

# Article Performance Study of a Bladeless Microturbine

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**Abstract:** The paper presents a comprehensive numerical and experimental analysis of the Tesla turbine. The turbine rotor had 5 discs with 160 mm in diameter and inter-disc gap equal to 0.75 mm. The nozzle apparatus consisted of 4 diverging nozzles with 2.85 mm in height of minimal cross-section. The investigations were carried out on air in subsonic flow regime for three pressure ratios: 1.4, 1.6 and 1.88. Maximal generated power was equal to 126 W and all power characteristics were in good agreement with numerical calculations. For each pressure ratio, maximal efficiency was approximately the same in the experiment, although numerical methods proved that efficiency slightly dropped with the increase of pressure ratio. Measurements included pressure distribution in the plenum chamber and tip clearance and temperature drop between the turbine's inlet and the outlet. For each pressure ratio, the lowest value of the total temperature marked the highest efficiency of the turbine, although the lowest static temperature was shifted towards higher rotational speeds. The turbine efficiency could surpass 20% assuming the elimination of the impact of the lateral gaps between the discs and the casing. The presented data can be used as a benchmark for the validation of analytical and numerical models.

**Keywords:** microturbine; bladeless turbine; radial turbine; Tesla turbine; experimental data; numerical simulation

# 1. Introduction

One of the biggest challenges in power engineering is to provide a continuous power supply to all-electric grid users. The power units have to be on the one hand reliable and efficient, but on the other hand must not pollute the environment. The growing share of renewable energy units in the energy mix is a positive trend, but certain limitations (e.g., a limit of the share of wind energy in the total amount of electricity generated in the system [1] and the unpredictability of photovoltaics and wind turbines [2]) do not allow to fully replace power plants burning fossil fuels. One of the ways of improving this situation is a recuperation of waste heat. More than 50% of the energy used in the world is wasted as low-temperature heat [3] (e.g., 60% of fuel energy in internal combustion engines is lost as waste heat [4]), so retrieving a part of this energy is vital to increase the efficiency of a process and thereby decrease the fuel consumption. The organic Rankine cycle is a technology suitable for this aim. It is similar to a classic Rankine cycle, but the working medium is an organic fluid characterized by the low temperature of boiling. One of the most important parts of such a unit is the expander. Dumont et al. [5] and Zywica et al. [6] conducted a comparative analysis of expanders suitable for small-scale ORC systems. The Tesla turbine seems to be an interesting alternative to the expanders used so far.

The Tesla turbine is a bladeless radial turbine invented by Nicola Tesla in 1913 [7]. Contrary to conventional turbines, its principle of operation is based on viscous effects in a fluid. Despite its unsophisticated construction, phenomena in the turbine are quite complex. The supply system is responsible for the increase of the kinetic energy of the working medium and its orientation in relation to the discs. It may consist of a plenum



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). chamber and a set of nozzles or guide vanes [8]. The supply system is the major reason for low turbine efficiency [9,10]. Despite the fact that nozzles can work efficiently, there is a problem with phenomena like shock waves or the interaction between the inlet apparatus and rotor [11]. More details regarding the selection of the inlet shape can be found in [9,12]. The rotor of the Tesla turbine is its most characteristic component. It is comprised of a set of thin discs mounted co-axially on a shaft. Fluid entering the rotor creates stresses on the discs' walls. The circumferential component of the stress can be considered as a driving factor of the turbine, contrary to conventional turbines, where the wall stresses contribute to the efficiency drop. It can be utilized to formulate an alternative approach to efficiency determination. Allen [13] proposed the turbine performance estimation as a ratio of circumferential stresses generated on the disc wall to the sum of radial and circumferential components. The rotor theoretical efficiency calculated based on the adiabatic enthalpy drop under specific conditions (laminar flow and small mass flow rate) can be higher than 95% [10]. However, due to the non-uniformity of the flow and kinetic energy loss in the outlet system, it is not an easy task to reach that level of efficiency. Many researchers have carried out numerical and analytical investigations concerning flow in the rotor. Boyd and Rice [14] derived a 2D analytical model of incompressible flow. Carey [15] developed a one-dimensional model of the rotor, where viscous shear forces were substituted by body forces. The research proved that the Tesla turbine applied in a small-scale Rankine system can achieve isentropic efficiency above 75%. The model was confirmed to give results in line with numerical simulations of a real turbine [16]. Further improvement of this model was presented in [17,18], where the flow was treated as turbulent and compressible with the consideration of the radial pressure gradient. The model agreed well with experimental data within the low and medium range of rotational speeds but differed for high velocities. Guha and Sengupta [19] developed a three-dimensional model of the flow in the rotor, in which the impact of inertial, viscous, Coriolis and centrifugal forces on the fluid dynamics and generated power was explained. Based on the models presented in [15,19,20], Manfrida and Talluri [21] presented a two-dimensional analytical model, which took into account real fluid properties depending on local variables. In [22,23], the model was upgraded with the stator impact and detailed treatment of the losses and tests on refrigerant and hydrocarbon fluids were carried out. The model was also validated using Computational Fluid Dynamics CFD analysis and gave a correct distribution of kinematic and thermodynamic parameters [24]. Talluri et al. [25] performed an analysis concerning the possibility of the application of a Tesla turbine in ORC systems.

One of the most important parameters is the quality and shape of the disc surface. For micro-channels, where a relative roughness is higher than 0.05, the constriction of flow becomes an important factor [26]. Rusin et al. [11] proved by means of numerical simulation that the appropriate value of roughness can lead to a rise in generated power by 35%. Romanin and Carey presented in [27] an analytical model which took into account roughness. Another important factor influencing the turbine efficiency is disc spacing. Qi et al. [28] investigated an optimal spacing for two sets of inlet system configurations: one-to-one, where one nozzle supplies exactly one gap and one-to-many where one nozzle supplies all gaps. Research presented in [29] concerns the impact of the shape of the disc tips (blunt, sharp, circular and elliptic tips) and inlet nozzle configuration on the turbine performance. According to this research, a blunt tip is recommended for one-to-many inlet while a sharp tip for one-to-many tips. A similar scientific problem was a topic of investigation of Sengupta and Guha [30]. They concluded that small disc thickness, blunt disc tips, full nozzle opening, optimal radial clearance and uniform inlet condition were necessary factors to achieve high turbine performance.

An important part of the Tesla turbine analysis is the experimental investigations. One of the early works concerning this subject was the thesis of Armstrong [31]. He performed a series of investigations on a Tesla turbine with steam as the working medium. Steidel and Weiss [32] performed an experimental campaign on a 0.32 m Tesla turbine utilizing geothermal fluids. They obtained an efficiency below 7%. Leaman [33] investigated a

turbine whose rotor consisted of 4 discs with 10 cm in diameter and obtained 8.6% maximal efficiency. Davydov and Sherstyuk [34] carried out investigations on a turbine with 40 mm in diameter and air as a working medium. In the comprehensive study, they determined the influence of admission degree, gap spacing, number of discs and type of outlet on turbine performance. Works [35,36] provided design recommendations for scaling Tesla turbines to the millimetre scale and an experimental campaign. The maximal efficiency of such a turbine with water as the medium was equal to 36%. A new technique for measurement turbine net torque called the angular acceleration method was presented in [37]. Lemma et al. [38] investigated a 50 mm turbine whose plenum chamber was 100 mm in diameter. Maximum obtained power and efficiency were equal to around 220 W and 25%, respectively. It was concluded that the turbine could reach over 38% of efficiency if parasitic losses, like losses in the bearing, viscous losses in the end walls, and other dissipative losses in the plenum chamber, were eliminated. Talluri [39] investigated two turbines: the first one with 125 mm rotor diameter and air as the medium and the second one with 216 mm rotor diameter intended for ORC unit with R1233zd(E) as the medium. The maximum achieved efficiency of the air turbine was 11.2% and 9.62% for the ORC turbine. Rusin et al. [40] presented an investigation of a Tesla turbine, whose rotor had 73 mm in diameter. The maximum power obtained for a pressure ratio equal to 4 was 71.5 W and the maximum efficiency totalled 8.9%. Renuke et al. [41] manufactured two turbines: 64.5 mm and 120 mm in diameter and investigated the influence of inter-disc gap, disc thickness and exhaust area. Maximal efficiency was 23% for the larger turbine. Renuke et al. [42] investigated also 3 kW air Tesla expander with the focus on the influence of the number of nozzles. The authors concluded that the turbine worked better with two nozzles rather than with one or four nozzles. Experimental and numerical study concerning the impact of the disc thickness, gap between discs, radius ratio, and the outlet area of the exhaust are shown in [43].

Another important field of research concerning the Tesla turbine is its dynamic behaviour. Due to extremely high rotational speeds, often exceeding 20,000 rpm, the arising vibrations may constitute a serious problem. Bagiński and Jędrzejewski [44] carried out the strength and modal analysis of the rotor model of the Tesla turbine. Silvestri et al. [45] carried out extensive numerical and experimental investigations devoted to the dynamic behaviour of the rotor. A single disc and the whole assembled turbine rotor were examined. The developed models are of great utility in the design process of the turbine.

Research presented in this paper pertains to the experimental analysis of the 160 mm air Tesla turbine. The study fills in the gap of the lack of experimental data concerning the thermodynamic parameters within the turbine components. The investigations provide the performance characteristics, but also the pressure in the plenum chamber, in the radial rotor clearance and in the outlet, for different operating conditions, which makes it possible to divide the pressure drop between the turbine components. Measurement of the temperature drop between the inlet and the outlet is carried out as well. The experimental results are also used for the validation of the numerical model created using Ansys CFX software.

The paper is organized as follows: a new Tesla turbine prototype with the test stand is presented in the first part. The second part concerns the set-up of the numerical model and the last part is devoted to a comprehensive analysis of the results obtained.

# 2. Materials and Methods

#### 2.1. Tesla Turbine Prototype

The object of investigation was the new construction of a Tesla turbine (Figures 1 and 2), which was designed in the Department of Power Engineering and Turbomachinery at the Silesian University of Technology. Its rotor in nominal configuration consisted of five disks of 160 mm in diameter with inter-disc gaps equal to 0.75 mm. The surfaces of the discs were polished (arithmetical mean deviation of roughness heights  $R_a < 1$ ) to avoid consideration of the roughness in numerical calculations. The lateral clearance between

the rotor and the turbine casing was equal to 0.75 mm and the disc tip clearance totalled 0.5 mm. The flexible construction made it possible to change the inter-disc gap between 0.3 mm to 1.2 mm while keeping constant lateral clearance by the addition of stationary discs with appropriate thickness mounted to the casing. The rotor discs were made out of aluminium alloy and their thickness was equal to 2 mm, which prevented the discs from vibration at high rotational speeds. The inlet apparatus consisted of a plenum chamber and 4 converging inlet nozzles. The inlet nozzles were in configuration one-to-many [28], which means that the nozzles spread across the rotor width. This approach is less efficient compared to the one-to-one approach solution as the fluid hitting the disc tips generates losses. Nevertheless, it was necessary to maintain the flexibility of the rotor construction and to avoid misalignment of the nozzles and the inter-disc gaps. The dimensions of the nozzle's minimal cross-section were 2.85 mm in height and width depended on the rotor configuration (i.e., inter-disc gap width and the number of discs). The aim of the plenum chamber was to decrease the total pressure losses [9] and to organize the tangential inflow of the medium at the nozzles. The working medium was supplied to the chamber by two pipes, each 12.4 mm in diameter and 200 mm in length. There were 8 taps in the rotor casing intended to measure flow parameters in the plenum chamber and the rotor tip clearance. The outlet comprised 5 ellipsoidal holes in the rotor's discs close to the shaft and the collecting chamber. The total area of the set of holes in the disc was approximately equal to 350 mm<sup>2</sup>, which was enough to prevent the flow from choking at the outlet for the investigated pressure ratios. The collecting chamber gathers the working medium and enables outflow.



Figure 1. Internal flow system of the turbine (blue arrows-flow direction).



Figure 2. Cross-section of the turbine.

#### 2.2. Test Stand

The test stand was located in the Laboratory of Flow Machines of the Department of Power Engineering and Turbomachinery at the Silesian University of Technology. It consisted of the air installation, the measurement equipment and the Tesla turbine. The air installation (Figure 3) works in under-pressure and its main parts are the roots blower, air tank and piping. The roots blower generates under pressure (up to 50 kPa) in the tank to which the Tesla turbine was connected by the piping of 110 mm in diameter. Maximum output of the roots blower totalled 12.3 m<sup>3</sup>/s, which was enough to work in the continuous mode. The aim of the 3.4 m<sup>3</sup> tank was to stabilize the pressure pulsation and to make it possible to regulate pressure at the turbine outlet by the control valves. Closing the valves made airflow only through the turbine, thereby decreasing its operating pressure. Fully open valves made the roots blower suck air from the ambient, omitting the turbine. Due to pressure losses in the piping, the static pressure at the turbine outlet was slightly higher than the pressure in the tank. For this reason and the fact that there was a pressure drop in the turbine's rotor which increased pressure at the outlet section of the nozzles, those conditions were not enough to produce transonic flow at the nozzle's outlet. The turbine's inlet pipes were equipped with two confusors. Their aim was to minimize pressure losses, prevent vena contracta from occurring and level out the velocity profile. The measurement equipment (Figure 4) made it possible to acquire most of the important turbine parameters. Figure 5 presents the location of the selected measurement sensors. There were six pressure transducers: two in the middle of the plenum chamber (separated by 90°), two in the rotor tip clearance, one in the outlet section of the turbine, and one at the inlet. The temperature was measured utilizing J-type thermocouples with the sensors located in the ambient and at the outlet section. The turbine was loaded by means of a 1 kW brushless motor acting as the generator connected to the electric load. This configuration allowed setting the generator load at a demanded level. Torque transducer model HBM T21WN placed between the turbine shaft and the motor enabled the measurement of the rotational speed up to 20,000 rpm and torque up to 2 nm with the accuracy of  $\pm 0.2\%$ . It gave an opportunity to directly measure the shaft power of the turbine which could be later compared to the

results from the numerical modelling. All signals were gathered by the data acquisition system and processed using in-house build software programed in LabView.



Figure 3. Scheme of air installation.



Figure 4. Scheme of measurement unit.



Figure 5. Location of sensors: temperature (red dots) and pressure (blue dots).

The measurement procedure started with setting a demanded load on the generator. Then, the pressure level was adjusted by means of the control valves in the air tank. Due to the high inertia of the rotor, the time to obtain steady-state, i.e., small variability of power and rotational speed, for a given load was between 3–5 min. Data were sampled with a frequency of 100 Hz and a single measurement point was obtained by averaging the samples over 30 s.

#### 2.3. Numerical Model

Numerical simulations were performed using Ansys CFX commercial software, which is based on the finite volume method. The software uses an implicit finite volume scheme to solve discretized set of governing equations, which are mass, momentum and energy conservation laws [46]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{1}$$

$$\frac{\partial(\rho \boldsymbol{U})}{\partial t} + \nabla \cdot (\rho \boldsymbol{U} \otimes \boldsymbol{U}) = -\nabla \mathbf{p} + \nabla \cdot \boldsymbol{\tau}$$
<sup>(2)</sup>

$$\frac{\partial(\rho h_{tot})}{\partial t} - \frac{\partial p}{\partial t} + \nabla \cdot (\rho \boldsymbol{U} h_{tot}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\boldsymbol{U} \cdot \boldsymbol{\tau})$$
(3)

Term  $\nabla \cdot (\boldsymbol{U} \cdot \boldsymbol{\tau})$  represents the internal heating by viscosity in the fluid and due to the turbine's principle of operation, it cannot be neglected. Spatial discretization of the equations was carried out using the high-resolution scheme. Shear stress transport with intermittency was selected as the turbulence closure.

The geometry of the 3D model reflected all details of the turbine prototype. The only difference was the outlet ducts from the turbine, which were prolonged to obtain undisturbed parameters at the outlet. Details of the boundary conditions are highlighted in Figure 6.



Figure 6. Details of boundary conditions.

Ambient pressure and total temperature equal to 296 K were set at the inlet sections with the medium (5%) turbulence intensity. Static pressure averaged over the surfaces was applied at the outlet. These boundary conditions corresponded to the quantities measured in the experiment, which made it easier to compare the results. The upper parts of the inter-disc gaps were modelled as stationary, but with adiabatic, smooth and rotating walls. The bottom part of the rotor, which contained the spacers, was a rotating domain and was connected to the upper part using the frozen rotor interface. The no-slip condition was applied on all walls of the inter-disc gaps, the plenum chamber and inlets to let the boundary layer develop and account for pressure losses. The free slip was adopted on the outlet ducts' walls to attenuate swirling and prevent air from re-entering the computational

domain at the outlet's surface. All analyses were undertaken in a steady state. A single calculation was considered as converged when the standard deviation of power, mass flow rate and efficiency of the last 200 iterations did not exceed 0.01 W,  $10^{-4}$  kg/s and  $10^{-3}$ , respectively.

Mesh study was performed to assure that the obtained results were independent of the grid size. All meshes were generated using Ansys Meshing tool. The study was divided into two parts. In the first part, a global mesh independence study was performed and the second part pertained to boundary layer discretization. Power was used as an indicator of the mesh quality as it was calculated based on the global thermodynamic and kinematic flow parameters [40]:

Ν

$$=\omega \cdot M$$
 (4)

Torque was calculated using the integral of tangential stresses over the disc surface, which in turn was based on dynamic and eddy viscosities and velocity gradients. The study was carried out for inlet total pressure 3 bar, outlet static pressure 1 bar and rotational speed 20,000 rpm. Figure 7 presents a global mesh independence study. Six unstructured, hexahedral meshes were created and in each the maximal value of the dimensionless distance  $y^+ = 1$ . Power obtained for the finest one was considered as the benchmark for other cases. It can be seen that all results asymptotically approached the benchmark value. The relative difference between the coarsest and the finest mesh was 3.5%. Increasing the node number over 6 M did not improve the results, but to make sure that the increase of the flow parameters would not force additional refinement, the mesh counting ~8 M nodes was selected for the second stage of the study.



Figure 7. Global mesh study.

Six meshes differing on the height of the first element were created in the second stage of the study. In all cases, the growth rate between consecutive cells was equal to 1.25 and a variable number of layers provided that the same distance from the wall was covered with an inflating mesh. The heights were:  $5 \cdot 10^{-5}$  mm,  $10^{-5}$  mm,  $5 \cdot 10^{-4}$  mm,  $10^{-3}$  mm,  $4 \cdot 10^{-3}$  mm and  $8 \cdot 10^{-3}$  mm. The four finest grids ensured maximal  $y^+ \leq 1$ . It is shown in Figure 8 that results within 99.8% of accuracy were obtained for these meshes. Power obtained from the last two, for which  $y^+ > 16$  and  $y^+ > 30$  differed substantially. For the coarsest mesh, the power was more than two times underestimated, which proved a great significance of boundary layer discretization for the Tesla turbine modelling. The mesh with the first element height of  $10^{-3}$  mm was chosen.



Figure 8. Boundary layer mesh study.

The last performed analysis concerned the growth rate between cells in the inflation layer. It turned out that there was no difference between 1.05, 1.15 and 1.25 values, therefore, the latter was selected.

Finally, the mesh selected for the numerical test campaign counted approximately 8 M nodes.

#### 3. Results and Discussion

Investigations were carried out on the inter-disc gap  $\delta = 0.75$  mm for three different pressure ratios:  $\pi_1 = 1.88$ ,  $\pi_2 = 1.6$  and  $\pi_3 = 1.4$  (labelled on charts also in "ratio\_source" convention). The results were compared with data obtained from the numerical calculations.

Figure 9 presents the power characteristics as a function of rotational speed. For CFD calculations, curves were created out of 5 sampling points as it was enough to create a precise approximation due to the lack of random error. In the case of the experiment, all data points gathered during the investigations were placed on the chart. It can be seen that the higher the pressure ratio, the higher the generated power. Pressure ratios also shifted the peak values toward higher rotational speeds. In the case of the experiment, the maximal values were: 126 W (at 10,800 rpm), 93.5 W (at 9300 rpm) and 59 W (at 8160 rpm), respectively for  $\pi_1$ ,  $\pi_2$  and  $\pi_3$ . Due to the occurrence of high vibrations at 9650 rpm, which disturbed torque measurement, it was impossible to gather reliable data in the vicinity of this rotational speed. For this reason, the power peak was not measured directly in the case of  $\pi_2$ , but it was interpolated instead. All curves had high coefficients of determination and the lowest one occurred for  $\pi_3$ . One of the possible explanations can be the fact that small power values were measured with lower precision due to low torque (in the range of 0.02–0.1 nm) and, therefore, the standard deviation was higher than in other cases. For the same reason, the measuring precision dropped with the rotational speed in the case of all curves as the torque decreased. Nevertheless, all approximation curves were symmetrical about the peak values. Comparison with the numerical data shows that the agreement was quite good for low revolutions, but the results gradually diverged with the increase of rotational speed. Peaks were slightly higher than in the case of the experiment; for  $\pi_1$ : 140 W (11,700 rpm), for  $\pi_2$ : 105.5 W (10,400 rpm) and for  $\pi_3$ : 71.1 W (8800 rpm). It can be said that the maximal values of power were overestimated by approximately 10% and optimal rotational speed was also higher by about 10% in relation to the experimental ones. This trend indicates that one of the possible explanations regarding the discrepancies was the different pressure conditions in the turbine components.



Figure 9. Power as a function of rotational speed for different pressure ratios.

Figure 10 presents pressure in the plenum chamber as a function of rotational speed. Data obtained from CFD were calculated as an area average over surfaces located in the middle of the plenum chamber. In the case of the experiment, one sample point was calculated as an average of two values obtained from pressure transducers located in these locations (see Figure 5). Second-order polynomials were used for approximation of the data with the determination coefficients on the level of 0.9. As the rotational speed increased, pressure in the plenum chamber rose. This tendency was visible in both CFD and experiment. The difference in CFD between values for 5000 rpm and 12,500 rpm was equal to 3 kPa for each pressure ratio. Experimental curves were approximately 10 kPa ( $\pi_3$ ), 11 kPa ( $\pi_2$ ) and 16 kPa ( $\pi_1$ ) lower than their numerical counterparts. Moreover, their slope angles were higher, which means that the pressure increment for a unit of rotational speed was higher, e.g., between 5000 rpm and 12,500 rpm it was: 5 kPa, 6 kPa and 6.75 kPa for  $\pi_3$ ,  $\pi_2$  and  $\pi_1$ , respectively.



**Figure 10.** Pressure in the plenum chamber as a function of rotational speed for different pressure ratios.

Figure 11 shows pressure distribution in the tip clearance. It can be seen that the differences between CFD and the experiment were much smaller than in the case of the plenum chamber, but the general tendencies were the same. The static pressure drop in the nozzles became smaller as the pressure drop in the rotor rose. This can be explained by the fact that the outwardly acting centrifugal force resulting from the rotational speed rose and had to be countered by the inwardly acting pressure force. For this reason, the pressure gradient in the rotor rose as well and pressure at the rotor entrance had to be sufficiently high. Curves obtained from CFD and experiment were parallel for each pressure ratio. Pressure in the tip clearance can be considered as the outlet pressure from the inlet nozzles, therefore its increase would result in a smaller pressure drop in the nozzles. As none of the investigated pressure ratios was critical, the nozzles were not choked and the change in the backpressure influenced the mass flow rate. It can be seen in Figure 12, which presents the mass flow rate for different pressure ratios as a function of rotational speed.



Figure 11. Pressure in the tip clearance as a function of rotational speed for different pressure ratios.



Figure 12. Mass flow rate as a function of rotational speed for different pressure ratios.

The highest values of mass flow rate equal to about 0.033 kg/s were obtained for the biggest pressure ratio and in all cases decreased gradually in the range of 5000 rpm–12,500 rpm by about 0.003 kg/s. Another way of explaining the drop of mass flow rate with the rotational speed concerns the rotor. As it was already mentioned, fluid particles are subjected to the action of the centrifugal forces, which decreases the radial component of velocity. As a consequence, the mass flow rate has to drop to conserve the continuity equation. In all cases, the mass flow rate was underpredicted in the numerical computations, which can be elucidated by the higher pressure in the tip clearance in CFD than in the experiment, i.e., higher nozzle's backpressure and minor leakages in the test stand working in negative gauge pressure. Nevertheless, as was already shown, a bigger mass flow rate did not translate into higher generated power. This leads to the conclusion that the turbine efficiency differed between the numerical calculations and the experiment. The efficiency was based on a total to static enthalpy drop in the turbine [40]:

$$\eta_i = \frac{N_i}{\dot{m}c_p T_{in,tot} \left(1 - \left(\frac{p_{out,stat}}{p_{in,tot}}\right)^{\frac{\kappa-1}{\kappa}}\right)}$$
(5)

The efficiency as a function of rotational speed can be seen in Figure 13. Data points for the experiment were calculated based on the approximation of the mass flow rate and power. Estimation of an error was performed using the propagation of uncertainty [47]:

$$u(\eta) = \sqrt{\left(\frac{\partial\eta(m,N)}{\partial m}\right)^2 \cdot \sigma^2(m) + \left(\frac{\partial\eta(m,N)}{\partial N}\right)^2 \cdot \sigma^2(N)} \tag{6}$$



Figure 13. Efficiency as a function of rotational speed for CFD and experiment.

Numerical calculations lack random error, therefore no error analysis was performed for CFD data.

It can be seen that the numerical calculations proved that the smaller the pressure ratio, the higher the efficiency. This can be explained by the fact that a consequence of the low pressure ratio is also low mass flow rate which in turn impacts the flow regime. Shear-force turbomachinery works efficiently only in the laminar flow regime, therefore a decrease in mass flow rate and thereby Reynolds number will increase efficiency. The highest predicted efficiencies in CFD were equal to 10.30% (for  $\pi_3$ ), 9.95% (for  $\pi_2$ ) and 9.60% (for  $\pi_1$ ) obtained for the same optimal rotational speeds as in the case of peak values of power. The curves were getting flattened as the pressure ratio increased. In the case of the experimental investigations, the trends were different. For each pressure ratio, the highest efficiency was on the same level of about 8.3%. The peak values were located for the same rotational speeds as in the case of the peak values of power. Data gathered for  $\pi_3$  were burdened with the highest uncertainty and the highest precision was obtained for  $\pi_2$ .

Laminar flow is not the only condition of high Tesla turbine efficiency. As can be seen in Figure 14, which presents the efficiency as a function of mass flow rate, for each pressure ratio there was an optimal value of throughput. Moreover, the efficiency was strongly sensitive to the change of the mass flow rate. The optimal values in the case of CFD were: 0.0254 kg/s (for  $\pi_3$ ), 0.0282 kg/s (for  $\pi_2$ ) and 0.0305 kg/s (for  $\pi_1$ ). A 10% change in mass flow rate would result in approximately 6 times, 3 times and 2 times lower efficiency, respectively for pressure ratios 1.88, 1.6 and 1.4. In the case of the experiment, optimal values of mass flow rate totaled 0.0264 kg/s (for  $\pi_3$ ), 0.03 kg/s (for  $\pi_2$ ) and 0.0319 kg/s (for  $\pi_1$ ). Sensitivity to the change was also strong as 10% alteration would decrease the efficiency 2.5 times, 5 times and 3.5 times for pressure ratios 1.88, 1.6 and 1.4, respectively.



Figure 14. Efficiency as a function of mass flow rate for CFD and experiment.

The explanation of such a strong sensitivity could be a variation of the tangential velocity ratio. This kinematic quantity is defined as a ratio between the circumferential fluid velocity and a disc velocity at a given radius. According to the Tesla turbomachinery theory, it should tend to 1. In this ideal situation, the fluid neither possesses an excess of unconverted kinetic energy nor energy is transmitted to the fluid, like in a compressor. In

reality, the optimal value of this parameter may vary between 1.1–1.5, depending on several factors, e.g., the number of nozzles or the uniformity of flow admittance. An increase in the mass flow rate will result in the change of the velocity components in the rotor, the rotational speed of the discs and, thereby, the tangential velocity ratio in the whole rotor. This information is essential from the design point of view as the usual operational constraints involve rotational speed (depends on the generator) and throughput. In this case, the mass flow rate has to be divided between a sufficiently high number of inter-disc gaps in such a way that the tangential velocity ratio will be kept at an optimal level and the Reynolds number as low as possible.

Interesting insights can be obtained by analyzing CFD results for individual gaps. Table 1 shows the power and mass flow rate in the individual gaps related to the total mass flow rate and power generated by the whole turbine for different pressure ratios and rotational speed n = 10,000 rpm. Gaps between the rotor and the turbine's casing are labelled as  $G_0$  and  $G_{00}$  (c.f. Figure 1).  $G_1$ ,  $G_2$ ,  $G_3$  and  $G_4$  symbolize inter-disc gaps of the rotor. It can be seen that a substantial portion of the mass flow rate flowed through the lateral gaps, but they were responsible for the generation of a small fraction of the total power. In the case of  $\pi_3 = 1.4$ , 57.3% of mass flow rate generated 12.2% of the total power, for  $\pi_2 = 1.6, 54.5\%$  of the total throughput contributed only to 11.4% of total power and for the highest pressure ratio  $\pi_1$  = 1.88, 51% of the flow generated merely 11% of the power. Those numbers allowed it to be stated that the flow in the lateral gaps can be considered as a loss and should be avoided. There are several ways of dealing with this effect. As can be seen, this effect was getting slightly weaker with the increase in the pressure ratio. Another way is to increase the number of inter-disc gaps to decrease their resistance to flow. The last method concerns minimizing the size of the lateral gaps, so the tendency of flowing that way will drop.

π	Parameter	G <sub>0</sub>	<b>G</b> <sub>1</sub>	G <sub>2</sub>	G <sub>3</sub>	$G_4$	G <sub>00</sub>
1.4	N <sub>gap</sub> /N <sub>total</sub>	0.112	0.244	0.223	0.209	0.202	0.01
	m <sub>gap</sub> /m <sub>total</sub>	0.282	0.103	0.109	0.11	0.105	0.291
1.6	N <sub>gap</sub> /N <sub>total</sub>	0.109	0.245	0.227	0.211	0.202	0.005
	m <sub>gap</sub> /m <sub>total</sub>	0.267	0.109	0.113	0.119	0.113	0.278
1.88	N <sub>gap</sub> /N <sub>total</sub>	0.108	0.248	0.232	0.216	0.194	0.002
	m <sub>gap</sub> /m <sub>total</sub>	0.252	0.118	0.121	0.127	0.12	0.263

Table 1. Power and mass flow rate for individual gaps for 10,000 rpm.

The difference in power generation was also visible between the inter-disc gaps. It can be said that the closer the gap from the outlet, the bigger the share in the power generation. The maximal differences between the individual gaps were on the level of 5 percentage points (e.g., 24.8% vs. 19.4% for  $\pi_1$  = 1.88). Gaps G<sub>2</sub> and G<sub>3</sub> were characterized by a higher total share in the mass flow rate than gaps G<sub>1</sub> and G<sub>4</sub>. It can be explained by the presence of gaps G<sub>00</sub> and G<sub>0</sub>. The working medium flowing in the tip clearance was sucked into the lateral gaps and prevented from entering G<sub>1</sub> and G<sub>4</sub> gaps.

Table 2 shows the efficiency for the individual gaps obtained for 10,000 rpm by the two methods. The first method, which utilizes an enthalpy drop, was already described by Equation (5). The second method compares circumferential and radial stresses generated on the disc walls, as proposed in [37]:

$$\eta_s = \frac{\int \tau_{\theta} dA}{\int \tau_{\theta} dA + \int \tau_r dA} \tag{7}$$

π	Parameter	G <sub>0</sub>	G <sub>1</sub>	G <sub>2</sub>	G <sub>3</sub>	G <sub>4</sub>	G <sub>00</sub>
1.4	$\begin{array}{l} \eta_i \\ \eta_s \end{array}$	$0.041 \\ 0.118$	0.245 0.561	0.212 0.519	0.196 0.517	0.199 0.526	0.030 0.110
1.6	$\eta_i$ $\eta_s$	$0.040 \\ 0.141$	0.221 0.568	0.199 0.551	0.175 0.547	0.177 0.556	0.010 0.098
1.88	$\eta_i$ $\eta_s$	0.040 0.155	0.195 0.571	0.177 0.565	0.158 0.559	0.140 0.558	0.010 0.096

Table 2. Efficiency for individual gaps for 10,000 rpm obtained from two methods.

This equation has a strong foundation in the Tesla turbine principle of operation. Radial stresses are considered as a loss whereas circumferential stresses propel the turbine. Efficiency equal to 0% would be obtained only if there are no circumferential stresses, ergo the flow is carried out only in the radial direction so the turbine is not revolving. Efficiency equal to 100% is impossible to obtain as the radial stresses would have to be equal to 0, which implies the lack of the radial component and thereby flow.

It can be seen that both methods yielded significantly different results. In the case of the enthalpy method, the efficiency was at the level of 20%, but in the case of the stress method, it was equal to ~50% on average. The efficiency was getting smaller as the pressure ratio increased in the enthalpy method, but the trend was reversed in the stress method. However, the relation between lateral gaps and inter-disc gaps was sustained: lateral gaps had approximately five times smaller efficiency than the inter-disc gaps.

Figure 15 presents the temperature drop in the turbine in the case of the experiment. It was calculated as the difference between the temperature at the inlet and the outlet. It is important to note that the chart should be treated rather as a qualitative assessment of the thermal processes taking place in the turbine and cannot be used to calculate, e.g., the enthalpy drop or other thermodynamic quantities. The reason for this is the temperature measurement at the outlet. The probe was located in the axis of the outlet duct (see Figure 5), in the stream of flowing air and measurement of static temperature was not possible. On the other hand, the total temperature cannot be measured precisely because the fluid coming to rest at the probe's wall would not convert the kinetic energy into temperature adiabatically due to frictional heating. For these reasons, it was not possible to directly compare the experiment with numerical calculations.



Figure 15. Temperature drop in the turbine from the experiment.

Taking it all into account, it can be said that there was a relation between the turbine performance and the temperature drop. The curves were of parabolic shape and resembled the efficiency and power characteristics, with peaks obtained for the same rotational speeds. The highest drop occurred for the ratio  $\pi_1 = 1.88$  and totalled roughly 3.5 K. Peak values for other ratios were equal to 2.55 K for  $\pi_2 = 1.6$  and 1.75 K for  $\pi_3 = 1.4$ . The determination coefficients were between  $R^2 = 0.72-0.76$ , which was slightly lower than in the case of previous measurements. The explanation can be the very long time that is required to obtain the thermal equilibrium between the fluid and the probe, greatly surpassing the

time to reach the steady-state for power and rotational speed. An additional difficulty was a small variation of the temperature at the level of a few kelvin. It can be concluded that in the case of a lack of precise tools measuring efficiency, a pair of thermo-probes would be enough to locate the optimal conditions of the turbine's operation.

Figure 16 presents the total and static temperatures at the outlet section of the turbine as a function of the rotational speed obtained from CFD. It can be seen that the static temperatures were decreasing with rotational speed up to a certain point and then the trend reversed as a result of two contradictive thermodynamic phenomena taking place. On the one hand, the static enthalpy of the flow was being converted into mechanical energy in the rotor, which decreased the static temperature. On the other hand, viscous forces were dissipating energy into heat and thereby increasing the static temperature. The heating effect was getting stronger with rotational speed as the higher rotation, the longer fluid particles' path from the inlet to the outlet and the contact between discs' walls and the fluid. These two effects were competing and for each pressure ratio, it was possible to determine a point at which the trend reversed. In the case of  $\pi_1$  it was n = 16,000 rpm, for  $\pi_2$  it was n = 15,000 rpm and for  $\pi_3$  it was 12,300 rpm. In all cases, however, the static enthalpy drop was positive and contributed to the power generation. Total temperature curves had similar trends. It can be seen that the lowest points of the curves corresponded to the optimal working condition of the turbine. The difference between the total and static curves for a given pressure ratio and rotational speed was due to the fluid velocity, which at this point can be considered as a kinematic loss. Rotational speed at which the total outlet temperature was predicted to be equal to the total inlet temperature coincided for each pressure ratio with the idle turbine operation approximated from power curves (c.f. Figure 9). In this case, the total temperature drop would be equal to 0, although the static temperatures at the inlet and the outlet would differ.



Figure 16. Total and static temperature at the outlet from CFD.

# 4. Conclusions

The manuscript presents an experimental and numerical analysis of a 160 mm Tesla turbine propelled by air. Experimental investigations were conducted on air for three pressure ratios  $\pi_1 = 1.88$ ,  $\pi_2 = 1.6$  and  $\pi_3 = 1.4$ . The highest obtained power was equal to 126 W in the experiment and 140 W in numerical calculations. Comparison between CFD and the experiment showed good agreement for low rotational speed. The highest efficiency was 10.3% in CFD and 8.3% in the experiment. For each pressure ratio, the maximal efficiency was approximately the same in the experiment, although numerical methods proved that the efficiency slightly dropped with the increase of pressure ratio. The efficiency of an individual gap was between 14–24%, but the efficiency of the lateral gaps between the rotor and the casing was within 1–4%, which ultimately decided about the overall turbine efficiency was highly sensitive to the change in mass flow rate: a 10% alteration of flow could result in a 6 fold decrease in efficiency. The mass flow rate distribution over the gaps pointed out the important role of the highly efficient sealing in

the external gaps. This problem will be less significant when the number of discs in the turbine increases.

Pressure distributions in the plenum chamber and rotor tip clearance changed linearly with rotational speed. Measurement of the temperature drop between the inlet and the outlet can be helpful in search of optimal working conditions of the turbine, although the determination of the precise values might be difficult in the case when the dynamic temperature drop is of the same order of magnitude as the static temperature drop.

All data gathered during the experimental campaign were used for the validation of the numerical model. The mass flow rate was predicted within 10% accuracy for all examined pressure ratios. Power was slightly overestimated in CFD with respect to the experiment and the difference rose with the rotational speed. Pressure in the rotor radial clearance calculated in CFD was in line with the experimental values; however, the differences in the plenum chamber were bigger. It allows concluding that the numerical model captured the most important flow phenomena, but high rotational speeds require special treatment, especially in turbulence modelling.

In conclusion, it can be said that to work efficiently, a Tesla turbine has to have a sufficiently high number of discs so it works in a laminar flow regime and the mass flow rate is divided between inter-disc gaps optimally. Experimental data presented in the paper may be used for the validation of the analytical and numerical models of the turbine components. Research also gives insights on the mutual relationships between the turbine parameters and their importance to the turbine performance. The presented conclusions may be also useful in the design process of a Tesla turbine.

Future works will concern the influence of the supply system parameters (number of nozzles, angle of admittance, nozzle cross-section) on the turbine performance.

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## Nomenclature

cp	Heat capacity at constant pressure	${ m J}{ m kg}^{-1}{ m K}^{-1}$
h	Enthalpy	kJ
Μ	Torque	Nm
m	Mass flow rate	${ m kgs^{-1}}$
Ν	Power	W
n	Rotational speed	$\rm rad~s^{-1}$
р	Pressure	Pa
T	Temperature	Κ
U	Velocity	${ m m~s^{-1}}$
Greek symbols	-	
η	Efficiency	-
λ	Heat conductivity	$\mathrm{W}\mathrm{m}^{-1}\mathrm{K}^{-1}$
к	Heat capacity ratio	-
π	Pressure ratio	-
ρ	Density	$ m kgm^{-3}$
σ	Standard deviation	-
τ	Tangential stress	Pa
ω	Angular velocity	$s^{-1}$

Subscripts	
in	Inlet
out	outlet
r	Radial direction
ref	Reference case value
s	Stress
tot	Total

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