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The Catenary Method as an Alternative to the Horizontal Directional Drilling Trajectory Design in 2D Space

Rafał Wiśniowski, Krzysztof Skrzypaszek, Paweł Łopata * and Grzegorz Orłowicz *

Faculty of Drilling Oil and Gas, Department of Drilling and Geoengineering, AGH University of Science and Technology, Mickiewicza 30 Av., 30–059 Kraków, Poland; wisniows@agh.edu.pl (R.W.); varna@agh.edu.pl (K.S.)
* Correspondence: lopatapaw@gmail.com (P.Ł.); orlowiczg@gmail.com (G.O.)

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Abstract: An increase in demand for energy natural resources has stimulated the development of gas pipeline networks in Europe, as well as globally. Also, in Poland in recent years there has been a significant increase in natural gas consumption. Therefore, it is necessary to build new pipelines networks using dig and no–dig techniques. Horizontal Directional Drilling is one of the most popular trenchless technologies. The aim of this article is to present a new approach in the design of HDD trajectories in two–dimensional space (2D). A review of the trajectories used so far has been provided, offering calculation algorithms to determine their well path. Then, the original catenary method is proposed, taking into account natural deflection of casing pipes. Applicable formulas and computational algorithms have been given, together with a computational example which enables comparison of the classical design methodology with the new one. According to the authors, due to natural stress distribution, the catenary method allows the use of smaller pulling forces during installation and ensures longer pipeline life. Therefore, it should be used in industrial practice as an alternative to current designing methods.

Keywords: horizontal directional drilling (HDD); trenchless technologies; pipeline installation; well design; mining and environmental engineering

1. Introduction

Horizontal Directional Drilling (HDD) is one of the most popular trenchless technologies. The HDD technology has revolutionized geoengineering drilling in cities and under natural obstacles where the use of classical dig methods is impossible. It enables the ability to perform underground pipeline installations, while overcoming natural or artificial obstacles such as streets, buildings, railways, rivers and lakes, as well as minimizing a negative impact on the natural environment.

Taking into account both environmental and economic factors, the HDD technology can currently be considered one of the few alternatives to the classical dig methods in the construction of underground installations for the transmission of natural resources; in particular, natural gas. Increasing development and interest in this technology is in line with current trends in Polish energy policy and changes taking place in the natural gas market, the consumption of which in Poland increases year by year. The increase in demand is caused, among other reasons, by newly built power plants such as the Combined Cycle Gas Turbine in the Żerań CHP in Warsaw, which will burn approximately 0.65 billion m^3 of natural gas annually [1]. In addition to the energy sector, other industries also consume more and more gas: steel mills, refineries, ceramic factories, glassworks, and nitrogen plants. In 2018, Poland consumed about 18 billion m^3 of natural gas, including 4 billion m^3 from the national resources. The remaining quantity was imported but the import direction changed in comparison to previous

years. Gas imported from Russia plays a smaller role (a decrease of 0.62 billion m^3 compared to 2017). However, the import of LNG (liquefied natural gas) transported to the terminal in Świnoujście by sea is growing. In 2018, gas carriers from the USA, Qatar and Norway supplied over 2.71 billion m^3 of LNG. This is an increase of 58.2% compared to 2017. At the end of 2022, also, the Baltic Pipe gas pipeline will be completed, ensuring transport of the natural gas extracted in Norway [2,3]. The above–described changes make it necessary to construct new and modernize the existing pipeline networks, taking into account, at the same time, urban infrastructure (roads, buildings) and the natural environment (rivers, protected areas).

The Horizontal Directional Drilling method dates back to the 1960s when Martin Cherrington built his own drill rig and formed Titan Contractors, a company specializing in utility road boring in Sacramento, California. The first Horizontal Directional Drilling application in history was made in 1971 under the Pajero river near Wotsonville, California, United States. A 180-meter long pass with the diameter of 100 mm was carried out by Titan Contractor under the leadership of Martin Cherrington [4–7].

Nowadays, wellbores with diameters ranging from 50 to 1200 mm and a length of up to 2000 m are the most popular. Polyethylene (PE), high density polyethylene (HDPE), PVC, and steel pipes are used [8–10].

Parallel to the conventional HDD method, the Intersect Drilling method is being developed, which involves drilling pilot holes from the start, and end points simultaneously. It is used for long boreholes drilled in difficult conditions, where classical drilling from the entry point to the exit point would be impossible [11–13].

Drilling equipment categories are presented in the Table 1 below:

Table 1. Horizontal Directional Drilling (HDD) equipment categories [14,15].

Category	Type of Equipment	Pulling Force [kN]	Torque [Nm]	Power [kW]	Maximum Mud Flow Rate [l/min]	Type of Horizontal Drilling
1	very small	<100	<2500	<75	100	Small
2	small	100–250	<2500–15000	75–150	500	Medium
3	medium	250–500	15000–25000	150–300	1000	
4	large	500–1000	25000–50000	300–600	1500	Large
5	extra large	>1000	>50000	>600	>1500	

Horizontal Directional Drilling consists of three stages [14,16–18]:

1. Pilot hole drilling
2. Reaming
3. Casing installation

Drilling a pilot hole is the most important part. The pilot hole is made according to the assumed trajectory design. Depending on the rock hardness, drilling works are performed with the use of an asymmetrical hydromonitor bit, a cutter bit, or a cogged bit. The rock is drilled hydraulically and mechanically. The trajectory of the well axis is constantly or periodically measured; the location of the probe in an antimagnetic connection (disposed right after the bit) is verified. Surface or subsurface radiometric cable or cableless telemetric navigation systems are used. The direction of drilling can be corrected by rotating the drill pipe by a predefined angle. In order to obtain the predicted final diameter, the borehole is widened one or more times [15,18,19].

After making a pilot borehole, the drilling bit is substituted in the borehole with a cutter or a milled reamer on the drill pipes. The reamer is introduced into the pilot borehole to enlarge the diameter of the existing well. In the process of reaming, drill pipes are successively added to proceed with reaming operations or to trip the casing [15,18,19].

The final stage of HDD is casing pullback. To do this, a reamer, rotary sub, and a head (tripped with the pipe) are disposed on the string. The rotary sub (swivel) prevents the system against rotations and torque of the drill pipes on the tripped casing [15,18,19].

The key element affecting the effectiveness and success of HDD drilling is the correct well trajectory design. A decision about the trajectory design is made based on the design data.

The following options are considered [20]:

- design of Horizontal Directional Drilling trajectory in two–dimensional space, with the following variations:
    - trajectory being a combination of straight and curvilinear sections
    - chain curve trajectory (catenary)
    - an irregular curve trajectory
- design of Horizontal Directional Drilling trajectory in three–dimensional space

This article is dedicated to the Horizontal Directional Drilling trajectory design in two–dimensional space. A combination of straight and curvilinear sections and the chain curve trajectory (catenary) design will be examined.

2. Mathematical Foundations of a Horizontal Trajectory Design in a Plane Perpendicular to the Terrain Surface

2.1. Trajectories Being a Combination of Straight and Curvilinear Sections

One of the most commonly applied concepts of the Horizontal Directional Drilling trajectory design is a combination of straight and curvilinear sections. The simplest variant is a trajectory consisting of one curvilinear section with a constant radius of curvature. Next variants are a combination of different numbers of straight and curvilinear sections [18,20]. Figure 1 shows the most common types of trajectories.

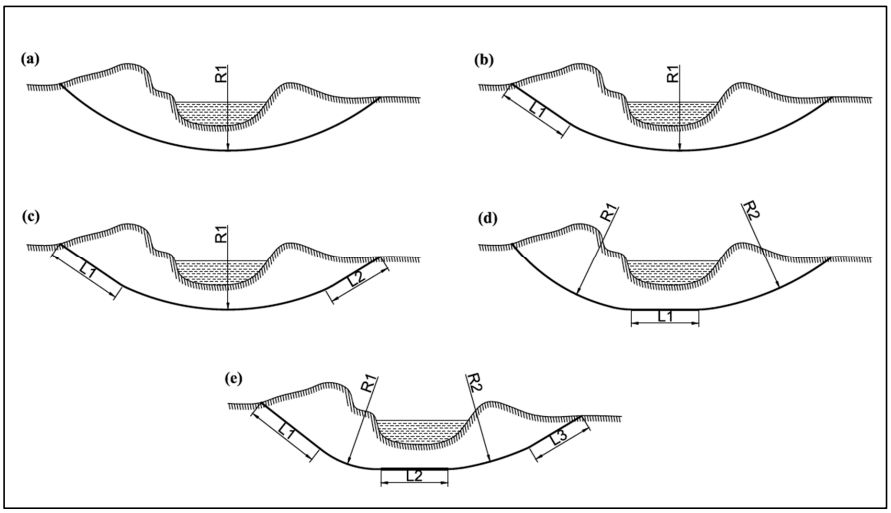

Figure 1. Horizontal directional drilling trajectories as a combination of straight and curvilinear sections: (**a**) one curvilinear section; (**b**) straight section and one curvilinear section; (**c**) two straight sections and one curvilinear section; (**d**) straight section and two curvilinear sections; (**e**) three straight sections and two curvilinear sections.

Assuming a reference system (see Figure 2) in a plane perpendicular to the surface of the terrain, it is possible to determine general dependencies for the types of trajectories presented in Figure 1 [18].

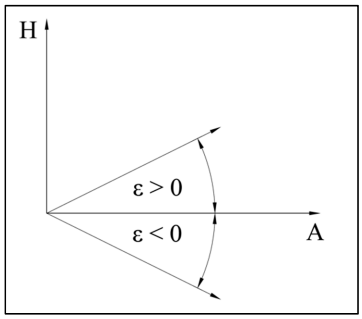

Figure 2. Reference system used when designing HDD trajectories in two–dimensional space.

General dependencies:

- vertical displacement of the end point relative to the start point:

$$H = \sum_{j=1}^{n} H_j \tag{1}$$

- horizontal displacement of the end point relative to the start point:

$$A = \sum_{j=1}^{n} A_j \tag{2}$$

- trajectory length:

$$L = \sum_{j=1}^{n} L_j \tag{3}$$

- azimuth.

Graphic interpretation of general dependencies is shown in Figure 3.

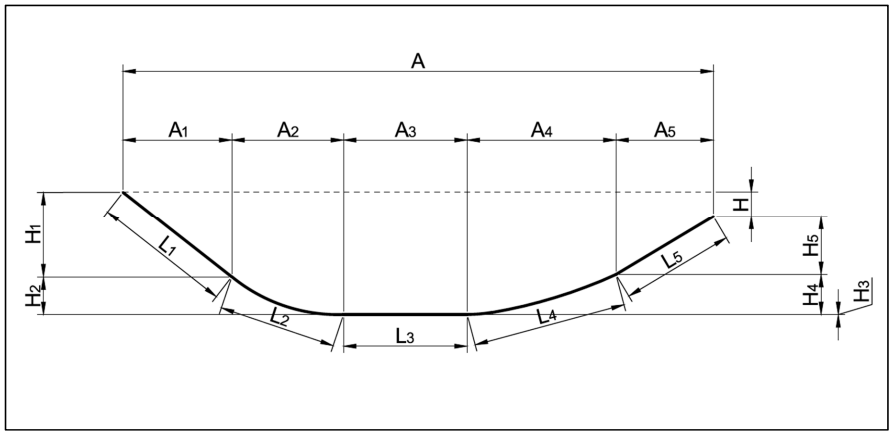

Figure 3. General geometric dependencies.

Curvilinear sections are defined by the following parameters:

- radius of curvature:

$$R_j = \frac{180}{\pi DLS_j}, \tag{4}$$

- angle of curvature:

$$\delta_j = \varepsilon_j - \varepsilon_{j-1} \text{ for } \varepsilon_j > \varepsilon_{j-1}, \tag{5}$$

$$\delta_j = \varepsilon_{j-1} - \varepsilon_j \text{ for } \varepsilon_j < \varepsilon_{j-1} \tag{6}$$

- projection of a trajectory section on a vertical plane:

$$H_j = R_j(\cos\varepsilon_{j-1} - \cos\varepsilon_j), \text{ for } \varepsilon_j > \varepsilon_{j-1}, \tag{7}$$

$$H_j = R_j(\cos\varepsilon_j - \cos\varepsilon_{j-1}), \text{ for } \varepsilon_j < \varepsilon_{j-1}\,, \tag{8}$$

- projection of a trajectory section on a horizontal plane:

$$A_j = R_j(\sin\varepsilon_j - \sin\varepsilon_{j-1}), \text{ for } \varepsilon_j > \varepsilon_{j-1}, \tag{9}$$

$$A_j = R_j(\sin\varepsilon_{j-1} - \sin\varepsilon_j), \text{ for } \varepsilon_j < \varepsilon_{j-1}, \tag{10}$$

- section length:

$$L_j = \frac{\pi}{180}\cdot\delta_j\cdot R_j\,, \tag{11}$$

Straight sections are determined by the following parameters:

- projection of a trajectory section on a vertical plane:

$$H_j = L_j\cdot\sin\varepsilon_j\,, \tag{12}$$

- projection of a trajectory section on a horizontal plane:

$$A_j = L_j\cdot\cos\varepsilon_j\,, \tag{13}$$

- section length:

$$L_j = \sqrt{A_j^2 + H_j^2}\,, \tag{14}$$

At the AGH University of Science and Technology in Krakow at the Drilling and Geoengineering Department, of Faculty Drilling, Oil and Gas, algorithms that enable calculation of characteristic points P_j (A_j, H_j) of any type of trajectory with different input data variants were created. The Department of Drilling and Geoengineering also presented an algorithm enabling the determination of intermediate Pi characteristic points P_i (A_i, H_i, ε_i, β_i) with a given calculation step ΔL [18,20].

The most commonly used trajectory is a combination of straight and curvilinear sections consisting of 5 sections (see Figure 1e). Table 2 presents steps to determine characteristic values of a trajectory with the first variant of input data.

Table 2. The order of calculating the characteristic values for a trajectory consisting of 5 sections with the given input parameters: A, H, L_1, ε_1, ε_3, R_2, R_4 [20].

Variant I: A, H, L_1, ε_1, ε_3, ε_5, R_2, R_4			
Step	Value to be calculated	Step	Value to be calculated
1	H_1	11	DLS_4
2	A_1	12	L_4
3	H_2	13	L_3
4	A_2	14	H_3
5	δ_2	15	A_3
6	DLS_2	16	H_5
7	L_2	17	A_5
8	H_4	18	L_5
9	A_4	19	L
10	δ_4	–	–

Based on the calculated characteristic values of trajectory sections, it is possible to determine spatial coordinates of intermediate points. To do this, the algorithm (see Figure 4) with the following assumptions should be used:

- global left–handed Cartesian coordinate system with the origin at the entry point of the wellbore. Axis orientation: OX—east geographical, OY—north geographical, OZ—vertical
- specified azimuth of the plane (β) constant for the whole trajectory ($\beta_L = \beta$)

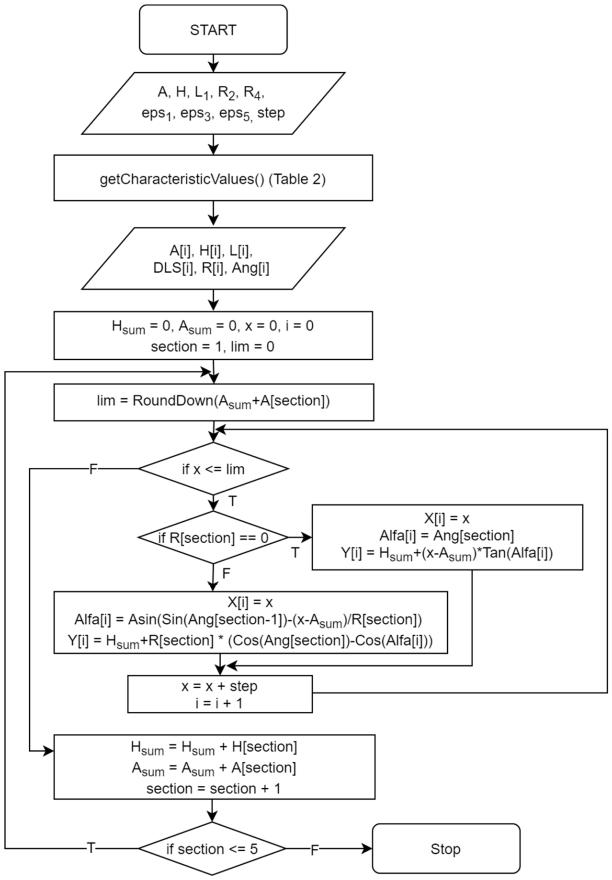

Figure 4. Simplified algorithm for determining the spatial coordinates of trajectories being a combination of straight and curvilinear sections.

The value of the angle of deviation from the horizontal plane ε_L tangent to the trajectory at point P_L lying between the characteristic points P_{j-1} and P_j at the distance L_L, measured from the beginning of the wellbore:

- rectilinear section $\varepsilon_j = \varepsilon_{j-1}$

$$\varepsilon_L = \varepsilon_j \,, \tag{15}$$

- curvilinear section $\varepsilon_j > \varepsilon_{j-1}$

$$\varepsilon_L = \varepsilon_{j-1} + (L_L - \sum_{i=1}^{j-1} L_i) \cdot DLS_j \,, \tag{16}$$

Spatial coordinates of point PL lying between characteristic points P_{j-1} and P_j:

- rectilinear section $\varepsilon_j = \varepsilon_{j-1}$

$$X_L = \left(\sum_{i=1}^{j-1} A_i + \left(L_L - \sum_{i=1}^{j-1} L_i\right) \cdot \cos \varepsilon_j\right) \cdot \sin \beta \,, \tag{17}$$

$$Y_L = \left(\sum_{i=1}^{j-1} A_i + \left(L_L - \sum_{i=1}^{j-1} L_i\right)\cdot\cos\varepsilon_j\right)\cdot\cos\beta\ , \tag{18}$$

$$Z_L = \sum_{i=1}^{j-1} H_i + \left(L_L - \sum_{i=1}^{j-1} L_i\right)\cdot\sin\varepsilon_j\ , \tag{19}$$

- curvilinear section $\varepsilon_j > \varepsilon_{j-1}$

$$X_L = \left\{\sum_{i=1}^{j-1} A_i + R_j\cdot\left[\sin\left(\varepsilon_{j-1} + \left(L_L - \sum_{i=1}^{j-1} L_i\right)\cdot DLS_j\right) - \sin\varepsilon_{j-1}\right]\right\}, \tag{20}$$

$$Y_L = \left\{\sum_{i=1}^{j-1} A_i + R_j\cdot\left[\sin\left(\varepsilon_{j-1} + \left(L_L - \sum_{i=1}^{j-1} L_i\right)\cdot DLS_j\right) - \sin\varepsilon_{j-1}\right]\right\}, \tag{21}$$

$$Z_L = \left\{\sum_{i=1}^{j-1} H_i + R_j\cdot\left[\cos\varepsilon_{j-1} - \cos\left(\varepsilon_{j-1} + \left(L_L - \sum_{i=1}^{j-1} L_i\right)\cdot DLS_j\right)\right]\right\}, \tag{22}$$

The most important steps of the algorithm determining the trajectory being a combination of straight and curvilinear sections:

1. Determination of input data (A, H, L_1, R_2, R_4, ε_1, ε_3, ε_5) and a calculation step.
2. Calculation of the characteristic points according to the steps in Table 2.
3. Transfer of calculation results from point 2 and definition of additional variables.
4. Beginning of the iteration block associated with the hole section number.
5. Calculation of the current abscissa value rounded down to an integer.
6. Beginning of the nested iteration block. Check if variable X has not exceeded the range, if so, go to point 9.
7. Calculation of spatial coordinates X, Y and the alpha angle depending on the type of section.
8. Incrementation of loop variables and return to point 6.
9. Add to H_{sum} and A_{sum} values corresponding to a given section. Increment the section.
10. Check if the section scope has not been exceeded, if not, return to step 4.
11. End of algorithm.

2.2. Chain Curve Trajectory (Catenary)

The second concept for designing Horizontal Directional Drilling trajectories is related to the natural deflection of the casing described by a chain curve (see Figure 5) [18,20]. A catenary curve is a natural curve that a cable, chain, or any other line of uniform weight assumes when suspended between two points. A similar suspension of a drill string would also form a catenary curve. A common example is a curve formed by a telephone line hanging between two utility poles. The catenary method was first introduced by R.T. McClendon and E.O. Anders [21]. In the oil and gas industry, the catenary method is often used to perform extended–reach wells. The application of this method has been described in detail in the literature [22–24]. The authors point out the benefits of casing natural deflection.

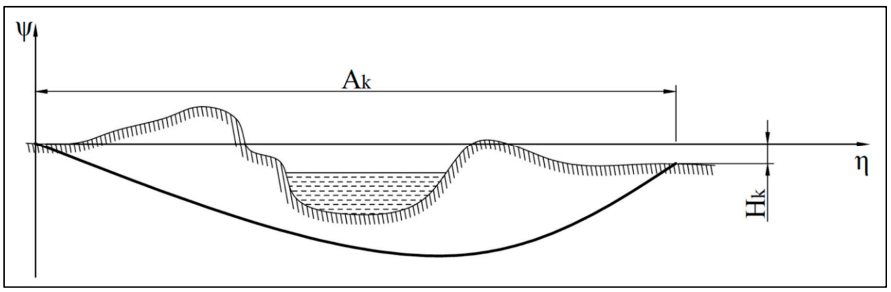

Figure 5. Chain curve trajectory.

As casings used in HDD have diameters from several centimeters to a meter, and their length is up to over a thousand meters, it can be stated that the length of a pipe column is disproportionately larger than its diameter, which is why such a construction can be treated as a rope hanging between two poles.

In the model adopted in this way, there may be slight deviations associated with the assumption that the transverse bending and torsional stiffness is negligible in relation to longitudinal stiffness. Figure 6 shows a fragment of a free–hanging column of casing. In order to determine the relationship between the unit weight of the casing and its forces, section AB of the ds length, which is in equilibrium, is considered.

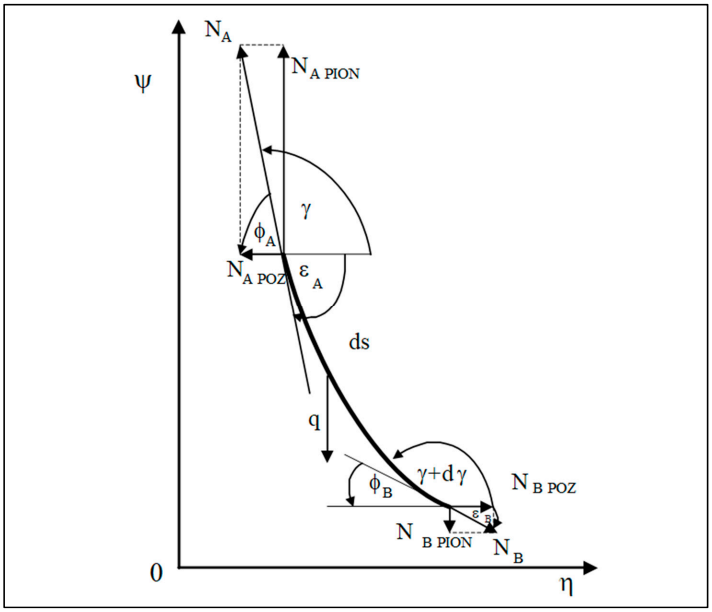

Figure 6. Geometric dependencies for the section of 9a free hanging casing [20].

General dependencies:

Assuming that the casing is not susceptible to longitudinal deformations and its unit weight q is constant over the entire length, the chain curve equation in the system from Figure 6 is as follows:

$$\psi = \frac{N_{poz}}{q}\cosh\left(\frac{q}{N_{poz}}(\eta + C_1)\right) - \frac{N_{poz}}{q}\cosh\left(\frac{q}{N_{poz}}(C_1)\right), \quad (23)$$

The relationship between the value of force N_{poz} and the constant C_1 can be determined from the formula (23), using the condition: f(AK) = H_k (see Figure 6), which gives:

$$\frac{q}{N_{poz}}H_k = \cosh\left(\frac{q}{N_{poz}}(A_k + C_1)\right) - \cosh\left(\frac{q}{N_{poz}}(C_1)\right), \quad (24)$$

Assuming the value of force N_{poz} equal to the pulling force of the drilling device, it is possible to determine the value of constant C_1. Equation (24) is an implicit equation of variable N_{poz}. This type equation can be solved using numerical methods, e.g. bisection methods. The equation transforms into the form:

$$g(x) = \cosh\left(\frac{q}{N_{poz}}(A_k + x)\right) - \cosh\left(\frac{q}{N_{poz}}(x)\right) - \frac{q}{N_{poz}}H_k, \quad (25)$$

and then the zero of the function is determined.

Using the physical sense of the derivative of function Y at point 0 and the relationship between angles γ_0 and ε_0, the formula for constant C_1 can be obtained:

$$C_1 = \frac{N_{poz}}{q} \operatorname{arcsinh}(\operatorname{tg}(\varepsilon_0)) , \tag{26}$$

To determine spatial coordinates and the angle of deviation from the horizontal plane of the trajectory described by a chain curve, the algorithm (see Figure 7) and the following assumptions should be used:

- global right–handed Cartesian coordinate system with the origin at the entry point of the wellbore. Axis orientation: OX—east geographical, OY—north geographical, OZ—vertical.
- specified azimuth of the plane (β) constant for the whole trajectory ($\beta_L = \beta$)

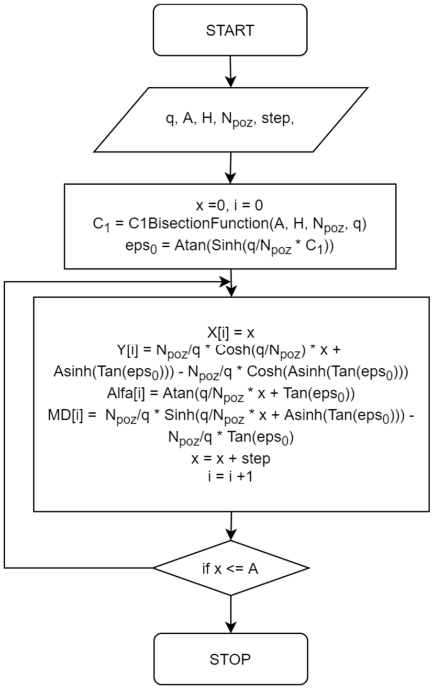

Figure 7. Simplified algorithm for calculating spatial points of the catenary trajectory.

The value of the angle of deviation from the horizontal plane ε_L tangent to the trajectory at point P_L lying at a distance L_L measured from the beginning of the wellbore:

$$\varepsilon_L = \operatorname{arctg}\left(\frac{q}{N_{poz}} L_L + \sinh\left(\frac{q}{N_{poz}} C_1\right)\right), \tag{27}$$

Total length of the trajectory ($n = A_k$):

$$L_K = \frac{N_{poz}}{q} \sinh\left(\frac{q}{N_{poz}} (A_k + C_1)\right) - \frac{N_{poz}}{q} \sinh\left(\frac{q}{N_{poz}} (C_1)\right), \tag{28}$$

The spatial coordinates of point PL with a given length step (ΔL) can be determined using the transformed equations (27) and (28):

$$\eta = \frac{N_{poz}}{q} \arcsin\left(\frac{q}{N_{poz}} L_L + \sinh\left(\frac{q}{N_{poz}} C_1\right)\right) - C_1 , \tag{29}$$

$$\psi = \frac{N_{poz}}{q}\cosh\left(\operatorname{arcsinh}\left(\frac{q}{N_{poz}}L_L + \sinh\left(\frac{q}{N_{poz}}C_1\right)\right)\right) - \frac{N_{poz}}{q}\cosh\left(\frac{q}{N_{poz}}C_1\right), \quad (30)$$

and:

$$X_L = \eta_L \cdot \sin\beta\,, \quad (31)$$

$$Y_L = \eta_L \cdot \cos\beta\,, \quad (32)$$

$$Z_L = \psi_L, \quad (33)$$

3. Calculation Example

Based on the presented equations and algorithms, an example of trajectory calculations was made.

3.1. Assumptions:

The distance between the entry point and the exit point in a horizontal projection: A = 1000 m.
Vertical displacement between the entry point and the exit point in a vertical projection: H = −15 m.
Azimuth: $\beta = 0^o$

3.1.1. Chain Curve Trajectory (Catenary):

Casing weight: 500 N/m
Drilling rig pulling force: N_{poz} = 250000 kgf = 2451662.5 N

3.1.2. Trajectory Being a Combination of Straight and Curvilinear Sections:

Radius of the 1st curve section: R_2 = 2500 m
Radius of the 2nd curve section: R_4 = 2500 m
Entry angle: $\varepsilon_1 = -8.33°$
Exit angle: $\varepsilon_5 = 7°$
First section length: L_1 = 100 m

3.2. Results

The calculations were made using the designed computational library which implements the proposed algorithms. The simplified results were presented in tabular (see Tables 3 and 4) and graphical (see Figures 8–10) formats.

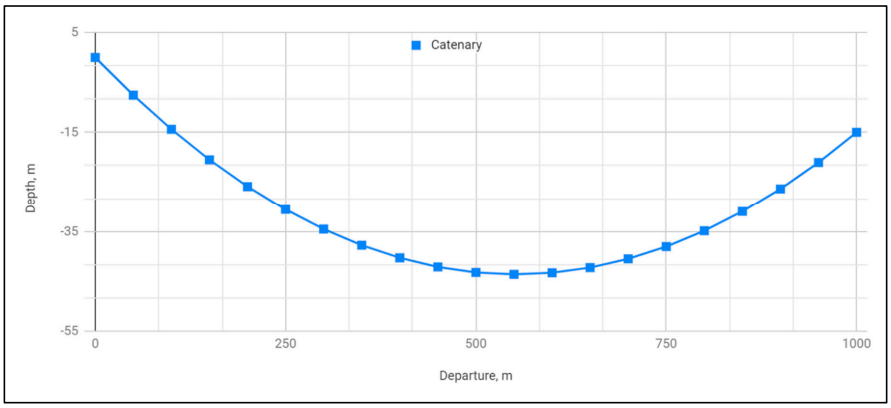

Figure 8. Catenary trajectory.

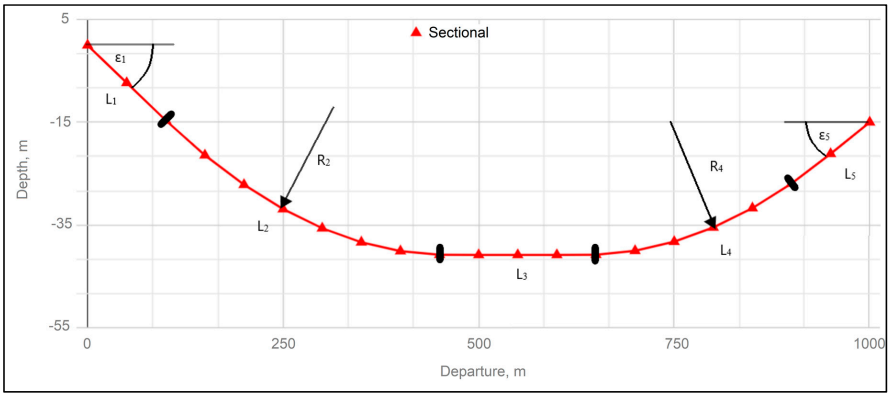

Figure 9. Trajectory being a combination of straight and curvilinear sections.

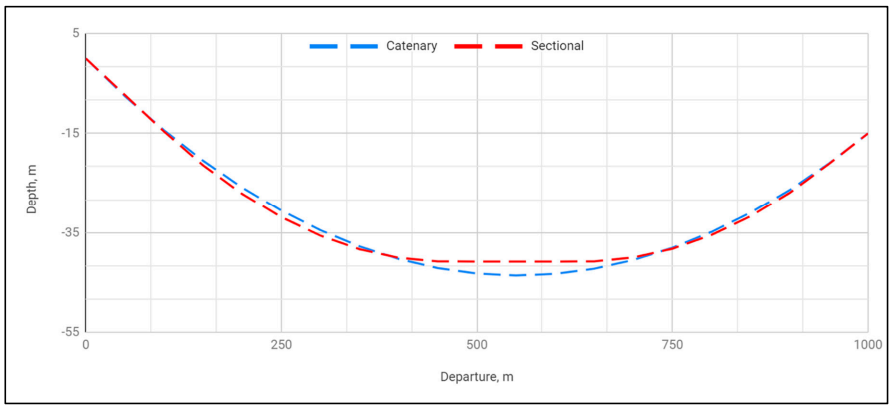

Figure 10. Comparison of the catenary trajectory and the trajectory being a combination of straight and curvilinear sections.

Table 3. Catenary trajectory results.

[i]	X	Y	ε
0	0	0	−9
1	50	−7.55	−8.2
2	100	−14.39	−7.39
3	150	−20.5	−6.59
4	200	−25.9	−5.78
5	250	−30.58	−4.97
6	300	−34.54	−4.16
7	350	−37.79	−3.34
8	400	−40.32	−2.53
9	450	−42.14	−1.71
10	500	−43.24	−0.89
11	550	−43.63	−0.08
12	600	−43.31	0.74
13	650	−42.27	1.56
14	700	−40.52	2.38
15	750	−38.06	3.19
16	800	−34.88	4.01
17	850	−30.98	4.82
18	900	−26.37	5.63
19	950	−21.05	6.44
20	1000	−15	7.24

Table 4. Results for the trajectory being a combination of straight and curvilinear sections.

[i]	X	Y	ε
0	0	0	−8.33
1	50	−7.32	−8.33
2	100	−14.64	−8.3
3	150	−21.42	−7.15
4	200	−27.17	−5.99
5	250	−31.91	−4.84
6	300	−35.64	−3.69
7	350	−38.37	−2.54
8	400	−40.09	−1.4
9	450	−40.81	−0.25
10	500	−40.83	0
11	550	−40.83	0
12	600	−40.83	0
13	650	−40.8	0.3
14	700	−40.03	1.45
15	750	−38.26	2.6
16	800	−35.49	3.75
17	850	−31.72	4.89
18	900	−26.93	6.05
19	950	−21.14	7
20	1000	−15	7

4. Conclusions

1. HDD is a dynamically developing technology for constructing underground pipelines This is due to the fact that this type of drilling can be performed in urban and hard–to–reach areas. Horizontal directional drilling is an alternative to microtunneling, direct pipe and jacking. At the same time, the HDD technology enables minimization of a negative impact on the environment compared to conventional methods.
2. A very attractive alternative to standard design solutions is the concept of a chain curve trajectory (catenary), which enables easier insertion of the pipeline into a wellbore and ensures its longer life due to the natural stress distribution along the length.
3. The algorithms developed by the authors of the article at the AGH University of Science and Technology at the Faculty of Drilling Oil and Gas should be used in practice, as they reduce costs, failures, and complications during well drilling using the HDD technology.
4. Based on given equations and project methodologies, the authors see the potential for expanding the topic in the area of trajectories optimization that will result in a trajectory that combines advantages of both design conceptions.

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Nomenclature

A_j	projection of the jth section on a horizontal plane	[m]
β	azimuth	[°]
DLS_j	dogleg severity	[°/m]
δ_j	angle of curvature at the jth point of the hole	[°]
ε_j	angle of deviation from horizontal plane at the jth characteristic point	[°]
H_j	projection of the jth section/segment on a vertical plane	[m]
L_j	length of jth section/segment	[m]
N_{poz}	drilling rig pulling force	[N]

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