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Heat Transmission of Engine-Oil-Based Rotating Nanofluids Flow with Influence of Partial Slip Condition: A Computational Model

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Abstract: This particular research was conducted with the aim of describing the impact of a rotating nanoliquid on an elasting surface. This specific study was carried out using a two-phase nanoliquid model. In this study engine oil is used as the base fluid, and two forms of nanoparticles are used, namely, titanium oxide and zinc oxide (TiO₂ and ZnO). Using appropriate similarity transformations, the arising system of partial differential equations and the related boundary conditions are presented and then converted into a system of ordinary differential equations. These equations are numerically tackled using powerful techniques. Graphs for nanoparticle rotation parameter and volume fraction for both types of nanoparticles present the results for the velocity and heat transfer features. Quantities of physical significance are measured and evaluated, such as local heat flux intensity and local skin friction coefficients at the linear stretching surface. Numerical values for skin friction and local heat flux amplitude are determined in the presence of slip factor.

Keywords: rotating flow; linear stretching surface; partial slip; ZnO and TiO₂ nanoparticles

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1. Introduction

In different fields of industrial processing (e.g., material processing), the cooling of a metal plate in a cooling bath, the extrusion of plastic sheets, and the movement of a rotational fluid over enlarging surfaces are very common. Besides, revolving flow over a stretching surface is often used in glass driving and in the spinning phase of fibers. The heat transmission rate on the stretching surface is mutually linked to the quality of the final product, and it has been the subject of intensive research as it relates to the production process. Many researchers have concentrated on this particular field of study, bearing in mind this essential characteristic of the industrial process. By explaining the fundamental flow occurrence over an extending surface, Crane [1] took the initiative. Siddappa et al. [2] developed the concept of flow over a stretching surface in their study.

An initial research on three-dimensional (3D) movement over an enlarging flat surface was proposed by Wang [3]. Rajagopal et al. [4] described the flow of a viscoelastic fluid above a widening sheet. Wang et al. [5] discovered nanofluids' thermal conductivity in their work. Numerous researchers have followed this revolutionary work, in which a complete variety of physical complications were extracted. Nath and Kumari [6] operated the characteristics of MHD flow and temperature transfer flow on an extending sheet with a magnetic field. Narahari et al. [7] presented the notion of unstable MHD (magnetohydrodynamics) boundary layers in a rotational flow. Wang [8] primarily inspected the movement of a revolving fluid above a widening surface. Vajravelu et al. [9] conducted remarkable research on the arithmetic and logical clarifications of a nonlinear system resulting from a three-dimensional rotating flow.

Nanofluids are a significant exploration target that is extremely respected as it is commonly used in a range of greatly progressive biomedical and scientific applications, such as nuclear reactors, microprocessor cooling, and radiators. Nanofluids are known to be highly effective cooling agents owing to their lesser molecular masses and extreme thermal conductivity. Buongiorno [10] studied the heat convection of nanofluids. Pak and Cho [11] explored the hydrodynamic and temperature transmission features of scattered fluids using metal oxide particles. Kakac et al. and Xuan et al. [12,13] studied heat transfer phenomena in nanofluids. Wong et al. [14] discussed the potential applications of nanofluids. Bachok et al. [15] expanded their experiments on a porous widening/shrinking surface for nanofluids. Kumari et al. [16] illuminated the intermittent spinning flow of an MHD power-law fluid above a broadening sheet. Zhou et al. [17] studied the wear effects and key oil constituents on low-viscosity engine oils.

Recently, research was carried out by Abbas et al. [18] to overcome the unsteady nanofluid magnetohydrodynamic flow through a porous medium past a moving plate in a revolving device, taking heat and mass transfer into account. Wróblewski et al. [19] discussed the configuration of an engine with increased isochoric pressure. Nadeem et al. [20] investigated heat transmission on an exponentially stretchable surface for water-based nanofluids. Bahiraei et al. [21] presented the competence of hybrid nanofluids. Bagherzadeh et al. [22] described the thermal conductivity of different nanofluids analytically instead of numerically. Ahmadi et al. [23] discussed the viscosity of a CuO/water nanofluid. Bagherzadeh et al. [24] discussed the efficacy of a hybrid nanofluid in a new microchannel. Bahmani et al. [25] studied the turbulent heat transfer in binary pipe exchangers. Indeed, many researchers have recently studied problems associated with nanofluids [26–33]. Noghrehabadi et al. [34] prescribed the slip effect at constant wall temperature. To the best of our understanding, no one has identified the joints of the rotation or the partial slip of the nanofluid above the extending surface. This type of flow could have potential uses in a range of current products and applications, including electronic chip cooling, hot rollers, plasma flow, and geothermal engineering.

The main aspect of this analysis is the numerical examination of the velocity and heat transfer properties of a rotating nanofluid above a linear extending surface using a partial slip effect. In this analysis, a physical model (i.e., a two-phase nanofluid physical model) is used to address this issue. Numerical outcomes and diagrams are obtained using the BVP4C technique. The effects of the dimensionless slip parameter K , volume fraction φ , and rotation parameter λ on temperature, velocity, and skin friction coefficients are tabularized, graphically presented, and discussed. ZnO and TiO₂ nanoparticles are used for this comparative study because they have higher and lower density, respectively, and to the best of our knowledge there has not yet been a demonstration of this combination with base engine oil.

2. Mathematical Formulation

The following Figure 1 describes the geometry of the problem. Consider viscous and incompressible engine oil-based nanofluid with constant density lying above a linearly stretching surface in region $z \geq 0$ which rotates along the z -axis with definite fixed angular velocity Ω . Along the x -axis, two equal and opposing forces are exerted such that the surface is extended at a speed $U(x)$ which is directly proportional to the distance from the origin at $x = 0$. The nanofluid is susceptible to slip-flow conditions and there are no external forces acting upon it. After neglecting the pressure gradient and the viscous dissipation, the arising energy and momentum equations [35] are as given below.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 2\Omega v + \frac{\mu_{nf}}{\rho_{nf}} \nabla^2 u \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -2\Omega u + \frac{\mu_{nf}}{\rho_{nf}} \nabla^2 v \quad (3)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}} \nabla^2 w \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2}. \quad (5)$$

Figure 1. Geometry of the problem.

Here u is the x component, v is the y component, and w is the z component of velocity Ω ; μ_{nf} is the fluid's dynamic viscosity, ρ_{nf} is the nanofluid density, α_{nf} is the thermal diffusivity of the nanofluid, T is the nanofluid temperature, $(\rho C_p)_{nf}$ is the volumetric heat capacity of the nanofluid, $(\rho C_p)_f$ is the volumetric heat capacity of the base fluid, and $(\rho C_p)_s$ is the volumetric heat capacity of the solid nanoparticles. All of these are related to the particle [35] volume fraction of nanoparticles φ as follows.

$$\left\{ \begin{array}{l} \rho_{nf} = \rho_f(1 - \varphi) + \varphi\rho_s, \quad \mu_{nf} = \frac{\mu_n}{(1 - \varphi)^{5/2}}, \quad (\rho C_p)_{nf} = \frac{k_{nf}}{\alpha_{nf}}, \\ (\rho C_p)_{nf} = (\rho C_p)_f(1 - \varphi) + \varphi(\rho C_p)_s, \quad \frac{k_f}{k_{nf}} = \frac{k_s + 2k_f + 2\varphi(k_f - k_s)}{k_s + 2k_f - 2\varphi(k_f - k_s)}. \end{array} \right\} \quad (6)$$

Here, k_f is the thermal conductivity of the base fluid, k_s is the thermal conductivity of the solid nanoparticles, ρ_s is the density of the solid nanoparticles, and ρ_f is the density of the base fluid.

The corresponding boundary conditions are given below.

$$\left\{ \begin{array}{l} u(x, y, 0) - U = kv \frac{\partial u}{\partial z}(x, y, 0), \\ v(x, y, 0) = kv \frac{\partial v}{\partial z}(x, y, 0), \\ w(x, y, 0) = kv \frac{\partial w}{\partial z}(x, y, 0). \end{array} \right\} \quad (7)$$

Here $U = ax$ is the velocity of the surface, " a " is the proportionality constant representing the surface extension rate, k is the slip distance, and v is the base fluid's kinematic viscosity.

3. Transformation Methodology

By using the following similarity transformation [35], that is,

$$\left\{ \begin{array}{l} u = axf'(\eta), \quad v = axh(\eta), \quad w = -\sqrt{av}f(\eta), \\ \eta = z\sqrt{\frac{a}{v}}, \quad \theta(\eta) = \frac{T - T_\infty}{T - T_w}. \end{array} \right\} \quad (8)$$

Here η is the match space parameter, and T_∞ and T_w are the temperatures at the wall and free streams, respectively.

By using Equation (8) in Equation (1), we observe that it remains identical (i.e., mass remains conserved). Equations (2)–(5), using values from Equations (6) and (8), are transformed into ordinary differential equations as follows.

$$\frac{1}{(1-\varphi)^{5/2}} f''' + \left(1 - \varphi + \varphi \frac{\rho_s}{\rho_{bf}}\right) (ff'' - f'^2 + 2h\lambda) = 0 \quad (9)$$

$$\frac{1}{(1-\varphi)^{5/2}} h'' + \left(1 - \varphi + \varphi \frac{\rho_s}{\rho_{bf}}\right) (fh' - hf' - 2\lambda f') = 0, \quad (10)$$

$$\frac{k_{nf}}{k_f} \theta''(\eta) + Pr \left[1 - \varphi + \varphi \frac{(\rho C_p)_s}{(\rho C_p)_f}\right] f\theta'(\eta) = 0 \quad (11)$$

In these Equations (9)–(11), λ is the rotation parameter and Pr denotes Prandtl number which is defined as

$$\lambda = \frac{\Omega}{a}, \quad Pr = \frac{(\mu C_p)_f}{k_f} \quad (12)$$

From the above-given equations, Ω is the fixed angular fluid's velocity.

Using Equation (8) in Equation (7), the corresponding boundary conditions take the form

$$\left\{ \begin{array}{l} f(0) = 0, \quad f'(0) = 1 + Kf''(0), \quad f'(\infty) = 0, \\ h(0) = Kh'(0), \quad h(\infty) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0. \end{array} \right\} \quad (13)$$

Here $K = K\sqrt{av}$ is the slip parameter.

4. Method of Solution

The combined system of ordinary differential Equations (9)–(11) is vastly non-linear and has been tackled numerically using the BVP4C technique built-in MATLAB. The procedure transforms the leading differential equations into an ordinary differential equation system of the first order that is iteratively addressed. Numerical tables and graphical results are acquired using the very capable tool MATLAB. The physical quantities we evaluate are the skin frictions Cf_x , Cf_y along the x -axis and y -axis respectively and the Nusselt number Nu , where

$$\left\{ \begin{array}{l} Cf_x = \frac{\tau_{xz}}{\rho(ax)^2}, \\ Cf_y = \frac{\tau_{yz}}{\rho(ax)^2}, \\ Nu = \frac{xq_w}{k_f(T-T_\infty)}. \end{array} \right\} \quad (14)$$

Here q_w is the heat flux, and τ_{xz} and τ_{yz} [36] are shear stresses of the surface given by

$$\left\{ \begin{array}{l} \tau_{xz} = \mu_{nf} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)_{z=0'}, \\ \tau_{yz} = \mu_{nf} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)_{z=0'}, \\ q_w = -k_{nf} \left(\frac{\partial T}{\partial z} \right)_{z=0}. \end{array} \right\} \quad (15)$$

By substituting Equations (8) and (15) in Equation (14), we get

$$\left\{ \begin{array}{l} (Re_x)^{1/2} Cf_x = \frac{1}{(1-\varphi)^{5/2}} f''(0), \\ (Re_x)^{1/2} Cf_y = \frac{1}{(1-\varphi)^{5/2}} h'(0), \\ (Re_x)^{-1/2} Nu = \frac{-k_{nf}}{k_f} \theta'(0), \end{array} \right\} \quad (16)$$

where the local Reynolds number is $Re_x = \frac{(ax)_x}{v_f}$, skin frictions $\frac{1}{(1-\phi)^{5/2}} f''(0)$, $\frac{1}{(1-\phi)^{5/2}} h'(0)$, and heat flux $\frac{-k_{nf}}{k_f} \theta'(0)$ are presented together for both nanofluids via different variations of volume fraction ϕ , rotation parameter λ , and slip parameter K .

5. Results and Graphical Interpretation

This section addresses the effect of different physical constraints (e.g., rotation parameter λ , volume fraction ϕ , and slip constraint K) on the temperature and velocity profiles of the TiO_2 and ZnO engine oil nanofluids. Graphical outcomes are presented to determine their real effect on these physical quantities. The following figures are plotted for this purpose.

The plots in Figure 2a–d are given to determine the effect on the velocity profile $f'(\eta)$ of the rotation parameter λ and slip parameter K for ZnO and TiO_2 engine oil nanofluids. In Figure 2a,b it can be seen that an increase of the rotation parameter λ results in a decreasing behavior of the velocity profile $f'(\eta)$ for both nanoliquids. This is because the existence of the slip velocity factor overcomes the motion of the fluid associated with its increasing rotation. By increasing the slip parameter K , Figure 2c,d shows that the horizontal velocity $f'(\eta)$ increases for both ZnO and TiO_2 engine oil nanofluids. This results from the existence of slip—the velocity of the fluid near the surface does not remain similar to the extension of the surface. Therefore, slip velocity also increases by increasing the slip parameter K . This subsequently reduces the fluid velocity, since in this case the drawing of the extending surface can be partially explained by the existence of the slip factor. As the slip factor K increases, the frontier layer thickness decreases. However, it is fascinating to know that the insertion of ZnO nanoparticles leads to a decrement in the velocity profile $f'(\eta)$, and the behavior is opposite in the case of TiO_2 nanoparticles (Figure 2c,d). In Figure 2a–d the lines on the graph are very close because of the base fluid. Engine oil has a higher viscosity than that of water. So, when rotation starts there is a very small change in the velocity profile $f'(\eta)$, and when the rotation rate increases, the velocity lines separate. Similar arguments can be made regarding the slip parameter ZnO . The influence of the partial slip factor K and rotation constraint λ on the vertical velocity constituent $h(\eta)$ is presented in Figure 3a–d for both ZnO and TiO_2 engine oil nanofluids. Figure 3a,b shows that the vertical velocity $h(\eta)$ decays with increasing rotation constraint λ for both ZnO and TiO_2 engine oil nanofluids. Additionally, $h(\eta)$ decreases as the slip parameter K increases near the wall. This specific behavior can be noted for both ZnO and TiO_2 engine oil nanofluids (Figure 3c,d). In Figure 4a–d, it can be determined that the velocity profile $f(\eta)$ increases with increasing slip parameter K and decreases with increasing rotation parameter λ . Furthermore, with the addition of ZnO nanoparticles, the velocity profile $f(\eta)$ decreases (Figure 4a,c), and it increases with the addition of TiO_2 nanoparticles (Figure 4b,d). A distinction in the temperature profile $\theta(\eta)$ with varying slip parameter K and volume fraction ϕ for ZnO and TiO_2 engine oil nanofluids can be seen in Figure 5a–d. We can determine from Figure 5a,b that the temperature profile $\theta(\eta)$ significantly increases with increasing volume fraction ϕ and it decreases as slip parameter K increases (Figure 5c,d). It is interesting to note that the rotation parameter λ does not affect the temperature profile $\theta(\eta)$.

Figure 2. The impact of rotation parameter λ (a,b) and slip parameter K (c,d) on the velocity profile $f'(\eta)$ for ZnO and TiO₂ engine oil nanofluids.

Figure 3. The impact of rotation parameter λ (a,b) and slip parameter K (c,d) on the velocity profile $h(\eta)$ for ZnO and TiO₂ engine oil nanofluids.

Figure 4. The impact of rotation parameter λ (a,b) and slip parameter K (c,d) on the velocity profile $f(\eta)$ for ZnO and TiO₂ engine oil nanofluids.

Figure 5. The impact of volume fraction ϕ (a,b) and slip parameter K (c,d) on the temperature profile $f(\eta)$ for ZnO and TiO₂ engine oil nanofluids.

To verify the dissimilarities of fascinating physical constraints on the local skin friction coefficient and local Nusselt number, the following tables are presented. Table 1 represents the thermophysical properties of the nanofluid components. Table 2 illustrates that for a specific value of slip parameter K and fixed volume fraction ϕ , the local skin friction (i.e., $\frac{1}{(1-\phi)^{5/2}} f''(0)$) increases with increasing rotation parameter λ . The trend is similar if we specify the value of λ and change ϕ . However, for a permanent pair (ϕ, λ) , the absolute skin friction owing to ZnO is permanently higher than that owing to TiO₂, which can be explained by the comparatively higher density of ZnO particles. Local skin friction (i.e., $\frac{1}{(1-\phi)^{5/2}} h'(0)$) is calculated for different pairs of (ϕ, λ) . It can be seen from the tables that an increase in ϕ and λ increases the skin friction along the y -axis. Remarkably, for any pair (ϕ, λ) , the skin friction due to ZnO is higher than that for other nanofluids. Similarly, the local heat flux $\left(\frac{-k_{nf}}{k_f} \theta(0)\right)$ decreases as the rotation parameter λ increases. Meanwhile, increasing the nanoparticle volume fraction ϕ results in an increased heat transfer rate due to the greater thermal conductivity and lower specific heat of the nanofluid as compared to the base fluid. Further, for any fixed pair (ϕ, λ) , the local heat flux due to ZnO is always higher than that due to TiO₂. This is because TiO₂ has a lower thermal conductivity as compared with ZnO, which allows for the rapid removal of heat. In Table 3, the previously published data is given and compared to the present results. It is interesting to observe that the present values for ZnO are in better agreement with those obtained previously.

Table 1. Thermo-physical properties of nanoparticles (ZnO, TiO₂) and base fluid engine oil.

Component	Density (ρ)	Thermal Conductivity (K)	Specific Heat (C_p)
ZnO	7140	120	389
TiO ₂	4250	8.96	686.2
Engine oil	884	0.144	1910

Table 2. Variation of distinct parameters on Cf_x (skin friction along x -axis) (a,b), Cf_y (skin friction along y -axis) (c, d) and Nusselt number Nu_x (e, f).

(a) TiO ₂ engine oil nanofluid for $Pr = 6450$				
ϕ	$\lambda = 0.0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
0.0	1.0000	1.1384	1.3252	1.6523
0.01	1.0291	1.1715	1.3635	1.7004
0.1	1.3138	1.4956	1.7408	2.1708
0.2	1.6991	1.9342	2.2513	2.8075
(b) ZnO engine oil nanofluid for $Pr = 6450$				
ϕ	$\lambda = 0.0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
0.0	1.0000	1.1388	1.3255	1.6529
0.01	1.0525	1.1982	1.3946	1.7389
0.1	1.5290	1.7408	2.0262	2.5265
0.2	2.1282	2.4228	2.8199	3.5164

Table 2. *Cont.*

(c) TiO ₂ engine oil nanofluid for $Pr = 6450$.				
φ	$\lambda = 0.0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
0.0	0.0000	0.5130	0.8372	1.2873
0.01	0.0000	0.5277	0.8614	1.3247
0.1	0.0000	0.6737	1.0998	1.6912
0.2	0.0000	0.8713	1.4223	2.1872
(d) ZnO engine oil nanofluid for $Pr = 6450$.				
φ	$\lambda = 0.0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
0.0	0.0000	0.5132	0.8375	1.3243
0.01	0.0000	0.5401	0.8817	1.3599
0.1	0.0000	0.7844	1.2818	1.9691
0.2	0.0000	1.0915	1.7832	2.7399
(e) TiO ₂ engine oil nanofluid for $Pr = 6450$.				
φ	$\lambda = 0.0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
0.0	1.7710	1.7257	1.6603	1.5335
0.01	1.7863	1.7393	1.6716	1.5402
0.1	1.9265	1.8648	1.7752	1.6033
0.2	2.0932	2.01254	1.9025	1.6898
(f) ZnO engine oil nanofluid for $Pr = 6450$.				
φ	$\lambda = 0.0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
0.0	1.7923	1.7294	1.6802	1.5340
0.01	1.8201	1.7726	1.6723	1.5345
0.1	1.9823	1.8932	1.7832	1.5521
0.2	2.1671	2.0991	1.8303	1.5743

Table 3. Comparison of absolute values of $f''(0)$ and $h'(0)$.

λ	Wang [8]		Nazar et al. [37]		Kumari et al. [6]		Hayat et al. [36]		Present Results	
	$f''(0)$	$h'(0)$	$f''(0)$	$h'(0)$	$f''(0)$	$h'(0)$	$f''(0)$	$h'(0)$	$f''(0)$	$h'(0)$
0.0	1.0000	0.0000	1.000	0.0000	0.9999	0.0000	1.0000	0.0000	1.0000	0.0000
0.5	1.1384	0.5128	1.1384	0.5128	1.1382	0.5128	1.1383	0.5127	1.1385	0.5130
1.0	1.3250	0.8371	1.3250	0.8371	1.3251	0.8371	1.3250	0.8370	1.3252	0.8372
2.0	1.6523	1.2873	1.6523	1.2873	1.6535	1.2873	1.6523	1.2872	1.6525	1.2874

6. Graphs of Skin Friction and Nusselt Number

Figure 6 shows the impact of rotation parameter on skin friction along the x -axis. Figure 7 demonstrated the influence of rotation parameter on skin friction along the y -axis. Figure 8 described the effect of rotation parameter on Nusselt number.

Figure 6. The impact of rotation parameter λ (a,b) on skin friction along the x-axis.

Figure 7. The impact of rotation parameter λ (a,b) on skin friction along the y-axis.

Figure 8. The impact of rotation parameter λ (a,b) on the Nusselt Number (Nu).

7. Conclusions

In this study, the partial slip effect of a rotating engine-oil-based nanofluid's flow on a linear extending surface is thoroughly analyzed. Further deductions can be made as follows:

1. Partial slip increases and rotation decreases the velocity of both nanofluids.
2. The numerical values of skin friction for $\phi = 0$ in the case of a viscous fluid match those already published, confirming the accuracy of the present results.
3. The temperature of the nanofluid decreases with increasing partial slip.

4. Volume fraction ϕ and rotation λ both increase the temperatures of both nanofluids, but this effect is more prominent in the case of ZnO as compared to TiO₂ engine oil nanofluid.
5. Local skin resistances increase with increasing nanoparticle angular velocity and particle volume fraction. This rise is higher in the case of the ZnO-based nanofluid.
6. The entire heat transfer of the surface increases with increasing volume fraction and decreases with increasing rotation.
7. The ZnO-based engine oil is proved to be a good heat carrier as compared to the TiO₂ engine oil nanofluid.

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Nomenclature

a	surface stretching rate
Cf_x, Cf_y	skin friction coefficients along x - and y -axis.
C_p	specific heat at constant pressure
f, h	dimensionless components of velocity
T, T_w, T_∞	fluid temperature, wall temperature, and free stream temperatures, respectively
u, v, w	velocity components of x, y , and z directions
V	velocity field
x, y, z	Cartesian coordinates
Pr	Prandtl number
K	slip parameter
Greek Symbols	
η	dimensionless space variable
θ	dimensionless temperature
ϕ	volume fraction of nanoparticles
λ	rotation parameter
Ω	constant angular velocity
k_s, k_f, k_{nf}	thermal conductivities of solid nanoparticle, base fluid, and nanofluid, respectively
$\rho_s, \rho_f, \rho_{nf}$	density of solid nanoparticle, base fluid, and nanofluid, respectively
μ_f, μ_{nf}	dynamic viscosities of the base fluid and nanofluid, respectively
ν_f, ν_{nf}	kinematic viscosities of the base fluid and nanofluid, respectively
α_f, α_{nf}	thermal diffusivities of the base fluid and nanofluid, respectively
$(\rho C_p)_f$	volumetric heat capacity of the base fluid
$(\rho C_p)_{nf}$	volumetric heat capacity of the nanofluid
Subscripts	
w, ∞	conditions at wall and infinity, respectively
nf, f	nanofluid and base fluid, respectively
s	solid nanoparticles
Superscripts	
${}', {}'', {}'''$	1st, 2nd, and 3rd derivative for η

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