structures are to improve the gain and power-conversion efficiency (PCE), as well as to reduce the number of switches and flying capacitors.

Studies of PCE of charge pump is an important topic and have been carried out for many decades [7–11]. Flying and output capacitances are the considerations of optimization of PCE because smaller chip size can be achieved upon minimizing on-chip flying capacitances and production cost can be reduced when smaller and fewer off-chip flying capacitors are used. Switching frequency impacts PCE and output-voltage ripple. There are many factors towards PCE optimization. Modeling of output-voltage ripple for PCE optimization is one of the approaches [7]. However, the modeling is not accurate enough and has room to further enhance. In this paper, a piecewise modeling of output-voltage ripple of linear charge pump is proposed. Based on the proposed piecewise modeling, the expressions of PCE of 1-stage linear charge pump, dual-branch 1-stage linear charge pump and 4× cross-coupled charge pump are found. The PCE expressions can be used to select appropriate flying and output capacitances, as well as switching frequency.

2. Methods

2.1. Proposed Piecewise Modeling of Output-Voltage Ripple for 1-Stage Linear Charge Pump

The two-phase switching operation of a 1-stage linear charge pump is shown in Figure 1. Two complementary clock signals $\Phi 1$ and $\Phi 2$, which have approximately half of a switching period (T), are used to control the ON and OFF of switches. Deadtime is inserted in transitions between $\Phi 1$ and $\Phi 2$ to avoid short-circuit loss of the switches. V_i , V_o and I_o represent input voltage, output voltage and output current. C_f and C_o are flying and output

capacitors. Assuming the lump resistances of switches are the same, the lump sum of the resistances from switches, routing and bond-wire could be noted as R_l , which is around two times of a single switch. Figure 1a shows the case for $\Phi_1 = 1$ and $\Phi_2 = 0$. V_i and C_f are connected in series to generate approximately two times of V_i for V_o . The corresponding modeling is shown on the right-hand side of Figure 1a. Similarly, Figure 1b shows the case for $\Phi_1 = 0$ and $\Phi_2 = 1$. C_f is charged by V_i , while C_o maintains V_o and supply current to load. The right-hand-side figure of Figure 1b shows the modeling. When $T/2$ is larger than about $6R_lC_f$, the voltage across C_f is close to V_i . However, usually C_f could not be fully charged since a large T leads to large output voltage drop due to the long discharging time of C_o .

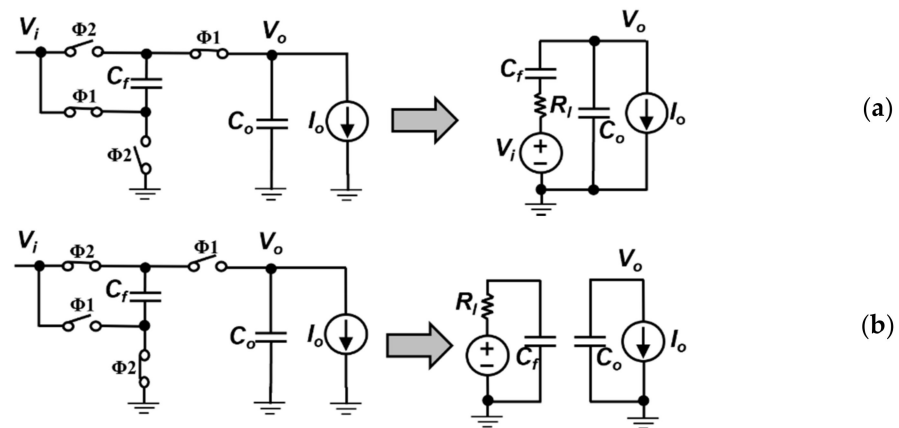


Figure 1. Two-phase switching operation of 1-stage linear charge pump (a) $\Phi_1 = 1$; $\Phi_2 = 0$ (b) $\Phi_1 = 0$; $\Phi_2 = 1$.

Figure 2 shows the output-voltage ripples of a 1-stage linear charge pump. The actual output-voltage ripple is represented by the solid black line. The green line shows the modeling proposed in [7]. There is no charging time of C_f and no charge re-distribution between C_f and C_o in this modeling so that V_o is sharply increased at nT in the n -th switching cycle. Since the average V_o (i.e., $\bar{V}_o$) is used to evaluate PCE in [7], inaccurate modeling of the output-voltage ripples results in inaccurate $\bar{V}_o$ and PCE.

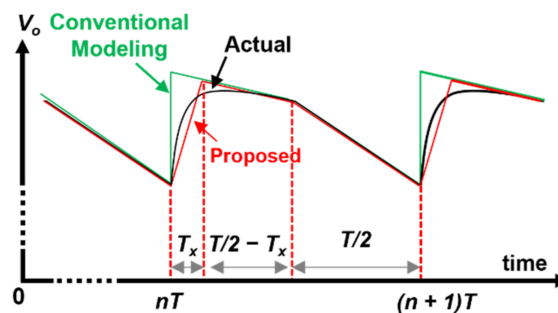


Figure 2. Output-voltage ripples of 1-stage linear charge pump.

The proposed modeling of the output-voltage ripples is shown by the red line in Figure 2. There are three segments within a switching period, and each segment is represented by a straight line. A short period of T_x is used to model the situation when V_o raises at nT . Comparing the proposed modeling with the actual output-voltage ripple shows that the proposed piecewise method can better represent the ripple voltage at output. Thus, $\bar{V}_o$ and PCE can be predicted more accurately.

Figure 3 shows more details of the output waveform and the corresponding RC circuits (from Figure 1) of a 1-stage linear charge pump. V_o starts at V_{o3} at nT and reaches V_{o1} within T_x . Then, it decreases from V_{o1} to V_{o2} within $(T/2 - T_x)$. Finally, it further decreases

from V_{o2} to V_{o3} to complete a cycle. More details will be provided below to investigate the up and down of V_o within a switching cycle to generate the voltage ripple.

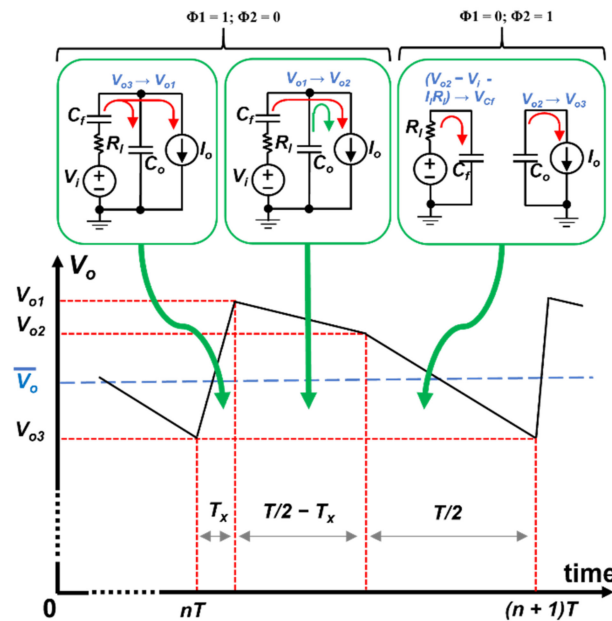


Figure 3. Output waveform and corresponding RC circuits of 1-stage linear charge pump.

In the previous cycle, C_f is previously charged to V_{Cf} , which is close to V_i , while C_o is lightly discharged by the load and the voltage across C_o is less than $2V_i$. At nT , the series-connected combination of the voltage source V_i and C_f has a sum of voltage of $V_i + V_{Cf}$. Thus, C_f is discharged itself to provide charges to C_o and the load I_o . As the voltage across C_o is increasing, V_o is increased from V_{o3} to V_{o1} , and the required time to complete this operation is T_x .

From the RC circuits in Figure 3, at nT , the voltage of C_f is V_{Cf} , and thus the charge of C_f is $C_f V_{Cf}$. Moreover, the charge of C_o is $C_o V_{o3}$. Then, at $(nT + T_x)$, the voltage across C_f is dropped to $(V_{o1} - V_i - I_o R_1)$, so that the charges of C_f is $C_f (V_{o1} - V_i - I_o R_1)$. I_{R1} has the same value as I_o , since the current going into C_o should be zero at $(nT + T_x)$. Meanwhile, the charge of C_o is $C_o V_{o1}$. The charge supplied to the load is $I_o T_x$. By principle of conservation of charges [7], the following relationship is achieved.

$$C_f V_{Cf} + C_o V_{o3} = C_f (V_{o1} - V_i + I_o R_1) + C_o V_{o1} + I_o T_x \quad (1)$$

V_{Cf} is the voltage obtained by capacitor C_f when charging with an ideal voltage supply V_i within a time period $T/2$, and the initial voltage is V_{o2} . Therefore, V_{Cf} is given by

$$V_{Cf} = \left(1 - 2e^{-\frac{T}{2R_1 C_f}} \right) V_i + \frac{I_o C_f R_1}{C_f + C_o} e^{-\frac{T}{2R_1 C_f}} + V_{o2} e^{-\frac{T}{2R_1 C_f}} \quad (2)$$

At $(nT + T/2)$, the voltage of C_o is V_{o2} . Since the charge redistribution of C_f and C_o is complete, the current passing through C_f and C_o is in constant ratio. Therefore, the current of C_f should be $C_f I_o / (C_f + C_o)$. As a result, the following relationship is obtained.

$$C_f (V_{o1} + I_o R_1) + C_o V_{o1} = C_f \left(V_{o2} + \frac{I_o C_f R_1}{C_f + C_o} \right) + C_o V_{o2} + I_o (T/2 - T_x) \quad (3)$$

Between $(nT + T/2)$ and $(n + 1)T$, the voltage across C_o drops from V_{o2} to V_{o3} . The change of charges of C_o is $C_o(V_{o2} - V_{o3})$. These charges supply current to the load to give

$$C_o(V_{o2} - V_{o3}) = \frac{I_o T}{2} \quad (4)$$

Assume that $T = m_1 R_1 C_f$, $C_o = m_2 C_f$, and $T_x = m_3 R_1 C_f$. It should be noted that $T/2 > 6R_1(C_f // C_o)$, i.e., $m_1 > \frac{12m_2}{m_2+1}$, so that the charge redistribution between C_f and C_o is complete. By solving Equations (1)–(4), V_{o1} , V_{o2} and V_{o3} can be found, respectively.

$$V_{o1} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} - \frac{m_2 I_o R_1}{(m_2 + 1)^2} + \frac{(m_1 - 2m_3) I_o R_1}{2(m_2 + 1)} \quad (5)$$

$$V_{o2} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} \quad (6)$$

$$V_{o3} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} - \frac{m_1 I_o R_1}{2m_2} \quad (7)$$

From Equations (5)–(7), as well as the durations of each segment within a switching cycle, the average value of V_o is found and given by:

$$\overline{V_o} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} - \left(\frac{1}{4} + \frac{m_3}{2m_1}\right) \frac{m_1 I_o R_1}{2m_2} - \frac{m_2 I_o R_1}{4(m_2 + 1)^2} + \frac{(m_1 - 2m_3) I_o R_1}{8(m_2 + 1)} \quad (8)$$

For m_3 in Equation (8), it can be evaluated by the following. Refer to Figure 4 for the currents and voltages of C_f and C_o during the period from nT to $(nT + T_x)$, it can be found that

$$V_i + v_{Cf}(t) = i_R(t)R_l + v_{co}(t) \quad (9)$$

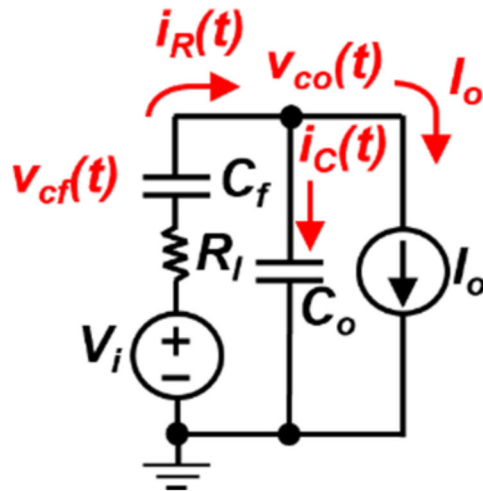


Figure 4. Currents and voltages of C_f and C_o between nT and $(nT + T_x)$.

By considering the charges in capacitors and differentiating in Equation (9) on both sides with respect to time, it gives

$$\frac{dV_i}{dt} + \frac{1}{C_f} \frac{dQ_{Cf}(t)}{dt} = R_l \frac{di_R(t)}{dt} + \frac{1}{C_o} \frac{dQ_{Co}(t)}{dt} \quad (10)$$

where $Q_{Cf}(t)$ and $Q_{Co}(t)$ are the charges stored in C_f and C_o at t . From Figure 4, it can be found that $\frac{dQ_{Cf}(t)}{dt} = -i_R(t)$ and $\frac{dQ_{Co}(t)}{dt} = i_C(t) = i_R(t) - I_o$. Based on these relationships

and substituting into Equation (9), the following expression is obtained. It is noted that $\frac{dV_i}{dt} = 0$ as V_i is a dc voltage.

$$\frac{di_R(t)}{dt} = -\frac{i_R(t)}{R_l} \left(\frac{1}{C_f // C_o} \right) + \frac{I_o}{R_l C_o} \quad (11)$$

Solving the above differential equation, and determining the constant of integration by the initial condition of the circuit, the expression of $i_R(t)$ is given by

$$i_R(t) = \frac{I_o C_f}{C_f + C_o} + \left(I_o - \frac{I_o C_f}{C_f + C_o} \right) e^{\frac{-t}{R_l(C_f // C_o)}} \quad (12)$$

where $C_f // C_o = \frac{C_f C_o}{C_f + C_o}$. The capacitor current of C_o is given by

$$i_C(t) = i_R(t) - I_o = \frac{-I_o C_f}{C_f + C_o} + \left(I_{C00} - \frac{I_o C_f}{C_f + C_o} \right) e^{\frac{-t}{R_l(C_f // C_o)}} \quad (13)$$

where $I_{C00} = \frac{V_i + V_{Cf} - V_{o3}}{R_l}$ is the initial current of C_o at nT .

Referring to Figure 2, the peak voltage of V_o (i.e., the peak voltage across C_o) occurs at about $(nT + T_x)$. Therefore, T_x can be found by differentiating Equation (11) with respect to time to find the maximum point. As a result, T_x is given by

$$T_x = R_l (C_o // C_f) \ln \left(\frac{I_{C00} (C_o + C_f) - I_o C_f}{I_o C_o} \right) \quad (14)$$

By solving Equation (14), we have

$$m_3 = \frac{m_2}{m_2 + 1} \left(\ln(m_1) + \ln \left(1 + \frac{1}{m_2} \right) + \ln \left(1 + \frac{1}{2m_2} \right) \right) \quad (15)$$

Figure 5 shows the charge transfer in both phases. In Phase 2, the total charges from C_o to load is Q_a , and so $Q_a = I_o(T/2)$. Since the output-voltage waveform is periodic, the net charges leaving C_o in Phase 2 equals to the net charges inputted into C_o in Phase 1. Thus, the injected charges to C_o in Phase 1 is also Q_a . Assuming the net charges to load in Phase 1 is Q_b where $Q_b = I_o(T/2)$, the charges from C_f in Phase 1 becomes $(Q_a + Q_b)$. For the series connections of V_i and C_f in Phase 1, the charges from V_i is also $(Q_a + Q_b)$ in Phase 1. Since, again, the output-voltage waveform is periodic, the net charges leaving C_f in Phase 1 equals to the net charges inputted into C_f in Phase 2. As such, the charges from V_i to C_f in Phase 2 is $(Q_a + Q_b)$. The total charges from V_i is equal to $(Q_a + Q_b)$ in Phase 1 plus $(Q_a + Q_b)$ in Phase 2, which is $2(Q_a + Q_b) = 2I_o T$. Therefore, the input current (I_i) from V_i is given by two times of I_o (i.e., $I_i = 2I_o$), which is two times the load current.

The PCE of a 1-stage linear charge pump is the ratio of output power (P_o) to input power (P_i) and is given by

$$PCE = \frac{P_o}{P_i} = \frac{\overline{V_o} I_o}{\overline{V_i} I_i} = \frac{\overline{V_o}}{2\overline{V_i}} \quad (16)$$

where $\overline{V_o}$ is the expression shown in Equation (8).

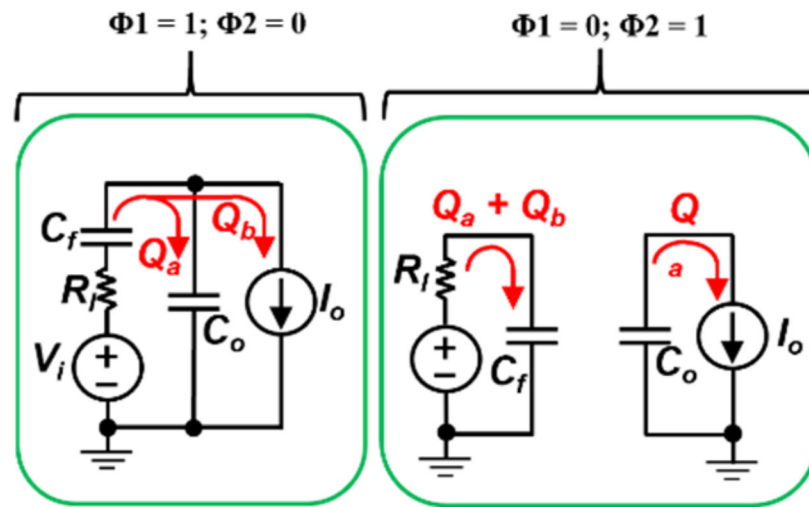


Figure 5. Charge transfer in both phases: Phase 1 (left) and Phase 2 (right).

2.2. Proposed Piecewise Modeling of Output-Voltage Ripple for Dual-Branch 1-Stage Linear Charge Pump/Cross-Coupled Voltage Doubler

In this section, the proposed piecewise modeling of output-voltage ripple is applied to dual-branch 1-stage linear charge pump. It is applicable to cross-coupled voltage doubler, since the ON and OFF arrangements of switches of both dual-branch 1-stage linear charge pump and cross-coupled voltage doubler are the same. Figure 6 shows the switching of a dual-branch 1-stage linear charge pump or cross-coupled voltage doubler, where C_{fA} and C_{fB} are flying capacitors. Similarly, R_l is used to denote the lump sum of the resistances from switches, routing, and bond-wire. The parallel structure enables the load supplied by the flying and output capacitors simultaneously when $\Phi_1 = 1; \Phi_2 = 0$ and $\Phi_1 = 0; \Phi_2 = 1$, except that only C_o provides charges to the load at deadtime (i.e., $\Phi_1 = 0; \Phi_2 = 0$). In fact, T_d is much shorter than T .

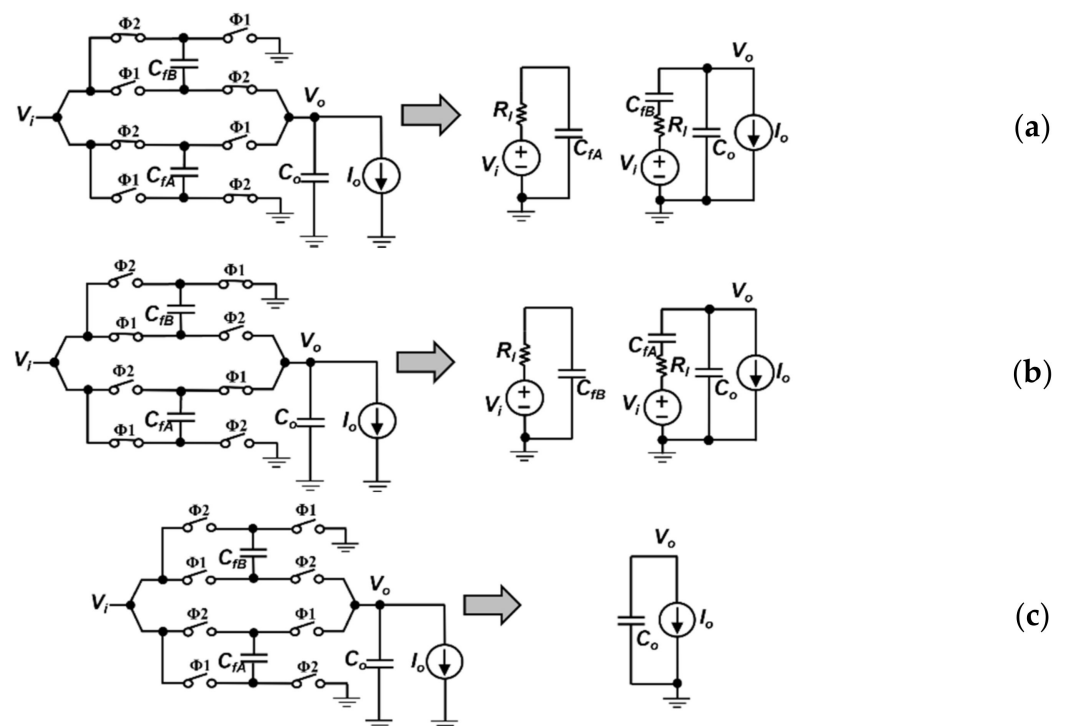


Figure 6. Switching of dual-branch 1-stage linear charge pump/cross-coupled voltage doubler (a) $\Phi_1 = 1; \Phi_2 = 0$ (b) $\Phi_1 = 0; \Phi_2 = 1$ (c) $\Phi_1 = 0; \Phi_2 = 0$ (i.e., deadtime).

Figure 6a shows the case for $\Phi_1 = 1$ and $\Phi_2 = 0$. V_i and C_{fA} are connected in series to generate approximately two times of V_i for V_o , and C_{fB} is charged by V_i . The corresponding modeling is shown on the right-hand side of Figure 6a. R_l , same as before, is the lump sum of the resistances from switches, routing, and bond-wire. Similarly, Figure 6b shows the case for $\Phi_1 = 0$ and $\Phi_2 = 1$. V_i and C_{fB} are connected in series to provide about two times of V_i for V_o , and C_{fA} is charged by V_i . The right-hand-side figure of Figure 6b shows the modeling. Finally, Figure 6c shows the moment of deadtime (i.e., the case for $\Phi_1 = 0$ and $\Phi_2 = 0$), where all switches are turned off. Only C_o maintains about $2V_i$ and supplies charges to the load.

Figure 7 shows more details of the output waveform of one switching cycle and the corresponding RC circuits (from Figure 6) of a dual-branch 1-stage linear charge pump and cross-coupled voltage doubler. In the previous cycle, C_{fA} is previously charged to V_{Cf} , which is close to V_i , while C_o is lightly discharged by the load and the voltage across C_o is less than $2V_i$. At nT , the series-connected combination of the voltage source V_i and C_{fA} has a sum of voltage of $V_i + V_{Cf}$. Thus, C_{fA} is discharged itself to provide charges to C_o and the load. C_{fB} is connected with V_i for re-charging. As the voltage across C_o is increasing, V_o is increased from V_{o3} to V_{o1} , and the required time to complete this operation is T_x . After T_x , where the output voltage achieves the highest value, both C_{fA} and C_o discharge themselves to provide charges to the load. The duration is $(T/2 - T_x - T_d)$, and V_o drops to V_{o2} finally. At $(nT + T/2 - T_d)$, all switches are turned off in the deadtime period. Only C_o supplies charges to the load. Thus, the drop of V_o is more rapid than before. At $(nT + T/2)$, V_o reaches V_{o3} to complete half of a cycle. Between $(nT + T/2)$ and $(n + 1)T$, the operation of the first half switching cycle repeats. The only difference is that another half of the circuit enables C_{fB} to supply charges to the load. In the above analysis, it is assumed that $T/2$ is longer than $6R_l(C_f/C_o)$ (where $C_f = C_{fA} = C_{fB}$) to ensure that the redistribution of C_f (C_{fA} and C_{fB}) and C_o is complete when the capacitors are connected.

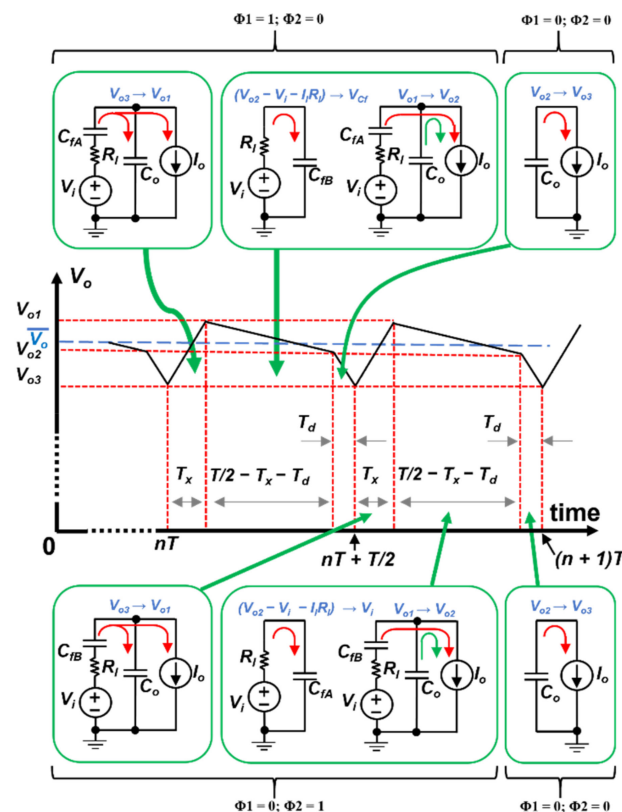


Figure 7. Output waveform and corresponding RC circuits of dual-branch 1-stage linear charge pump/cross-coupled voltage doubler.

As shown in Figure 7, the output-voltage waveforms in the first half and second half of a switching cycle are the same. The analysis below takes a period of $T/2$ into account. C_{fA} and C_{fB} are considered to have the same value, such that $C_{fA} = C_{fB} = C_f$. From the RC circuits in Figure 8, at nT , the voltage of C_{fA} is V_{Cf} , and thus the charges of C_{fA} is $C_{fA}V_{Cf}$. Moreover, the charge of C_o is C_oV_{o3} . Then, at $(nT + T_x)$, the voltage across C_{fA} is dropped to $(V_{o1} - V_i - I_oR_1)$, so that the charge of C_{fA} is $C_{fA}(V_{o1} - V_i - I_oR_1)$. Meanwhile, the charge of C_o is C_oV_{o1} . The charge supplied to the load is I_oT_x . By the principle of conservation of charges [7], the following relationships are achieved.

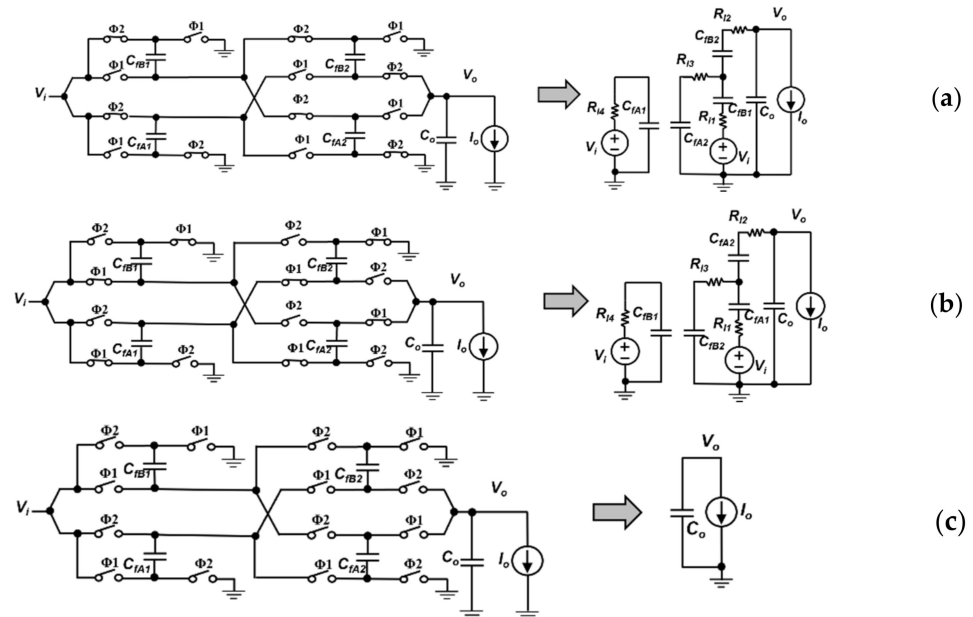


Figure 8. Switching of 2-stage cross-coupled voltage doubler (a) $\Phi_1 = 1$; $\Phi_2 = 0$ (b) $\Phi_1 = 0$; $\Phi_2 = 1$ (c) $\Phi_1 = 0$; $\Phi_2 = 0$ (i.e., deadtime).

$$C_f V_{Cf} + C_o V_{o3} = C_f (V_{o1} - V_i - I_o R_1) + C_o V_{o1} + I_o T_x \quad (17)$$

V_{Cf} is the voltage obtained by capacitor C_{fA} when charging with an ideal voltage supply V_i within a time period $T/2$, and the initial voltage is V_{o2} . Therefore, V_{Cf} is given by

$$V_{Cf} = \left(1 - 2e^{-\frac{T-2T_d}{2R_1C_f}} \right) V_i + \frac{I_o C_f R_1}{C_f + C_o} e^{-\frac{T-2T_d}{2R_1C_f}} + V_{o2} e^{-\frac{T-2T_d}{2R_1C_f}} \quad (18)$$

At $(nT + T/2 + T_x - T_d)$, the voltage of C_o is V_{o2} . Since the charge redistribution of C_{fA} and C_o is complete, the current passing through C_{fA} and C_o is in constant ratio. Therefore, the current of C_{fA} should be $C_f I_o / (C_f + C_o)$. As a result, the following relationship is obtained.

$$C_f (V_{o1} + I_o R_1) + C_o V_{o1} = C_f \left(V_{o2} + \frac{I_o C_f R_1}{C_f + C_o} \right) + C_o V_{o2} + I_o (T/2 - T_x - T_d) \quad (19)$$

Between $(nT + T/2 + T_x - T_d)$ and $(nT + T/2)$, the voltage across C_o drops from V_{o2} to V_{o3} . The change of charges of C_o is $C_o (V_{o2} - V_{o3})$. These charges supply current to the load to give

$$C_o (V_{o2} - V_{o3}) = I_o T_d \quad (20)$$

Assume that $T = m_1 R_1 C_f$, $C_o = m_2 C_f$, $T_x = m_3 R_1 C_f$, and $T_d = m_4 R_1 C_f$. It should be noted that $T/2 > 6R_1 (C_f / C_o)$, i.e., $m_1 > \frac{12m_2}{m_2+1}$, so that the charge redistribution

between C_f and C_o is complete. By solving Equations (17)–(20), V_{o1} , V_{o2} and V_{o3} can be found, respectively.

$$V_{o1} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-(m_1/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} - \frac{m_2 I_o R_1}{(m_2 + 1)^2} + \frac{(m_1 - 2m_3 - 2m_4) I_o R_1}{2(m_2 + 1)} \quad (21)$$

$$V_{o2} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-(m_1/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} \quad (22)$$

$$V_{o3} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-(m_1/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} - \frac{m_4 I_o R_1}{m_2}. \quad (23)$$

From Equations (21)–(23), as well as the durations of each segment within half of a switching cycle, the average value of V_o is found and given by

$$\bar{V}_o = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-(m_1/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} - \frac{m_4(m_3 + m_4) I_o R_1}{m_1 m_2} - \frac{(m_1 - 2m_4) m_2 I_o R_1}{2m_1(m_2 + 1)^2} + \frac{(m_1 - 2m_4)(m_1 - 2m_3 - 2m_4) I_o R_1}{4m_1(m_2 + 1)} \quad (24)$$

The conditions to evaluate T_x is same as before, and T_x has the same expression as stated in Equation (13). Thus, we have:

$$m_3 = \frac{m_2}{m_2 + 1} \left(\ln(m_1) + \ln\left(1 + \frac{1}{m_2}\right) + \ln\left(\frac{1}{2} + \frac{m_4}{m_1 m_2}\right) \right) \quad (25)$$

Similar to the analysis of Equation (16), the input energy is given by a simple expression, as below.

$$E_i = 2V_i I_o \quad (26)$$

Finally, the PCE can be easily derived by the ratio of E_o to E_i .

$$PCE = \frac{E_o}{E_i} = \frac{\bar{V}_o I_o}{2V_i I_o} = \frac{\bar{V}_o}{2V_i} \quad (27)$$

where $\bar{V}_o$ is found as shown in Equation (24).

2.3. Proposed Piecewise Modeling of Output-Voltage Ripple for 2-Stage Cross-Coupled Voltage Doubler

To verify the application of proposed piecewise modeling, the analysis is extended to 2-stage cross-coupled voltage doubler. Figure 8 shows the switching behaviors of a 2-stage cross-coupled voltage doubler. Figure 8a shows the case for $\Phi_1 = 1$ and $\Phi_2 = 0$, Figure 8b illustrates the condition for $\Phi_1 = 0$ and $\Phi_2 = 1$ and Figure 8c reveals the situation of deadtime when $\Phi_1 = 0$ and $\Phi_2 = 0$. Since the number of switches in each branch is different, the lump sums of the resistances from switches, routing and bond-wire are noted as R_{l1} , R_{l2} , and R_{l3} . It is noted again that T_d is much shorter than T . C_{fA1} and C_{fB1} are the flying capacitors in the first stage, and C_{fA2} and C_{fB2} are the flying capacitors in the second stage.

Basically, the operations for $\{\Phi_1 = 1 \text{ and } \Phi_2 = 0\}$ and $\{\Phi_1 = 0 \text{ and } \Phi_2 = 1\}$ are the same due to the parallel structure. Thus, the corresponding modeling of both cases are the same, except different flying capacitors are used to complete the operation of the circuit. Ideally, from the switching operations, C_{fA1} and C_{fB1} are charged to V_i , while C_{fA2} and C_{fB2} are charged to $2V_i$. Therefore, V_o is $4V_i$ theoretically, which is the sum of the source voltage and the voltages across C_{fA1} (or C_{fB1}) and C_{fA2} (or C_{fB2}). During the deadtime, all switches are turned off to disconnect the output from the flying capacitors and V_i . Thus, the load is supplied.

Figure 9 shows the details of the output waveform of one switching cycle and the corresponding RC circuits (from Figure 8) of a 2-stage cross-coupled voltage doubler. The operations at nT and $(nT + T/2)$ are the same, except another half circuit operates

alternatively. Thus, the analysis can be conducted for half of a switching cycle. At nT , C_{fB1} and C_{fB2} are previously charged to V_{Cf} and about $2V_i$, respectively, at $((n-1)T + T/2)$ (i.e., the same operation at $(nT + T/2)$). C_{fB1} provides charges to C_{fA2} to re-charge it to about $2V_i$. Similarly, they supply charges to C_o such that the output increases from V_{o3} to V_{o1} . The required time to complete this operation is T_x . Then, at $(nT + T_x)$, the source V_i , C_{fA2} , C_{fB1} , C_{fB2} and C_o supply charges to the load. The discharges of flying capacitors and output capacitor decrease the output from V_{o1} to V_{o2} . At $(nT + T/2 - T_d)$, all switches are turned off. Only C_o supplies charges to the load, and thus the output drops more rapidly than before from V_{o2} to V_{o3} . In the above analysis, it is assumed that $T/2$ is longer than $6R_lC_{f2}$ (where $C_{f1} = C_{fA1} = C_{fB1}$ and $C_{f2} = C_{fA2} = C_{fB2}$) to the charge redistribution during T to T_x , $T/2$ to $T/2 + T_x$, is complete, while C_{fA2} and C_{fB2} are charged to about $2V_i$ at the end of half of a switching cycle.

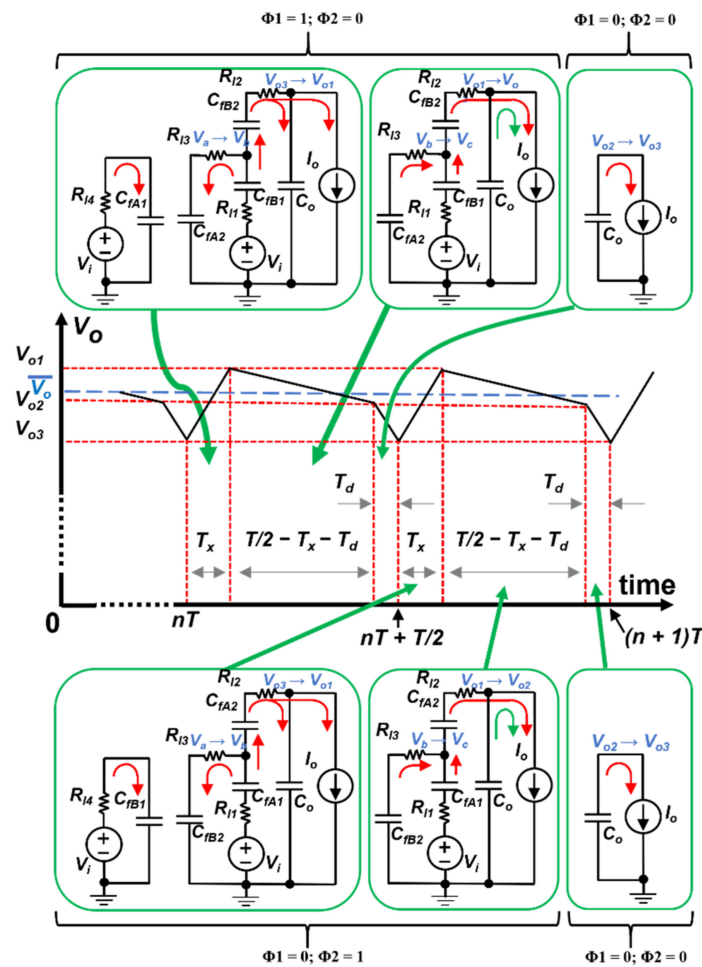


Figure 9. Output waveform and corresponding RC circuits of 2-stage cross-coupled voltage doubler.

Based on the RC circuits in Figure 9, at $(nT - T_d)$, the charge redistribution between C_{fB1} , C_{fA2} , and C_{fB2} is completed. Considering the voltage at the output of the first stage of charge pump is a constant value, the current passing through C_{fB2} and C_o is in constant ratio. Hence, the current passing through C_{fA2} and C_o can be approximated as $C_{fA2}I_o/(C_{fA2} + C_o)$ and $C_oI_o/(C_{fA2} + C_o)$. Similarly, the current passing through C_{fB2} and C_{fA1} is $C_{fA2}C_{fB2}I_o/(C_{fA2} + C_o)/(C_{fB2} + C_{fA1})$ and $C_{fA2}C_{fA1}I_o/(C_{fA2} + C_o)/(C_{fB2} + C_{fA1})$. At nT , the charges stored in C_{fA2} and C_{fB1} are $C_{fA2}(V_{o2} + C_{fA2}I_oR_{l2}/(C_{fA2} + C_o))$ and $C_{fB1}V_{Cf1}$, respectively. It is noted that the negative charges at the bottom plate of C_{fB2} should also be considered. Assuming that $C_{f1}/C_{f2} = C_{f2}/C_o = m_2$, the highest output voltage of the first and second stage of the charge pump achieves at T_{x1} and T_{x2} . According to similar

analysis of Equations (14) and (15), it could be seen that T_{x1} and T_{x2} have close values due to the log relationship of T_x and m_1 . Hence, it is reasonable to assume that V_b and V_{o1} both achieve at $(nT + T_x)$. Then, at $(nT + T_x)$, the current passing through C_{fA2} and C_o is 0. Therefore, the charges remaining in C_{fA2} , C_{fB1} and C_{fB2} are $C_{fA2}V_b$, $C_{fB1}(V_b - V_i + I_oR_{l1})$ and $C_{fB2}(V_{o1} - V_b + I_oR_{l2})$, respectively. By the principle of conservation of charges [7],

$$C_{f2}(V_{o2} + I_oR_{l2}\frac{C_{f2}}{C_{f2}+C_o} - V_c) + C_{f1}V_{Cf1} - C_{f2}\left(V_c + I_oR_{l3}\frac{C_{f2}}{C_{f2}+C_o}\frac{C_{f1}}{C_{f1}+C_{f2}}\right) = C_{f2}V_b + C_{f1}(V_b - V_i + I_oR_{l1}) - C_{f2}(V_{o1} - V_b + I_oR_{l2}) \quad (28)$$

T_x could be approximated by T_{x2} , which satisfies the following equation.

$$T_x = R_{l2}\left(C_o / C_{f2}\right) \ln \left(\frac{I_{Co0}(C_o + C_{f2}) - I_oC_o}{I_oC_o} \right), \quad (29)$$

with $I_{Co0} = \frac{V_{i2} \times 2 - V_{o3}}{R_{l2}}$. For simplification of the calculation, the average output voltage value for the first stage V_{i2} is approximated as V_c , which could be calculated without T_x , and the related calculation of V_c is given in the following part.

V_{Cf1} is the voltage obtained by capacitor C_{fA1} when charging with an ideal voltage supply V_i within a time period $T/2$. Therefore, V_{Cf1} is given by

$$V_{Cf} = \left(1 - 2e^{-\frac{T-2T_d}{2R_{l1}C_{f1}}}\right)V_i + V_c e^{-\frac{T-2T_d}{2R_{l1}C_{f1}}} + \frac{I_oC_{f1}C_{f2}R_{l1}}{(C_{f1} + C_{f2})(C_{f2} + C_o)}e^{-\frac{T-2T_d}{2R_{l1}C_{f1}}} \quad (30)$$

At nT , the charges in C_{fB2} obtained before the deadtime in the previous half of a cycle is $C_{fB2}[V_c + I_oR_{l3}C_{fA2}C_{fA1}I_o/(C_{fA2} + C_o)/(C_{fB2} + C_{fA1})]$, and the charges in C_o is C_oV_{o3} . At $(nT + T_x)$, the charges in C_{fB2} and C_o are $C_{fB2}(V_{o1} - V_b + I_oR_{l2})$ and C_oV_{o1} , respectively. The charges to the load are I_oT_x . Thus, the following relationship is obtained.

$$C_{f2}\left(V_c + I_oR_{l3}\frac{C_{f2}}{C_{f2}+C_o}\frac{C_{f1}}{C_{f1}+C_{f2}}\right) + C_oV_{o3} = C_{f2}(V_{o1} - V_b + I_oR_{l2}) + C_oV_{o1} + I_oT_x \quad (31)$$

Between $(nT + T_x)$ and $(nT + T/2 - T_d)$, some charges in C_{fA2} and C_{fB1} go to C_{fB2} . Since the overall charges in the connection of three capacitors are constant, the following equation is obtained.

$$C_{f2}V_b + C_{f1}(V_b - V_i + I_oR_{l1}) - C_{f2}(V_{o1} - V_b + I_oR_{l2}) = C_{f2}\left(V_c + I_oR_{l3}\frac{C_{f2}}{C_{f2}+C_o}\frac{C_{f1}}{C_{f1}+C_{f2}}\right) + C_{f1}\left(V_c - V_i + I_oR_{l1}\frac{C_{f2}}{C_{f2}+C_o}\frac{C_{f1}}{C_{f1}+C_{f2}}\right) - C_{f2}(V_{o2} + I_oR_{l2}\frac{C_{f2}}{C_{f2}+C_o} - V_c) \quad (32)$$

Moreover, the net charges of C_{fA2} and C_o will supply the load so that the following expression is obtained.

$$C_{f2}(V_{o1} - V_b + I_oR_{l2}) + C_oV_{o1} = C_{f2}\left(V_{o2} - V_c + I_oR_{l2}\frac{C_{f2}}{C_{f2}+C_o}\right) + C_oV_{o2} + I_o(T/2 - T_d - T_x) \quad (33)$$

Finally, during the deadtime, the charges from C_o supplies to the load gives the following relationship.

$$(V_{o2} - V_{o3})C_o = I_oT_d \quad (34)$$

Solving Equations (28)–(34), V_{o1} , V_{o2} and V_{o3} could be calculated, and the average value of V_o is given by

$$\overline{V_o} = \frac{T_x}{T}(V_{o1} + V_{o3}) + \frac{T_d}{T}(V_{o2} + V_{o3}) + \frac{T - 2T_x - 2T_d}{2T}(V_{o1} + V_{o2}) \quad (35)$$

Similar to the analysis of Equation (16), PCE can be easily derived by the ratio of E_o to E_i .

$$PCE = \frac{E_o}{E_i} = \frac{\bar{V}_o I_o}{4V_i I_o} = \frac{\bar{V}_o}{4V_i} \quad (36)$$

where $\bar{V}_o$ is found as shown in Equation (35).

3. Results

Simple simulations were conducted to prove the analysis. Ideal switch models with non-zero on-resistance are used [12]. The supply voltage V_i is 1.6 V, and the load current I_o is 50 mA.

The values of switching frequency, flying capacitors and output capacitor are chosen according to the load, which are highly related to the ripple voltage at the output. With the condition fulfilled and the assumptions presented, the proposed analysis is applicable to different values of switching frequency, flying capacitors and output capacitor. In the simulations, for 1-stage linear charge pump and dual-branch charge pump, the switch resistance is 1 Ω and the clock frequency is 36 kHz. The capacitance of the flying capacitor C_f and the load capacitor C_o is 4.7 μ F and 1.5 μ F, respectively. The calculation results according to [7] and this work are shown in Tables 1 and 2. The simulation is conducted by Hspice, and the result is also given in Tables 1 and 2. The error value, which is the percentage of the difference between calculation and simulation over the simulation result, is also given for better insight.

Table 1. The simulation result for 1-stage linear charge pump.

Parameter	[7]		This Work		Simulation
	Result (V)	Error	Result (V)	Error	Result (V)
V_{o1}	3.02	8.72%	2.77	−0.10%	2.77
V_{o2}	2.90	5.90%	2.74	−0.05%	2.74
V_{o3}	2.44	7.11%	2.28	−0.05%	2.28
ΔV_o	0.58	16.14%	0.49	−0.34%	0.50
$\bar{V}_o$	2.82	7.98%	2.57	−1.56%	2.61

Table 2. The simulation result for dual-branch 1-stage linear charge pump.

Parameter	[7]		This Work		Simulation
	Result (V)	Error	Result (V)	Error	Result (V)
V_{o1}	3.16	5.78%	2.99	0.00%	2.99
V_{o2}	3.05	4.08%	2.93	−0.02%	2.93
V_{o3}	3.05	4.09%	2.93	−0.01%	2.93
ΔV_o	0.12	85.07%	0.06	0.48%	0.06
$\bar{V}_o$	3.11	4.66%	2.96	−0.29%	2.97

For 4 $\times$ cross-coupled charge pump, the on-resistance of each switch is 0.5 Ω and the clock frequency is 20 kHz. The capacitance of the flying capacitor C_{f1} , C_{f2} , and the load capacitor C_o is 8 μ F, 4 μ F and 2 μ F, respectively. The simulation and calculation results are shown in Table 3.

Table 3. The simulation result for $4\times$ cross-coupled charge pump.

Parameter	[7]		This Work		Simulation
	Result (V)	Error	Result (V)	Error	Result (V)
V_{o1}	5.67	1.58%	5.59	0.04%	5.58
V_{o2}	5.46	1.28%	5.39	−0.08%	5.39
V_{o3}	4.84	−10.26%	5.39	−0.08%	5.39
ΔV_o	0.83	333.78%	0.20	3.44%	0.19
$\overline{V_o}$	5.36	−2.64%	5.49	−0.31%	5.50

According to the tables, the prediction error is reduced to below 0.1% for V_{o1} , V_{o2} and V_{o3} in all three cases, which shows a more reliable method when choosing optimal capacitor values for charge pumps.

4. Conclusions

By properly modeling the charge pump circuits, this paper demonstrates a better way for analyzing charge pumps. Hspice simulation is conducted to prove the reliability of the analysis method. According to the equation given, the PCE of charge pumps is closely related to the value of clock period, lumping resistance, flying capacitor and load capacitor. Hence, optimization of charge pump is possible based on careful choosing of the component values. Additionally, the analysis could be extended to other charge-pump circuits such as exponential, Fibonacci and Cockcroft–Walton charge pumps.

Author Contributions: The authors contributed to different parts of the paper preparation as follows: Conceptualization, Y.L., J.Y., T.-W.M. and Y.Z.; methodology and software, Y.L., J.Y., T.-W.M. and Y.Z.; validation and analysis, Y.L., J.Y., T.-W.M. and Y.Z.; writing—original draft preparation, Y.L., T.-W.M. and K.-N.L.; writing—review and editing, Y.L., T.-W.M. and K.-N.L.; supervision, K.-N.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work was funded by the Research Grant Council of Hong Kong SAR Government under Project CUHK 14204917.

Conflicts of Interest: The authors declare no conflict of interest.

References

- Dickson, F. On-chip high-voltage generation in MNOS integrated circuits using an improved voltage multiplier technique. *IEEE J. Solid-State Circuits* **1976**, *11*, 374–378. [\[CrossRef\]](#)
- Palumbo, G.; Pappalardo, D. Charge pump circuits with only capacitive loads: Optimized design. *IEEE Trans. Circuits Syst. II Express Briefs* **2006**, *53*, 128–132. [\[CrossRef\]](#)
- Lee, H.; Mok, P.K.T. Switching noise and shoot-through current reduction techniques for switched-capacitor voltage doubler. *IEEE J. Solid-State Circuits* **2005**, *40*, 1136–1146. [\[CrossRef\]](#)
- Ying, T.R.; Ki, W.H.; Chan, M. Area-efficient CMOS charge pumps for LCD drivers. *IEEE J. Solid-State Circuits* **2003**, *38*, 1721–1725. [\[CrossRef\]](#)
- Ueno, F.; Inoue, T.; Oota, I.; Harada, I. Emergency power supply for small computer systems. *IEEE Trans. Circuits Syst.* **1991**, *2*, 1065–1068.
- Cockcroft, J.D.; Walton, E.T. Production of high velocity positive ions. *Proc. Roy. Soc. A* **1932**, *136*, 619–630.
- Ki, W.H.; Su, F.; Tsui, C.Y. Charge redistribution loss consideration in optimal charge pump design. *IEEE Circuits Syst. Mag.* **2005**, *2*, 23–26.
- Seeman, M.D.; Sanders, S.R. Analysis and optimization of switched-capacitor DC-DC converters. *IEEE Trans. Power Electron.* **2008**, *23*, 841–851. [\[CrossRef\]](#)
- Ki, W.H.; Lu, Y.; Su, F.; Tsui, C.Y. Design and analysis of on-chip charge pumps for micro-power energy harvesting applications. In Proceedings of the 2011 IEEE/IFIP 19th International Conference on VLSI and System-on-Chip, Hong Kong, China, 3–5 October 2011; pp. 374–379.
- Makowski, M.S. A note on resistive models of switched-capacitor DC-DC converters: Unified incremental-graph-based formulas given. In Proceedings of the 2012 International Conference on Signals and Electronic Systems (ICSES), Wroclaw, Poland, 18–21 September 2012; pp. 42–46.

-
11. Cheung, C.K.; Tan, S.C.; Tse, C.K.; Ioinovici, A. On energy efficiency of switched-capacitor converters. *IEEE Trans. Power Electron.* **2013**, *28*, 862–876. [[CrossRef](#)]
 12. Locorotondo, E.; Pugi, L.; Corti, F.; Becchi, L.; Grasso, F.; Makowski, M.S. Analytical Model of Power MOSFET Switching Losses due to Parasitic Components. In Proceedings of the 2019 IEEE 5th International forum on Research and Technology for Society and Industry (RTSI), Florence, Italy, 9–12 September 2019; pp. 331–336.