

Article

Multi-Area Distribution System State Estimation Using Decentralized Physics-Aware Neural Networks

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Abstract: The development of active distribution grids requires more accurate and lower computational cost state estimation. In this paper, the authors investigate a decentralized learning-based distribution system state estimation (DSSE) approach for large distribution grids. The proposed approach decomposes the feeder-level DSSE into subarea-level estimation problems that can be solved independently. The proposed method is decentralized pruned physics-aware neural network (D-P2N2). The physical grid topology is used to parsimoniously design the connections between different hidden layers of the D-P2N2. Monte Carlo simulations based on one-year of load consumption data collected from smart meters for a three-phase distribution system power flow are developed to generate the measurement and voltage state data. The IEEE 123-node system is selected as the test network to benchmark the proposed algorithm against the classic weighted least squares and state-of-the-art learning-based DSSE approaches. Numerical results show that the D-P2N2 outperforms the state-of-the-art methods in terms of estimation accuracy and computational efficiency.

Keywords: distribution system state estimation (DSSE); pruned physics-aware neural network (P2N2); phasor measurement unit (PMU); data-driven modeling



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1. Introduction

State estimation (SE) is an essential function for active distribution grids. The estimated voltage from SE is used for different applications such as: voltage control [1], energy management [2,3], or scheduling electric vehicle charging [4]. The massive integration of renewable energy into distribution systems, however, renders the systems unpredictable and volatile; therefore, there is a critical need for SE in modern distribution systems. Several major differentiating issues between the distribution system and the transmission system were discussed in [5], including the observability, unbalanced operation, low x/r value, network configuration, renewable energy integration, cyber-security, and communications issues. Many studies have primarily focused on analyzing the observability in SE for distribution systems. Traditionally, weighted least square (WLS) estimators have been used to estimate the system state (e.g., voltage magnitude, voltage angle) in transmission systems, where grids have redundant measurements. On the other hand, distribution systems often have a limited number of measurements, and, hence distribution systems are inherently unobservable [5,6]. This renders the WLS formulation not applicable to a more extensive distribution system because the singularity of the the gain matrix hinders the solvability for the state variables [7].

A practical solution of the unobservable grid is to use pseudo measurements [8], which are often forecasted from historical data or calculated by interpolating observed measurement data. In [9], an algebraic-based method is used to determine the observability of the electric grid. Then, in [10] an improved measurement placement algorithm is developed that prevents the iterative addition of measurements. In [11], a game theoretic-based, data-driven technique is studied with the purpose of generating pseudo measurements

in distribution system state estimation (DSSE). In addition, a machine learning model is developed to learn load patterns and to then generate active power pseudo measurements. For the same purpose, in [12], a frequency-based clustering algorithm is proposed that determines the load patterns and estimates the daily energy consumption. On the other hand, in [13] a probabilistic data-driven method is used to generate time-series pseudo measurements for an unmeasured photovoltaic systems.

In addition to the unobservability issue, a particularly important issue for DSSE is the significant computational complexity arising from the large number of nodes and the unbalanced three-phase structure. Further, data sharing among different distribution system operators is not always available because of information privacy concerns [14]. To tackle these issues, the concept of decentralized state estimation (DSS) has been proposed [15–17], where as most approaches have focused on multi-area state estimation. In [15], the authors proposed a complete DSS based on a multi-agent system that can obtain an accurate solution compared to WLS; however, the method was tested for a highly meshed network with significant redundancy in measurements. In addition, reliable communications were required for the computations of the multi-agent estimation problem. A hybrid DSS framework combining a two-level model-based and a data-driven approach was presented in [16], with a decentralized model based at the lower level to provide the necessary data for a the data-driven model at the upper level. The method offers a DSS model without requiring communications infrastructure. In [17], the authors proposed DSS based on phasor measurement units (PMUs). This DSS framework for active distribution grids was designed based on a sparse l_1 relaxation models and can operate independently and handle bad data. Recently, a multi-timescale data-driven approach was proposed in [14] where daily consumption patterns of customers were determined based on three machine learning models to enhance distribution system observability.

Overall, data-driven techniques present a very promising solution to improve grid distribution system monitoring and control. Motivated by these approaches, the authors of [18] proposed a physics-aware neural network model. The idea is to embed the physical connection of the distribution system into the neural network model; however, the method used fixed embedding of the network graph at all neural network layers, which adds significant unnecessary connections and weight to the neural network. Thus, our recent work in [19] proposed the pruned physics-aware neural network (P2N2) model to prune unnecessary connections; however, for large distribution systems, the model still requires a high computational cost. To this end, this paper proposes a decentralized pruned physics-aware neural network (D-P2N2) to provide a novel, decentralized, data-driven DSSE solution that uses knowledge about the physical network model.

A graphic summary of the proposed approach is shown in Figure 1. First, Monte Carlo simulations are set up with 35,040 data points from smart meters (one-year of collected data). Then, the load consumption data are fed into the power flow simulation. The voltage magnitude and measurement data are then segregated into different data packages corresponding to subareas; then, the data will be used for the DSSE and will be fed to different D-P2N2 models. Three different simulation cases are carried out with the modified IEEE-123 node test system. The simulation result demonstrates the efficacy of the D-P2N2 approach.

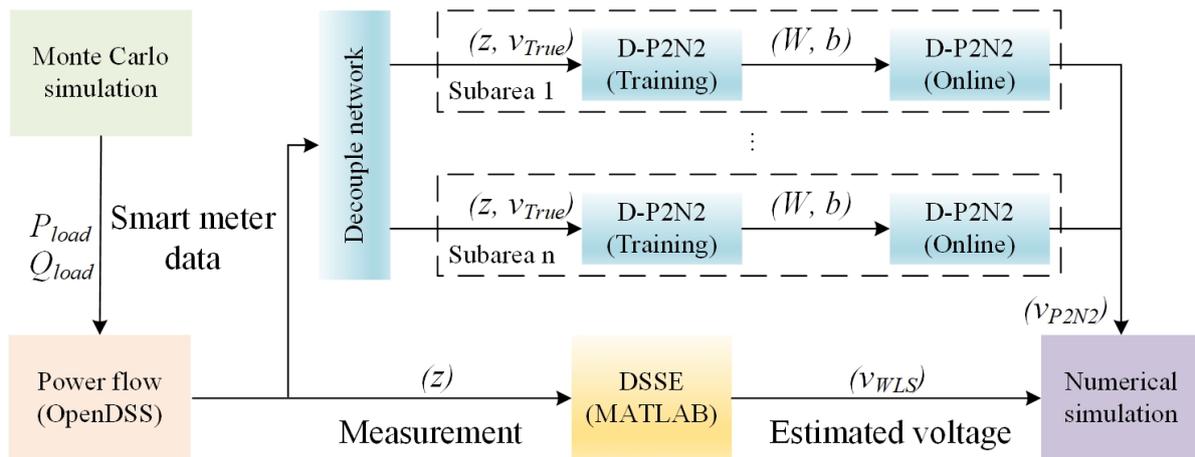


Figure 1. The methodology of the proposed approach.

2. Distribution System State Estimation

In this section, the three-phase estimator is presented in the first subsection, which is in the rectangular voltage-based DSSE. In the second subsection, different types of measurements in the distribution system are discussed in detail.

2.1. Three-Phase Estimator

SE is a well-known problem that aims to estimate the system's state variables from measurements, using the physical expressions relating the measurements to the system state. The synthesizing function can be written as follows:

$$z = h(x) + e \quad (1)$$

where z is a measurement vector obtained from grid-measuring devices and pseudo measurements; $h(x)$ is a vector function from state variables x to measurements z ; and e is a measurement noise vector with elements assumed to be independent zero-mean Gaussian random variables. In general, the WLS formulation minimizes the sum of squares of the residuals where the matrix R denotes the covariance matrix:

$$J(x) = [z - h(x)]^T R^{-1} [z - h(x)]. \quad (2)$$

The Gauss–Newton method is often used to obtain the solution. The iterative process stops when the number of iterations is higher than the limited value or when the changes in residuals are below a prespecified threshold:

$$x^k = x^{k-1} + G(x^k)^{-1} H(x^k)^T R^{-1} [z - h(x)] \quad (3)$$

where $H(x)$ and $G(x)$ are:

$$H(x) = \frac{\partial h(x)}{\partial x} \quad (4)$$

$$G(x^k) = H(x^k)^T R^{-1} H(x^k) \quad (5)$$

Different from transmission systems, distribution networks are highly unbalanced systems. This leads to singularity of the gain matrix $G(x^k)$, and, hence the single-phase SE model used for transmission SE is often not applicable for DSSE. This work uses, the three-phase state estimator from [20], which is based on the rectangular voltage. The state variables of the network are represented by three-phase rectangular form (i.e., the real part and the imaginary part) at every phase.

2.2. The Used Measurements

Because distribution systems are highly unobservable, the application of the WLS algorithm needs additional pseudo measurements to remedy the low-observability issue. The measurements used in this work are:

- PMUs: three-phase synchronized measurements. Normally, they are located near the step-down transformer, which is used to measure the voltage phasor at the node and the current phasor of the connected branches. With PMUs, the maximum error is 1% for the magnitude and 10^{-2} rad for the phase angle.
- Smart meters: these measurements are installed at the household (customer measurements). The power consumption of customers is obtained normally every 15 min. With smart meters, the maximum error is 2% for power measurements.
- Pseudo measurements: historical data are used at the buses where no measurement device is installed as pseudo measurements. These data include three-phase active and reactive power injections, and the maximum error of a pseudo measurement can be up to 50% for active and reactive power absorbed from loads.
- Zero injection buses: at the buses without any loads or generators connected, these buses are considered zero injection buses. The active and reactive power injection measured at these buses is zero with a maximum error equal to 0.001% (to be able to invert the covariance matrix).

3. Decentralized Pruned Physics-Aware Neural Network

In this section, the concept of P2N2 and the proposed method of D-P2N2 are discussed in detail. The background of the decentralized DSSE based on the PMU location is explained. To this aim, we present an example of a 6-bus system. We show the way we design the D-P2N2 based on the physical distribution grid and the PMU location. Then, in Section 3.3, we present the implementation details.

3.1. Pruned Physics-Aware Neural Network

As mentioned, our earlier papers [18,19] study the use of PMU measurements to design a physics-aware neural network model. The PMU is a three-phase synchronized measurement of the real-time measured value with very high accuracy. Taking advantage of the high accuracy of the PMU measurements, the estimated voltage at a specified bus does not require the information of all available measurements in the network. Specifically, estimating the voltage at a specific node does not require measurements at the buses where any path between these buses and the specific node has a bus equipped with a PMU. This means that, with an accurate measurement at a the PMU-equipped buses, other measured quantities behind this bus can be neglected. As an example, Figure 2 shows a simple 6-bus system with a PMU installed at Bus 4. Applying the concept of vertex-cut, the system can be divided into three different partitions, which are shown in Figure 3:

- Partition 1 in Figure 3a: consists of buses 1, 2, 3 and 4
- Partition 2 in Figure 3b: consists of buses 4 and 5
- Partition 3 in Figure 3c: consists of buses 4 and 6.

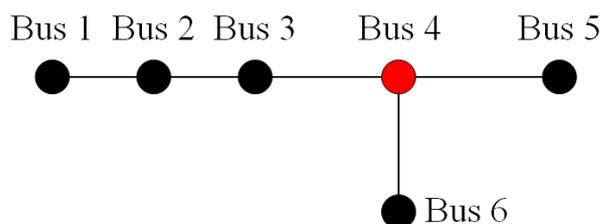


Figure 2. Example of a 6-bus system with PMU located at Bus 4.

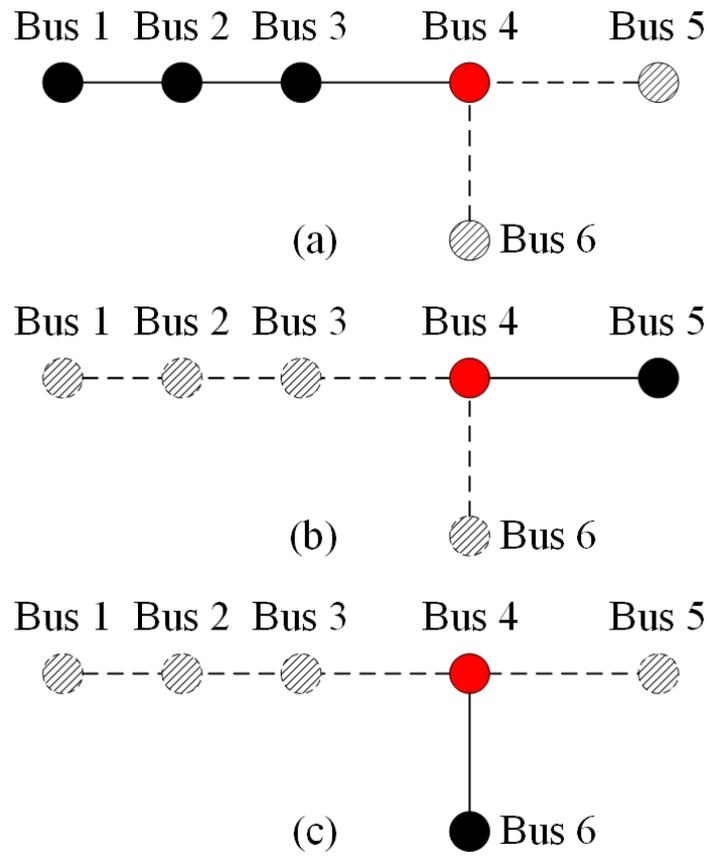


Figure 3. Vertex-cut partitioning example with PMU located at Bus 4. (a) partition 1 with buses 1, 2, 3, and 4; (b) partition 2 with buses 4 and 5; (c) partition 3 with buses 4 and 6.

The P2N2 model [19] is designed with multiple layers, which are built based on the physical connections of the distribution network. The required number of layers is the maximum diameter of each partition. In this example, the number of hidden layers in this case is three because the maximum diameter of all partitions is three. Then, the connections between layers are designed based on the physical connection of the network. This is the idea behind the physics-aware technique, which prunes the connections that are not present in the physical network. Figure 4 shows the results of the designed connections between the layers of P2N2 for the 6-bus system. Figure 4a shows exactly the structure of the network admittance matrix of the 6-bus network.

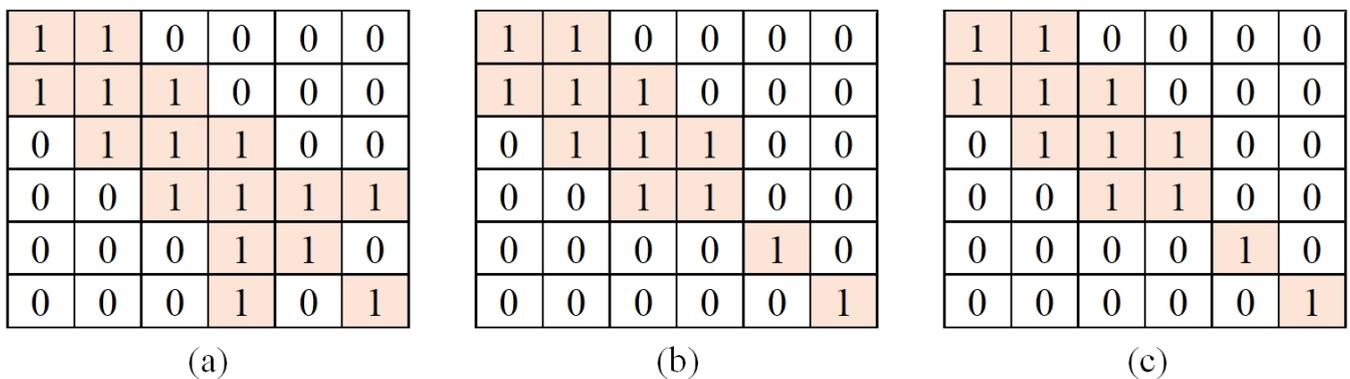


Figure 4. The designed connection between layers for the 6-bus system. (a) The connection between the input layer and Layer 2; (b) The connection between layers 2 and 3; (c) The connection between Layer 3 and the output layer.

Let the input layer of the P2N2 model be denoted by x , and y is the output layer of P2N2. The output vector y represents the voltage at all the phases in the network. Let $k(i)$ denote the intermediate output at i -th layer. Then, we have:

$$k_{i+1} = \sigma_i(W_i k_i) \tag{6}$$

where σ_i is a point-wise nonlinearity, w , which has a size of $N \times N$, is weight matrix, where N is the number of output y . The matrix W is designed the same as the connection shown in Figure 4a. The (i, j) element in the matrix W is pruned if the nodes i and j are not connected. Partitions 2 and 3 have the same diameter of 2, meaning that we can get the voltage value of buses 5 and 6 after the second layer. To this end, the P2N2 is designed to reduce unnecessary connections between layers. As shown from Figure 4b, which shows the connection between Layer 2 and Layer 3, and the four unnecessary connections of (4,5), (4,6), (5,4), and (6,4) are zeroed out. Similarly, the new structure of the connection between Layer 3 and the output layer is shown in Figure 4c. Then, three different weight matrices are used for the P2N2 model, which is shown in Figure 5. Therefore, the output of the P2N2 can be written as:

$$y_i = \begin{cases} \sigma_3(W_3\sigma_2(W_2\sigma_1(W_1x + b_1) + b_2) + b_3) & \text{if } i = 1, 2, 3, 4 \\ \sigma_2(W_2\sigma_1(W_1x + b_1) + b_2) & \text{if } i = 5, 6 \end{cases} \tag{7}$$

where b_1, b_2, b_3 are the bias vectors of the P2N2 model.

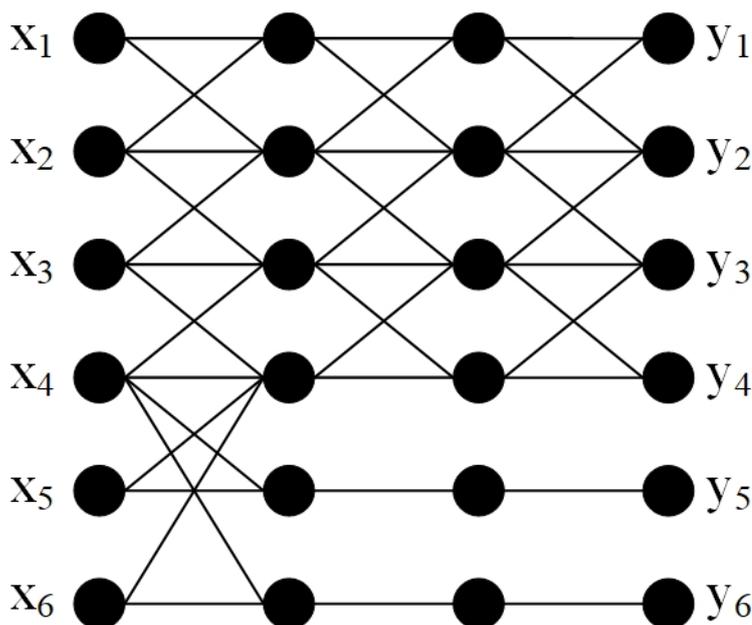


Figure 5. The graph-pruned structure of the P2N2 model.

3.2. Decentralized Pruned Physics-Aware Neural Network Design Considerations

The main goal of decentralized state estimation is to reduce the computational complexity and to reduce the communications requirements in distribution grids. In particular, in large distribution networks, different DSOs can independently monitor their feeders. To this aim, the distribution network is decoupled into different subareas depending on the PMU location, where local state estimators based on P2N2 are designed, which we call *D-P2N2*. A summary of the *D-P2N2* design is schematically shown in Figure 6. Again, we consider the 6-bus system with the PMU installed at Bus 4. In the first step, the network configuration and the measurement data of the whole system are collected. Then, the network is divided into different subareas. All the measurement data are also segregated

into different data packages corresponding with subareas. Finally, the neural network model based on the P2N2 method in Section 3.1 are designed locally for each subarea. Therefore, the output of the D-P2N2 for the 6-bus system can be rewritten as two different models for subarea 1 as:

$$y_i = \sigma_3(W_3\sigma_2(W_2\sigma_1(W_1x + b_1) + b_2) + b_3) \text{ if } i= 1, 2, 3, 4, \tag{8}$$

subarea 2 as:

$$y_i = \sigma_1(W_1x + b_1) \text{ if } i= 4, 5, \tag{9}$$

and for subarea 3 as:

$$y_i = \sigma_1(W_1x + b_1) \text{ if } i= 4, 6. \tag{10}$$

This way, the state estimation problem of the whole system can be solved locally at each partition. Because the number of buses in each subarea is less than the number of buses of the network, the designed D-P2N2 has fewer layers and fewer neurons in each layer. In addition, fewer biases and weights are needed, which effectively reduce the computational cost in the training phase and estimation time in the execution phase. Moreover, the model needs minimal data exchange between different subareas.

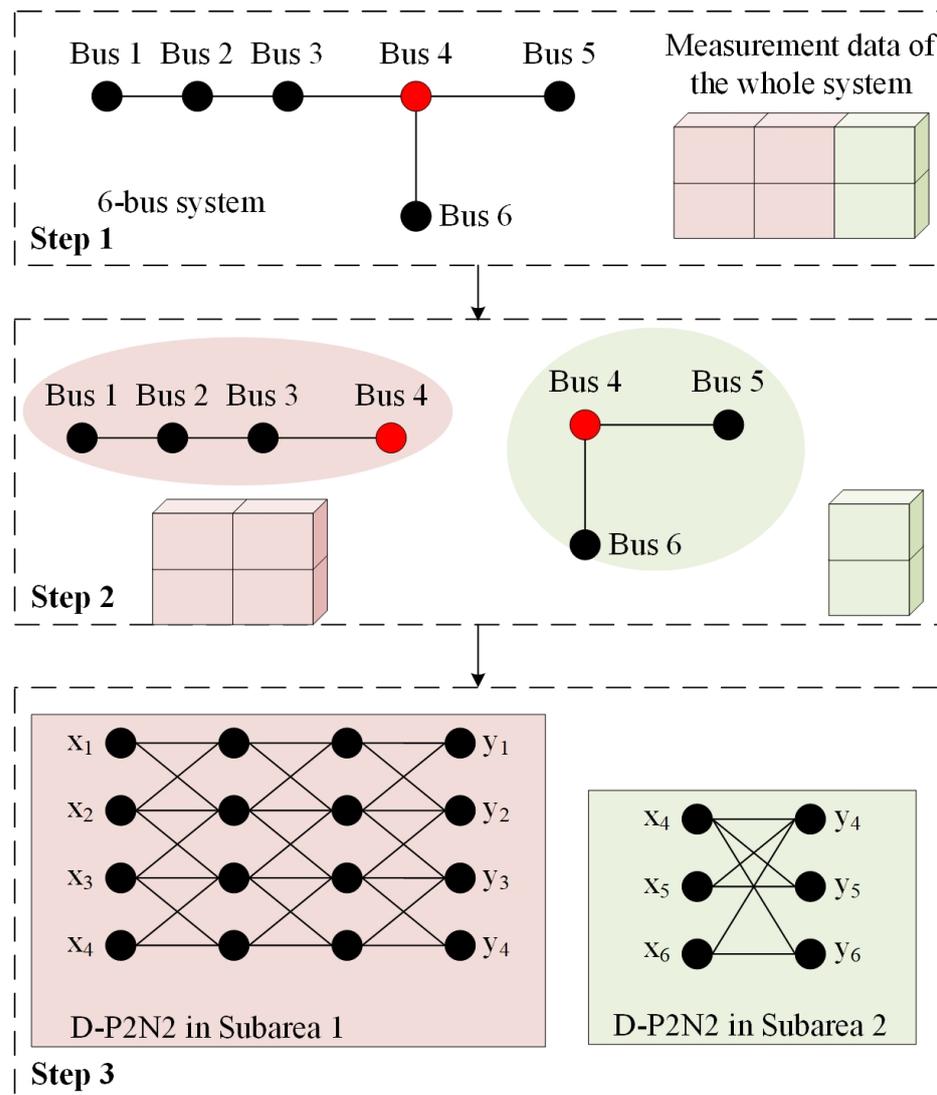


Figure 6. Decentralized training and execution of D-P2N2 estimators.

3.3. Model Implementation and Validation

In this work, TensorFlow [21] was used to train the model. The data of the whole network were divided into data packages for subareas, where each data package was divided into 90% training and 10% testing. The models were trained in parallel for different subareas based on the ADAM optimizer [22], and the optimization function is formulated as follows:

$$\min_{\{b_t, W_t\}_{t=1}^T} \sum_j \|v^j - g_T(z^j; \{b_t, W_t\}_{t=1}^T)\|_2^2 \quad (11)$$

where v^j and z^j are the true state and the measurement in the j -th training sample, respectively. g_T is the j -th mapping realized by the T -th layer of the model parameterized by $\{b_t, W_t\}_{t=1}^T$. The network structure is imposed on the D-P2N2 model; hence, the number of neurons in each layer is proportional to the number of buses. Finally, we used the average estimation to calculate the accuracy of each algorithm as follows:

$$v = \frac{1}{N} \sum_{i=1}^N \|\hat{v}^i - v_{true}^i\|_2^2 \quad (12)$$

where \hat{v}^i is the estimated voltage.

4. Simulation and Results

This section presents the test case and the simulation results. First, the IEEE 123-node test system is described. Then, the methodologies explained in Sections 2 and 3 are applied. Three different scenarios were carried out to assess the performance of the proposed model.

4.1. Test Network

This work uses the IEEE 123-node test system; the grid topology is shown in Figure 7. The IEEE 123-node system is a radial distribution grid with single-phase and two-phase loads; thus, the grid is a highly unbalanced network. The feeder has four different voltage regulators and different voltage levels. The detailed grid parameters are available in [23]. There are four switches (13–152, 60–160, 97–197, and 18–135) that have been modified as connection buses. Further, voltage regulators are excluded in this work. Generally, these modifications are common for this kind of study [16,18,24] without affecting the generality of the study. The DSSE algorithm is built in the MATLAB environment, and OpenDSS is used for the power flow calculation. In addition, we assumed that the system has two PMUs installed, one each at Bus 149 and Bus 60. Then, the whole system can be split into two subareas (This PMU placement partitions the network into three subareas; however, one partition is very small, and, hence, we decided to combine it with a larger partition). The first subarea in the dashed line has one PMU at Bus 149. By exporting the maximum diameter of the possible partitions in this area, the diameter is thus 14. Similarly, in the second subarea, the diameter is 11. Then, we have two D-P2N2 models with 14 and 11 layers for Subarea 1 and Subarea 2, respectively.

4.2. Simulation Scenarios

To assess the performance of the data-driven estimator, one-year time series data collected from smart meters are used with 35,040 data points. Then, $M = 35,040$ possible operation conditions (normally, a set of 10,000 data points is sufficient to ensure the quality of the results) is fed into the power flow model as the load consumption. By extracting from each power flow simulation, the measurements and the true voltage magnitude value at the buses are collected. Hence, we have the set of 35,040 measurements (z) for the WLS and the sets of 35,040 measurements (z) and system state (voltage magnitude, v_{true}) for the P2N2. In addition, 90% of the 35,040 data points are used for the training, and the rest are used for the testing. The process of the simulation and model evaluation is shown in Figure 8. Then, the performance of each algorithm is calculated using Equation (12).

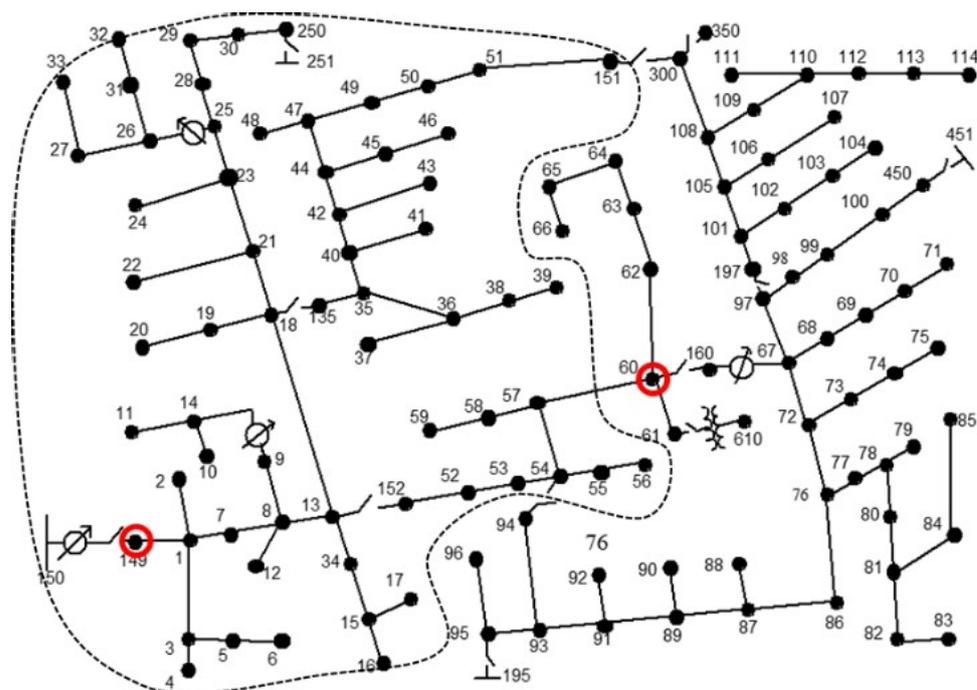


Figure 7. The IEEE 123-node test system.

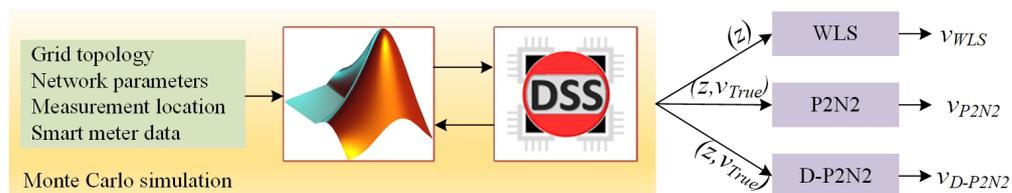


Figure 8. The simulation and model evaluation process.

This whole process is tested with three different scenarios:

- Scenario 1: the algorithms are tested with a large number of measurements.
- Scenario 2: we kept the same amount of measurements and we increased the error of the pseudo measurements from 30% to 50%.
- Scenario 3: limited measurements are used for this scenario, i.e., 14 pseudo measurements are removed.

In the first scenario, the network has two voltage measurements and two current injection measurements at Bus 149 and Bus 60. Further, 118 pseudo measurements are used, which consist of 85 load power measurements and 33 zero injection measurements. As an example, we present only the voltage magnitudes at Phase A of all buses. Figure 9 depicts the estimated voltage magnitudes in the first scenario. The results show the robustness of the WLS in case of redundant measurements; however, the P2N2 and D-P2N2 also show the high accuracy of the estimated voltage magnitudes. To show the performance of the proposed method, we increased the error of the pseudo measurements from 30% to 50% while keeping the same number of measurements. As shown in Figure 10, the P2N2 and D-P2N2 show a better result compared to the WLS. This means that the neural network model with a large set of training data can provide reliable estimation performance in case of measurement noise.

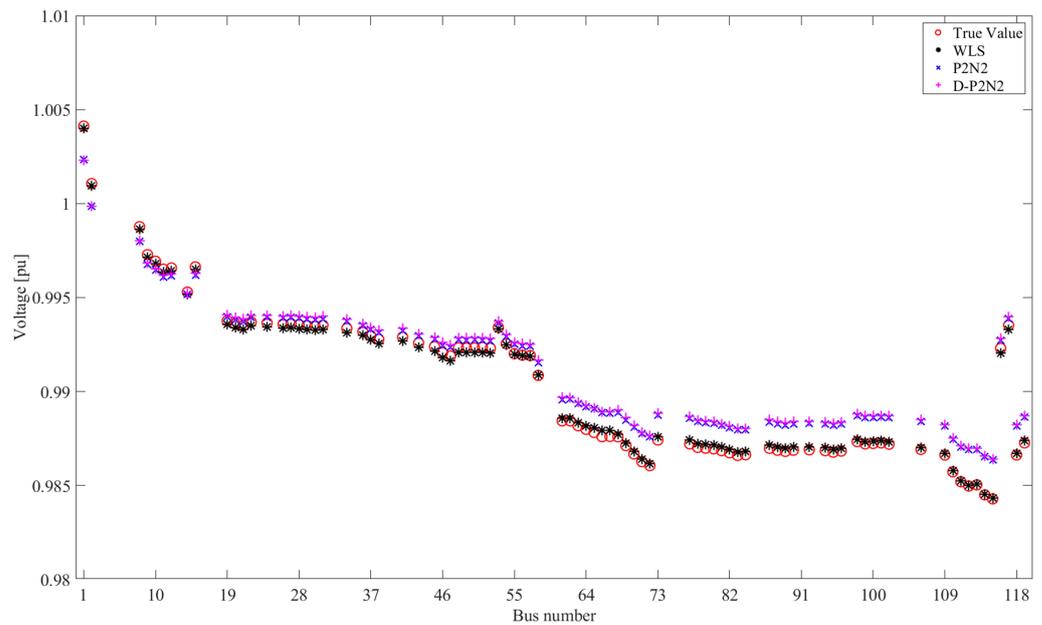


Figure 9. Estimated voltage magnitude at Phase A in Scenario 1.

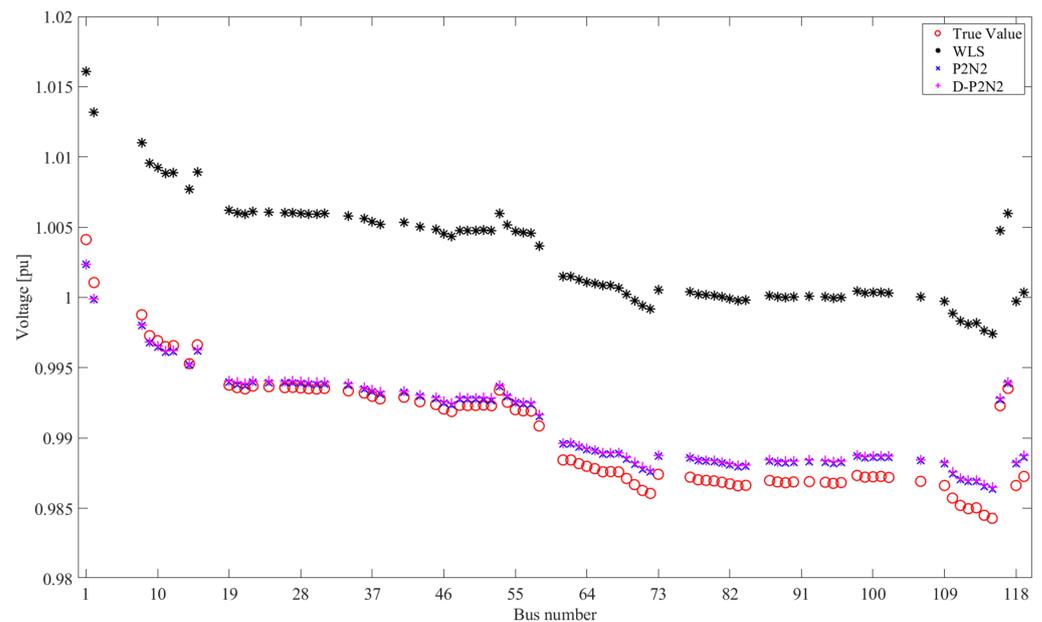


Figure 10. Estimated voltage magnitude at Phase A in Scenario 2.

The third scenario is carried out with a limited number of measurements; 14 load power measurements are neglected (compared with the first scenario). In this case, the network is unobservable because of the limited number of measurements. Thus, the WLS cannot obtain estimates for the voltage magnitudes; however, Figure 11 shows the effectiveness of the P2N2- and D-P2N2-based neural network even with the unobservable distribution system.

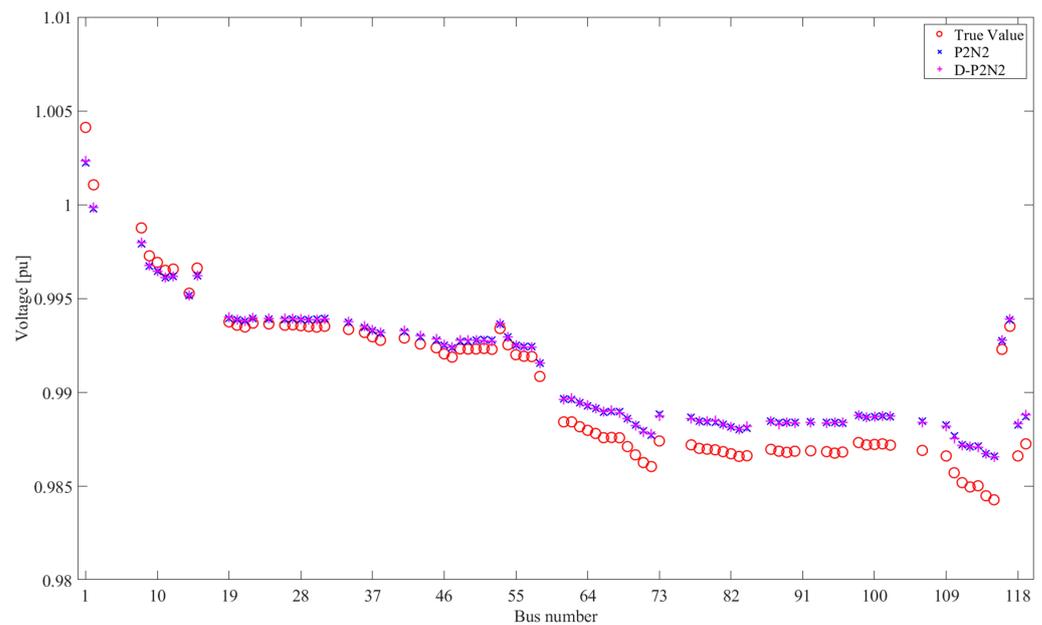


Figure 11. Estimated voltage magnitude at Phase A in Scenario 3.

Further, the average estimation errors of the different scenarios are shown in Table 1. It shows the accuracy of the WLS in the case of the observable distribution network with noiseless measurements; however, the better estimation result is achieved by the P2N2 and D-P2N2 in the case of higher noise from the measurement or when the network is unobservable. Table 2 summarizes the estimation time of each time step; it shows the robustness of the proposed D-P2N2 with approximately 600, and it is four times faster than the WLS and P2N2, respectively. Therefore, the D-P2N2 can significantly reduce the computational cost for large distribution grids.

Table 1. The average estimation accuracy.

Scenario	WLS	P2N2	D-P2N2
1	0.0019	0.0346	0.0346
2	0.1188	0.0346	0.0348
3	-	0.0347	0.0346

Table 2. The estimation time of each time step.

Scenario	WLS	P2N2	D-P2N2
1	2.7299 s	0.0173 s	0.0042 s
2	2.8212 s	0.0171 s	0.0042 s
3	-	0.0156 s	0.0041 s

5. Conclusions

This paper introduced a new decentralized, data-driven state estimations based on the physical connections of the distribution network for large distribution grids. The model was designed based on the physical connections of the distribution network and the position of the PMU, which pruned the unnecessary connections between layers. One-year of smart meter data were used to generate training data sets by performing the power flow analysis. Then, the set of 35,040 data points was collected for the training and testing phase. The simulation results in the IEEE 123-node test network verify that our algorithm has the same estimation accuracy as the centralized algorithms in WLS and P2N2. Compared with the classic WLS algorithm, our algorithm shows higher estimation performance in the case

of measurement noise and low-observability scenarios. Moreover, our D-P2N2 is a fully decentralized algorithm that significantly reduces the computational cost for the training and execution phases and limits the required communication between subareas.

Author Contributions: M.-Q.T.: conceptualization, methodology, modeling, coding, and writing; A.S.Z. and P.H.N.: conceptualization, review, editing, and supervision; G.P.: review and supervision. All authors have read and agreed to the published version of the manuscript.

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