

Article

A Multimodal Smart Quantum Particle Swarm Optimization for Electromagnetic Design Optimization Problems

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Abstract: Electromagnetic design problems are generally formulated as nonlinear programming problems with multimodal objective functions and continuous variables. These can be solved by either a deterministic or a stochastic optimization algorithm. Recently, many intelligent optimization algorithms, such as particle swarm optimization (PSO), genetic algorithm (GA) and artificial bee colony (ABC), have been proposed and applied to electromagnetic design problems with promising results. However, there is no universal algorithm which can be used to solve engineering design problems. In this paper, a stochastic smart quantum particle swarm optimization (SQPSO) algorithm is introduced. In the proposed SQPSO, to tackle the premature convergence problem in order to improve the global search ability, a smart particle and a memory archive are adopted instead of mutation operations. Moreover, to enhance the exploration searching ability, a new set of random numbers and control parameters are introduced. Experimental results validate that the adopted control policy in this work can achieve a good balance between exploration and exploitation. Finally, the SQPSO has been tested on well-known optimization benchmark functions and implemented on the electromagnetic TEAM workshop problem 22. The simulation result shows an outstanding capability of the proposed algorithm in speeding convergence compared to other algorithms.

Keywords: smart quantum particle; particle swarm optimization; design optimization; electromagnetic problem



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1. Introduction

Optimization of high dimensional design problems with a multimodal objective function in electromagnetics has attracted more attention for exploiting stochastic approaches as deterministic methods are not capable of finding the global optimum solution to these problems. In general, there is no unique solution to such an optimal problem, and most of the techniques and algorithms are problem-oriented. Therefore, the intensification of global searching ability stands essential in the optimization problems. In order to effectively tackle this issue, scholars have developed many algorithms [1,2], such as a self-adaptive penalty approach genetic algorithm [3], an artificial bee colony [4] and cuckoo search [5] for finding solutions to optimization problems. Moreover, in the last decade particle swarm optimization (PSO) has gained increasing popularity due to its better performances in optimizing design problems [6]. Many theoretical analyses have been performed on the PSO algorithm, focusing on the behavior of individual particles to understand the search mechanism and parameter settings of the algorithm [7,8]. On the other hand, to solve an engineering inverse problem such as super conducting magnetic energy storage (SMES), optimization benchmark TEAM problem 22 [9] is used to check the robustness and output of various optimization algorithms [10–12]. Much effort has been spent in the past few years applying a stochastic approach, instead of a deterministic one, to find global optimal solutions [13]. S.L. Ho et al. enhanced the convergence speed of conventional PSOs by introducing age variables, but the premature convergence still creates a serious problem

in finding the global minima [14]. However, premature convergence is still a key issue in the QPSO algorithm, specifically when it is used in complicated design problems. To address this problem, numerous modifications have been made by researchers in various fields such as power-systems [15], control systems [16], antenna design [17], “internet of things” [18], and electromagnetics [19,20]. In fundamental PSO, the basic equation comprises the classical mechanics terminology (velocity $v(t)$ and position $x(t)$) of a particle in the search space to solve the optimization problem. However, in quantum mechanics, waves are used instead of particles. Therefore, researchers switched and upgraded the Newtonian mechanical PSO to a quantum mechanical PSO, known as a quantum particle swarm optimization (QPSO). QPSO has shown great potential in the optimization of design problems [21–23].

QPSO can demonstrate a more specific and rich global searching ability in the search space. Schrodinger worked to unify the wave and energy equations, known as the Schrodinger equation. In [18], the author presents a delta potential well model by using the time-dependent Schrodinger equation, as given by

$$i\hbar\Psi(x, t) = \frac{-\hbar}{2m} \frac{d^2\Psi(x, t)}{dx^2} + V(x)\Psi(x, t) \quad (1)$$

where \hbar is the Planck’s constant, $V(x)$ is the potential energy and $\Psi(x, t)$ is a quantum state known as the normalized wave state vector $\Psi(x, t)$, which is similar to a particle in mechanics expressed as

$$\Psi(x) = \frac{1}{\sqrt{L}} e^{(-\frac{|z-x|}{L})} \quad (2)$$

where z is a convergence point in search space. Max Born gives the interpretation of a particle appearance in the search space by using a probability density function of the quantum state, as given by

$$\text{Probability density function} = |\Psi(x_t)|^2 = \frac{1}{L} e^{(-\frac{2|z-x_t|}{L})} \quad (3)$$

Subsequently, the position function obtained by the Monto Carlo stochastic model is given as follows:

$$X_i(t+1) = \begin{cases} p(t) + \beta \times |M_{best} - X_i(t)| \times \ln \frac{1}{u}, & \text{if } u \geq 0.5 \\ p(t) - \beta \times |M_{best} - X_i(t)| \times \ln \frac{1}{u}, & \text{otherwise} \end{cases} \quad (4)$$

where β is a contraction–expansion coefficient and M_{best} is the mean best, as given by

$$M_{best} = \frac{1}{M} \sum_{i=1}^M P_{best_i}(t) \quad (5)$$

From our earlier work we identified that the traditional QPSOs have a premature convergence problem due to the diversity loss at the final stages of the evolution process and the unbalancing between the global and local searches of the particle. In order to address the aforementioned issues, we introduced a novel global smart best particle to the basic QPSO process and developed a new mechanism for the contraction–expansion coefficient. The main proposal of the novel strategies is to control the premature convergence process of the basic QPSO method.

The remainder of this paper is organized as follows: The proposed framework for a new variant of QPSO is presented in Section 2. In Section 3, the proposed QPSO are compared with the standard QPSO, GQPSO, LIQPSO and MQPSO and applied on different benchmark functions followed by result and discussion in Section 4. Section 5 reports our experimental testing and validation of the proposed QPSO on TEAM-22 optimization benchmark problems of superconducting magnetic energy stored (SMES). Finally, our conclusion is presented in Section 6.

2. The Proposed Work

The standard QPSO has good searching ability; however, its drawbacks manifest as low convergence speed and an unexpected premature convergence without enough exploration of the search space. As aforementioned, clinching the low convergence speed and premature convergence are the core focusses of improvements to QPSOs. The proper mechanism of the proposed framework will be described below.

2.1. Process Analysis of Smart Particle of the Swarm

In this section, we analyze the process of the best particle nomination in the swarm to lead and increase the searching ability of the algorithm. Unlike the traditional QPSO, which is focused to select the global best particle g_{best} amongst the p_{best} of the current iteration of the whole population, the proposed algorithm uses a memory bank called “the archive” to store the current and previous p_{best} for better selections of the global best particle g_{best} . Mathematically this process is expressed as

$$P_{best_i}(t) = \begin{cases} X_i & \text{if } f(X_i(t)) < f(P_{X_{i-1}}(t)) \\ P_{X_{i-1}}(t) & \text{otherwise} \end{cases} \quad (6)$$

In this process, the current p_{best} position of the particle is compared with its own previously store p_{best} position. If it is better than the previous one, it will replace the previous p_{best} , otherwise it will retain the current one for future use. A pseudocode of the archive phenomenon is given below:

Pseudocode of updating rule of p_{best} in the archive

```

If  $f(X_{i+1}) < f(P_{X_i})$  do
  Clear the previous  $p_{best}$  of  $P_{X_i}$ 
  Store the new  $p_{best}$  as  $X_{i+1}$  in the archive
elseif  $f(X_{i+1}) > f(P_{X_i})$ 
  Ignore the new  $p_{best}$  and upheld the previous one
end if
declare the global best  $g_{best}$  from the updated archive.

```

2.2. Optimal Strategy for Parameter Setting

We have intensified the exploration capability of the proposed algorithm to obtain a better convergence speed and to avoid premature convergence by a proposed strategy to update the contraction–expansion coefficient β by

$$\beta = u(0, 1) \times N(\mu, \sigma^2) \quad (7)$$

where $u(0, 1)$ is a random number and $N(\mu, \sigma^2)$ is the cumulative distribution function with $\sigma = 1$ and $\mu = 0$.

With updated version of β , the proposed algorithm used also revised the sets of random numbers φ and phi ρ with 0.5 offset instead of a pure random number, as given below in Equations (8) and (9).

$$\varphi = u + n_{offset} \quad (8)$$

$$\rho = u + n_{offset} \quad (9)$$

After using the revised set of-random numbers, the position-updating equation becomes

$$X_i(t+1) = \begin{cases} p(t) + \beta \times |M_{best} - X_i(t)| \times \ln \frac{1}{\rho}, & \text{if } u \geq 0.5 \\ p(t) - \beta \times |M_{best} - X_i(t)| \times \ln \frac{1}{\rho}, & \text{otherwise} \end{cases} \quad (10)$$

$$p(t) = \varphi \times p_{best}(t) + (1 - \varphi) \times g_{best}(g) \quad (11)$$

Considering the new set of random numbers and the proposed strategy, it is possible to observe that this approach enhances the global search ability in the early stage of the optimization and encourages the particles to converge quickly towards the global optimal solution.

3. Numerical Result Analysis

In this section, to elaborate the performance of our proposed algorithm, it is compared with some well-known optimization algorithms, including standard QPSO proposed by J. Sun et al. [24], GQPSO by L. dos S. Coelho [21], LIQPSO proposed by S. Jiang et al. [23], and MQPSO in [22]. The comparison conditions and benchmark functions listed in Table 1 are taken as the same for all algorithms: population sizes were set to be 40, corresponding to the dimension 30, and maximum iterations were set to 2000.

Table 1. High dimensional classical benchmark functions.

Modal	Name	Benchmark Functions	Search Space	$f(x^*)$
Unimodal	Sphere	$f_1(x) = \sum_{i=1}^n x_i^2$	$[-100, 100]^D$	0
	Schwefel's 2.22	$f_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	$[-100, 100]^D$	0
Multimodal	Rosenbrock	$f_3(x) = \sum_{i=1}^n [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-100, 100]^D$	0
	Griewank	$f_4(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-100, 100]^D$	0
Complex	Schwefel's Problem 1.2	$f_5(x) = \sum_{i=1}^D \left(\sum_{j=1}^n z_j \right)^2 + f_{bias_1}$ $z = x - o$ and $f_{bias_1} = -450$	$[-100, 100]^D$	0
	Griewank	$f_6(x) = \frac{1}{4000} \sum_{i=1}^n z_i^2 - \prod_{i=1}^n \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f_{bias_2}$ $z = x - o$ and $f_{bias_2} = -180$	$[-100, 100]^D$	0

All algorithms are executed with the same number of function evolutions to make judicious comparison when analyzing the statistical data of these benchmark functions, as shown in Table 2.

Table 2. High dimensional classical benchmark functions results.

Sphere f_1					
	QPSO	GQPSO	LIQPSO	MQPSO	SQPSO
Max	3.00	2.00	0.00	-4.00	-26.66
Min	-32.60	-14.40	-41.50	-160.00	-282.40
Std	13.95	4.43	12.69	60.38	72.92
Mn	-13.43	-2.88	-27.17	-78.82	-230.27
Schwefel's 2.22 f_2					
	QPSO	GQPSO	LIQPSO	MQPSO	SQPSO
Max	1.00	0.00	5.00	-39.80	-8.58
Min	1.00	-96.30	-111.67	-138.01	-352.19
Std	0.00	29.18	34.63	25.14	99.77
Mn	1.00	-43.47	-75.96	-125.62	-175.87

Table 2. Cont.

Rosenbrock f_3					
	QPSO	GQPSO	LIQPSO	MQPSO	SQPSO
Max	0.75	0.40	1.40	−0.60	1.59
Min	0.75	−2.42	−2.60	−3.70	−10.33
Std	0.00	0.36	0.48	0.78	2.92
Mn	0.75	−0.19	−1.06	−3.46	−7.30
Griewank f_4					
	QPSO	GQPSO	LIQPSO	MQPSO	SQPSO
Max	1.20	1.60	1.80	1.20	−7.83
Min	−4.20	−7.20	−5.30	−12.01	−36.04
Std	2.17	1.74	1.19	2.79	6.47
Mn	−1.91	−6.71	−4.21	−11.22	−33.61
Schwefel's Problem $1.2f_5$					
	QPSO	GQPSO	LIQPSO	MQPSO	SQPSO
Max	0.10	0.05	0.12	−1.70	6.04
Min	−7.48	−3.20	−5.90	−7.10	−8.38
Std	2.89	0.95	1.80	1.32	4.31
Mn	−3.09	−0.89	−1.48	−6.59	−5.14
Complex Griewank f_6					
	QPSO	GQPSO	LIQPSO	MQPSO	SQPSO
Max	0.25	0.15	0.25	−4.80	1.38
Min	−6.30	−1.50	−3.10	−6.20	−6.31
Std	2.61	0.58	1.30	0.21	1.68
Mn	−2.84	−0.41	−1.35	−6.17	−3.42

In Table 2, four indicators: the minimum (best), the maximum, the mean and the SD, are used to measure the performance of a SQPSO in comparison with other algorithms. Each algorithm runs ten times to attain the average value of each indicator for a fair comparison.

4. Result and Discussion

Based on these corresponding data indicators, it is noted that SQPSO shows better performance. In Table 2, the statistical optimized results are highlighted for our proposed algorithm on f_1 and f_2 functions, which are unimodal benchmark problems. Similarly, the best results are also highlighted for complicated multimodal functions f_3, f_4 and complex functions with various premature convergences and global solutions for other variants of algorithms tested on f_5 and f_6 . The presented functions are more dynamically challenging and complex and therefore research experts commonly utilize them as benchmark problems for computing algorithm tests. Consequently, the tabulations depict that our proposed novel smart SQPSO excels in performance compared to other well-known modified algorithms on the presented optimization problems.

Moreover, to clarify the convergence effect of the SQPSO over time and speed, Figures 1–6 present the convergence curves for all benchmark problems. SQPSO converges more rapidly with the optimal global region than the basic QPSO, GQPSO, LIQPSO and MQPSO, specifically in the earlier variants, in f_1, f_2 , as presented by the test functions' graphical comparison curves. In the same way, the convergence trajectory for other test functions show the proposed method's computational superiority in comparison to other state of the art algorithms. To conclude, our modified algorithm finds all the test functions' global optimal solution, highlighting that the proposed algorithm is more robust and efficient.

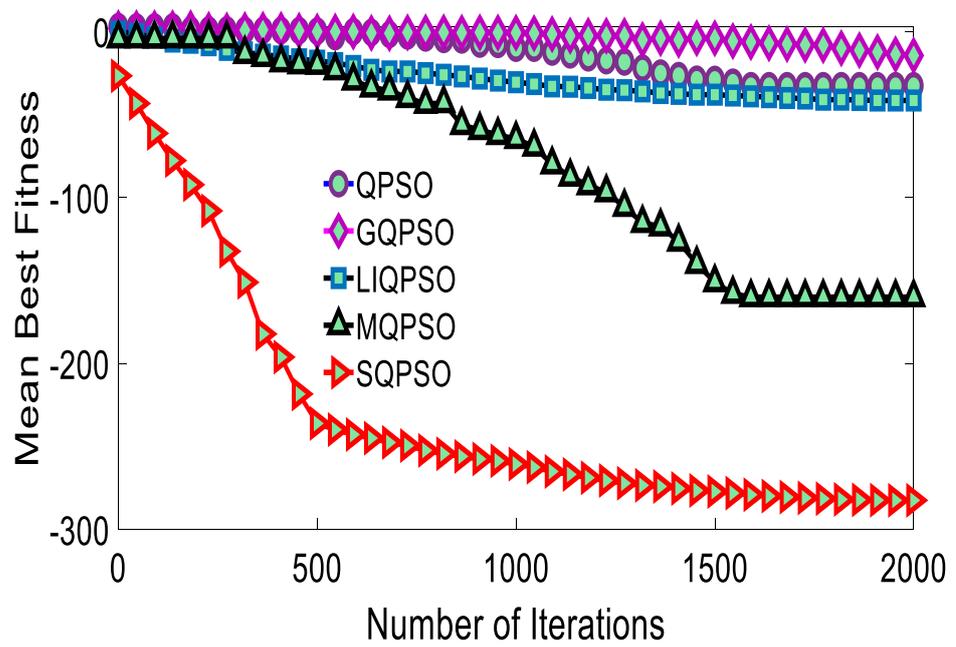


Figure 1. The convergence curve of algorithms on f_1 .

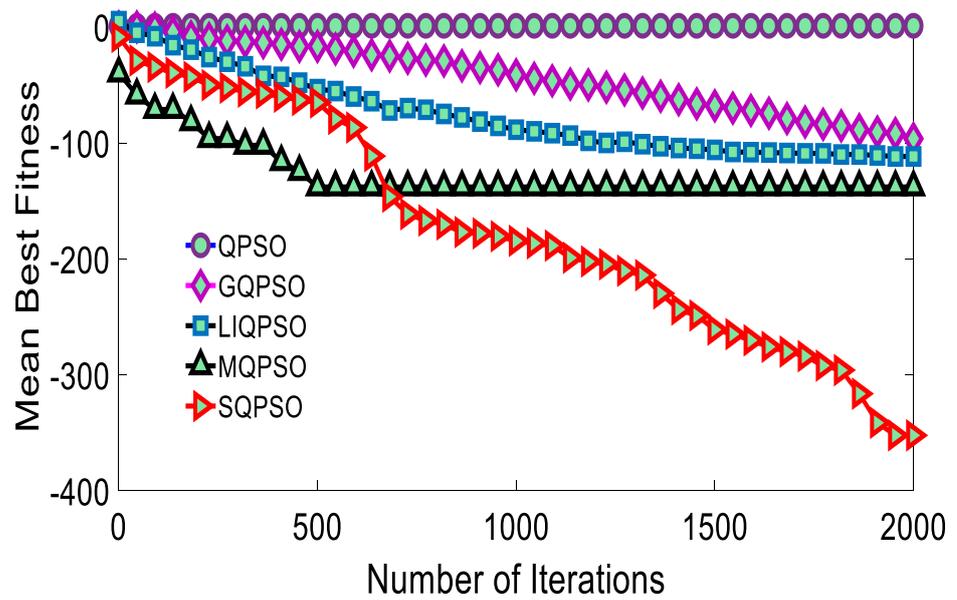


Figure 2. The convergence curve of algorithms on f_2 .

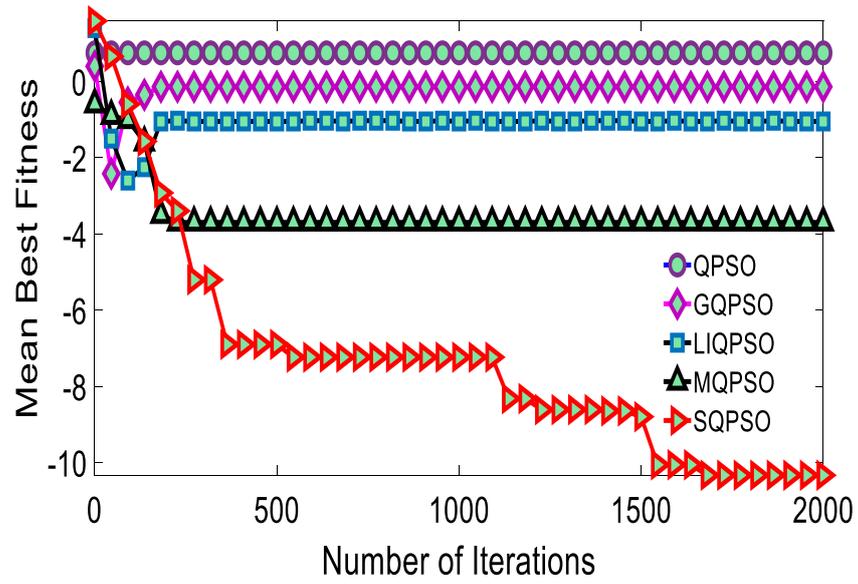


Figure 3. The convergence curve of algorithms on f_3 .

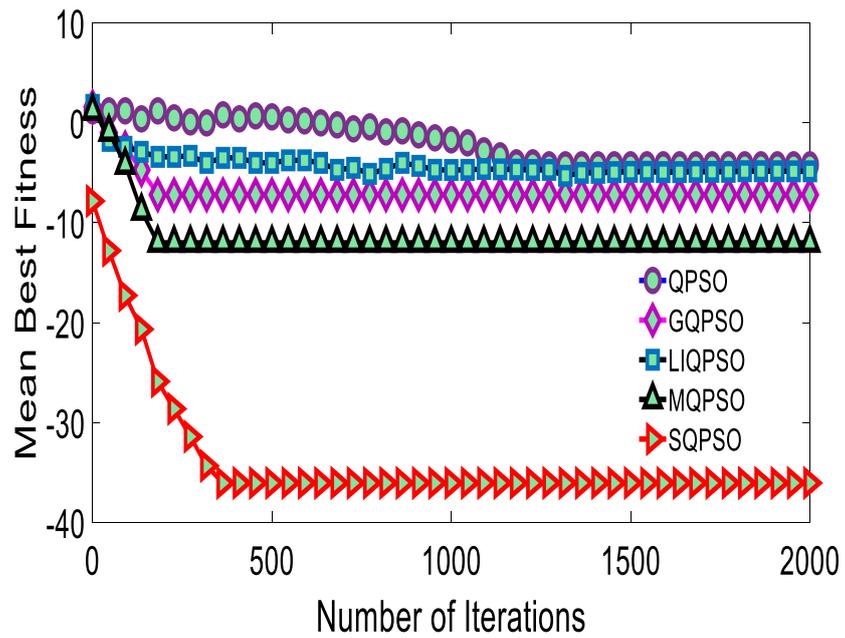


Figure 4. The convergence curve of algorithms on f_4 .

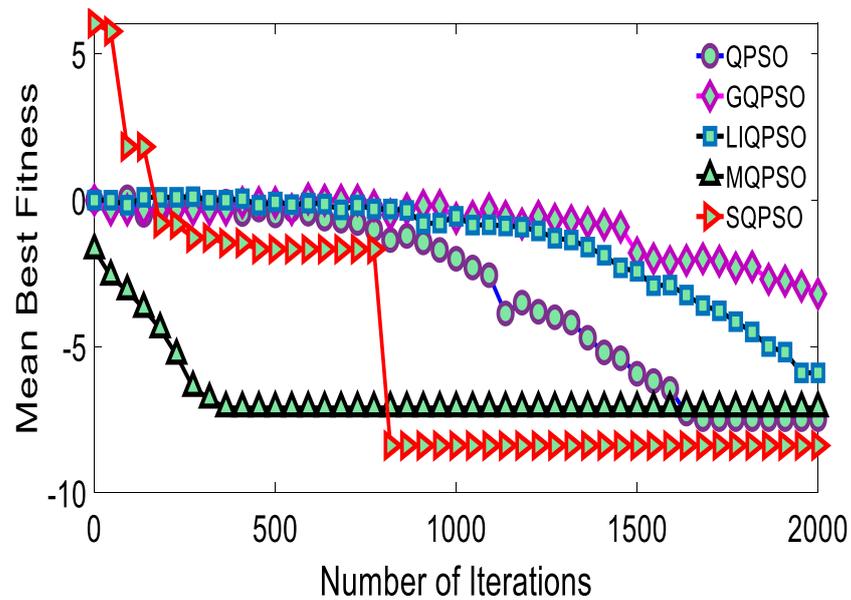


Figure 5. The convergence curve of algorithms on f_5 .

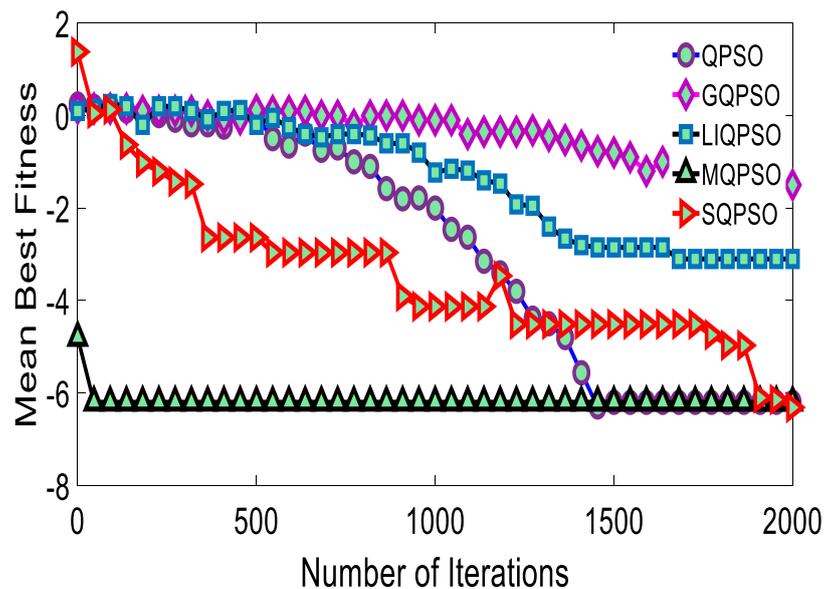


Figure 6. The convergence curve of algorithms on f_6 .

From these computational results and statistical analyses, it is clear that the proposed algorithm's convergence curves and corresponding results are better than the standard QPSO, GQPSO and LIQPSO, and MQPSO and it converges to the global minimum using fewer iterations. Moreover, the execution time of SQPSO is shorter than other versions of QPSOs.

5. Numerical Validation for Engineering Problems

The efficacy of the proposed SQPSO control algorithm has been already validated by benchmark functions. Thus, to make further validation of the SQPSO, we implemented the algorithm on the TEAM workshop problem 22, using this as a test suite for electromagnetic optimization problems [25]. We leveraged the same aforementioned evaluation parameters used for QPSO, GQPSO and LIQPSO and MQPSO. In Figure 7, a sample reward of a SMES device consists of two superconducting concentric coils carrying currents in the opposite

direction, with the corresponding radius, heights, thickness and search space of the stray field [26].

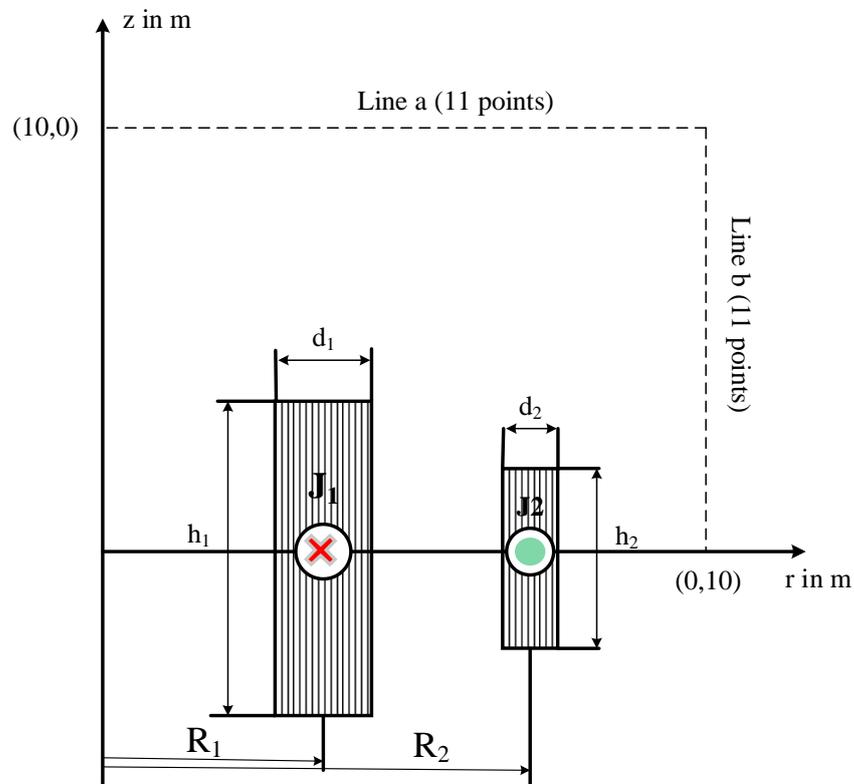


Figure 7. Schematic diagram of SMES optimization TEAM problem 22.

Objective Function of the TEAM Problem 22

In this paper, we consider the stray field as the objective function with three design parameters: radius, height and thickness of the SMES device, i.e.,

$$OF = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|E - E_{ref}|}{E_{ref}} \quad (12)$$

where $E_{ref} = 180MJ$, $B_{norm} = 3 \times 10^{-3}T$ and B_{stray}^2 is defined as:

$$B_{stray}^2 = \frac{\sum_{i=1}^{22} |B_{stray,i}|^2}{22} \quad (13)$$

The main aim of this work was to find the global optimal solution to the problem examined above. The problem was solved by considering three design variables of the outer coil in continuous states with a dynamic current density following through the quench condition, which guarantees that the superconducting material should work safely. This linked the value of the current density in the coils with the maximum value of the generated flux density according to the TEAM workshop problem 22.

Hence, the experimental results in Table 3 show the SQPSO stray field minimum compared with that of the other algorithms.

Table 3. Performance comparison of different optimal methods on Team problem 22.

Algorithm	R_2	$h_2/2$	d_2	OF
QPSO	3.0786	0.2414	0.3795	0.1077
GQPSO	3.1723	0.2319	0.3892	0.1222
LI-QPSO	3.0214	0.2732	0.3419	0.0959
MQPSO	3.1396	0.3160	0.2871	0.0716
SQPSO	3.0245	0.2561	0.2871	0.0278

6. Conclusions

In this paper, to improve the reliability of QPSO in solving electromagnetic optimization problems, a SQPSO was used to encode particles using memory adaptation of smart behavior. A replacement of the particles' best position in the whole swarm is no longer required to update the optimal position; instead, individual experience is used for this purpose. Consequently, several significant features are now possessed by the developed SQPSO: (1) a smart particle; (2) a memory archive; and (3) a new set of random variables and control parameters to reach the global minima without premature convergence in a shorter execution time. The numerical experimental results show that this algorithm effectively improves the global search ability and earlier convergence rapidity compared with other modified optimal QPSO algorithms. Moreover, the computational results verified its comprehensive applications for multimodal objective functions of electromagnetic optimization problems.

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