Design of a Finite-Time Terminal Sliding Mode Controller for a Nonlinear Hydro-Turbine Governing System

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Abstract: We focus on the finite-time control of a hydro-turbine governing system (HGS) in this paper. First, the nonlinear mathematical model of the hydro-turbine governing system is presented and is consistent with the actual project. Then, based on the finite-time stability theory and terminal sliding mode scheme, a new finite-time terminal sliding mode controller is designed for the hydro-turbine governing system and a detailed mathematical derivation is given. Only three vector controllers are required, which is less than the HGS equation dimensions and is easy to implement accordingly. Furthermore, numerical simulations for the proposed scheme and an existing sliding mode control are presented to verify the validity and advantage of improving transient performance. The approach proposed in this paper is simple and provides a reference for relevant hydropower systems.

Keywords: nonlinear control; finite-time control; terminal sliding mode; hydro-turbine governing system

1. Introduction

As people pay more attention to the use of renewable resources, many countries give priority to the development of hydropower, and China is no exception. In recent years, with the increase in hydropower stations, the security and stability of hydropower systems face more challenges [1–6]. A hydro-turbine governing system (HGS) is a coupling, nonlinear, and non-minimum phase system as it is an integration of a hydraulic, mechanical, and electrical system. An HGS significantly affects the stable operation of the generator and the whole power grid system [7,8]. The stability analysis and reliable control of an HGS have become a hot topic for hydraulic mechanical systems [9–13]. In practical application, due to non-linear vibration of hydroelectric generating units, external disturbances and the aging of generating equipment, stability problems of hydropower stations are common [14,15]. Therefore, it is very meaningful to design a controller for the stable operation of an HGS.

Some scholars introduced fractional calculus into the modeling of HGS and carried out dynamic analysis on it [16,17]. What is more, the linear hydro-turbine model and the first order fundamental power generator model [18–20] of an HGS have also been adopted for dynamical analysis and simulation. The linear approximation simplification of an HGS may be acceptable for small fluctuations, but it is not reasonable for all operating conditions. However, the precise model of an HGS is the basis of subsequent stability analysis and controller design. Therefore, establishing the exact nonlinear model of an HGS is very important. In this paper, the nonlinear model of an HGS is considered.
Recently, nonlinear system control has attracted the attention of many scholars. At present, most control methods of nonlinear systems are mainly asymptotically stable, requiring infinite time, theoretically, to achieve the control objectives [21–24]. However, the adequacy of the steady-state performance does not represent the transient performance quality. Particularly for the rapid transition process of an HGS, the control system requires a faster response [25–27]. Finite-time control can improve the transient characteristics of the system, including reducing the overshoot and stabilization time [28–32]. Terminal sliding mode (TSM) is an advanced finite-time control algorithm, which enables nonlinear systems to be stable in a finite-time. However, there are few results using TSM for a nonlinear HGS. Can TSM be applied to the nonlinear HGS control? If so, how can the terminal sliding surface be designed? What are the finite-time controllers and the rigorous mathematical derivations? Since these questions have yet to be answered, research in this area should be interesting and challenging.

The main contribution of this study is as follows. First, based on the finite-time stability theory and terminal sliding mode scheme, a terminal sliding surface is presented, and a novel finite-time TSM controller is designed for the HGS. Second, compared with the relevant research results, our approach needs only three vector controllers, which is one less than the HGS equation dimensions and easier to achieve. Finally, numerical simulations for the proposed TSM and an existing sliding mode control are presented to demonstrate the effectiveness and superiority of the proposed scheme. Besides, this study may provide some theoretical guidance for the stable operation of the actual hydropower station.

The remainder of the paper is organized as follows: In Section 2, the nonlinear model of the hydro-turbine governing system is presented; in Section 3, based on the finite-time control theory, a new finite-time terminal sliding mode controller is designed and the rigorous theoretical analysis is given; numerical simulations are given in Section 4; and conclusions are presented in Section 5.

2. A Nonlinear Model of HGS

The structure of the hydro-turbine governing system is shown in Figure 1.

![Figure 1. Structure of the hydro-turbine governing system.](image)

The second-order nonlinear model of the generator is given as [32]

\[
\begin{align*}
\dot{\delta} &= \omega_0 \omega \\
\dot{\omega} &= \frac{1}{J_0} [m_t - m_e - D \omega]
\end{align*}
\]

where \( \delta \) is the generator’s rotor angle, \( \omega_0 = 2\pi f_0 \) and \( f_0 = 50 \text{Hz} \), \( \dot{\omega} \) is the relative deviation of the rotational speed of the generator and \( D \) is the damping coefficient of the generator. When analyzing dynamic features of the generator, if the impact of the generator speed vibration on the torque is included in the generator damping coefficient, the electromagnetic torque and electromagnetic power are equal.

\[
m_e = P_e
\]
The terminal active power of a salient pole synchronous generator is written as

$$ P_e = \frac{E'_q V_s}{x_{d\Sigma}} \sin \delta + \frac{V_s^2}{2} \left( \frac{x'_{d\Sigma} - x_{q\Sigma}}{x_{d\Sigma} x_{q\Sigma}} \right) \sin 2\delta $$

(3)

where $E'_q$ is the transient electromotive force along the $q$ axis and $V_s$ is the infinite system bus voltage of the power system.

$$ x'_{d\Sigma} = x'_d + x_T + \frac{1}{2}x_L $$

(4)

$$ x_{q\Sigma} = x_q + x_T + \frac{1}{2}x_L $$

(5)

where $x'_d$, $x_q$, $x_T$, $x_L$ are the transient reactance along the $d$ axis, the synchronous reactance along the $q$ axis, the transformer short circuit reactance, and the transmission line reactance, respectively.

Considering that the disturbance of the hydro-turbine unit at the rated condition occurs, the main servomotor differential equation is given as

$$ \frac{dy}{dt} = (U - y) \frac{1}{T_y} $$

(6)

where $y$ is the incremental deviation of the guide vane opening, $U$ is the regulator output, and $T_y$ is the engager relay time constant.

The regulator output can be described as

$$ U = -k_p \omega - k_d \dot{\omega} - k_i \frac{\delta}{\omega_0} $$

(7)

The typical hydro-turbine system model is illustrated in Figure 2. Here, $x$ denotes the rotating speed deviation, $y$ is the incremental deviation of the guide vane opening, and $z$ represents the paddle servomotor stroke deviation, $m_1$ is the hydro-turbine output incremental torque deviation, $q$ is the incremental turbine flow deviation, and $h$ is the incremental turbine head deviation. Assume $z = 0$, $x = 0$ [33], one can easily obtain

$$ q(s) = e_{qq} y(s) $$

(8)

$$ h(s) = G_h(s)(q(s) + e_{qh}h(s)) $$

(9)

$$ \begin{cases} m_{11}(s) = h(s)e_h \\ m_{12}(s) = y(s)e_y \end{cases} $$

(10)

where $m_1(s) = m_{11}(s) + m_{12}(s)$.

![Figure 2. The hydro-turbine system model.](image-url)
From Equations (8)–(10), the transfer function between the incremental guide vane deviation \(y\) and the incremental torque deviation \(m_t\) can be written as

\[
G_t(s) = \frac{m_t(s)}{y(s)} = \frac{m_1(s) + m_2(s)}{1 + e_g G_h(s) e_y} \frac{1}{y(s)} = \frac{m_t(s)}{y(s)} = \frac{m_t_1(s)}{y(s)} + \frac{m_t_2(s)}{y(s)}
\]

(11)

where \(G_h(s)\) is the water hammer transfer function, \(e_g\) is the first-order partial derivative value of the flow rate with respect to the water head, \(e_y\) is the first-order partial derivative value of the torque with respect to the wicket gate, and \(e\) is the intermediate variable, which can be represented as

\[
e = e_g e_y e_h - e_{gh}
\]

(12)

where \(e_h\) is the partial derivative of the torque on the turbine water head and \(e_{gh}\) is the partial derivative of the flow rate of the turbine gate opening.

If the water level does not change in the transient process, and there are no head losses, it can be assumed that the penstock model is an ideal model. Then, the water hammer transfer function for the input incremental turbine flow deviation \(q\) and the output incremental turbine head deviation \(h\) can be described as [34]

\[
G_h(s) = -2h_w \tanh(0.5T_s)
\]

(13)

where \(h_w\) is the characteristic coefficient of the penstock and \(T_s\) is the length of the phase of the water hammer wave.

According to the Taylor formula, the transfer function Equation (13) can be rewritten as follows:

\[
G_h(s) = -2h_w \sum_{i=0}^{n} \frac{(0.5T_s)^{2i+1}}{(2i+1)!} \sum_{i=0}^{n} \frac{(0.5T_s)^{2i}}{(2i)!}
\]

(14)

For small and medium sized hydropower stations, the penstock is very short. Therefore, the rigid water hammer model is the most suitable model to describe the penstock system, thus, letting \(i = 0\):

\[
G_h(s) = -T_w s
\]

(15)

where \(T_w\) is the inertia time constant of the pressure water diversion system.

By substituting (15) into (11), the transfer function of the hydro-turbine can be rewritten as

\[
G_t(s) = e_y \frac{1 - eT_w s}{1 + e_{gh} T_w s}
\]

(16)

According to (6), (11), and (16), the output incremental torque deviation of the hydro-turbine can be described as

\[
m_t = \frac{1}{e_{gh} T_w} [-m_t + e_y y - \frac{e e_y T_w}{T_y} (U - y)]
\]

(17)
By combining Equations (1)–(17), the nonlinear dynamic model of the hydro-turbine governing system can be represented as

\[
\begin{aligned}
\dot{\delta} &= \omega_0 \omega \\
\dot{\omega} &= \frac{1}{m_0} \left[ m_1 - D_2 \omega - \frac{E_1 V_t}{x_0 \omega} \sin \delta + \frac{V_t^2}{2} \frac{x_2^2}{x_0^2} \omega \sin 2\delta \right] \\
\dot{m}_t &= \frac{1}{e_{yp}} \left[ -m_t + e_y y - e_{yp} T_w ( -k_p \omega - k_d \delta_{\omega} - k_i \frac{\delta}{\omega_0} - y) \right] \\
\dot{y} &= \frac{1}{T_y} \left( -k_p \omega - k_d \delta_{\omega} - k_i \frac{\delta_{\omega}}{\omega_0} - y \right)
\end{aligned}
\]

(18)

where \( \delta, \omega, m_t, y \) are dimensionless variables. According to reference [35], the parameters are selected as \( \omega_0 = 314, T_{ab} = 9.0s, D = 2.0, E_1 = 1.2, T_w = 0.8s, T_y = 0.1s, x_2 = 1.15, x_0 = 1.47, V_s = 1.0, e = 1.0, e_{yp} = 0.5, e_y = 1.0, k_p = 2.0, k_i = 1.0, \) and \( k_d = 6.5 \). Now, to facilitate the mathematical analysis, \( x, y, z, \) and \( w \) are used to take the place of \( \delta, \omega, m_t, \) and \( y \), respectively.

The time-domain of system (18) is shown in Figure 3. We can see that each state variable is unstable and in nonlinear vibration. Therefore, it is necessary to design controllers to ensure the stable operation of an HGS (18).

![Figure 3. State time domain of hydro-turbine governing system (HGS) (18). (a) \( x - t \); (b) \( y - t \); (c) \( z - t \); (d) \( w - t \).](image)

3. Controller Design

To meet the demands of controller design, first, the lemma of the finite-time stability is given.

**Lemma 1** [36]. Assume that a continuous positive definite function satisfies the following differential inequality:

\[
\dot{V}(t) \leq -c V^\eta(t), \forall t \geq t_0, V(t_0) \geq 0
\]

(19)

where \( c > 0, 0 < \eta < 1 \). For any given initial time \( t_0 \), if \( V(t) \) can meet the following inequality form:

\[
V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1 - \eta)(t - t_0), t_0 \leq t \leq t_1
\]

(20)
and \( V(t) \equiv 0, \forall t \geq t_1 \) with \( t_1 \) given by

\[
t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{21}
\]

Thus, the system can be stabilized in a finite-time \( t_1 \).

Consider the following nonlinear systems, which represent the hydro-turbine governing system \((18)\):

\[
\begin{aligned}
x_1' &= A_{11} x_1 + A_{12} x_2 \\
x_2' &= A_{21} x_1 + A_{22} x_2 + f_2(x_1, x_2) + B_2 u
\end{aligned} \tag{22}
\]

where \( x_1 \in R^m \) and \( x_2 \in R^{(n-m)} \) are the system state variables, \( m = 1, n = 4 \), and \( B_2 \in R^{(n-m) \times (n-m)} \), \( u \in R^{(n-m)} \), \( f_2(x_1, x_2) \) are the full rank matrix, control input to be designed and the nonlinear terms of the system, respectively. For convenience, system \((22)\) is written as

\[
x' = Ax + Bu + f(x) \tag{23}
\]

where

\[
A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ B_2 \end{pmatrix}, \quad f(x) = \begin{pmatrix} 0 \\ f_2(x) \end{pmatrix} \tag{24}
\]

Sat, sign, tanh, and other nonlinear functions are commonly used in the design of the sliding mode controller. The sign function is generally used to find the zero point of a function and discuss the problem of absolute value. Tanh is the information transfer function or activation function in an artificial neural network. Considering the chattering problem of the sliding mode control method, this paper chooses the saturation function with better chattering suppression effect to construct the sliding mode surface and the controller. To make the system state \( x(t) \) reach stabilization in a finite-time, the following fast terminal sliding mode switching surface is selected:

\[
s = C_1 x_1 + C_2 x_2 + C_3 sat(x_1) = Cx + C_3sat(x_1) \tag{25}
\]

where \( C_1 \in R^{(n-m) \times m}, C_2 \in R^{(n-m) \times (n-m)} \), \( C_3 \in R^{(n-m) \times m}, C = [C_1, C_2], sat(x) = \begin{cases} \text{sign}(x), & |x| > 1 \\ \text{sign}(x) \sqrt{|x|}, & |x| \leq 1 \end{cases} \), \( C_2 \) is a reversible matrix, and \( C_1 \) and \( C_3 \) are pending parameters matrices.

**Theorem 1.** If the terminal sliding mode switching surface is selected in formula \((25)\), the sliding mode control of the controller can be designed as

\[
u = -(CB)^{-1} \left[ CAx + Cf(x) + (\mu + \eta)|s|^{\beta-1})s + C_3d(sat(x_1))/dt \right] \tag{26}
\]

where \( 0 < \beta < 1, \mu, \beta > 0 \). The control law \((26)\) can make the state trajectories of system \((23)\) converge to the sliding surface \( s = 0 \) in a finite-time.

**Proof.** The Lyapunov function is selected as \( V_1 = \frac{1}{2} s^T s = \frac{1}{2}||s||^2 \), and from Equations \((23)\) and \((25)\), its first derivative with respect to time is

\[
\begin{aligned}
\dot{V}_1(t) &= \frac{1}{2} s^T \dot{s} + \frac{1}{2} s^T \dot{s} \\
&= s^T \dot{s} \\
&= s^T (CAx + Cf(x) + (\mu + \eta)|s|^{\beta-1})s + C_3d(sat(x_1))/dt
\end{aligned} \tag{27}
\]
First case: When $|x_1| > 1$, $\text{sat}(x_1) = \text{sign}(x_1), \frac{d(\text{sat}(x_1))}{dt} = 0$, therefore,

$$
\dot{V}_1(t) = s^T(C\dot{x} + C_3\frac{d(\text{sat}(x_1))}{dt})
= s^TC\dot{x}
= s^T(C(Ax + Bu + f(x))
= s^T(CAx - CAx - Cf(x) - (\mu + \eta\|s\|^{\beta-1})s + Cf(x) - C_3d(\text{sat}(x_1))/dt)
= s^T\left[ -((\mu + \eta\|s\|^{\beta-1})s \right]
= -\mu\|s\|^2 - \eta\|s\|^{\beta+1}
\leq -2^{\frac{\beta+1}{\beta+2}}\eta V_1^{\frac{\beta+1}{\beta+2}}
$$

Second case: When $|x_1| \leq 1$, $\text{sat}(x_1) = \text{sign}(x_1) \sqrt{|x_1|}$, thus,

$$
\dot{V}_1(t) = s^T(C\dot{x} + C_3\frac{d(\text{sat}(x_1))}{dt})
= s^TC(Ax + Bu + f(x)) + C_3\frac{d(\text{sat}(x_1))}{dt})
= s^T(CAx - CAx - Cf(x) - (\mu + \eta\|s\|^{\beta-1})s + Cf(x) - C_3d(\text{sat}(x_1))/dt + C_3\frac{d(\text{sat}(x_1))}{dt})
= s^T\left[ -((\mu + \eta\|s\|^{\beta-1})s \right]
= -\mu\|s\|^2 - \eta\|s\|^{\beta+1}
\leq -2^{\frac{\beta+1}{\beta+2}}\eta V_1^{\frac{\beta+1}{\beta+2}}
$$

Therefore, according to the previous lemma 1, the controller (26) can make the state trajectories converge to the sliding surface $s = 0$ in a finite-time. \(\square\)

**Theorem 2.** If there exist general pending parameter matrices $C_1$ and $C_3$ and reversible matrix $C_2$, which make the following matrix equation and inequality hold

$$
\begin{align*}
(A_{11} - A_{12}C_2^{-1}C_1) + (A_{11} - A_{12}C_2^{-1}C_1)^T &\leq 0 \\
A_{12}C_2^{-1}C_3 &\geq \Lambda
\end{align*}
$$

where $\Lambda$ is a positive definite diagonal matrix, i.e., $\Lambda = \text{diag}\{\Lambda_j(j = 1, \cdots, m)\}$, $\Lambda_j > 0$, then the system (22) can be stabilized in a finite-time under the designed controller (26).

**Proof.** On the sliding surface, $s = C_1x_1 + C_2x_2 + C_3\text{sat}(x_1) = 0$, one obtains

$$
x_2 = -C_2^{-1}[C_1x_1 + C_3\text{sign}(x_1)]
$$

Substituting Equation (31) into (22), we can transfer the equation into the sliding mode equation as follows:

$$
x_1 = A_{11}x_1 - A_{12}x_2
= (A_{11} - A_{12}C_2^{-1}C_1)x_1 - A_{12}C_2^{-1}C_3\text{sat}(x_1)
$$

The Lyapunov function is selected as $V_2 = \frac{1}{2}x_1^T x_1 = \frac{1}{2}\|x_1\|^2$ and using the sliding mode Equation (25), one can obtain

$$
\dot{V}_2(t) = \frac{1}{2}x_1^T x_1 + \frac{1}{2}x_1^T \dot{x}_1
= \frac{1}{2}x_1^T (A_{11} - A_{12}C_2^{-1}C_1) x_1 - A_{12}C_2^{-1}C_3\text{sat}(x_1)
= \frac{1}{2}x_1^T (A_{11} - A_{12}C_2^{-1}C_1) x_1 - x_1^T A_{12}C_2^{-1}C_3\text{sat}(x_1)
\leq -x_1^T A_{12}C_2^{-1}C_3\text{sat}(x_1)
$$
First case: When $|x_1| > 1$, $sat(x_1) = sign(x_1)$, therefore,

$$
\dot{V}_2(t) \leq -x_1^2 A_{12} C_2^{-1} C_3 sat(x_1) \\
= -x_1^2 A_{12} C_2^{-1} C_3 sign(x_1) \\
= -x_1^2 A_{12} C_2^{-1} C_3 \lambda \langle x_1 \rangle \\
= -\lambda (|x_1| + |x_1| + \cdots + |x_1|) \\
\leq -\lambda \sqrt{x_{11}^2 + x_{12}^2 + \cdots + x_{1m}^2} \\
= -\lambda \|x_1\| \\
= -\sqrt{2} \lambda V_2^{\frac{1}{2}}
$$

Second case: When $|x_1| \leq 1$, $sat(x_1) = sign(x_1) \sqrt{|x_1|}$, thus,

$$
\dot{V}_2(t) \leq -x_1^2 A_{12} C_2^{-1} C_3 sat(x_1) \\
= -x_1^2 A_{12} C_2^{-1} C_3 \lambda \sqrt{|x_1|} \\
= -\lambda (|x_1| + |x_1| + \cdots + |x_1|) \\
\leq -\lambda \sqrt{x_{11}^2 + x_{12}^2 + \cdots + x_{1m}^2} \\
= -\lambda \|x_1\|^2 \\
= -2^2 \lambda V_2^{\frac{2}{4}}
$$

where $\lambda = \min\{\lambda_j (j = 1, 2, \cdots, m)\}$. By Lemma 1 and Theorem 1, we know that the state variable $x_1$ will converge to stabilization in a finite-time along the sliding surface. Similarly, the state variable $x_2$ will also converge to stabilization in a finite-time. Therefore, system (22) can be stabilized in finite-time under the designed controller (26), i.e., the controller (26) can stabilize the hydro-turbine governing system (18) in a finite-time. □

4. Numerical Simulations

The controlled hydro-turbine governing system (18) can be described as

$$
\begin{align*}
\dot{x} &= 314y \\
\dot{y} &= -\frac{2}{9}y + \frac{1}{2}z - \left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) + u_1 \\
\dot{z} &= -\frac{7}{137}x + \frac{70}{9}y + \frac{125}{13}z + 16.5w - 91\left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) + u_2 \\
\dot{w} &= -\frac{5}{137}x - \frac{50}{9}y - \frac{65}{9}z - 10w + 65\left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) + u_3
\end{align*}
$$

(36)

where $u_1, u_2, u_3$ are the control input determined by the controller (26). One can obtain $A_{11} = (0)$, $A_{12} = \left(\begin{array}{cc} 314 & 0 \\ 0 & 0 \end{array}\right)$, $A_{21} = \left(\begin{array}{ccc} 0 & 0 & \frac{5}{7} \\ \frac{7}{137} & \frac{125}{13} & \frac{120}{9} \end{array}\right)$, $A_{22} = \left(\begin{array}{cc} 16.5 & 0 \\ \frac{50}{9} & \frac{65}{9} \end{array}\right)$, $f(x) = \left(\begin{array}{c} 0 \\ \frac{8}{99} \sin x + \frac{320}{30429} \sin 2x \\ -91\left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) \\ 65\left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) \end{array}\right)$. 

\[A_{21} = \left(\begin{array}{ccc} 0 & 0 & \frac{5}{7} \\ \frac{7}{137} & \frac{125}{13} & \frac{120}{9} \end{array}\right), \quad A_{22} = \left(\begin{array}{cc} 16.5 & 0 \\ \frac{50}{9} & \frac{65}{9} \end{array}\right), \quad f(x) = \left(\begin{array}{c} 0 \\ \frac{8}{99} \sin x + \frac{320}{30429} \sin 2x \\ -91\left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) \\ 65\left(\frac{8}{99} \sin x + \frac{320}{30429} \sin 2x\right) \end{array}\right)\]
and the parameter matrix in the terminal sliding surface is selected as:

\[
C = [C_1, C_2, C_3] = \begin{bmatrix}
1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]  

(37)

In addition

\[
\begin{aligned}
(A_{11} - A_{12}C_2^{-1}C_1) + (A_{11} - A_{12}C_2^{-1}C_1)^T &= -628 < 0 \\
A_{12}C_2^{-1}C_3 &= 314 > 0
\end{aligned}
\]  

(38)

The conditions of Theorem 2 are satisfied. Therefore, the terminal sliding surface can be designed as

\[
s = \begin{bmatrix}
s_1 \\
s_2 \\
s_3
\end{bmatrix} = \begin{bmatrix}
x + y + \text{sat}(x) \\
z \\
w
\end{bmatrix}
\]  

(39)

According to Theorem 1, the finite-time stabilization controllers is presented as follows:

\[
u = \begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix} = -\begin{bmatrix}
\frac{2824}{9}y + \frac{1}{9}z - \frac{8}{9} \sin x + \frac{320}{30429} \sin 2x + (\mu + \eta\|s_2\|^{\beta-1})s_1 + \\
\frac{7}{157}x + \frac{70}{9}y + \frac{137}{59}z + 16.5w - 91\left(\frac{8}{9} \sin x + \frac{320}{30429} \sin 2x\right) + \\
(\mu + \eta\|s_2\|^{\beta-1})s_2 + \\
\frac{7}{157}x + \frac{50}{9}y - \frac{65}{9}z + 65\left(\frac{8}{9} \sin x + \frac{320}{30429} \sin 2x\right) + \\
(\mu + \eta\|s_2\|^{\beta-1})s_3
\end{bmatrix}
\]  

(40)

The parameters of the controller are selected as \(\mu = \eta = 1, \beta = 0.5\), and the initial value of the system state variable is \([x, y, z, w] = [0, 0, \pi/6, 0]\).

When adding the Controller (40) at the time of 10 s, the state trajectories of the controlled HGS (18) are shown in Figure 4. It can be seen from Figure 4 that when the controller was not added, state trajectories presented irregular repeated oscillations and the system stability was destroyed. After adding the controller at 10 seconds, the system converged rapidly in a short time, showing good control performance.

Referring to the design method for the existing sliding mode control proposed in reference [37], the sliding surface is selected as

\[
S = KE - \int_0^\tau K(A - BL)E(\tau)d\tau
\]  

(41)

where the additional matrix \(K \in \mathbb{R}^{4 \times 4}\) satisfies the condition \(\det(KB) \neq 0\), and for convenience, \(K = \text{diag}(1, 1, 1, 1)\), the additional matrix \(L \in \mathbb{R}^{4 \times 4}\) and \(A - BL\) is a negative definite matrix. For the HGS (18), the proportional integral sliding surface is presented as:

\[
\begin{aligned}
S_1 &= e_1 + \int_0^\tau 5e_1(\tau)d(\tau) \\
S_2 &= e_2 + \int_0^\tau 5e_2(\tau)d(\tau) \\
S_3 &= e_3 + \int_0^\tau 5e_3(\tau)d(\tau) \\
S_4 &= e_4 + \int_0^\tau 5e_4(\tau)d(\tau)
\end{aligned}
\]  

(42)
and the control inputs are designed as follows:

\[
\mathbf{u} = \begin{pmatrix}
  u_4 \\
  u_5 \\
  u_6 \\
  u_7
\end{pmatrix} = \begin{bmatrix}
  -(5e_1 + 314e_2) - \varepsilon \text{sat}(e_1 / 0.01) \\
  \left[ \left( \frac{8}{69} \sin x + \frac{320}{30429} \sin 2x \right) + \frac{23}{7} e_1 + \frac{1}{9} e_3 \right] - \\
  \left( \varepsilon + \sqrt{\left( \frac{8}{69} \right)^2 + \left( \frac{320}{30429} \right)^2} \right) \text{sat}(e_2 / 0.01) \\
  -\left[ -91 \left( \frac{8}{69} \sin x + \frac{320}{30429} \sin 2x \right) + \frac{7}{157} e_1 + \frac{70}{9} e_2 + \frac{227}{18} e_3 + 16.5 e_4 \right] - \\
  \left( \varepsilon + 91 \sqrt{\left( \frac{8}{69} \right)^2 + \left( \frac{320}{30429} \right)^2} \right) \text{sat}(e_3 / 0.01) \\
  \left[ 65 \left( \frac{8}{69} \sin x + \frac{320}{30429} \sin 2x \right) - \frac{8}{59} e_1 - \frac{50}{9} e_2 - 65 e_3 + 5e_4 \right] - \\
  \left( \varepsilon + 65 \sqrt{\left( \frac{8}{69} \right)^2 + \left( \frac{320}{30429} \right)^2} \right) \text{sat}(e_4 / 0.01)
\end{bmatrix}.
\]

(43)

![Figure 4](image_url)

**Figure 4.** State trajectories of the HGS (18) with Controller (40). (a) \( x - t \); (b) \( y - t \); (c) \( z - t \); (d) \( w - t \).

In the following simulation, the finite-time terminal sliding mode controller (40) and the existing sliding mode controller (40) are used for the control of the HGS (18) at the time of 10 s. Under the action of the controller, the controlled variable will converge to the sliding surface and approach to the equilibrium point in a finite-time and finally, achieve stability. The corresponding simulation results of the state responses in the presence of the control law are shown in Figure 5, which shows that the controller can stabilize the HGS (18). In addition, it shows that the HGS (18) converges to stabilization in a finite-time after adding the controller (40) at time 10 s, and it is clear that the stabilization time is within 13.10 s. According to Equation (21), the calculated finite-time is \( t_1 = 13.22 \) s, which confirms the validity of the proposed method.

Additionally, after comparing the existing sliding mode with the proposed finite-time terminal sliding mode, it is clear that the stabilization time is longer for the existing sliding mode, especially in Figure 5a, and the maximum overshoot is larger as shown in Figure 5b. It can be seen from Figure 5c,d that the stability process of the proposed finite-time terminal sliding mode method is smoother without additional oscillation.
In addition, the following active controller (44) is added for a comprehensive comparison.

\[
\begin{align*}
    u_1 &= -x - 314y \\
    u_2 &= -7y - \frac{1}{5}z \\
    u_3 &= -\frac{165}{18}z - 16.5w \\
    u_4 &= 9w
\end{align*}
\]  

(44)

Corresponding simulation results of the state responses in the presence of the control law (44) are shown in Figure 6, which shows that the controller can stabilize the HGS (18). Besides, from the comparison of active control and the propose finite-time terminal sliding mode in Figure 6, we can clearly see that the maximum overshoot of active control is larger, especially in Figure 6c,d, and the stabilization time is longer.

From the above simulation results, it can be easily deduced that the proposed method can stabilize the HGS in a finite-time. The maximum overshoot is smaller, and the stabilization time is shorter, showing the validity and advantages of the proposed scheme.
5. Conclusions

In this paper, a new finite-time terminal sliding mode method based on the finite-time stability theory and the terminal sliding mode scheme was studied for the stability control of a nonlinear HGS. Numerical simulations, including the derivations for the proposed finite-time terminal sliding mode and the existing sliding mode, were presented to demonstrate the effectiveness and rapidity of the proposed scheme. The change law of the controller and its action mechanism for HGS were studied. The controller is simple and easy to implement and can be applied to other relevant hydropower systems.

Although the control method designed in this paper only uses three controllers, it still has great limitations in practical application. Therefore, we will continue to optimize the control method in later work, to realize the stable operation of the whole HGS through the control of the servomotor part.

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Nomenclature

- $\delta$: Generator rotor angle.
- $\omega$: Rotational speed relative of the generator.
- $m_t$: Hydro-turbine output incremental torque.
- $y$: Incremental of the guide vane opening.
- $T_{ab}$: Inertia time constant of the rotating part.
- $D$: Damping coefficient of the generator.
- $E'_q$: Transient electromotive force of $q$ axis.
- $V_s$: Infinite system bus voltage of the power system.
- $x'_d$: Transient reactance of $d$ axis.
- $x'_q$: Synchronous reactance of $q$ axis.
- $e_{qh}$: Transfer coefficient of turbine flow on the head.
- $T_w$: Water inertia time constant.
- $e_y$: Transfer coefficient of turbine torque on the main servomotor stroke.
- $T_y$: Transfer time of turbine servomotor stroke.
- $k_p$: Proportional adjustment coefficient.
- $k_d$: Differential adjustment coefficient.
- $k_i$: Integral adjustment coefficient.

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