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Distributed Event-Based Control of Hierarchical Leader-Follower Networks with Time-Varying Layer-To-Layer Delays

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Abstract: Compared with the traditional multi-agent models, the hierarchical leader–follower network (HLFN) can describe some real-world multi-agent systems more precisely due to its layered properties. The distributed event-based consensus control problem of HLFNs with layer-to-layer delays, namely, communication delays among agents of different layers, is presented in this essay. In order to solve the aforementioned problem, several innovative hierarchical event-based control (HEC) algorithms are proposed. The sufficient conditions on the control parameters and event-triggered mechanism were derived to undertake the reliability of the closed-loop dynamics. Moreover, it is shown that the zeno-behaviors of the presented HEC algorithms can be excluded. Finally, there are some numerical examples that verify the availability of the results.

Keywords: event-based consensus; hierarchical leader–follower network; hierarchical event-based control; layer-to-layer delays

1. Introduction

In recent years, coordination problems of multi-agent systems (MAS) have attracted research interests, which cover a wide area such as flocking [1,2], formation control [3–5], consensus tracking of a robot [6], and rendezvous in distributed robotics [7–9]. On these issues, many remarkable and significant results have emerged one after another.

In recent years, the research on the consensus of the leader-following multi-agent system has been increasing, and a large number of achievements have emerged. Wang et al. [10] investigated the leader-following consensus by a self-triggered algorithm. At the same time, considering the existence of a disturbance in reality, the input delay was considered. Shen et al. [11] investigated the leader-following consensus problem of multi-agent systems with two distributed adaptive control schemes. Ning [12] investigated the problem of leader-following consensus for multiple wheeled mobile robots. Based on the above research, the multi-layer hierarchical structure on the basis of the two-layer structure was investigated. The hierarchical structure exists widely in nature. In reality, birds show a hierarchical structure in the process of migration, and each layer is both a leader layer and follow layer, in addition to the leader of the first layer and the last follow layer, this model is called a hierarchical leader–follower network (HLFN). Nagy [13] showed the hierarchical piloting behavior of a group of pigeons during a long flight for the first time. Through the way of recording the

experimental data, it can be found that the pigeons have established a hierarchical leadership structure to interact with each other during the flight, so as to complete the whole long-distance flight mission effectively. He and Xu [14] solved the coordination problem of MAS with two-layer leaders. Based on the previous work, we investigated the consensus of HLFNs.

In the actual communication network, due to the limitations of distance, bandwidth, and sensors, time delays between nodes are inevitable. Based on the passivity theory, the state estimation of recurrent neural networks (MRNNs) with time-varying delays is studied in [15]. Li [16] investigated the leader-following consensus of MASs with time delays and switching topology. Zhao [17] studied the consensus problem for MASs with communication delays. Jiang [18] solved the consensus problem of networked multiple robotic manipulators with and without time-varying delays. However, in large hierarchical networks, the communication delays between layers (layer-to-layer delays) are often different, and the different types of delays between different layers have not been solved.

In practical applications, continuous communication will lead to high communication costs and high requirements for equipment. Therefore, the event-based control has received widespread attentions as a method which can effectively reduce the cost of communication. In event-based control, the controller update only depends on the designed trigger conditions, and the trigger conditions are often related to the state of the system. The bipartite consensus problem of multi-agent system with connected structure balanced symbol graph based on event trigger was studied in paper [19]. Yi [20] has investigated the global consensus problem for first-order continuous-time multi-agent systems with input saturation, in which the triggering law does not require any a priori knowledge of global network parameters. The paper [21] investigates the bipartite leader-following consensus of second-order multi-agent systems with signed digraph topology. The paper [22] showed that event trigger control has lower communication times and controller update times and better control effect than periodic sampling control on the premise of completing control tasks. The event-based control technology attracted much attention because of the characteristics of resource saving, and related research also achieved more results [23–30]. However, the consensus problem of HLFNs remains unsolved.

Inspired by the preceding discussions, it is desirable to design economical and efficient hierarchical event-based control (HEC) algorithms for consensus problems of HLFNs with layer-to-layer delays. The main contributions of our paper are twofold: (1) Compared with the existing research achievements [30–34], this essay deals with the consensus problem of HLFNs with layer-to-layer delays. The existing literature considers the time-delays between different agents at the same layer. In order to better study the hierarchical structure, we considered layer-to-layer delays between different layers. (2) Based on the particularity of hierarchical structure, a new HEC algorithm was proposed to solve the previous problem. The algorithm is different from the general event-based method, as the trigger conditions of each layer were different.

In the rest of this paper is structured as follows: Section 2 introduces several fundamental theories of graph and matrix, and the three-layer MAS is described. In Section 3, the convergence of the designed control law is analyzed in detail. A numerical example is given to present the effectiveness of designed event-triggered mechanism in Section 4. Section 5 concludes this paper.

2. Preliminaries

2.1. Basic Theory on Graphs and Matrices

A directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ consists of a node set $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, an edge set $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij})$. If there is a edge between nodes i and j , then $(i, j) \in \mathcal{E} \Leftrightarrow a_{ij} > 0$. There are no self-loops, i.e., $a_{ii} = 0$. The neighbor set of agent i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}, j \neq i\}$. Besides, the Laplacian matrix \mathcal{L} is defined as $\mathcal{L} = [l_{ij}]$, where $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. Further, $D_k = \text{diag}(d_1, d_2, \dots, d_{r_k}), k = 2, 3, \dots, l, r_k$ is the number of

nodes of k th layer. $1 + r_2 + \dots + r_l = N$. If the node of k th layer can receive information from the r_{k-1} , then $d_{r_k} > 0$, otherwise $d_{r_k} = 0$.

2.2. System Description

Without loss of generality, the HLFNs contain three-layer network in this paper. The first layer contains only one leader (as node 0), the second layer contains r_2 minor leader, the third layer contains m groups, r_3 followers, and each group can receive information from the minor leader layer. Let the topological relationship between the minor leaders be \mathcal{G}_2 , its corresponding Laplacian matrix is \mathcal{L}_2 . Followers are divided into m subgroups, and \mathcal{G}_p consists of r_p number of nodes and satisfies $N = 1 + r_2 + r_3$, $p = 1, 2, \dots, m$. The Laplacian matrix of the graph \mathcal{G}_p is recorded as \mathcal{L}_p . Including a major leader and minor leaders, an augmented topology diagram $\bar{\mathcal{G}}_2$ which consists of diagram \mathcal{G}_2 , node 0, and the directed edges of some nodes to node 0 is considered. An augmented topology diagram $\bar{\mathcal{G}}_p$ is composed of diagram \mathcal{G}_p , nodes from minor leader layer, and the directed edges of some node-to-node from the minor leader layer. Define set $v1 = \{1, 2, \dots, r_2\}$, $v2 = \{r_2 + 1, r_2 + 2, \dots, r_2 + r_3\}$.

The linear dynamics of major leader without input can be described as

$$\dot{x}_0(t) = Ax_0(t), \quad (1)$$

the dynamics of minor leaders and followers are given as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i \in v1 \cup v2, \quad (2)$$

where $x_0(t) \in \mathbb{R}^n$, $x_i(t) \in \mathbb{R}^n$ are the states of major leader, minor leaders and followers, respectively, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are constant matrices, and $u(t) \in \mathbb{R}^m$ is the control input.

Firstly, some basic assumptions and lemmas are given as follows:

Assumption 1. Node 0 is globally accessible in a graph $\bar{\mathcal{G}}_2$ that contains a major leader and r_2 minor leaders. The node from the minor leader layer is globally accessible in a graph $\bar{\mathcal{G}}_p$ that contains a minor leader and some followers.

Assumption 2. The matrix pair (A, B) is stabilizable.

Assumption 3. Each row sum of l_{ij} in \mathcal{L}_1 is zero; each row sum of l_{ij} in \mathcal{L}_p is zero.

Lemma 1. The Laplacian matrix has a simple zero eigenvalue and all the other eigenvalues have positive real parts if and only if graph \mathcal{G} contains a directed spanning tree.

Definition 1. The consensus of the HLFN is considered achieved if

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_0(t)\| = 0, i \in v1 \cup v2. \quad (3)$$

Remark 1. These assumptions are necessary for MAS with three-layer networks. If there is no Assumption 1, some minor leaders may be isolated from the major leader and unable to receive information from the major leader, thus detaching from the entire MAS, and several isolated smaller groups or individual isolated followers may be formed in the same follower subgroup, so that some followers cannot receive control information from the corresponding minor leaders. This obviously fails to achieve the desired control objectives.

3. Main Results

3.1. Event-Based Control for Consensus of HLFNs

In this section, the consensus of uncoupled followers under the influence of dynamic major leader is analyzed.

For the minor leaders, the control goal is to enable the states of the minor leaders follow the major leader states. In order to achieve the consensus, the event-triggered control law for agent $i (i \in v1)$ is designed:

$$u_{i_1}(t) = K \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j(t) - \hat{x}_i(t)) + d_i (x_0(t) - \hat{x}_i(t)) \right], i \in v1, \quad (4)$$

where $K \in \mathbb{R}^{m \times n}$ is a feedback matrix, which is described in a later section. For the follower $i (i \in v2)$ in the follower group, the corresponding control protocol is designed as

$$u_{i_2}(t) = K \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j(t) - \hat{x}_i(t)) + d_i (\hat{x}_p(t) - \hat{x}_i(t)) \right], i \in v2. \quad (5)$$

where $x_p(t)$ is the state of the minor leader to node i . Here, $\hat{x}_i(t)$ is defined as $x_i(t_k^i), t \in [t_k^i, t_{k+1}^i)$.

In order to analyze the consensus problem, we define:

$$\delta_i(t) = x_i(t) - x_0(t), i \in v1 \cup v2, \quad (6)$$

$$\varepsilon_i(t) = \hat{x}_i(t) - x_i(t), i \in v1 \cup v2. \quad (7)$$

Then the system of Equation (2) can be expressed as:

$$\begin{aligned} \dot{\delta}_{i1}(t) &= \dot{x}_i(t) - \dot{x}_0(t) \\ &= A\delta_i(t) + BK \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\varepsilon_j(t) + \delta_j(t) - \varepsilon_i(t) - \delta_i(t)) \right. \\ &\quad \left. - d_i (\varepsilon_i(t) + \delta_i(t)) \right], i \in v1, \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{\delta}_{i2}(t) &= \dot{x}_i(t) - \dot{x}_0(t) \\ &= A\delta_i(t) + BK \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\delta_j(t) + \varepsilon_j(t) - \delta_i(t) - \varepsilon_i(t)) \right. \\ &\quad \left. + d_i (\varepsilon_p(t) + \delta_p(t) - \delta_i(t) - \varepsilon_i(t)) \right], i \in v2. \end{aligned} \quad (9)$$

where $\delta_{i1}(t)$ is the error of minor leaders, and $\delta_{i2}(t)$ is the error of followers. Let $\delta(t) = (\delta_1^T(t), \delta_2^T(t), \dots, \delta_r^T(t))^T$, $\varepsilon(t) = (\varepsilon_1^T(t), \varepsilon_2^T(t), \dots, \varepsilon_r^T(t))^T$, the system of Equations (8) and (9) becomes

$$\begin{aligned} \dot{\delta}_1(t) &= [(I_{r_2} \otimes A) - (\mathcal{L}_2 + \mathcal{D}_2) \otimes BK] \delta_1(t) \\ &\quad - [(\mathcal{L}_2 + \mathcal{D}_2) \otimes BK] \varepsilon(t), i \in v1, \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{\delta}_2(t) &= [(I_{r_p} \otimes A) - (\mathcal{L}_p + \mathcal{D}_p) \otimes BK] \delta_2(t) - [(\mathcal{L}_p \\ &\quad + \mathcal{D}_p) \otimes BK] \varepsilon(t) + (\mathcal{D}_p \otimes BK) (\delta_p(t) + \varepsilon_p(t)), i \in v2. \end{aligned} \quad (11)$$

Theorem 1. Suppose that Assumptions 1, 2 and 3 hold. Consider the HLFN Equations (1) and (2) with the control in Equations (4) and (5). The consensus problem can be solved if:

$$f_{i_1}(t) = \|\varepsilon_i(t)\| - \|BKq_i(t)\| - \beta e^{-\gamma(t-t_0)}, i \in v1. \quad (12)$$

$$\begin{aligned} f_{i_2}(t) &= \|\varepsilon_i(t)\| - \beta e^{-\gamma(t-t_0)} \\ &\quad - \frac{\|BKq_i(t)\| - 2 \left\| \sum_{j \in \mathcal{N}_i} d_i (\hat{x}_p(t) - x_0(t)) \right\|}{2 \|(L_p + D) \otimes BK\|}, i \in v2. \end{aligned} \quad (13)$$

and

$$q_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + d_i(x_0(t) - \hat{x}_i(t)), i \in v1,$$

$$q_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + d_i(\hat{x}_p(t) - \hat{x}_i(t)), i \in v2.$$

For any $\beta > 0$, and $K = B^T P$, P is a positive-definitive matrix.

Proof of Theorem 1. Using the variation of parameter formula, Equation (10) becomes

$$\begin{aligned} \delta(t) &= e^{[(I_r \otimes A) - (\mathcal{L}_2 + \mathcal{D}_2) \otimes BK](t-t_0)} \delta(t_0) \\ &+ \int_{t_0}^t e^{[(I_r \otimes A) - (\mathcal{L}_2 + \mathcal{D}_2) \otimes BK](t-\theta)} \\ &\times (-((\mathcal{L}_2 + \mathcal{D}_2) \otimes BK)) \varepsilon(\theta) d\theta. \end{aligned}$$

In reality, if (A, B) is stabilizable and the communication topology of these agents in the leader–follower structure has a directed spanning tree, it can be obtained from Lemma 1 that all the eigenvalues of $I_r \otimes A - (\mathcal{L}_2 + \mathcal{D}_2) \otimes BK$ have non-positive real parts. Therefore, positive constants a and ρ for $t \geq t_0$ are existed, such that (refer to [35])

$$\left\| e^{[I_r \otimes A - (\mathcal{L}_2 + \mathcal{D}_2) \otimes BK](t-t_0)} \right\| \leq a e^{-\rho(t-t_0)}.$$

Furthermore, we can get

$$\begin{aligned} \|\delta(t)\| &\leq \alpha_1 e^{-\gamma_1(t-t_0)} \|\delta(t_0)\| \\ &+ \alpha_1 \int_{t_0}^t e^{-\gamma_1(t-\theta)} \|(\mathcal{L}_2 + \mathcal{D}_2) \otimes BK\| \|\tilde{x}(\theta)\| d\theta, \end{aligned} \quad (14)$$

Forced by Equation (12), we can get

$$\|\varepsilon(t)\| \leq \tilde{\alpha} \|\delta(t)\| + \tilde{\beta} e^{-\gamma(t-t_0)}, \quad (15)$$

Combine Equation (14) with Equation (15), it holds

$$\begin{aligned} \|\delta(t)\| &\leq \alpha_1 \|\delta(t_0)\| e^{-\rho(t-t_0)} + \alpha_1 \int_{t_0}^t e^{-\rho(t-\theta)} \\ &\times \|(\mathcal{L}_2 + \mathcal{D}_2) \otimes BK\| (\tilde{\alpha} \|\delta(\theta)\| + \tilde{\beta} e^{-\gamma(\theta-t_0)}) d\theta. \end{aligned} \quad (16)$$

Then, we prove that

$$\|\delta(t)\| < \eta \alpha_1 \|\delta(t_0)\| e^{-\gamma(t-t_0)}. \quad (17)$$

If Equation (17) does not hold, then $t^* > t_0$ for $\|\delta(t^*)\| = v(t^*)$ and $\|\delta(t)\| < v(t)$ for $t < t^*$. Then by Equation (16), one has

$$\begin{aligned}
v(t^*) &= \|\delta(t^*)\| \\
&< \eta\alpha_1 e^{-\rho(t^*-t_0)} \|\delta(t_0)\| + \eta\alpha_1 \int_{t_0}^{t^*} e^{-\rho(t^*-\theta)} \\
&\quad \times (\tilde{\alpha} \|\delta(\theta)\| + \tilde{\beta} e^{-\gamma(\theta-t_0)}) d\theta \\
&< \eta\alpha_1 e^{-\rho(t^*-t_0)} \|\delta(t_0)\| + \eta\alpha_1 (\tilde{\alpha}\alpha_1 \|\delta(t_0)\| + \tilde{\beta}) \\
&\quad \times (e^{-\gamma(t^*-t_0)} - e^{-\rho(t^*-t_0)}) \\
&< \eta\alpha_1 \|\delta(t_0)\| e^{-\gamma(t^*-t_0)} \\
&= v(t^*).
\end{aligned} \tag{18}$$

The conflict of Equation (18) demonstrates that Equation (17) is well-founded, which implies that the consensus of the system in Equation (1), Equation (2) can be achieved exponentially.

Then using the variation of parameter formula, Equation (11) can be rewritten as

$$\begin{aligned}
\varepsilon(t) &= e^{[(I_{rp} \otimes A) - (\mathcal{L}_p + \mathcal{D}_p) \otimes BK](t-t_0)} \varepsilon(t_0) \\
&\quad + \int_{t_0}^t e^{[(I_{rp} \otimes A) - (\mathcal{L}_p + \mathcal{D}_p) \otimes BK](t-\theta)} \\
&\quad \times [(-((\mathcal{L}_p + \mathcal{D}_p) \otimes BK))e(\theta) \\
&\quad + (\mathcal{D}_p \otimes BK)(\tilde{x}(\theta) + \delta(\theta))] d\theta.
\end{aligned} \tag{19}$$

It follows from Equation (19)

$$\begin{aligned}
\|\varepsilon(t)\| &\leq \alpha_2 e^{-\rho(t-t_0)} \|\varepsilon(t_0)\| \\
&\quad + \alpha_2 \int_{t_0}^t e^{-\rho(t-\theta)} [\|(\mathcal{L}_p + \mathcal{D}_p) \otimes BK\| \|\varepsilon(\theta)\| \\
&\quad + \| \mathcal{D}_p \otimes BK \| \|\varepsilon(\theta) + \delta(\theta)\|] d\theta.
\end{aligned} \tag{20}$$

Replacing Equation (15) in Equation (20),

$$\begin{aligned}
\|\varepsilon(t)\| &\leq \alpha_2 e^{-\rho(t-t_0)} \|\varepsilon(t_0)\| \\
&\quad + \alpha_2 \int_{t_0}^t e^{-\rho(t-\theta)} [\|(\mathcal{L}_p + \mathcal{D}_p) \otimes BK\| \|\varepsilon(\theta)\| \\
&\quad + r\beta e^{-\gamma(\theta-t_0)}] d\theta.
\end{aligned} \tag{21}$$

Then, we prove that

$$\|\varepsilon(t)\| < \eta\alpha_2 \|\varepsilon(t_0)\| e^{-\gamma(t-t_0)}. \tag{22}$$

This part of the proof is equivalent to the previous process, which is omitted here.

To avoid zero-behavior, we need to get the lower boundary of the positive constant for the interval between any two events Δt_i , suppose agent i triggered at the moment $t_i^* \geq 0$, then $e_i(t_i^*) = 0$. For between the time t_i^* and the time t_i which the next event is triggered, we have

$$\dot{e}_i(t) = -\dot{x}_i(t) = -[Ax_i(t) + Bu_i(t)]. \tag{23}$$

By integrating the two sides of the above equation, we can get

$$|e_i(t)| = \int_{t_i^*}^t |Ax_i(s) + Bu_i(s)| ds. \tag{24}$$

we know

$$\begin{aligned}
 |Ax_i(t) + Bu_i(t)| &\leq \|(I_r \otimes A)\| \|x(t)\| \\
 &\quad + \|(\mathcal{L}_p + \mathcal{D}_p) \otimes BK\| (\|\varepsilon(t)\| + \|e(t)\|) \\
 &\quad + \|\mathcal{D}_p \otimes BK\| \|\delta(t)\| \\
 &= c.
 \end{aligned} \tag{25}$$

where c is a normal value.

By Equations (24) and (25), $|e_i(t)| \leq c(t - t^*)$ can be obtained. Assume $c(t - t^*) = c_1$, when the error vector decreases from $|e_i(t)| = c_1$ to $|e_i(t)| = 0$, the next event trigger will occur. Therefore, there exists a strict positive lower bound $\Delta t_i = t_i - t_i^* = c_1/c$ between the triggering times of two events. Since there exists such a lower bound for arbitrary agent and any event time, we believe that zeno-behavior will not exist. The process of proof for Theorem 1 is thus completed. \square

3.2. Event-Based Control for Consensus of HLFNs with Layer-to-Layer Delays

In this part, we consider the result in case when the layer-to-layer delays exist. For the minor leaders, the following protocol for agent i is designed:

$$\begin{aligned}
 u_i(t) &= K \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j(t) - \hat{x}_i(t)) + d_i (x_0(t - \tau_1(t)) - \hat{x}_i(t)) \right], \\
 & \quad i \in v1,
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 u_i(t) &= K \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_j(t) - \hat{x}_i(t)) + d_i (\hat{x}_p(t - \tau_2(t)) - \hat{x}_i(t)) \right], \\
 & \quad i \in v2.
 \end{aligned} \tag{27}$$

In this part, we consider layer-to-layer delays in communication. Define that the communication between major leader and minor leaders is subject to a time delay $\tau_1(t)$, which satisfies $0 < \tau_1(t) < \mu_1$, the communication between minor leader and followers is subject to a time delay $\tau_2(t)$, which satisfies $0 < \tau_2(t) < \mu_2$.

Theorem 2. Suppose that Assumptions 1, 2 and 3 hold. Consider the HLFN Equations (1) and (2) with control, (26) and (27), the consensus problem can be solved if:

$$\begin{aligned}
 f_i(t) &= \|\varepsilon_i(t)\| - \beta e^{-\gamma(t-t_0)} \\
 &\quad - \frac{\|BKq_i(t)\| - 2 \left\| BK \sum_{q \in \mathcal{N}_p} d_i (\hat{x}_p(t) - x_0(t - \tau_1(t))) \right\|}{2 \|(\mathcal{L}_2 + \mathcal{D}_2) \otimes BK\|}, \\
 & \quad i \in v1,
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 f_i(t) &= \|\varepsilon_i(t)\| - \beta e^{-\gamma(t-t_0)} \\
 &\quad - \frac{\|BKq_i(t)\| - 2 \left\| BK \sum_{j \in \mathcal{N}_i} d_i (\hat{x}_p(t) - x_0(t - \tau_2(t))) \right\|}{2 \|(\mathcal{L}_p + \mathcal{D}_p) \otimes BK\|}, \\
 & \quad i \in v2.
 \end{aligned} \tag{29}$$

For any $\beta > 0$, and $K = B^T P$, P is a positive-definitive matrix.

Proof of Theorem 2. In this part, we divide the proof process into two parts, for minor leader $i (i = 1, 2, \dots, r_2)$, we can get

$$\begin{aligned} \|\delta(t)\| &\leq \alpha_1 e^{-\rho(t-t_0)} \|\delta(t_0)\| \\ &+ \alpha_1 \int_{t_0}^t e^{-\rho(t-\theta)} \|(\mathcal{L}_2 + \mathcal{D}_2) \otimes BK\| \|\varepsilon_i(\theta)\| \\ &- \|(\mathcal{D}_2 \otimes BK)\| \left\| (e^{A\theta} - e^{A(\theta-\tau_1(\theta))}) \right\| d\theta, \end{aligned} \quad (30)$$

forced by Equation (28), one can get

$$\|\delta(t)\| < \eta \alpha_1 e^{-\gamma(t-t_0)} \|\delta(t_0)\|, \quad (31)$$

For follower $i (i \in v_2)$, it holds

$$\begin{aligned} \|\delta(t)\| &\leq \alpha_2 e^{-\rho(t-t_0)} \|\delta(t_0)\| + \alpha_2 \int_{t_0}^t e^{-\rho(t-\theta)} \\ &\times \left[\|(\mathcal{L}_p + \mathcal{D}_p) \otimes BK\| \|\varepsilon(\theta)\| + \|\mathcal{D}_2 \otimes BK\| \right. \\ &\left. \times \left\| \delta(\theta - \tau_2(\theta)) + \varepsilon_p(\theta - \tau_2(\theta)) - (e^{A\theta} - e^{A(\theta-\tau_2(\theta))}) \right\| \right] d\theta, \end{aligned} \quad (32)$$

It can be concluded from Equation (29) that

$$\|\delta(t)\| < \eta \alpha_2 \|\delta(t_0)\| e^{-\gamma(t-t_0)}. \quad (33)$$

The proof of zeno-behavior is equivalent to that of Theorem 1, which is omitted here. The proof of Theorem 2 is thus completed. \square

Remark 2. The conclusion of this paper is that, based on the three-layer hierarchical structure, agents from the minor leader layer and the follower layer only interacts with the same layer agents and direct leaders, therefore, the analysis of three-layer network can be extended to multi-layer hierarchical MAS.

4. Simulation Results

In this section, two numerical example are shown to demonstrate the corresponding analysis of the results gained in the previous section. Consider the event-based consensus of HLFNs with thirteen agents, a major leader, three minor leaders, and nine followers. The communication topology is described as seen in Figure 1 and the connectivity weights are given as $a_{ij} = 1, d_{ij} = 1$, the other weights are all equal to zero. From the topology graph and the matrix relationship given, it can be seen that node 0 and nodes from the second layer are globally reachable in $\bar{\mathcal{G}}_2$ and $\bar{\mathcal{G}}_p$, respectively. Moreover, there is a directed spanning tree in the interconnection graph. Assume that

$$A = \begin{bmatrix} -6 & 3 & 3 \\ 3 & -3 & 0 \\ -3 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.2 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.3492 & 0.4488 & 0.2378 \\ 0.4488 & 0.9798 & 0.4697 \\ 0.2378 & 0.4697 & 0.4827 \end{bmatrix}$$

Thus (A,B) is stabilizable. Consider the hierarchical leader-follower networks with the interconnection graph given in Figure 1, for any initial condition, Figures 2–4 show the change of error of each agents. It can be seen from Figures 2–4 that the consensus can be achieved eventually under the proposed event-triggered strategy. The enlarged part of the figure shows that the system

reached consensus in about 11 s. Event time instants are shown in Figure 5, in which no Zeno triggering behaviors or even dense events for any agent are observed. Figures 6–8 shows the the change of error of agents with layer-to-layer delays, which shows the consensus can be achieved eventually under the proposed event-triggered strategy with layer-to-layer delays in about 15 s. Due to the interference of the layer-to-layer delays, the time to reach the consensus in Figures 6–8 is longer than that in Figures 2 and 3. Figure 9 shows the event time instants under the event-triggered law with layer-to-layer delays. As can be seen from Figures 5 and 9, leader 1–3 triggers less than other followers, which greatly saves the waste of system resources. From the simulation results, one can see that Theorem 2 does more controller updates than that in Theorem 1.

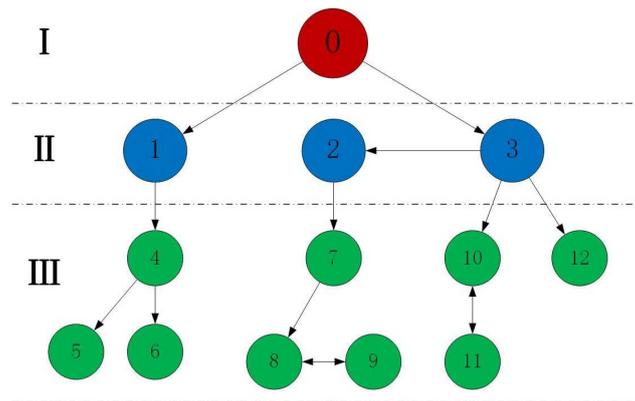


Figure 1. Topology of the hierarchical multi-agent network with uncoupled follower groups.

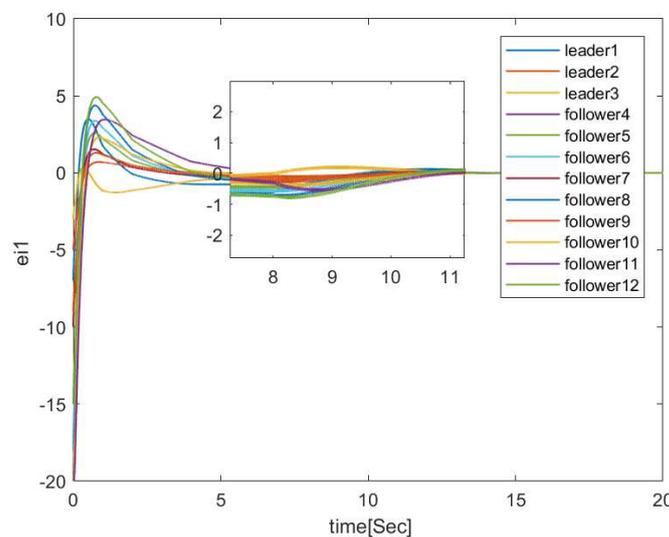


Figure 2. Position State Component 1 of Agent without delay.

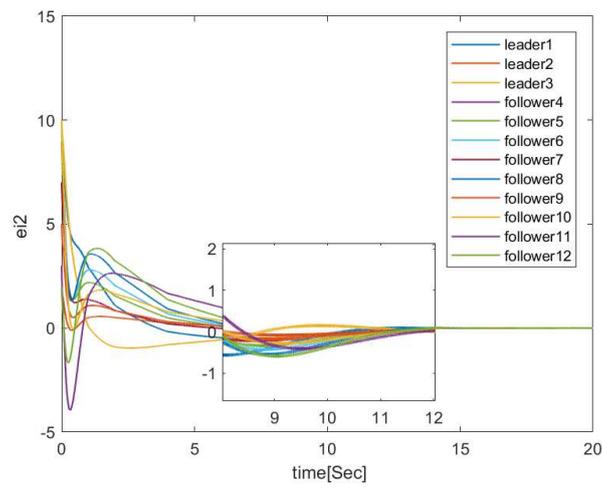


Figure 3. Position State Component 2 of Agent without delay.

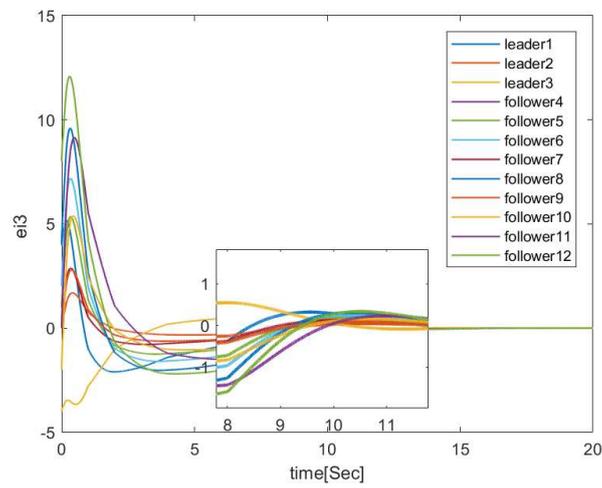


Figure 4. Position State Component 3 of Agent without delay.

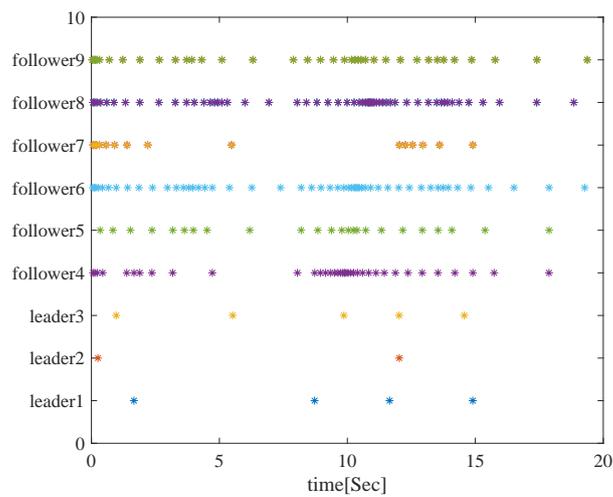


Figure 5. Event-trigger numbers under the event-based control law of Equation (4).

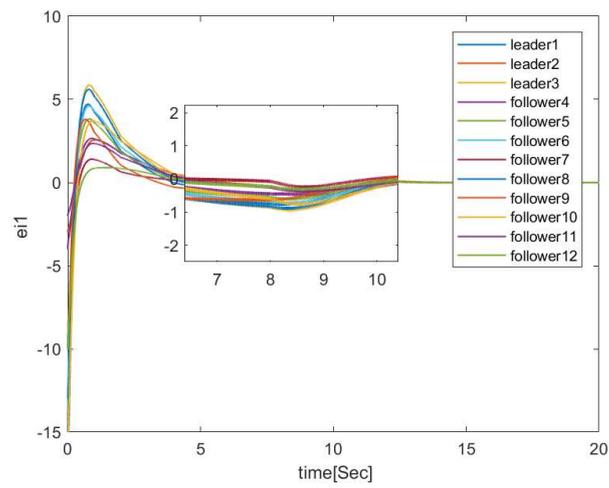


Figure 6. Position State Component 1 of Agent with hierarchical delay.

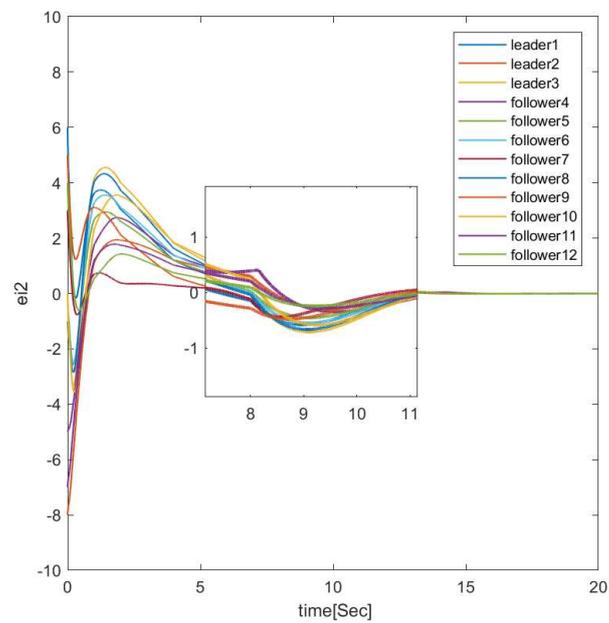


Figure 7. Position State Component 2 of Agent with hierarchical delay.

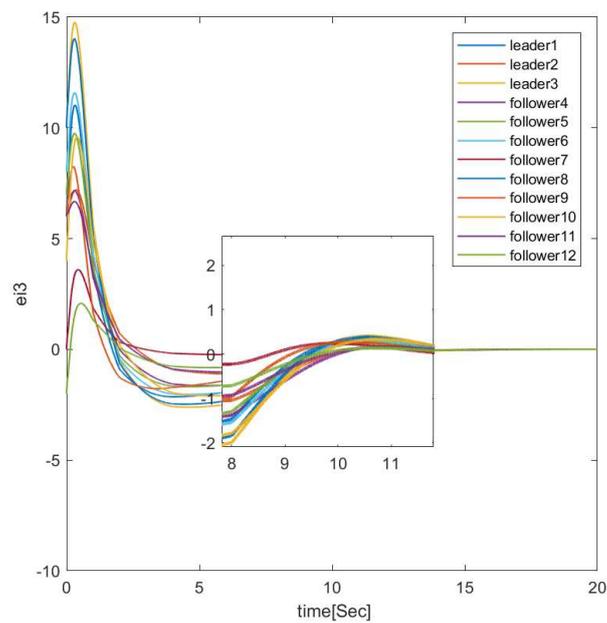


Figure 8. Position State Component 3 of Agent with hierarchical delay.

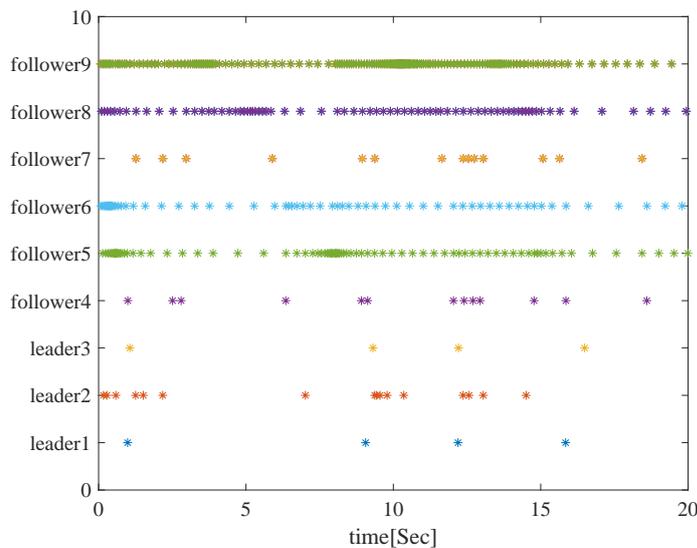


Figure 9. Event-trigger numbers under the event-based control law of Equation (5) with layer-to-layer delay.

Remark 3. The state of the agents in this paper is three-dimensional, and Figures 2–4 describe the state changes of the three dimensions, respectively.

5. Conclusions

In this paper, a novel hierarchical event-based control for hierarchical leader-follower networks is proposed. Based on matrix theory, algebraic graph theory, Lyapunov theory, and under the given event-triggered condition, it is proven that the protocols can realize consensus. Moreover, we show that the system can reach consensus with the layer-to-layer delays. Finally, simulation results are presented to support the theorems. In the future, more effort will be made towards relaxing the connectivity

condition on the topology of the multi-agent network. The restrictions on the time delays may be lifted. How to extend consensus to multi-consensus is another topic in the future.

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