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A Review and Evaluation of Predictive Models for Thermal Conductivity of Sands at Full Water Content Range

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Abstract: The effective thermal conductivity (λ_{eff}) of sands is a critical parameter required by applications in geothermal energy resources, geo-technique and geo-environment and in science disciplines. However, the availability of the reliable λ_{eff} data is not sufficient and predictive models are usually used in practice to estimate λ_{eff} . These predictive models may vary in complexity, flexibility, accuracy and applications. There is no universal model that can be applied to all soil types and full water content range. The choice of different models may result in distinctive estimates of λ_{eff} . The objectives of this study were to conduct an extensive review of the thermal conductivity models of sands and evaluate their performance with a large dataset consisting of various sand types from dry to saturation. A total of 14 models to predict λ_{eff} of sands were evaluated with a large compiled dataset consisting of 1025 measurements on 62 sands from 20 studies. The results show that the models of Chen 2008 (CS2008) and Zhang et al. 2016 (ZN2016) give the best estimates of thermal conductivity of sands, with Nash–Sutcliffe efficiency = 0.9 and $RMSE = 0.3 \text{ W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$. These two models are potentially applied to accurately estimate thermal conductivity of sands of different types.

Keywords: soil thermal conductivity models; sands; model evaluation; transient heat pulse method

1. Introduction

The effective thermal conductivity (λ_{eff} , $\text{W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$) of sands is of great interest because it is required by petroleum or natural gas production from underground oil- or methane-bearing sands [1,2]. It is also the critical parameter to determine heat exchange between the surrounding environments and the borehole heat exchanger [3–6], nuclear waste disposal [7] or energy storage [8], buried power cables and steam/hot water pipelines [9], and other thermo-active ground structures (e.g., energy pile and tunnel linings). The value of λ_{eff} is often an input for calculating the ground thermal regimes or land-atmosphere energy balance and associated soil physical, chemical and biological processes [10,11].

The values of λ_{eff} can be either measured or indirectly predicted with mathematic models. The available λ_{eff} data of sands are not sufficient to meet the demands, although great progress has been achieved in the measurement technologies of λ_{eff} [12–14]. In addition, reliable λ_{eff} data can only be measured by the transient method (e.g., heat pulse method) at full water content range, because the transient method is less likely to be affected by the water redistribution or phase change compared

to the steady-state method (e.g., guarded hot plate). On the other hand, modeling of λ_{eff} is a highly thriving field, because there is no single model for universal applications. Previous studies generally applied models of other applications to estimate λ_{eff} of sands, while some developed predictive models for sands but based on a small size dataset [15,16] or unreliable measurements [17]. The complexity, flexibility, accuracy and applications of the predictive models largely determine the accuracy of their applications [18,19]. Therefore, it is critical to evaluate the performance of the predictive models for sands for accurate applications in engineering and science.

The objectives of this study were: (1) to conduct an extensive review of the predictive thermal conductivity models of sands; (2) to compile a large and reliable dataset consisting of measurements on various sands at full water content range with the transient heat pulse methods; and (3) to evaluate the performance of the collated models with the compiled dataset.

2. A Review of STC Models for Sands

2.1. Overview of Soil Thermal Conductivity Models for Sands

An extensive literature search gives 14 models that were originally developed to predict thermal conductivity of sands. The 14 models are divided into three categories, including six mixing or analytical models, four models based on the normalized concept and four linear/non-linear regression models. The models of Woodside and Messmer [1] (WM1961I, WM1961II) and Tarnawski and Leong [20] (TL2016) are original or modified geometric mean models and that of Lu et al. [21] (LY2018) is a mixed series and parallel model. The Haigh [16] (HS2012) and Rubin and Ho [22] (RH2018) models are physically based models. The Ewen and Thomas [23] (ET1987), Zhang et al. [24] (ZN2015), Zhang et al. [25] (ZN2016) and Zhao et al. [26] (ZY2019) models are normalized thermal conductivity models. The Becker et al. [27] (BB1992), Chen [15] (CS2008), Li et al. [28] (LY2012) and Alrtimi et al. [17] (AA2016) models were developed empirically.

2.2. Analytical or Mixing Models

2.2.1. Woodside and Messmer 1961 Model (WM1961I)

Woodside and Messmer [1] presented the classical weighted geometric mean model to calculate λ_{eff} of 2-phase saturated sands. The geometric mean model was extended for a three-phase soil system [29,30]

$$\lambda_{eff} = \lambda_s^{1-P} \lambda_w^\theta \lambda_{air}^{P-\theta} \quad (1)$$

where P is porosity ($\text{cm}^3 \text{cm}^{-3}$), θ is water content ($\text{cm}^3 \text{cm}^{-3}$), λ_s , λ_w and λ_{air} are thermal conductivity of soil solids, water and air ($\text{W m}^{-1} \text{ }^\circ\text{C}^{-1}$). $\lambda_w = 0.56 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ and $\lambda_{air} = 0.025 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ are assumed.

2.2.2. Woodside and Messmer 1961 Model (WM1961II)

Woodside and Messmer [1] also presented another model similar to the geometric mean model [31–33]

$$\lambda_{eff} = \left[(1-P) \sqrt{\lambda_s} + \theta \sqrt{\lambda_w} + (P-\theta) \sqrt{\lambda_{air}} \right]^2 \quad (2)$$

2.2.3. Tarnawski and Leong 2016 Model (TL2016)

Kiyohashi and Deguchi [30] were among the first to extend Equation (1) of Woodside and Messmer [1] to unsaturated solid rocks (e.g., sandstones, zeolites, tuff, etc.) with $0.05 < n < 0.51$ (with accuracy of $\lambda_{eff} = \pm 0.20 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$)

$$\lambda_{eff} = [1/(1-P)]^{1-S_r} [(1-P)\lambda_s + P\lambda_w]^{1-P} \lambda_w^{PS_r} \lambda_{air}^{P(1-S_r)} \quad (3)$$

Tarnawski and Leong [20] found Equation (3) overestimates λ_{eff} of unsaturated soils due to the model is lack of physical characteristics for unsaturated soil applications. They introduced the inter-particle thermal contact resistance factor (α)

$$\lambda_{eff} = (\alpha \lambda_s)^{1-P} \lambda_w^{PS_r} \lambda_{air}^{P(1-S_r)} \quad (4)$$

where α is expressed as [20,34,35]

$$\alpha = \begin{cases} [\varepsilon + (1 - \varepsilon) \lambda_s / \lambda_{air}]^{-1} & 0 < S_r \leq S_{r-cr} \quad (a) \\ 1 & S_{r-cr} < S_r \leq 1 \quad (b) \end{cases} \quad (5)$$

where S_{r-cr} is the degree of saturation of a miniscule pore space, $S_{r-cr} = (0.22f_{clay} + 0.01)/P$ [20,36], ε is a dimensionless interface contact coefficient ($0 \leq \varepsilon \leq 1$, $\varepsilon = 1$ indicates an ideal contact between soil particles). ε depends on soil compaction, soil specific surface area, and grain size distribution. Tarnawski and Leong [20] obtained ε value by reverse modeling of experimental λ_{eff} data of 40 Canadian soils; the resulting ε varied between 0.988 and 0.994 for coarse and fine soils, respectively. For dry soils, $\alpha \approx 1$ due to the highest λ reduction arising from the thermal contact resistance.

2.2.4. Haigh 2012 Model (HS2012)

Based on dataset of Chen [15], Haigh [16] developed an analytical model based on unidirectional heat flow through a three-phase soil system [17,37],

$$\begin{aligned} \lambda_{eff} = & \frac{2F\lambda'_w\lambda_s(1+a)^2}{(1-\lambda'_w)^2} \ln \left[\frac{(1+a) + (\lambda'_w - 1)b}{a + \lambda'_w} \right] \\ & + \frac{2F\lambda'_s\lambda'_{air}(1+a)^2}{(1-\lambda'_{air})^2} \ln \left[\frac{(1+a)}{(1+a) + (\lambda'_{air} - 1)b} \right] \\ & + \frac{2F\lambda_s(1+a)}{(1-\lambda'_w)(1-\lambda'_{air})} [(\lambda'_w - \lambda'_{air})b - (1 - \lambda'_{air})\lambda'_w] \end{aligned} \quad (6)$$

where $F = 1.58$ is a correction factor, λ'_w and λ'_{air} are thermal conductivity of water (λ_w , $W m^{-1} ^\circ C^{-1}$) and air (λ_{air} , $W m^{-1} ^\circ C^{-1}$), respectively, that are divided by λ_s

$$\lambda'_w = \lambda_w / \lambda_s, \quad \lambda'_{air} = \lambda_{air} / \lambda_s \quad (7)$$

where coefficients a and b are related to the thickness of water film and degree of saturation

$$a = (2e - 1) / 3 \quad (8)$$

$$b = \left(\frac{1+a}{2} \right) (1 + \cos A - \sqrt{3} \sin A) \quad (9)$$

$$\cos 3A = [2(1 + 3a)(1 - S_r) - (1 + a)^3] / (1 + a)^2 \quad (10)$$

where e is the void ratio, $e = P / (1 - P)$. The left side of Equation (10) can be simplified as $\cos 3A = 4(\cos A)^3 - 3\cos A$, and $(\cos A)^2 + (\sin A)^2 = 1$.

2.2.5. Lu Et Al. 2018 Model (LY2018)

Lu et al. [21] developed a parallel-series mixed model for predicting λ_{eff} of Aeolian sand on the Qinghai-Tibet Plateau

$$\lambda_{eff} = \begin{cases} (\phi_s \lambda_s + \phi_{air} \lambda_{air} + \phi_w \lambda_w)^a (\phi_s / \lambda_s + \phi_{air} / \lambda_{air} + \phi_w / \lambda_w)^{a-1} + b S_r & T \geq 0 \quad (a) \\ (\phi_s \lambda_s + \phi_{air} \lambda_{air} + \phi_{ice} \lambda_{ice})^a (\phi_s / \lambda_s + \phi_{air} / \lambda_{air} + \phi_w / \lambda_w)^{a-1} + b S_r & T < 0 \quad (b) \end{cases} \quad (11)$$

where a and b are fitting parameters. Lu et al. [21] recommended $a = 0.76$ and $b = 1$ for unfrozen soils and $a = 0.72$ and $b = 2.06$ for frozen soils.

2.2.6. Rubin and Ho 2018 Model (RH2018)

Rubin and Ho [22] showed that the Haigh [16] model underestimates λ_{eff} and they provided an alternative empirical equation for the correction factor F [38]

$$F = \begin{cases} 35.787x^{0.803} & S_r < 0.1 \quad (a) \\ 1.8387x^{-0.323} & S_r > 0.1 \quad (b) \end{cases} \quad (12)$$

where x is the estimated λ_{eff} with the Haigh [16] model when $F = 1$, multiplying the λ_{eff} calculating with Haigh [16] by the F gives the corrected λ_{eff} . The R^2 for Equation (12a,b) are 0.65 and 0.87, respectively.

2.3. Normalized Models

2.3.1. Ewen and Thomas 1987 Model (ET1987)

Ewen and Thomas [23] proposed a modified Johansen [39] model

$$\lambda_{eff} = (\lambda_{sat} - \lambda_{dry})K_e + \lambda_{dry} \quad (13)$$

where λ_{sat} and λ_{dry} are thermal conductivities when soil is saturated or dry, respectively. K_e is an exponential function that is dependent on degrees of saturation. K_e is calculated by [23]

$$K_e = 1 - \exp(-\xi S_r) \quad (14)$$

where ξ is a fitted parameter, $\xi = 8.9$ was recommended by Ewen and Thomas [23] which returns $K_e \approx 1$ at $S_r = 1$. $S_r = \theta/P$.

Values of λ_{sat} and λ_{dry} are calculated by a two-phase system (solid-air or solid-water) [39]

$$\lambda_{sat} = \begin{cases} \lambda_s^{1-P} \lambda_w^P & \text{Unfrozen soil} \quad (a) \\ \lambda_s^{1-P} \lambda_w^{\theta_l} \lambda_{ice}^{P-\theta_l} & \text{Frozen soil} \quad (b) \end{cases} \quad (15)$$

where θ_l is the unfrozen water content ($\text{cm}^3 \text{ cm}^{-3}$), λ_w is the thermal conductivity of water, $\lambda_w = 0.56 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ at above-zero temperature, $\lambda_w = 0.269 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ at sub-zero temperature [40], λ_{ice} is the thermal conductivity of ice ($2.2 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$), ρ_b is the bulk density (g cm^{-3}), λ_s is the thermal conductivity of the solid that can be calculated based on volumetric fraction and thermal conductivity of quartz (λ_{quartz}) and other minerals (λ_{other}),

$$\lambda_s = \lambda_q^{f_q} \lambda_{other}^{1-f_q} \quad (16)$$

where $\lambda_q = 7.7 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ and $\lambda_{other} = 2.0 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ for $f_q > 20\%$ and $\lambda_{other} = 3$ for $f_q \leq 20\%$ at $20 \text{ }^\circ\text{C}$.

$$\lambda_{dry} = \begin{cases} (0.135\rho_b + 0.0647)/(\rho_s - 0.947\rho_b) \pm 20\% & \text{Natural soils} \quad (a) \\ 0.039P^{-2.2} \pm 25\% & \text{Crushed rock} \quad (b) \end{cases} \quad (17)$$

2.3.2. Zhang Et Al. 2015 Model (ZN2015)

Zhang et al. [24] modified the Côté and Konrad [41] model by introducing a different set of parameters with $k = 6.0$, $\chi = 8.12 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$, and $\eta = 3.28$ for quartz sand. The modified model becomes [24]

$$\lambda_{eff} = \frac{6S_r(\lambda_s^{1-P} \lambda_w^P - 8.12 \times 10^{-3.28P})}{1 + 5S_r} + 8.12 \times 10^{-3.28P} \quad (18)$$

2.3.3. Zhang Et Al. 2016 Model (Z2016)

Zhang et al. [25] further modified the Côté and Konrad [41] model with experimental data measured on sand-kaolin clay mixtures with thermo-TDR method, the resulting formula is

$$\lambda_{eff} = \frac{[2.168 \times 10^{-5} e^{(f_q/0.07903)} + 1.252] S_r}{1 + [2.168 \times 10^{-5} e^{(f_q/0.07903)} + 0.252] S_r} \cdot \left\{ \lambda_w^P \left(\lambda_q^{f_q} \lambda_{kaolin}^{1-f_q} \right)^{1-P} - [1.216 \times 10^{-6} e^{(f_q/0.06599)} + 3.034] \times 10^{[-0.003 e^{(f_q/0.16452)} - 1.84] P} \right\} + [1.216 \times 10^{-6} e^{(f_q/0.06599)} + 3.034] \times 10^{[-0.003 e^{(f_q/0.16452)} - 1.84] P} \quad (19)$$

where λ_{kaolin} is thermal conductivity of kaolin ($2.9 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$), Kaolin is assumed to be zero in this study. Equation (16) is used to calculate λ_s .

2.3.4. Zhao Et Al. 2019 Model (ZY2019)

Zhao et al. [26] presented a closed-form thermal conductivity model for mixtures of sands and peat materials

$$\lambda_{eff} = A + B \ln(1 + S_r) \quad (20)$$

where A and B are parameters related to soil properties (composition, texture and structure)—they can be directly derived from the lower and upper water saturations. $\lambda_{eff} = \lambda_{dry} = A$ when $S_r = 0$ and $B = (\lambda_{sat} - \lambda_{dry})/\ln 2$ when $S_r = 1$. Equation (20) can be converted to the normalized form as $Ke = \log_2(1 + S_r)$

$$\lambda_{eff} = \lambda_{dry} + (\lambda_{sat} - \lambda_{dry}) \log_2(1 + S_r) \quad (21)$$

where Equations (15a), (16) and (17a) are retained for calculation of λ_{sat} , λ_s and λ_{dry} , respectively.

2.4. Linear or Nonlinear Regression Models

2.4.1. Becker 1992 Model (BB1992)

Becker et al. [27] developed an unified methodology for predicting λ_{eff} of gravel, sand, silt, clay and peat in both frozen and unfrozen states using literature data [42]

$$S_r = 0.01 a_1 [\sinh(6.9348 a_2 \cdot \lambda_{eff} + a_3) - \sinh(a_4)] \quad (22)$$

or

$$\lambda_{eff} = UC \left\{ \log \left[(100 S_r / a_1 + \sinh(a_4)) + \sqrt{(100 S_r / a_1 + \sinh(a_4))^2 + 1} \right] - a_3 \right\} / a_2 \quad (23)$$

where $UC = 0.1442$ is used to convert the SI unit ($\text{W m}^{-1} \text{ }^\circ\text{C}^{-1}$) back to “Btu in. ft⁻² h⁻¹ °F⁻¹” for consistence with the original study. a_1 to a_4 are coefficients depending on soil type. For unfrozen sand, a_1 to a_4 are 6.8, 0.4, -2.9 and -1.5, respectively.

2.4.2. Chen 2008 Model (CS2008)

Chen [15] proposed a simple empirical relationship for sandy soils

$$\lambda_{eff} = \lambda_s^{1-P} \lambda_w^P [(1-b) S_r + b]^{c \cdot P} \quad (24)$$

where $b = 0.0022$ and $c = 0.78$ are the fitting parameters obtained using DPS code with $R^2 = 0.996$, λ_s is calculated with Equation (16).

2.4.3. Li Et Al. 2012 Model (LY2012)

Li et al. [28] developed an empirical relationship for powdery sand while test soil thermal conductivity along a crude oil pipeline in tropical desert and grassland in southwest Sahara Desert in West Africa,

$$\lambda_{eff} = 0.337 + 0.301 \ln(100\theta + 0.5) \quad (25)$$

2.4.4. Alrtimi Et Al. 2016 Model (AA2016)

Alrtimi et al. [17] developed a new logarithmic expression for sands:

$$\lambda_{eff} = \begin{cases} 1.025\rho_b - 1.065 & \theta = 0 \quad (a) \\ (1 - P) \ln(\theta/\rho_b) - 7.75P + 6.83 & \theta > 0 \quad (b) \end{cases} \quad (26)$$

3. Data Compilation and Analysis

3.1. Experimental Data of Thermal Conductivity of Sand

A large dataset was collated by digitalizing literature data or by collecting through personal communications following several important criteria: (1) soil must be sands with zero or a negligibly small amount of organic matter. Full dataset of Chen [15] was included because it has been used by a few researchers to compare, develop, or test new models [16,18,22,38], additional soils contain sand content $\sim >90\%$; (2) λ_{eff} were measured on soil samples with the transient method (e.g., single/dual-probe heat pulse method, thermo-TDR) at room temperature; (3) sufficient sample size and a wide range of water contents or degrees of saturation (e.g., >5) and bulk densities should be provided for useful evaluation; and (4) detailed description of the sample preparation and complete soil information such as grain size distribution (f_{sand} , f_{silt} , and f_{clay}), P , ρ_b , and ρ_s should be available. The values of λ_s and f_q that were measured or recommended by original research were used, otherwise $f_q = f_{sand}$ is assumed and missed λ_s was calculated by Equation (16).

This screen procedure ended up with 1025 measurements on 62 sands from 20 studies [9,15,23,25,26,36,43–57], which are all published data. These data were mainly measured under controlled laboratory conditions to maintain high quality. The sand samples of this compiled dataset were from different regions around the world, including Germany [43], the UK [23], Greece [52], Australia [44], the USA [25,36,45,51,57], Canada [26,53–56], China [15,46], Japan [47,50] and South Korea [9]. Selected soil physical properties can be found in Table 1. Detailed information can be found in the respective studies.

Table 1. Selected Physical Properties of Soils for Model Evaluation.

Soil NO.	Literature Source	Soil Name	No of Meas.	Soil Texture ¹			f_q	λ_s (W·m ⁻¹ ·C ⁻¹)	ρ_s	ρ_b	P	θ cm ³ cm ⁻³	S_r
				Sand	Silt	Clay			g cm ⁻³				
1	Bachmann et al. [43]	Wettable quartz sand	24	1.00	0.00	0.00	1.00	7.66	2.65	1.41~1.70	0.36~0.47	0~0.42	0~1
2	Bachmann et al. [43]	Reellent quartz sand	24	1.00	0.00	0.00	1.00	7.66	2.65	1.46~1.67	0.37~0.45	0~0.45	0~1
3	Bachmann et al. [43]	Wettable humic soil	15	0.91	0.08	0.02	0.91	6.79	2.65	1.39~1.62	0.39~0.48	0~0.33	0~0.85
4	Bachmann et al. [43]	Wettable humic soil	15	0.92	0.06	0.02	0.92	6.89	2.65	1.3, 1.35	0.49~0.51	0~0.39	0~0.76
5	Barry-Macaulay et al. [44]	Brighton Group sand	14	0.97	0.03	0.00	0.92	6.91	2.61	1.22~1.80	0.30~0.53	0~0.30	0~0.97
6	Cass et al. [45]	Lysimeter sand	90	0.91	0.07	0.02	0.00	3.00	2.82	1.55	0.45	0~0.19	0~0.42
7	Chen [15]	Sand A	20	0.68	0.27	0.05	1.00	7.50	2.65	1.35~1.60	0.40~0.49	0~0.49	0~1
8	Chen [15]	Sand B	20	0.94	0.06	0.00	1.00	7.50	2.65	1.20~1.50	0.43~0.55	0~0.55	0~1
9	Chen [15]	Sand C	20	0.17	0.59	0.24	1.00	7.50	2.65	1.20~1.50	0.43~0.55	0~0.55	0~1
10	Chen [15]	Sand D	20	0.59	0.28	0.13	1.00	7.50	2.65	1.40~1.71	0.35~0.47	0~0.47	0~1
11	Chen et al. [46]	BJ0~10 (1)	9	0.87	0.12	0.01	0.44	6.21	2.53	1.41	0.44	0.04~0.44	0~1
12	Chen et al. [46]	BJ10~20 (1)	9	0.94	0.06	0.00	0.47	6.93	2.55	1.71	0.33	0~0.33	0~1
13	Chen et al. [46]	BJ20~30 (1)	9	0.92	0.07	0.00	0.46	6.79	2.42	1.53	0.37	0~0.37	0~1
14	Chen et al. [46]	BJ30~40 (1)	9	0.84	0.15	0.01	0.42	6.02	2.50	1.56	0.38	0~0.38	0~1
15	Chen et al. [46]	BJ40~50 (1)	9	0.90	0.10	0.00	0.45	6.69	2.58	1.69	0.35	0~0.35	0~1
16	Chen et al. [46]	BJ0~10 (2)	8	0.87	0.12	0.01	0.44	6.21	2.53	1.41	0.44	0~0.44	0~1
17	Chen et al. [46]	BJ10~20 (2)	8	0.94	0.06	0.00	0.47	6.93	2.51	1.67	0.33	0~0.33	0~1
18	Chen et al. [46]	BJ20~30 (2)	8	0.92	0.07	0.00	0.46	6.79	2.42	1.53	0.37	0~0.35	0~0.95
19	Chen et al. [46]	Dongsu0~10	8	0.92	0.08	0.00	0.46	6.87	2.57	1.67	0.35	0~0.34	0~0.98
20	Chen et al. [46]	Dongsu10~20	8	0.92	0.08	0.00	0.46	6.83	2.69	1.64	0.39	0~0.39	0~1
21	Chen et al. [46]	Dongsu20~30	8	0.93	0.07	0.00	0.47	6.95	2.34	1.52	0.35	0~0.35	0~0.99
22	Chen et al. [46]	Dongsu30~40	8	0.94	0.06	0.00	0.47	7.09	2.29	1.47	0.36	0~0.36	0~1
23	Chen et al. [46]	Dongsu0~10 (2)	8	0.92	0.08	0.00	0.46	6.95	2.55	1.61	0.37	0~0.37	0~1
24	Chen et al. [46]	Dongsu10~20 (2)	8	0.92	0.08	0.00	0.46	6.90	2.65	1.85	0.30	0~0.30	0~1
25	Chen et al. [46]	Dongsu20~30 (2)	8	0.93	0.07	0.00	0.47	7.01	2.64	1.78	0.33	0~0.33	0~1
26	Chen et al. [46]	Dongsu30~40 (2)	8	0.94	0.06	0.00	0.47	7.14	2.40	1.57	0.34	0~0.34	0~1
27	Ewen and Thomas [23]	Leighton Buzzard sand	22	1.00	0.00	0.00	0.96	7.30	2.70	1.50~1.76	0.35~0.44	0~0.35	0~1
28	Kasubuchi et al. [47]	Toyoura sand	7	1.00	0.00	0.00	0.87	5.26	2.70	1.62	0.40	0~0.40	0~1
29	Lu et al. [48]	S-001	10	0.94	0.01	0.05	0.74	5.21	2.71	1.60	0.41	0~0.41	0~0.99
30	Lu et al. [48]	S-008	14	0.93	0.01	0.06	0.51	4.06	2.71	1.60	0.41	0~0.43	0~1
31	Lu et al. [48]	S-010	10	0.92	0.07	0.01	0.74	5.21	2.71	1.58	0.42	0~0.37	0~0.90
32	Lu et al. [49]	S-001	10	0.94	0.01	0.05	0.74	5.21	2.71	1.50	0.45	0~0.30	0~0.66
33	McInnes [36]	Quincy	8	0.95	0.03	0.02	0.63	4.80	2.65	1.50	0.43	0~0.15	0~0.35
34	Mochizuki et al. [50]	Tottori dune sand	28	0.92	0.05	0.03	0.52	2.53	2.67	1.55	0.42	0~0.42	0~1
35	Park and Hartley [51]	Ottawa sand	39	1.00	0.00	0.00	1.00	7.70	2.65	0.95~1.75	0.32~0.64	0~0.64	0~1
36	Park and Hartley [51]	Masonry silica sand	10	1.00	0.00	0.00	1.00	7.70	2.65	1.00~1.64	0.38~0.62	0~0.62	0~1
37	Papadakis et al. [52]	Find sand	24	1.00	0.00	0.00	1.00	7.70	2.65	1.69	0.36	0.00	0.00
38	Sohn [9]	Silica sand	28	1.00	0.00	0.00	0.00	6.95	2.65	1.65~1.88	0.29~0.38	0~0.38	0~1
39	Sohn [9]	Quartzite sand	37	1.00	0.00	0.00	0.00	5.38	2.65	1.63~1.93	0.27~0.38	0~0.38	0~1
40	Sohn [9]	Limestone sand	36	1.00	0.00	0.00	0.00	3.09	2.74	1.68~1.99	0.27~0.39	0.27~0.39	0~1
41	Sohn [9]	Masonry sand	36	1.00	0.00	0.00	0.00	5.01	2.65	1.60~1.93	0.27~0.4	0.27~0.4	0~1
42	Tarnawski et al. [53]	C-109 (0.15~0.6 mm)	25	1.00	0.00	0.00	1.00	7.54	2.65	1.59~1.80	0.32~0.40	0.00	0.00
43	Tarnawski et al. [53]	C-190 (0.6~0.85mm)	20	1.00	0.00	0.00	1.00	7.55	2.65	1.59~1.75	0.34~0.40	0.00	0.00
44	Tarnawski et al. [53]	Toyoura sand (0.1~0.2mm)	25	1.00	0.00	0.00	0.75	5.43	2.63	1.42~1.60	0.39~0.46	0.00	0.00
45	Tarnawski et al. [54]	Ottawa sand C-109	30	1.00	0.00	0.00	1.00	7.57	2.65	1.59~1.80	0.32~0.40	0.00	0.00
46	Tarnawski et al. [54]	Toyoura sand	15	1.00	0.00	0.00	0.75	5.44	2.63	1.53~1.60	0.39~0.42	0.00	0.00
47	Tarnawski et al. [54]	Ottawa sand C-109	30	1.00	0.00	0.00	1.00	7.57	2.65	1.59~1.80	0.32~0.40	0.32~0.40	1.00
48	Tarnawski et al. [54]	Toyoura sand	15	1.00	0.00	0.00	0.75	5.44	2.63	1.53~1.60	0.39~0.42	0.39~0.42	1.00
49	Tarnawski et al. [55]	Ottawa sand C-109	16	1.00	0.00	0.00	1.00	7.70	2.65	1.59, 1.80	0.32, 0.4	0~0.4	0~1
50	Tarnawski et al. [55]	Ottawa sand C-190	16	1.00	0.00	0.00	1.00	7.70	2.65	1.59, 1.80	0.32, 0.4	0~0.4	0~1

Table 1. Cont.

Soil NO.	Literature Source	Soil Name	No of Meas.	Soil Texture ¹			f_q	λ_s (W·m ⁻¹ ·C ⁻¹)	ρ_s	ρ_b	P	θ cm ³ cm ⁻³	S_r
				Sand	Silt	Clay			g cm ⁻³				
51	Tarnawski et al. [55]	Toyoura sand	16	1.00	0.00	0.00	0.87	6.88	2.65	1.59, 1.64	0.38, 0.4	0~0.4	0~1
52	Tarnawski et al. [56]	NS-04-Sable sand	6	1.00	0.00	0.00	1.00	8.03	2.66	1.70	0.36	0~0.36	0~1
53	Tarnawski et al. [56]	QC-01-"Beach"-Sand	6	0.93	0.05	0.02	0.35	3.30	2.73	1.55	0.43	0~0.43	0~1
54	Zhang et al. [25]	Ottawa-type silica sand	15	1.00	0.00	0.00	1.00	7.50	2.65	1.55~1.65	0.36~0.40	0~0.40	0~1
55	Zhang et al. [25]	Sand-Kaolin mixture 1	20	0.95	0.00	0.05	0.95	7.15	2.65	1.55~1.70	0.34~0.41	0.34~0.41	0~1
56	Zhang et al. [25]	Sand-Kaolin mixture 2	20	0.90	0.00	0.10	0.90	6.82	2.64	1.55~1.75	0.34~0.41	0.34~0.41	0~1
57	Zhao et al. [26]	PS14 L	6	0.97	0.03	0.00	0.97	7.05	2.65	1.30	0.51	0~0.51	0~1
58	Zhao et al. [26]	PS14 H	6	0.97	0.03	0.00	0.97	7.04	2.65	1.45	0.45	0~0.46	0~1
59	Zhao et al. [26]	PS23 L	6	0.94	0.07	0.00	0.94	6.51	2.65	1.10	0.58	0~0.57	0~0.98
60	Zhao et al. [26]	PS23 H	6	0.94	0.07	0.00	0.94	6.47	2.65	1.30	0.51	0~0.53	0~1
61	Zhao et al. [26]	Sand L	6	1.00	0.00	0.00	1.00	7.66	2.65	1.45	0.45	0~0.45	0~0.99
62	Zhao et al. [26]	Sand H	6	1.00	0.00	0.00	1.00	7.65	2.65	1.60	0.40	0~0.39	0~0.98

¹ The texture class is sand (0.05 to 2 mm), silt (0.002 to 0.05 mm) and clay (<0.002 mm).

3.2. Performance Metrics

The similarity between the predicted and actual values was assessed by plotting them on a 1:1 diagram with $\pm 10\%$ line and two goodness-of-fit parameters were used, including (1) root mean square error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{j=1}^n (Y_j - \hat{Y}_j)^2}{n}} \quad (27)$$

and (2) Nash-Sutcliffe efficiency (NSE):

$$NSE = 1 - \frac{\sum_{j=1}^n (Y_j - \hat{Y}_j)^2}{\sum_{j=1}^n (Y_j - \bar{Y})^2} \quad (28)$$

where Y_j is the j^{th} measured value, $\hat{Y}_j$ is the j^{th} predicted value, $\bar{Y}$ is the mean of measured values, and n is the number of measured values. $NSE = 1$ indicates a perfect match between $\hat{Y}_j$ and Y_j ; $NSE = 0$ indicates that $\hat{Y}_j$ is as accurate as $\bar{Y}$; while $NSE < 0$ indicates that $\bar{Y}$ is a better predictor. $RMSE$ indicates the absolute difference between $\hat{Y}_j$ and Y_j —the closer the $RMSE$ value is to zero, the better the predictor.

4. Results and Discussion

4.1. Analytical or Mixing Models

The best performing model among the six thermal conductivity models of this category is TL2016 (Table 2 and Figure 1(3)) with $RMSE = 0.43 \text{ W m}^{-1} \text{ } ^\circ\text{C}^{-1}$ and $NSE = 0.79$. The HS2012 (Figure 1(4)) model retains a physical origin but it is relatively complex [17,37] and does not have a satisfactory performance on the dataset investigated. The models of TL2016 (Figure 1(3)) and WM1961I (Figure 1(1)) generally underestimate the measured λ_{eff} . The WM1961III (Figure 1(2)), LY2018 (Figure 1(5)) and RH2018 (Figure 1(6)) generally overestimate the measured λ_{eff} .

Table 2. Performance metrics of the 14 thermal conductivity models for sands.

No.	Models	Abbrev.	RMSE	NSE
1	Woodside and Messmer [1]	WM1961I	0.50	0.71
2	Woodside and Messmer [1]	WM1961II	1.53	−1.71
3	Tarnawski and Leong [20]	TL2016	0.43	0.79
4	Haigh [16]	HS2012	0.99	−0.14
5	Lu et al. [21]	LY2018	0.96	−0.06
6	Rubin and Ho [22]	RH2018	1.50	−1.58
7	Ewen and Thomas [23]	ET1987	0.42	0.80
8	Zhang et al. [24]	ZN2015	0.33	0.88
9	Zhang et al. [25]	ZN2016	0.30	0.90
10	Zhao et al. [26]	ZY2019	0.42	0.80
11	Becker et al. [27]	BB1992	0.70	0.44
12	Chen [15]	CS2008	0.30	0.90
13	Li et al. [28]	LY2012	0.71	0.41
14	Alrtimi et al. [17]	AA2016	0.69	0.45

Figure 1. Comparison of measured λ_{eff} and predicted λ_{eff} with six analytical or mixing models: (1) WM1961I [1], (2) WM1961II [1], (3) TL2016 [20], (4) HS2012 [16], (5) LY2018 [21] and (6) RH2018 [22].

The WM1961I and WM1961II were originally developed for water saturated soils with $\lambda_s/\lambda_w \approx 10$ [1]. Although the geometric mean model was extended for frozen soil study by Cosenza et al. [31] and Endrizzi et al. [32], and incorporated into the GEOtop program [33], the extension to three-phase system may not always be valid because this model lacks a physical basis for the heat conduction in granular materials [58,59]. The LY2018 model was developed based on the basic series-parallel model using a small dataset (28 measurements) of Aeolian sands from the Tibetan Plateau [21]. The RH2018 model (Figure 1(6)) is in fact the modified form of HS2012 (Figure 1(4)) model by multiplying an F factor based on Equation (12). However, it should be noted that Equation (12a) gave an unreasonably high value at $S_r < 0.1$ and only Equation (12b) was used for all degrees of saturation in this study. Rubin and Ho [22] found that the correction factor worked well for the dataset of Chen [15] and others (a total of 155 measurements), but a large variation was reported when applying this method to their own measurements (124 measurements). Therefore, it is not surprising to see that it showed a greater variation on a much larger dataset used in this study (a total of 1025 measurements).

4.2. Normalized Models

The four normalized thermal conductivity models generally performed better than the analytical or mixing models of Section 4.1. The ZN2016 (Table 2, Figure 2(3)) performed the best, with $RMSE = 0.3 \text{ W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$ and $NSE = 0.9$, followed by ZN2015 (Table 2, Figure 2(2)), with $RMSE = 0.33 \text{ W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$ and $NSE = 0.88$. It should be noted that both models, ZN2015 and ZN2016, are a modified form of the Côté and Konrad [41] model. The difference is that the ZN2015 model was developed for quartz sand while the ZN2016 was developed for a sand-clay mixture [24,25]. The normalized thermal conductivity models are among the most prevalent soil thermal conductivity models, because they generally give satisfactory estimates [19,60]. The ZN2015 and ZN2016 models extend the capability of the normalized thermal conductivity model to accurately predict λ_{eff} of sands at a high range of λ_{eff} (or water content range), but they still faced difficulties in λ_{eff} prediction at low to middle ranges of λ_{eff} . The ET1987 model (Figure 2(1)) generally overestimated the measured λ_{eff} , which agrees with the findings of other research [18,19]. The ZY2019 model (Figure 2(4)) generally slightly underestimated measured λ_{eff} , the same underestimates was reported by previous study [19]. The possible reason for the underestimates of the ZY2019 model is that it was developed for a sand-peat mixture, in which peat has a much smaller magnitude of thermal conductivity [26].

Figure 2. Comparison of measured λ_{eff} and predicted λ_{eff} with four normalized thermal conductivity models: (1) ET1987 [23], (2) ZN2015 [24], (3) ZN2016 [25] and (4) ZY2019 [26].

4.3. Linear or Non-Linear Regression Models

The CS2008 model (Figure 3(2)) is the best performing model in this category and performs equally well with model of ZN2016. The CS2008 model was developed empirically for estimating λ_{eff} of quartz sands, but it actually has a form similar to that of the geometric mean model [1,39,61]. The inclusion of λ_s may explain why it outperformed the other three empirical models (i.e., BB1992, LY2012 and AA2016) in this category. The BB1992 (Figure 3(1)) and LY2012 (Figure 3(3)) models have mixed performance, giving overestimates of λ_{eff} at ranges below $\sim 1 \text{ W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$ and underestimates of λ_{eff} at ranges greater than $\sim 1 \text{ W m}^{-1} \text{ }^{\circ}\text{C}^{-1}$. The LY2012 model was developed for powdery sand in tropical

desert and grassland in the southwest Sahara Desert in West Africa [28]—the sand characteristics may differ from sands from other parts of the world. This difference may result in the unsatisfactory performance of the LY2012 model. The AA2016 model (Figure 3(4)) generally overestimated the measured λ_{eff} as a whole, which may be attributed to the fact that AA2016 was developed based on steady-state measurements (i.e., guarded hot plates). Steady-state measurements are prone to water redistribution in unsaturated soils, which changes the thermal conductivity being measured [12,62]. This may explain why Alrtimi et al. [17] found that the models of de Vries [63], Johansen [39], Côté and Konrad [41], Lu et al. [48], Chen [15] and Haigh [16] could not be used to properly predict thermal conductivity of Tripoli sand measured with the steady-state measurements. Therefore, care should be taken when applying these models.

Figure 3. Comparison of measured λ_{eff} and predicted λ_{eff} with four linear or non-linear regression models: (1) BB1992 [27], (2) CS2008 [15], (3) LY2012 [28] and (4) AA2016 [17].

5. Conclusions

In this study, 14 models used to predict the thermal conductivity of sands were reviewed and evaluated with a large compiled dataset consisting of 1025 thermal conductivity measurements measured on 62 sands from 20 studies. The results show that most of the models either underestimated or overestimated the experimental measurements. Only the models of Chen [15] (CS2008) and Zhang et al. [25] (ZN2016) gave the best estimates, with Nash-Sutcliffe efficiency = 0.9 and RMSE = $0.3 \text{ W m}^{-1} \text{ } ^\circ\text{C}^{-1}$. These two models can potentially be used for accurately estimating the thermal conductivity of sands of different types with satisfactory performance. Although sands from various locations around the world were used in this study, care should be taken when applying the selected models for much wider applications beyond the sands tested due to the highly spatial variability in the nature of soil properties. This work would provide useful information pertaining to applications in geothermal energy resources, geo-techniques and geo-environments and in science disciplines that require the effective thermal conductivity of sands.

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