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A Numerical Model for Investigating the Steam Conformance along the Dual-String Horizontal Wells in SAGD Operations

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Abstract: Non-uniformity of the steam-assisted gravity drainage (SAGD) steam chamber significantly decreases the development of heavy oil reservoirs. In this study, to investigate the steam conformance in SAGD operations, a wellbore model is developed for fluid flow in dual-string horizontal wells. Then, a three-dimensional, three-phase reservoir model is presented. Next, the coupled wellbore and reservoir model is solved with a fully implicit finite difference method. Finally, the effects of the injector wellbore configuration, steam injection ratio and injection time on the steam conformance are investigated. The results indicate that under different injector wellbore configurations, the closer the differences between the pressure drop from the landing position of the short string to the heel of the wellbore and the pressure drop from the landing position of the short string to the toe of the wellbore, the better is the steam conformance. The smaller the difference in the steam injection rate between the long and short injection strings, the higher is the uniformity of the steam chamber. The injector annular pressure profile uniformity is consistent with the steam conformance. Creating a more uniform steam pressure in the annulus of the injector improves the uniformity of the steam chamber. The steam conformance decreases with increasing injection time, so the optimization method of steam chamber uniformity should be adjusted according to different injection times.

Keywords: heavy oil; SAGD; steam conformance; dual-string; numerical model

1. Introduction

Heavy oil is an important energy source [1–4], and Steam-assisted gravity drainage (SAGD) is an effective technology for the economic recovery of heavy oil [5–8]. The SAGD process usually employs two parallel horizontal wells. The horizontal injector is closely placed above the producer, which is usually near the base of the reservoir [9–12]. In traditional SAGD well pair operations, only the heel string is used for injection and production. The pressure drop along the wellbore usually leads to a non-uniform steam chamber growth (Figure 1a), which is not conducive to the efficient development of heavy oil reservoirs [13–15]. Therefore, it is important to improve the steam conformance. In recent years, the dual-string completion scheme has been adopted in SAGD wells. Successful field testing of the SAGD process has demonstrated that the dual-string completion scheme has the potential to

improve the steam chamber conformance [16]. This completion scheme offers two injection points in the injector and two production points in the producer to reduce the pressure gradients inside the liner, which leads to improvements in steam chamber conformance in comparison to injecting steam at a single point only (Figure 1b).

Figure 1. Schematic of steam chamber distribution: (a) traditional steam-assisted gravity drainage (SAGD); (b) dual-string SAGD.

Kumar et al. [17] optimized the length and positioning of multiple tubular strings within a liner for both injection and production wells. They found that the positioning of tubular strings in a multiple tubing completion SAGD well pair had a significant impact on the steam chamber growth and productivity. Rivas et al. [18] observed that with the dual-tubing-string configuration applied in the Lindbergh area in Alberta, Canada, a more uniform steam chamber could be achieved. In 2020, Dong et al. [19] developed a physical model to experimentally study the steam conformance with a single-string well configuration and a dual-string well configuration. They found that a better steam conformance could be obtained with the two-points steam injection method. The dual-string injection and production technique is an important method for improving the steam conformance and enhancing oil recovery. However, the best way to optimize the well configuration and control the operational parameters to improve the steam conformance in SAGD operations is not fully understood.

Numerical simulation is a good method to solve these problems. Sun et al. [20] and Lin et al. [21] proposed mathematical models for calculating the steam parameters in the long string and annulus of the injector with the toe-point injection technique. Sun et al. [22] and Wu et al. [23] proposed analytical models for predicting the steam quality and steam pressure in a horizontal injection well under the condition of simultaneous dual-string injection.

In these previous models, various transition criteria for gas/liquid two-phase flow were established to determine the flow pattern, and the annulus was treated as a pipe based on the equivalent hydraulic diameter concept. However, precisely dividing flow patterns is difficult, and the flow patterns in the annulus have been shown to be different from pipe-flow patterns [24,25]. Calculation of the pressure drop may be a rough approximation [26,27]. In addition, these analytical models only simulated the steam injection performance of a single horizontal well. In other words, the coupling of a reservoir model and a wellbore model was not considered, and the injection and production performance of dual-string dual horizontal wells has not been studied.

The focus of the study is to propose a numerical model to investigate the steam conformance along the dual-string horizontal wells in SAGD operations. First, based on multiphase flow theory, a wellbore model is developed for fluid flow in the string and annulus of dual-string injection and production horizontal wells. Second, a three-dimensional, three-phase reservoir model is proposed for fluid flow in heavy oil reservoirs. Then, the coupled wellbore and reservoir model is solved with a fully implicit finite difference method. Finally, after the validation of the proposed model, the effects of the injector wellbore configuration, steam injection ratio and injection time on the steam conformance are investigated. Moreover, a uniformity evaluation model is developed to estimate the uniformity of the steam chamber, and reasonable suggestions are given to improve the steam chamber conformance.

2. Model Description

2.1. General Assumptions

Figure 2 shows a schematic of fluid flow in dual-string horizontal wells during the SAGD process. The SAGD process usually consists of two stages, the circulation stage and SAGD operation. During the SAGD circulation stage, steam is injected through both the injector and the producer (Figure 2a). When the wells are converted to SAGD operation after circulation, steam is injected down the short and long strings in the injector, and both strings in the producer are used for production (Figure 2b). For clarity, assume that interfaces A, B, C and D are at the heel, the landing position of the short string, the steam convergence position and the toe, respectively. The major assumptions in deriving the numerical model are as follows:

1. The heavy oil reservoir is isotropic and homogeneous.
2. The injection parameters (e.g., injection pressure and steam quality) at the heel of the short and long strings in the injector and the production parameters (e.g., bottomhole pressure) at the heel of the short and long strings in the producer do not change with time.
3. The heat transfer in the wellbore is steady state, while the heat transfer in the reservoir is transient.
4. The flow of oil, water and steam in the reservoir obeys Darcy's law, the fluid flow in the reservoir only considers the laminar flow and oil and water are immiscible.
5. Heat convection and conduction are considered when the fluids flow in the reservoir.
6. The compressibilities of the rock and fluid are considered.

Figure 2. Schematic of the fluid flow in dual-string horizontal wells during the SAGD process: (a) circulation stage; (b) SAGD operation.

2.2. Wellbore Model of the Injector

2.2.1. Steam Flow Model in the Short and Long Strings

It is a constant mass flow process of steam flow in the short and long strings. The energy conservation equation can be expressed as:

$$\frac{dQ_1}{dl} = -m_1 \frac{d}{dl} \left(h_1 + \frac{v_1^2}{2} \right) \quad (1)$$

According to the principle of mass conservation, the auxiliary equation can be written as:

$$\frac{dm_1}{dl} = 0 \quad (2)$$

2.2.2. Steam Flow Model in the Annulus

When the steam flows from interface B to C and from interface D to C, for a random micro-segment in the BD section of the annulus, the energy conservation equation can be derived as:

$$\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + m_{af} \left(h_a + \frac{v_{ar}^2}{2} \right) = - \frac{d}{dl} \left[m_a \left(h_a + \frac{v_a^2}{2} \right) \right] \quad (3)$$

When the steam flows from interface B to A, there is heat exchange between the short and long injection strings and the annulus. For a random micro-segment in the AB section of the annulus, the energy conservation equation for steam flow in this micro-segment is:

$$\frac{dQ_a}{dl} - \frac{dQ_s}{dl} - \frac{dQ_l}{dl} + m_{af} \left(h_a + \frac{v_{ar}^2}{2} \right) = - \frac{d}{dl} \left[m_a \left(h_a + \frac{v_a^2}{2} \right) \right] \quad (4)$$

According to the principle of mass conservation, the subsidiary equation is given by:

$$\frac{dm_a}{dl} = -m_{af} \quad (5)$$

As stated above, the method for the calculation of the pressure gradient in the string and annulus of the injector may be a rough approximation in the previous models. Here, a simplified approach was first used to calculate the pressure drop in the steam injection horizontal wellbore [26–34], as derived in Appendix A. According to Equations (1), (3) and (4), the pressure drops in the string, the BD section of the annulus and the AB section of the annulus can be obtained, as shown in Equations (6)–(8), respectively:

$$\frac{dP_l}{dl} = \frac{\frac{dQ_l}{dl}}{m_l (C_{Jml} C_{pml} - C_{pml} \frac{dT_l}{dP_l} + \frac{v_l v_{sgl}}{P_l})} \quad (6)$$

$$\frac{dP_{a_BD}}{dl} = \frac{\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right)}{m_a (C_{Jma} C_{pma} - C_{pma} \frac{dT_a}{dP_a} + \frac{v_a v_{sga}}{P_a})} \quad (7)$$

$$\frac{dP_{a_AB}}{dl} = \frac{\frac{dQ_a}{dl} - \frac{dQ_s}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right)}{m_a (C_{Jma} C_{pma} - C_{pma} \frac{dT_a}{dP_a} + \frac{v_a v_{sga}}{P_a})} \quad (8)$$

When the steam flows in the horizontal injection wellbore, the steam may condensate into water. In this case, the value of steam quality is zero, and the detailed derivation of fluid pressure and temperature can be found in Appendix B.

2.3. Wellbore Model of the Producer

2.3.1. Liquid Flow Model in the Long String

When the fluid flows in the long string, the mass flow rate remains unchanged. Therefore, the energy and momentum conservation equations can be expressed as:

$$\frac{dQ_{pl}}{dl} = - \frac{d}{dl} \left[m_{pl} \left(h_{pl} + \frac{v_{pl}^2}{2} \right) \right] \quad (9)$$

$$\frac{dP_{pl}}{dl} - \frac{\tau_{pl}}{A_{pl} dl} = - \frac{d(\rho_{pl} v_{pl}^2)}{dl} \quad (10)$$

According to the principle of mass conservation, the auxiliary equation can be written as:

$$\frac{dm_{pl}}{dl} = 0 \quad (11)$$

2.3.2. Liquid Flow Model in the Annulus

When the fluid flows to the heel and toe from the liquid shunt position, for a random micro-segment in the annulus, the energy and momentum conservation equations can be derived as:

$$\frac{dQ_{pa}}{dl} - \frac{dQ_{pl}}{dl} - m_{paf} \left(h_{pa} + \frac{v_{pa}^2}{2} \right) = - \frac{d}{dl} \left[m_{pa} \left(h_{pa} + \frac{v_{pa}^2}{2} \right) \right] \quad (12)$$

$$\frac{dP_{pa}}{dl} - \frac{\tau_{pa}}{A_{pa} dl} = - \frac{d(\rho_{pa} v_{pa}^2)}{dl} \quad (13)$$

According to the principle of mass conservation, the subsidiary equation can be written as:

$$\frac{dm_{pa}}{dl} = -m_{paf} \quad (14)$$

2.4. Reservoir Model

In this Section, a three-dimensional, three-phase reservoir model is presented. The mass conservation equations of oil, water and steam are expressed as follows:

$$\nabla \cdot \left(\rho_o \frac{\alpha_1 k k_{ro}}{\mu_o} \nabla (P_o - \rho_o g D) \right) + \rho_o q_o = \frac{\partial}{\partial t} (\rho_o S_o \varphi) \quad (15)$$

$$\nabla \cdot \left(\rho_w \frac{\alpha_2 k k_{rw}}{\mu_w} \nabla (P_w - \rho_w g D) \right) + \rho_w q_w + m_c = \frac{\partial}{\partial t} (\rho_w S_w \varphi) \quad (16)$$

$$\nabla \cdot \left(\rho_g \frac{\alpha_3 k k_{rg}}{\mu_g} \nabla (P_g - \rho_g g D) \right) + \rho_g q_g - m_c = \frac{\partial}{\partial t} (\rho_g S_g \varphi) \quad (17)$$

The energy conservation equation is:

$$\begin{aligned} \nabla \cdot (\lambda_R \nabla T) + \nabla \cdot \left(\sum_{i=o,w,g} \rho_i H_i \frac{k k_{ri}}{\mu_i} \nabla (P_i - \rho_i g D) \right) - Q_{loss} + Q_H + \\ Q_a + Q_{pa} = \frac{\partial}{\partial t} \left[(1 - \varphi) \rho_R c_R T + \varphi \sum_{i=o,w,g} S_i \rho_i U_i \right] \end{aligned} \quad (18)$$

The equilibrium condition is:

$$T = T_s(P) \quad (19)$$

The subsidiary equations used to reduce the variables can be found in references [35–37].

2.5. Coupling Model

The reservoir model and wellbore model can be coupled through the sink/source terms [38]. The calculation of sink and source terms is shown in Equations (22) and (23), respectively:

$$m_g = \rho_a I_s (P_a - P) \frac{2\pi \sqrt{k_h/k_v} k_v dl}{[\ln(r_e/r_w) - 0.75 + S_k]} \left(\frac{k_{ro}}{B_o \mu_o} + \frac{k_{rw}}{B_w \mu_w} \right) \quad (20)$$

$$m_{liquid} = \rho_{pa} (P - P_{pa}) \frac{2\pi \sqrt{k_h/k_v} k_v dl}{[\ln(r_e/r_w) - 0.75 + S_k]} \left(\frac{k_{ro}}{B_o \mu_o} + \frac{k_{rw}}{B_w \mu_w} \right) \quad (21)$$

2.6. Uniformity Evaluation Model

The uniformity of the parameter along the horizontal well can be evaluated by choosing the non-uniformity coefficient as the evaluation criteria, and this equation is defined as:

$$\alpha_x = \sqrt{\sum_j (x_j - \bar{x})^2 / \bar{x}} \quad (22)$$

To evaluate the uniformity improvement effect of the parameter along the wellbore, the concept of the uniformity improvement coefficient is proposed:

$$\beta_x = \frac{\alpha_{x, \text{base scheme}} - \alpha_{x, \text{non-basescheme}}}{\alpha_{x, \text{base scheme}}} \times 100\% \quad (23)$$

The smaller the non-uniformity coefficient, the more uniform is the parameter distribution along the wellbore. The larger the uniformity improvement coefficient, the better is the uniformity improvement effect of the parameter along the wellbore (compared with the base scheme).

3. Solution Method of the Coupling Model

The wellbore model was solved fully implicitly with the reservoir model. This process was programmed with a computer, and the flow chart is presented in Figure 3. The schematic with discretized grids and major parameters is shown in Figure 4. The solution procedures are given as follows:

Step 1: Enter the initial values of the reservoir and wellbore. The well is discretized into n segments. The steam pressure drop ΔP_s , temperature drop ΔT_s and steam quality drop Δx_s are estimated for each segment.

Step 2: The changes in the reservoir pressure ΔP_o , temperature ΔT , water saturation ΔS_w and gas saturation ΔS_g are estimated for each grid block.

Step 3: Solution of the reservoir model

Combining Equations (16) and (17), the condensation term m_c was eliminated. The condensation rate is thus implicitly represented within a set of four equations. Therefore, the reservoir model can be discretized by using the fully implicit finite difference method. In the discretization process of the reservoir model, pressure-dependent density and viscosity of steam while solving fluid flow equations are considered, and the gravity induced movement of steam is incorporated. The detailed derivation of the reservoir model is provided in Appendix C.

Step 4: Solution of the wellbore model

The wellbore model is solved with the straightforward numerical method and iteration technique. The calculation is repeated until the limitations of pressure, temperature and steam quality condition are satisfied.

Step 5: The calculation results are output when the simulation time reaches the set time.

Figure 3. Calculation flow chart for the numerical model.

Figure 4. The schematic with discretized grids and major parameters.

4. Results and Discussion

4.1. Verification of the Developed Model

4.1.1. Validation Using the Experimental Data

The well configuration in the developed model is a dual-string well configuration; the fluid flow behaviour in the cases of the heel point injection mode and the toe point injection mode can be simulated by the developed model. The flow behaviour of the fluid in the cases of the heel point injection and the toe point injection modes in the developed model is similar to the flow behaviour of the fluid in the cases of the heel point injection and the toe point injection modes in Dong et al.'s experiment [19]. Therefore, the simulation results of the developed model can be compared with the experimental results of Dong et al. to validate the proposed model. The experimental data used for calculation are listed in Table 1. Major assumptions are made as follows: (1) the reservoir is homogeneous; (2) during the simulation, the steam pressure, steam injection rate and steam quality at the outlet of the string do not change with time; (3) the perforation diameter, density and phasing are the same in every micro-segment of the horizontal well. The results calculated using this model and the experimental results of Dong et al. are presented in Figure 5. Figure 5 illustrates that the average relative error of the pressure, which, in the cases of the heel point injection and toe point injection modes, is 2.52% and 2.81%, respectively. The average relative error of the temperature in the cases of the heel point injection and toe point injection modes is 1.80% and 1.99%, respectively. Results from the developed model and Dong et al.'s experiment have a relative error of <5%. The agreement between experimental results and the results obtained from the model is good, showing an acceptable average relative error. In addition, the simulation results are linear in the developed model, while the experimental results are non-linear in Dong et al.'s experiment. There are some small deviations between the simulation results of the developed model and the experimental results of Dong et al.'s experiment, because when the reservoir is assumed to be homogeneous, the experimental data input in the developed model for simulation are homogeneous, while the experimental data actually used in Dong et al.'s experiment may be not absolutely homogeneous due to the actual grain size of the quartz sand not being exactly the same.

Figure 5. Comparison of the simulated results and experimental data: (a) pressure; (b) temperature.

Table 1. Basic data used in the calculations.

Items	Values	Items	Values
Reservoir thickness, cm	15	Oil saturation, %	75
Porosity, %	0.38	Initial reservoir temperature, °C	25
Horizontal well length, cm	80	Reservoir permeability, $10^{-3} \mu\text{m}^2$	3000
Wellbore radius, cm	6	Steam temperature, °C	130
Steam injection rate, cm^3/min	40	Steam injection time, min	30

4.1.2. Verification against the Numerical Solution

In this section, to validate the proposed model, a traditional SAGD process was simulated based on the field data from the MacKay River heavy oil reservoir, Western Canada, and the field data are listed in Table 2. The relative permeability curves as well as the viscosity vs. temperature are shown in Figure 6. The 3D model has dimensions of $1100 \text{ m} \times 110 \text{ m} \times 20 \text{ m}$, and the well configuration is a conventional single-pipe well configuration.

During the traditional SAGD circulation stage, steam is injected through both the injector and the producer. The steam injection pressure, steam quality and injection rate at the heel of the wellbore are 2.0 MPa, 0.95 and $80 \text{ m}^3/\text{d}$, respectively. After 120 days of circulation, the well pair is converted to the SAGD operation. The steam injection pressure and steam quality at the heel of the horizontal injection well are 2.12 MPa and 0.95, respectively. The bottomhole pressure at the heel of the horizontal production well is 1.5 MPa. The calculated results of this model are compared with the commercial thermal simulator, Computer Modelling Group (CMG) simulation results, as shown in Figures 7 and 8.

Figure 7 illustrates that the average relative errors in the temperature and pressure values in each grid between the 3D present model and the CMG are 4.52% and 4.61% at the end of the circulation stage, respectively (outliers are not considered in error calculation). The visible differences between the CMG and the developed model in Figure 7 may be due to the different software utilizing different data processing methods. The calculation results of each grid by CMG are displayed in the form of a block fill in the CMG software interface. However, the calculation results of each grid using the developed model are displayed in the form of fill contours in the drawing software (Surfer 16) used herein. In addition, Figure 8 illustrates that the average relative error of the injector wellbore pressure at 160 days, 170 days and 180 days is 0.58%, 0.71% and 0.83%, and the average relative error of the cumulative oil production and oil production rate is 1.37% and 1.45%, respectively. Results from the developed model and CMG have a relative error of <5%. The calculation results of the developed model are consistent with the CMG simulated results, showing an acceptable average relative error. Thus, this model can predict the SAGD process.

Table 2. Reservoir data used in the calculations.

Items	Values	Items	Values
Initial oil saturation	0.75	Initial water saturation	0.25
Initial reservoir pressure, MPa	0.22	Vertical permeability, ($10^{-3} \mu\text{m}^2$)	1890
Initial reservoir temperature, °C	10	Horizontal permeability, ($10^{-3} \mu\text{m}^2$)	2700
Initial oil density, kg/m^3	1004.15	Initial water density, kg/m^3	1006.49
Oil heat capacity, $\text{KJ}/(\text{kg} \cdot ^\circ\text{C})$	2.2	Water heat capacity, $\text{KJ}/(\text{kg} \cdot ^\circ\text{C})$	4.2
Rock heat conductivity coefficient, $\text{W}/(\text{m} \cdot ^\circ\text{C})$	5.2	Rock compressibility coefficient, MPa^{-1}	0.002
Water compressibility coefficient, MPa^{-1}	5.21×10^{-4}	Oil compressibility coefficient, MPa^{-1}	2.44×10^{-4}
Rock density, kg/m^3	2060	Horizontal well length, m	850
Δx , m	10	Number of blocks along x	110
Δy , m	2	Number of blocks along y	55
Δz , m	1	Number of blocks along z	20
Time step, day	0.01	Porosity	0.33
Outer diameter of the slotted liner, mm	219.10	Inner diameter of the slotted liner, mm	201.20

Figure 6. Relative permeability curve and viscosity vs. temperature: (a) Relative permeability curves of oil and water; (b) Relative permeability curves of oil and gas; (c) Viscosities of oil and water.

Figure 7. Comparison of the steam chamber and reservoir pressure profile along the well pair axis at the end of the circulation stage (vertical exaggeration 10 times): (a) steam chamber calculated by the developed model; (b) steam chamber simulated by CMG; (c) reservoir pressure profile calculated by the developed model; (d) reservoir pressure profile simulated by CMG.

Figure 8. Comparison of the predicted results with CMG in the SAGD operation: (a) injector wellbore pressure; (b) oil production rate and cumulative oil production.

In addition, in the CMG wellbore model [39], various transition criteria for gas/liquid two-phase flow were established to determine the flow pattern, and the annulus was treated as a pipe based on the equivalent hydraulic diameter concept. However, precisely dividing flow patterns is difficult, and the flow patterns in the annulus have been demonstrated to be different from pipe-flow patterns [24,25]. The calculation of the pressure drop may be a rough approximation [26,27]. In the proposed model, a simplified approach was first used to calculate the pressure drop in the steam injection horizontal wellbore [26–28], as derived in Appendix A.

4.2. Sensitivity Analysis

To improve the steam conformance, the effects of the injector wellbore configuration, steam injection ratio and injection time on the steam conformance were investigated with dual-string horizontal wells in SAGD operations. The calculation parameters were based on the reservoir data in Table 2. The relative permeability curves as well as the viscosity vs. temperature are shown in Figure 6. For all of the studies, the short and long strings of the producer were landed at the heel and toe of the wellbore, respectively, and the bottomhole pressures at the heels of the short and long production strings are fixed values.

4.2.1. Effect of the Injector Wellbore Configuration

To study the effect of the injector wellbore configuration on the steam conformance, four schemes were designed. Namely, the short string of the injector was extended 0 m (heel of the wellbore), 150 m, 275 m and 425 m into the well. In each scheme, the total steam injection rate was 110 m³/d.

Figure 9 illustrates that the pressure drop in the annulus initially decreases and then increases as the landing position of the short string increases. When the short string was extended 0 m, 150 m, 275 m and 425 m into the horizontal section, the total annulus pressure drops are 45.49 kPa, 30.55 kPa, 141.29 kPa and 315.10 kPa, respectively. It can be seen that the pressure drop mainly occurs in the AB section of the annulus. Because the cross-sectional area of the annulus space of the AB section is smaller than that of the annulus space of the BD section, as the short string was extended 150 m, 275 m and 425 m into the well, the pressure drop in the AB section is 27.10 kPa, 132.87 kPa and 313.14 kPa, which accounts for 88.71%, 94.04% and 99.38% of the total annulus pressure drop, respectively; the pressure drop in the BD section is 30.55 kPa, 21.87 kPa and 9.37 kPa, which accounts for 100%, 15.48% and 2.98% of the total annulus pressure drop, respectively. The pressure drop in the AB section is the main factor affecting the uniform distribution of steam pressure in the annulus, and the closer the differences in the pressure drop between the AB section and BD section, the better is the uniformity of the distribution of steam pressure in the annulus. In this study, a relatively uniform injector annular pressure profile could be obtained when the short string was pushed 150 m into the horizontal section. In practical

situations, the steam pressure along the wellbore can be measured. Therefore, to obtain a uniform injector annular pressure profile under different injector wellbore configurations, it is recommended that production engineers monitor the pressure drop in the AB and BD sections. If the pressure drop in the AB section is larger than the pressure drop in the BD section, it is recommended that the landing position of the short string be shortened; if the pressure drop in the AB section is smaller than the pressure drop in the BD section, it is suggested that the landing position of the short string be extended. Finally, the pressure drop in the AB section is almost equal to the pressure drop in the BD section.

Figure 9. Effect of the injector wellbore configuration on the annulus pressure profiles of the injector.

Figure 10a shows that the steam flux rates are large at the landing position of the short injection string and at the toe of the wellbore. Because the annulus pressure is high at the landing position of the short injection string, the drainage area near the toe of the wellbore is large. Compared with the steam flux rate at the toe of the wellbore, the steam flux rate at the heel of the wellbore is small because the low annulus pressure at the heel restricts the advantage of the large drainage area around the heel. It can be observed that the distribution trends of the liquid production profiles and steam injection profiles are similar, as shown in Figure 10b. This phenomenon is due to the large amount of steam injected into the reservoir at a certain place contributing to a high fluid mobility and a high potential for fluid recovery. In addition, the fluctuation range of the injection and production profiles initially decrease and then increase as the landing position of the short string increases. When the short string was extended 0 m, 150 m, 275 m and 425 m into the well, the fluctuation range of the steam injection profile was $0.071 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.068 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.084 \text{ m}^3/(\text{d}\cdot\text{m})$ and $0.097 \text{ m}^3/(\text{d}\cdot\text{m})$, respectively. The fluctuation range of the liquid production profile was $0.073 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.071 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.088 \text{ m}^3/(\text{d}\cdot\text{m})$ and $0.115 \text{ m}^3/(\text{d}\cdot\text{m})$, respectively. Therefore, the steam breakthrough phenomenon around the heel of the wellbore could be mitigated, and the uniformity of the injection and production profiles could be improved as the short injection string was pushed 150 m into the horizontal section.

Figure 11 shows the steam chamber along the well pair axis at 210 days. It is observed that landing the short injection string at a certain position can effectively delay the occurrence of steam fingering around the heel section of the horizontal well. As the short string was pushed 150 m into the horizontal section, the steam conformance was improved, and a higher cumulative oil production was achieved. Table 3 shows that when the short injection string was pushed 150 m into the horizontal section, the uniformity of each parameter distribution along the wellbore was the highest, and the uniformity improvement coefficient of the steam chamber was 19.49% compared with the base scheme (the short string was landed at the heel of the well). Therefore, a more uniform steam conformance was created, and the oil recovery was enhanced as the short injection string was landed at 150 m in the well.

Figure 10. Effect of the injector wellbore configuration on the injection and production profiles: (a) steam flux rate; (b) liquid production rate.

Figure 11. Effect of the injector wellbore configuration on the steam chamber and cumulative oil production: (a) steam chamber; (b) cumulative oil production.

Table 3. Calculation results of the non-uniformity coefficients and uniformity improvement coefficient.

Items Schemes		Non-uniformity Coefficients				Uniformity Improvement Coefficients			
		Annulus Pressure, MPa	Steam Flux Rate, (m ³ ·d ⁻¹ ·m ⁻¹)	Liquid Production Rate, (m ³ ·d ⁻¹ ·m ⁻¹)	Height of Steam Chamber, m	Annulus Pressure, MPa	Steam Flux Rate, (m ³ ·d ⁻¹ ·m ⁻¹)	Liquid Production Rate, (m ³ ·d ⁻¹ ·m ⁻¹)	Height of Steam Chamber, m
Injector wellbore configuration	0 m	4.15×10^{-2}	0.812	0.854	0.395	—	—	—	—
	150 m	2.86×10^{-2}	0.692	0.774	0.318	31.08%	14.78%	9.37%	19.49%
	275 m	13.54×10^{-2}	0.815	0.947	0.432	−226.27%	−0.37%	−10.89%	−9.37%
	425 m	37.14×10^{-2}	1.074	1.552	0.487	−794.94%	−32.27%	−81.73%	−23.29%
Steam injection ratio	35:75	8.03×10^{-2}	0.866	0.981	0.491	−93.49%	−6.65%	−14.87%	−2.29%
	55:55	4.15×10^{-2}	0.812	0.854	0.480	—	—	—	—
	75:35	9.87×10^{-2}	0.868	0.982	0.494	−137.83%	−6.89%	−14.99%	−2.91%
Injection time	170 d	4.15×10^{-2}	0.812	0.854	0.394	—	—	—	—
	190d	7.17×10^{-2}	0.889	0.898	0.444	−72.77%	−9.18%	−5.15%	−12.69%
	210 d	9.90×10^{-2}	0.960	0.969	0.510	−138.55%	−18.23%	−13.47%	−29.44%

4.2.2. Effect of the Steam Injection Ratio

In this section, the steam injection ratios of the long and short injection strings are set to 35:75, 55:55 and 75:35 to study the influence of the steam injection ratio on the steam conformance. In each scheme, the total steam injection rate was $110 \text{ m}^3/\text{d}$. The short string landed at the heel, and the long string landed at the toe for both the injector and the producer.

Figure 12 illustrates that with an increase in the steam injection ratio of the long and short injection strings, the fluctuation range of the injector annular pressure profile decreases first and then increases. The maximum pressure drops in the annulus are 104.76 kPa, 45.49 kPa and 125.91 kPa as the steam injection ratios of the long and short injection strings are set to 35:75, 55:55 and 75:35, respectively. The smaller the difference in the steam injection rate between the long and short injection strings is, the more uniform the distribution of the annulus pressure of the injector. Therefore, when the steam injection ratio of the long and short injection strings is set to 55:55, the injector annular pressure profile uniformity can be improved.

Figure 12. Effect of the steam injection ratio on the annulus pressure profiles of the injector.

Figure 13a illustrates that with an increase in the steam injection ratio of the long and short injection strings, the “U”-shaped steam injection profile rotates counter-clockwise due to the distribution of the steam pressure in the annulus. It can be observed that the distribution trend of the liquid production profile is similar to the distribution trend of the steam injection profile, as shown in Figure 13b. The reason is discussed in Section 4.2.1.

In addition, the fluctuation ranges of the injection and production profiles initially decrease and then increase as the steam injection ratio of the long and short injection strings increases. When the steam injection ratio of the long and short injection strings is 35:75, 55:55 and 75:35, the fluctuation range of the steam injection profile is $0.0807 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.0705 \text{ m}^3/(\text{d}\cdot\text{m})$ and $0.0806 \text{ m}^3/(\text{d}\cdot\text{m})$, respectively; the fluctuation range of the liquid production profile is $0.0873 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.0724 \text{ m}^3/(\text{d}\cdot\text{m})$ and $0.0875 \text{ m}^3/(\text{d}\cdot\text{m})$, respectively. Consequently, to improve the uniformity of the injection and production profiles, setting the steam injection ratio of the long and short injection strings to 55:55 is suggested.

Figure 13. Effect of the steam injection ratio on the injection and production profiles: (a) steam flux rate; (b) liquid production rate.

Figure 14 shows the steam chamber along the well pair axis at 210 days, and it is found that the small difference in the steam injection rate between the long and short injection strings is beneficial for creating a more uniform steam chamber and improving cumulative oil production. Table 3 illustrates that in the base scheme (the steam injection ratio of long and short injection strings is set to 55:55), the uniformity of each parameter distribution along the wellbore is the highest. Thus, to establish a better steam conformance and ultimately improve oil recovery, using the same steam injection rate for the long and short injection strings is recommended.

Figure 14. Effect of the steam injection ratio on the steam chamber and cumulative oil production: (a) steam chamber; (b) cumulative oil production.

4.2.3. Effect of the Injection Time

The injection time is one of the main factors affecting the development of the steam chamber. However, previous dual-string horizontal injection well models are analytical models that fail to consider the influence of the injection time on the variations in reservoir properties. In this Section, the effect of the injection time on the steam conformance is discussed.

Figure 15 illustrates that the pressure drop in the annulus increases with increasing injection time. When the injection time is 170 days, 190 days and 210 days, the injector annular pressure drops are 45.49 kPa, 83.84 kPa and 112.63 kPa, respectively. The non-uniformity of the steam pressure in the annulus will become more serious as steam injection proceeds.

Figure 15. Effect of the injection time on the annulus pressure profiles of the injector.

Figure 16 illustrates that with the increase of the injection time, the steam flux rate and liquid production rate increase, and the fluctuation range of the injection and production profiles increases. When the injection time is 170 days, 190 days and 210 days, the fluctuation range of the steam injection profile is $0.071 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.085 \text{ m}^3/(\text{d}\cdot\text{m})$ and $0.995 \text{ m}^3/(\text{d}\cdot\text{m})$, respectively; the fluctuation range of the liquid production profile is $0.072 \text{ m}^3/(\text{d}\cdot\text{m})$, $0.085 \text{ m}^3/(\text{d}\cdot\text{m})$ and $0.107 \text{ m}^3/(\text{d}\cdot\text{m})$, respectively. The non-uniformity of the injection and production profiles increases as time proceeds.

Figure 16. Effect of the injection time on the injection and production profiles: (a) steam flux rate; (b) liquid production rate.

Figure 17 illustrates the effect of the injection time on the steam chamber growth. It is found that the development of the steam chamber is sensitive to the injection time. The uniformity of the steam chamber decreases with increasing injection time. In particular, the heel-toe effect plays a crucial role in this non-uniformity. In addition, Figures 11b and 14b indicate that with the increase of the injection time, the cumulative oil production increases in the early time stage of the SAGD operation but only slightly increases at a certain time. Consequently, to achieve a better steam conformance and recover more reserves, the optimization method of steam chamber uniformity should be adjusted according to different injection times.

In addition, as shown in Table 3, the more uniform the injector annular pressure profile, the more uniform the injection and production profiles and the steam chamber. In other words, the uniformity of the injection and production profiles and the steam conformance can be achieved by obtaining a uniform distribution of steam pressure in the annulus of the injector.

Figure 17. Effect of the injection time on the steam chamber.

Furthermore, under conditions of the same total steam injection rate ($110 \text{ m}^3/\text{d}$), the cumulative oil production is $12.27 \times 10^4 \text{ m}^3$ after simulation when the short string is pushed 150 m into the horizontal section (Figure 11b). The cumulative oil production is $11.59 \times 10^4 \text{ m}^3$ after simulation when the steam injection ratio of the long and short injection strings are set to 55:55 (Figure 14b). Compared with the steam injection ratio of the long and short injection strings set to 55:55, a higher cumulative oil production can be obtained as the short string is landed at a suitable position. Therefore, the short string landing at a suitable position is recommended to achieve high cumulative long-term oil production. The suggestion for production engineers to adjust the landing position of the short string to achieve a uniform injector annular pressure profile and better steam conformance is discussed in Section 4.2.1. To the best of our knowledge, the heel-toe effect is not beneficial for oil production. When the short string is landed at a suitable position, the steam breakthrough phenomenon around the heel of the wellbore can be mitigated. Thus, how to reduce the steam breakthrough phenomenon around the toe of the wellbore is the key to improve the steam conformance and cumulative oil production. To reduce the effect of the steam breakthrough phenomenon around the toe of the wellbore on the steam conformance and oil production, it is suggested that production engineers close the long string for steam injection or decrease the steam injection pressure of the long string when the monitoring pressure drop in the BD section is larger than the pressure drop in the AB section.

The model developed in this paper is limited to a solution for saturated steam injection with a dual-string well configuration. For the injector, the well configuration includes short string injection, long string injection and simultaneous dual-string injection. For the producer, the well configuration contains short string production, long string production and simultaneous dual-string production. Production engineers can use the proposed model to predict the distribution of the wellbore parameters and the reservoir properties during the entire lifetime of a simulation process with different well configurations, and adopt appropriate measures to improve the steam chamber uniformity and increase oil production based on the simulation results.

5. Conclusions

- (1) The predicted results of the developed model match well with the CMG calculation results and the experimental data. The proposed model is shown to be reliable.
- (2) Under different injector wellbore configurations, the closer the differences in the pressure drop in the AB and BD sections, the better is the uniformity of the injector annular pressure profile, injection and production profiles and steam conformance. It is recommended that production engineers measure the pressure drop in the AB section and BD section and adjust the landing position of the short string to obtain an equal pressure drop in the AB and BD sections.

- (3) The smaller the difference in the steam injection rate between the long and short injection strings, the more uniform is the distribution of the injection and production profiles and the steam chamber.
- (4) Creating a more uniform injector annular pressure profile is beneficial for improving the uniformity of the injection and production profiles and the steam conformance.
- (5) The steam conformance decreases as time proceeds. The appropriate optimization method should be adopted for different injection times to improve the steam conformance.

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Nomenclature

A_{pa}	effective cross-sectional area in the annulus [m^2]
A_{pl}	effective cross-sectional area in the string [m^2]
B_o	formation volume factor of oil [m^3/m^3]
B_w	formation volume factor of water [m^3/m^3]
c_R	specific heat capacity of rock [$J/(kg \cdot ^\circ C)$]
C_{Jma}	Joule–Thomson coefficient of steam in the annulus [$^\circ C/Pa$]
C_{Jml}	Joule–Thomson coefficient of steam in the string [$^\circ C/Pa$]
C_{pma}	heat capacity of steam in the annulus [$J/(kg \cdot ^\circ C)$]
C_{pml}	heat capacity of steam in the string [$J/(kg \cdot ^\circ C)$]
D	depth [m]
g	gravitational acceleration [m/s^2]
h_a	steam enthalpy in the annulus [J/kg]
h_l	steam enthalpy in the string [J/kg]
h_{pa}	fluid enthalpy in the annulus [J/kg]
h_{pl}	fluid enthalpy in the string [J/kg]
H_i	enthalpy of the phase i in the heat convection term [J/kg]
I_s	injection production ratio
k	absolute permeability [$10^{-3} \mu m^2$]
k_h	horizontal permeability [m^2]
k_{ro}	relative permeability of oil
k_{rw}	relative permeability of water
k_{rg}	relative permeability of gas
k_v	vertical permeability [m^2]
m_a	steam mass flow rate in the annulus [kg/s]
m_c	rate of steam condensation [$kg/(m^3 \cdot s)$]
m_{af}	steam mass injection rate from the annulus to the reservoir [$kg/(m \cdot s)$]
m_g	steam mass imbibition rate per unit length [kg/s]
m_l	steam mass flow rate in the string [kg/s]
m_{liquid}	liquid production rate per unit length [kg/s]
m_{pa}	fluid mass flow rate in the annulus [kg/s]
m_{paf}	fluid mass inflow rate from the reservoir to the annulus [$kg/(m \cdot s)$]
m_{pl}	fluid mass flow rate in the string [kg/s]

P	reservoir pressure [Pa]
P_a	steam pressure in the annulus of the injector [Pa]
P_{a_AB}	steam pressure in the AB section of the annulus [Pa]
P_{a_BD}	steam pressure in the BD section of the annulus [Pa]
P_l	steam pressure in the string [Pa]
P_o	oil pressure [Pa]
P_{pa}	wellbore pressure in the annulus [Pa]
P_{pl}	wellbore pressure in the string [Pa]
P_w	water pressure [Pa]
P_g	steam pressure [Pa]
q_o	volume flow rate of oil per unit volume [$m^3/(m^3 \cdot s)$]
q_w	volume flow rate of water per unit volume [$m^3/(m^3 \cdot s)$]
q_g	volume flow rate of steam per unit volume [$m^3/(m^3 \cdot s)$]
Q_a	heat loss rate of the annulus of the injector [J/s]
Q_l	heat loss rate of the long string of the injector [J/s]
Q_{pa}	heat loss rate of the annulus of the producer [J/s]
Q_{pl}	heat loss rate of the long string of the producer [J/s]
Q_{ps}	heat loss rate of the short string of the producer [J/s]
Q_s	heat loss rate of the short string of the injector [J/s]
Q_{loss}	heat loss rate to the surroundings [J/($m^3 \cdot s$)]
Q_H	enthalpy production rate [J/($m^3 \cdot s$)]
S_k	skin factor
S_o	oil saturation
S_w	water saturation
S_g	steam saturation
r_e	oil drainage radius [m]
r_w	wellbore radius [m]
t	time [s]
T	reservoir temperature [$^{\circ}C$]
T_a	steam temperature in the annulus [$^{\circ}C$]
T_l	steam temperature in the string [$^{\circ}C$]
U_i	internal energy of the phase i [J/kg]
v_a	steam flow velocity in the annulus [m/s]
v_{ar}	steam flow velocity into the reservoir [m/s]
v_l	steam flow velocity in the string [m/s]
v_{pa}	fluid flow velocity in the annulus [m/s]
v_{par}	fluid inflow velocity from the reservoir [m/s]
v_{pl}	fluid flow velocity in the string [m/s]
v_{sga}	superficial gas velocity in the annulus [m/s]
v_{sgl}	superficial gas velocity in the string [m/s]
$\bar{x}$	average value of parameter x
x_j	value of parameter x in the micro-segment j

Greek Symbols

α_1	unit conversion coefficient
α_2	unit conversion coefficient
α_3	unit conversion coefficient
α_x	non-uniformity coefficient of parameter x
β_x	uniformity improvement coefficient of parameter x
λ_R	thermal conductivity of the reservoir in the heat conduction term [$W/(m \cdot ^{\circ}C)$]
φ	porosity
μ_o	oil viscosity [Pa·s]
μ_w	water viscosity [Pa·s]
μ_g	steam viscosity [Pa·s]

ρ_a	steam density in the annulus of the injector [kg/m ³]
ρ_o	oil density [kg/m ³]
ρ_w	water density [kg/m ³]
ρ_g	steam density [kg/m ³]
ρ_R	rock density [kg/m ³]
ρ_{pa}	fluid density in the annulus of the producer [kg/m ³]
ρ_{pl}	fluid density in the string of the producer [kg/m ³]
τ_{pa}	friction when the fluid flows in the annulus of the producer [N]
τ_{pl}	friction when the fluid flows in the string of the producer [N]

Appendix A

Appendix A presents the calculation of pressure drop in the long string and annulus.

(1) The calculation of pressure drop in the CD segment of the annulus

When the steam flows from the interface D to C. For a random micro-segment in the CD segment of the annulus, the equations of mass conservation and energy conservation can be developed as follows:

$$\frac{dm_a}{dl} = -m_{af} \quad (A1)$$

$$\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + m_{af} \left(h_a + \frac{v_{ar}^2}{2} \right) = -\frac{d}{dl} \left[m_a \left(h_a + \frac{v_a^2}{2} \right) \right] \quad (A2)$$

For pressure drop in the annulus, Wu et al. [23] and Sun et al. [38] treated the annulus as pipes, namely, effective diameter of annulus (D_e) was replaced by equivalent hydraulic diameter, $D_e = 2(r_{li} - r_{io})$, where r_{li} is the inside radius of the long string and r_{io} is the inside radius of the slotted liners. As stated above, this may be a rough approximation. Here, a more rigorous method is presented to predict the steam pressure in the annulus [26–28].

Equation (A2) can be written as:

$$\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right) = -m_a \frac{dh_a}{dl} - m_a \frac{d}{dl} \left(\frac{v_a^2}{2} \right) \quad (A3)$$

The first term of the right side in Equation (A3) represents the enthalpy gradient, which can be written in terms of temperature and pressure gradients based on thermodynamic principles [28]:

$$\frac{dh_a}{dl} = C_{pma} \frac{dT_a}{dl} - C_{Jma} C_{pma} \frac{dP_a}{dl} \quad (A4)$$

To calculate Equation (A4), one of the most important tasks is to calculate the values of C_{pma} and C_{Jma} . The calculation methods for C_{pma} and C_{Jma} are presented in detail as flows:

For steam/water two-phase flow system, the total enthalpy of mixture fluid is the sum of the enthalpy of each phase [28]. Therefore, the enthalpy gradient in the random micro-segment in the CD segment of the annulus can be given by:

$$\frac{dh_a}{dl} = \frac{m_s}{m_a} \frac{dh_s}{dl} + \frac{m_w}{m_a} \frac{dh_w}{dl} \quad (A5)$$

where m_s and m_w are the mass flow rates of steam and water, respectively;
and:

$$\frac{dh_s}{dl} = C_{ps} \frac{dT_a}{dl} - C_{Js} C_{ps} \frac{dP_a}{dl} \quad (A6)$$

$$\frac{dh_w}{dl} = C_{pw} \frac{dT_a}{dl} - C_{Jw} C_{pw} \frac{dP_a}{dl} \quad (A7)$$

where C_{ps} and C_{pw} are the heat capacities of steam and water at constant pressure, respectively; C_{Js} and C_{Jw} are the Joule–Thompson coefficients of steam and water, respectively. C_{ps} , C_{pw} , C_{Js} and C_{Jw} are given by:

$$C_{ps} = \left(\frac{\partial h_s}{\partial T} \right)_P \quad (A8)$$

$$C_{pw} = \left(\frac{\partial h_w}{\partial T} \right)_P \quad (A9)$$

$$C_{Js} = \frac{1}{C_{ps}} \left\{ T \left[\frac{\partial}{\partial T} \left(\frac{1}{\rho_s} \right) \right]_P - \frac{1}{\rho_s} \right\} \quad (A10)$$

$$C_{Jw} = \frac{1}{C_{pw}} \left\{ T \left[\frac{\partial}{\partial T} \left(\frac{1}{\rho_w} \right) \right]_P - \frac{1}{\rho_w} \right\} \quad (A11)$$

If the compressibility of water is ignored, Equation (A11) can be reduced to.

$$C_{Jw} = -\frac{1}{C_{pw}\rho_w} \quad (A12)$$

Substituting Equations (A6) and (A7) into Equation (A5) gives:

$$\frac{dh_a}{dl} = \frac{m_s C_{ps} + m_w C_{pw}}{m_a} \frac{dT_a}{dl} - \left(\frac{m_s C_{Js} C_{ps}}{m_a} + \frac{m_w C_{Jw} C_{pw}}{m_a} \right) \frac{dP_a}{dl} \quad (A13)$$

Therefore, the heat capacity at constant pressure and the Joule–Thompson coefficient of mixture fluid are.

$$C_{pma} = \frac{m_s C_{ps} + m_w C_{pw}}{m_a} \quad (A14)$$

$$C_{Jma} = \frac{1}{C_{pma}} \left[\frac{m_s C_{Js} C_{ps}}{m_a} + \frac{m_w C_{Jw} C_{pw}}{m_a} \right] \quad (A15)$$

Substituting Equations (A10) and (A12) into Equation (A15), we have:

$$C_{Jma} = \frac{1}{C_{pma} m_a} \left\{ m_s T \left[\frac{\partial}{\partial T} \left(\frac{1}{\rho_s} \right) \right]_P - \frac{m_s}{\rho_s} - \frac{m_w}{\rho_w} \right\} \quad (A16)$$

The values of C_{pma} and C_{Jma} can be calculated by Equations (A14) and (A16), respectively.

According to the derivation method proposed by Beggs and Brill [29], the kinetic energy change per unit mass of annulus fluid per unit length of the wellbore represented by the second term of the right side in Equation (A2) can also be rewritten as:

$$\frac{d}{dl} \left(\frac{v_a^2}{2} \right) = \frac{v_a dv_a}{dl} = -\frac{v_a v_{sga}}{P_a} \frac{dP_a}{dl} \quad (A17)$$

Incorporating Equations (A4) and (A17) into Equation (A2) results in:

$$\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right) = m_a \left(C_{Jma} C_{pma} \frac{dP_a}{dl} - C_{pma} \frac{dT_a}{dl} \right) + m_a \frac{v_a v_{sga}}{P_a} \frac{dP_a}{dl} \quad (A18)$$

In this Section, the calculation of dT_a/dl in Equation (A18) is derived. A general relationship between temperature gradient and pressure gradient for saturated steam can be further created based on limit and derivative theories in mathematics. Figure A1 shows a random micro-segment in the CD segment of the annulus. Assuming that the steam temperature varies from $T_{a,i}$ to $T_{a,i+1}$ and the steam pressure varies from $P_{a,i}$ to $P_{a,i+1}$ when steam flows from the inlet to the outlet, then the temperature and the pressure changes over dl can be expressed as $\Delta T_a = T_{a,i+1} - T_{a,i}$ and $\Delta P_a = P_{a,i+1} - P_{a,i}$,

respectively. Consequently, the temperature gradient can be obtained from the definition of derivative and the algorithm of limit:

Figure A1. A random micro-segment in the CD segment of the annulus.

$$\frac{dT_a}{dl} = \lim_{\Delta l \rightarrow 0} \frac{\Delta T_a}{\Delta l} = \lim_{\Delta l \rightarrow 0} \left(\frac{\Delta T_a}{\Delta P_a} \frac{\Delta P_a}{\Delta l} \right) = \lim_{\Delta l \rightarrow 0} \frac{\Delta T_a}{\Delta P_a} \cdot \lim_{\Delta l \rightarrow 0} \frac{\Delta P_a}{\Delta l} = \lim_{\Delta l \rightarrow 0} \frac{\Delta T_a}{\Delta P_a} \cdot \lim_{\Delta l \rightarrow 0} \frac{dP_a}{dl} \quad (\text{A19})$$

In Equation (A19), there are two methods to estimate the value of $\lim_{\Delta l \rightarrow 0} \frac{\Delta T_a}{\Delta P_a}$. The first one is interpolation of steam tables, however, not only is interpolation itself inconvenient in computer procedure, but also this method for solving Equation (A19) is time- and memory-consuming. That is because in order to ensure that the interpolated results are precise enough, the wellbore must be divided into a large number of segments so that the length of each segment (Δl) is very small and $\lim_{\Delta l \rightarrow 0} \frac{\Delta T_a}{\Delta P_a}$ is approximately equal to $\frac{dT_a}{dP_a}$. Here, a more practical approach is recommended. According to steam tables, there is one-to-one correspondence between saturated temperature and pressure. By regression analysis and polynomial interpolation, Ejiogu et al. [34] and Tortike et al. [33] proposed different empirical correlations to describe this relationship. In this paper, the empirical correlation suggested by Tortike et al. [33] is adopted due to its high accuracy, and the expression is given by:

$$T_a = f(P_a) = \frac{dT_a}{dP_a} = 280.034 + 14.0856 \ln \frac{P_a}{1000} + 1.38075 \left(\ln \frac{P_a}{1000} \right)^2 - 0.101806 \left(\ln \frac{P_a}{1000} \right)^3 + 0.019017 \left(\ln \frac{P_a}{1000} \right)^4, \quad 611 \text{ Pa} \leq P_a \leq 2.212 \times 10^7 \text{ Pa} \quad (\text{A20})$$

Figure A2 shows a comparison of the results from Equation (A20) and those from steam tables. It is obviously found that the empirical correlation agrees very well with steam-table data, also, the maximum and the mean absolute residuals are only 0.13% and 0.06%, respectively [33]. Therefore, Equation (A20) can be used to simulate the relationship between saturated temperature and pressure in the computer procedure. More importantly, the proposed empirical function is differentiable, which lays a foundation for further simplification of Equation (A19).

Figure A2. Comparison of empirical correlation with steam-table data.

In Figure A2, $\Delta l \rightarrow 0$ is equivalent to $P_{a,i+1} \rightarrow P_{a,i}$, so $\lim_{\Delta l \rightarrow 0} \left(\frac{\Delta T_a}{\Delta P_a} \right)$ can be developed into another form:

$$\lim_{\Delta l \rightarrow 0} \frac{\Delta T_a}{\Delta P_a} = \lim_{\Delta l \rightarrow 0} \frac{T_{a,i+1} - T_{a,i}}{P_{a,i+1} - P_{a,i}} = \lim_{\Delta l \rightarrow 0} \frac{f(P_{a,i+1}) - f(P_{a,i})}{P_{a,i+1} - P_{a,i}} = \lim_{P_{a,i+1} \rightarrow P_{a,i}} \frac{f(P_{a,i+1}) - f(P_{a,i})}{P_{a,i+1} - P_{a,i}} = \frac{df(P_a)}{dP_a} = \frac{dT_a}{dP_a} \quad (\text{A21})$$

where dT_a/dP_a can be obtained from Equation (A20):

$$\frac{dT_a}{dP_a} = \frac{1}{P_a} \left[14.0856 + 2.7615 \left(\ln \frac{P_a}{1000} \right) - 0.305418 \left(\ln \frac{P_a}{1000} \right)^2 + 0.076068 \left(\ln \frac{P_a}{1000} \right)^3 \right] \quad (\text{A22})$$

Substituting Equation (A21) into Equation (A19) yields the general relationship between temperature gradient and pressure gradient for saturated steam:

$$\frac{dT_a}{dl} = \frac{dT_a}{dP_a} \cdot \frac{dP_a}{dl} \quad (\text{A23})$$

Incorporating Equation (A23) into Equation (A18) reduces to the governing differential equation for steam pressure in the CD segment of the annulus.

$$\frac{dP_{a_CD}}{dl} = \frac{\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right)}{m_a (C_{Jma} C_{pma} - C_{pma} \frac{dT_a}{dP_a} + \frac{v_a v_{sga}}{P_a})} \quad (\text{A24})$$

Similarly, the steam pressure in the BC segment of the annulus and the AB segment of the annulus can be obtained, as shown in Equations (A25) and (A26), respectively.

$$\frac{dP_{a_BC}}{dl} = \frac{\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right)}{m_a (C_{Jma} C_{pma} - C_{pma} \frac{dT_a}{dP_a} + \frac{v_a v_{sga}}{P_a})} \quad (\text{A25})$$

$$\frac{dP_{a_AB}}{dl} = \frac{\frac{dQ_a}{dl} - \frac{dQ_s}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right)}{m_a (C_{Jma} C_{pma} - C_{pma} \frac{dT_a}{dP_a} + \frac{v_a v_{sga}}{P_a})} \quad (\text{A26})$$

(2) The calculation of pressure drop in the long string

When steam flows in the long string, the steam mass flow rate remains unchanged. In this case, the equations of mass conservation and energy conservation for steam flow in the long string are derived as follows:

$$\frac{dm_l}{dl} = 0 \quad (\text{A27})$$

$$\frac{dQ_l}{dl} = -\frac{d}{dl} \left[m_l \left(h_l + \frac{v_l^2}{2} \right) \right] \quad (\text{A28})$$

The calculation method of pressure drop in the long string is similar to that in the annulus. The pressure drop in the long string can be given by:

$$\frac{dP_l}{dl} = \frac{\frac{dQ_l}{dl}}{m_l (C_{jml} C_{pml} - C_{pml} \frac{dT_l}{dP_l} + \frac{v_l v_{sgl}}{P_l})} \quad (\text{A29})$$

Appendix B

Appendix B presents modeling of steam flow in the string and annulus considering phase change.

(1) Calculation of pressure and temperature in the CD segment of the annulus

The steam may condense to water in the horizontal wellbore due to heat losses. Thus, the mass, momentum and energy balance equations for water flow in the annulus are given as follows:

Mass balance equation:

$$\frac{dm_a}{dl} = -m_{af} \quad (\text{A30})$$

Momentum balance equation:

$$A_a \frac{dP_a}{dl} = -\frac{\tau_a}{dl} - m_a \frac{dv_a}{dl} - v_a \frac{dm_a}{dl} \quad (\text{A31})$$

Energy balance equation:

$$\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + m_{af} \left(h_w + \frac{v_{ar}^2}{2} \right) = -\frac{d}{dl} \left[m_a \left(h_w + \frac{v_a^2}{2} \right) \right] \quad (\text{A32})$$

Equation (A32) can be written as:

$$\frac{dQ_a}{dl} - \frac{dQ_l}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right) = -m_a \frac{dh_w}{dl} - m_a \frac{d}{dl} \left(\frac{v_a^2}{2} \right) \quad (\text{A33})$$

where v_{ar} is the radial outflow velocity of water, m/s; m_a is the mass flow rate of water, kg/s; m_{af} is the mass flow velocity of water from annulus to reservoir, kg/(m·s); P_a is the annulus pressure, Pa; A_a is the cross-sectional area of the annulus, m²; v_a is the water flow velocity in the annulus, m/s; τ_a is the friction force in the annulus; and h_a is the water enthalpy in the annulus, J/kg.

$\frac{dh_w}{dl}$ is the enthalpy gradient of water, which can be expressed as Equation (A34).

The enthalpy drop of water can be calculated as:

$$\frac{dh_w}{dl} = -\mu_w C_w \frac{dP_a}{dl} + C_w \frac{dT_a}{dl} \quad (\text{A34})$$

Using the definition of thermal expansion coefficient of water:

$$\beta_w = \frac{dV}{VdT} = -\frac{d\rho_w}{\rho_w dT_a} \quad (\text{A35})$$

where ρ_a is the water density, kg/m³.

The density of water can be expressed as:

$$\rho_w = C_0 e^{-\beta_w T_a} \quad (\text{A36})$$

The coefficient C_0 in Equation (A36) can be obtained from the initial condition: $\rho_w|_{T=20} = C_0 e^{-\beta_w 20}$, then the coefficient C_0 can be given by.

$$C_0 = \rho_w|_{T=20} / e^{-\beta_w 20} \quad (\text{A37})$$

Substituting Equation (A37) into Equation (A36):

$$d\left(\frac{1}{\rho_w}\right) = \frac{\beta_w e^{\beta_w T_a} e^{-\beta_w 20}}{\rho_w|_{T=20}} dT_a \quad (\text{A38})$$

According to the equation $v_a = m_a / (\rho_w A_a)$, we can obtain:

$$\frac{dv_a}{dl} = \frac{d(m_a / (\rho_w A_a))}{dl} = \frac{1}{\rho_w A_a} \frac{dm_a}{dl} + \frac{m_a}{A_a} \frac{d(1/\rho_w)}{dl} \quad (\text{A39})$$

Substituting Equations (A34) and (A39) into Equations (A31) and (A33), the pressure and temperature in the micro-segment in the CD segment of the annulus can be expressed as:

$$\frac{dP_a}{dl} = -\frac{1}{A_a} \left[\tau_a + 2v_a \frac{dm_a}{dl} + \frac{m_a^2}{A_a} \frac{\beta_w e^{\beta_w T_a} e^{-\beta_w 20}}{\rho_w|_{T=20}} \frac{dT_a}{dl} \right] \quad (\text{A40})$$

$$T_a = \exp\left(-\frac{C_2}{C_1} l\right) \cdot \left[-\frac{C_3}{C_2} \exp\left(\frac{C_2}{C_1} l\right) + T_{a0} + \frac{C_3}{C_2} \right] \quad (\text{A41})$$

where:

$$C_1 = m_a \left(C_w + \frac{m_a v_a}{A_a} \frac{\beta_w e^{\beta_w T_a} e^{-\beta_w 20}}{\rho_w|_{T=20}} \right)$$

$$C_2 = \frac{1}{R_a}$$

$$C_3 = -\frac{T_e}{R_a} + \frac{dW_a}{dl} + \frac{dm_a}{dl} \left(\frac{v_a^2 - v_{ar}^2}{2} \right) + m_a \left(-\mu_w C_w \frac{dP_a}{dl} + \frac{v_a}{\rho_w A_a} \frac{dm_a}{dl} \right)$$

Similarly, the pressure and temperature in the BC segment of the annulus and the AB segment of the annulus can be obtained by using the same method.

(2) Calculation of pressure and temperature in the long string

The calculation method of pressure and temperature in the long string is similar to that in the annulus. The pressure and temperature in the long string can be given by.

$$\frac{dP_l}{dl} = -\frac{1}{A_l} \left[\tau_l + \frac{m_l^2}{A_l} \frac{\beta_w e^{\beta_w T_l} e^{-\beta_w 20}}{\rho_w|_{T=20}} \frac{dT_l}{dl} \right] \quad (\text{A42})$$

$$T_l = \exp\left(-\frac{C_2}{C_1} l\right) \cdot \left[-\frac{C_3}{C_2} \exp\left(\frac{C_2}{C_1} l\right) + T_{l0} + \frac{C_3}{C_2} \right] \quad (\text{A43})$$

where:

$$C_1 = m_l \left(C_w + \frac{m_l v_l}{A_l} \frac{\beta_w e^{\beta_w T_l} e^{-\beta_w 20}}{\rho_w|_{T=20}} \right)$$

$$C_2 = \frac{1}{R_l}$$

$$C_3 = -\frac{T_a}{R_l} + \frac{dW_l}{dl} - m_l \mu_w C_w \frac{dP_l}{dl}$$

Appendix C

Appendix C presents the linearization of reservoir equations.

The reservoir seepage and heat transfer model is a strong nonlinear model. To solve the non-linear equations, the equations need to be transformed into linear equations first.

The difference equations of the mathematical models of seepage and heat transfer in reservoirs can be derived as follows:

$$\Delta T_o^{l+1} \Delta P_o^{l+1} - \Delta T_o^{l+1} \Delta (\rho_o g D)^{l+1} + M_{oi,j,k}^{l+1} = \frac{V_B}{\Delta t} [(\rho_o \varphi S_o)_{i,j,k}^{l+1} - (\rho_o \varphi S_o)_{i,j,k}^l] \quad (A44)$$

$$\Delta T_w^{l+1} \Delta P_o^{l+1} - \Delta T_w^{l+1} \Delta (P_{cow} + \rho_w g D)^{l+1} + M_{wi,j,k}^{l+1} + \Delta T_g^{l+1} \Delta P_o^{l+1} + \Delta T_g^{l+1} \Delta (P_{cgo} - \rho_g g D)^{l+1} + M_{gi,j,k}^{l+1} = \frac{V_B}{\Delta t} [(\rho_w \varphi S_w)_{i,j,k}^{l+1} - (\rho_w \varphi S_w)_{i,j,k}^l] + \frac{V_B}{\Delta t} [(\rho_g \varphi S_g)_{i,j,k}^{l+1} - (\rho_g \varphi S_g)_{i,j,k}^l] \quad (A45)$$

$$\Delta T_R^{l+1} \Delta T^{l+1} + \Delta T_{ho}^{l+1} \Delta (P_o - \rho_o g D)^{l+1} + \Delta T_{hw}^{l+1} \Delta (P_o - P_{cow} - \rho_w g D)^{l+1} + \Delta T_{hg}^{l+1} \Delta (P_o + P_{cgo} - \rho_g g D)^{l+1} - Q_{lossi,j,k}^{l+1} + Q_{Hi,j,k}^{l+1} + Q_{welli,j,k}^{l+1} = \frac{V_B}{\Delta t} \left[((1-\varphi)M_R T)_{i,j,k}^{l+1} - ((1-\varphi)M_R T)_{i,j,k}^l + (\varphi(S_o \rho_o U_o + S_w \rho_w U_w + S_g \rho_g U_g))_{i,j,k}^{l+1} - (\varphi(S_o \rho_o U_o + S_w \rho_w U_w + S_g \rho_g U_g))_{i,j,k}^l \right] \quad (A46)$$

$$T^{l+1} = f(P)^{l+1} \quad (A47)$$

where i, j and k are the grid coordinates in the x, y, z directions, respectively; T_o, T_w and T_g are the mass transfer coefficients of oil, water and steam in the grid, respectively, $(\text{kg} \cdot \text{mD})/(\text{m}^2 \cdot \text{mPa} \cdot \text{s})$; M_o, M_w and M_g are the mass flow velocities of oil, water and steam in the grid (injection is “+”, and production is “−”), respectively, kg/s ; T_{ho}, T_{hw} and T_{hg} are the thermal conductivities related to the thermal convection of oil, water and steam in the grid, respectively, $(\text{kg} \cdot \text{mD})/(\text{m}^2 \cdot \text{mPa} \cdot \text{s})$; T_R is the thermal conductivity related to heat conduction, $(\text{kg} \cdot \text{mD})/(\text{m}^2 \cdot \text{mPa} \cdot \text{s})$; M_R is the volumetric heat capacity, $\text{J}/(\text{m}^3 \cdot ^\circ\text{C})$; V_B is the grid volume, m^3 ; Q_{well} is the heat exchange rate between the wellbore and reservoir, J/s .

Equations (A44)–(A47) contain five unknowns: P_o, S_o, S_w, S_g and T . As S_o can be calculated from the saturation equation $S_o + S_w + S_g = 1$ according to S_w and S_g , therefore, Equations (A44)–(A47) can be simplified into the form of $AX = B$ using a fully implicit finite difference method. The coefficient matrix A is a seven-diagonal 4×4 block matrix, and the unknowns $X = [\delta P_o, \delta T, \delta S_w, \delta S_g]^T$. Finally, the equation $AX = B$ can be solved by an iterative technique.

The relative permeability of the oil phase is a function of both water saturation and gas saturation, and the densities of oil and water are functions of both temperature and pressure.

$$k_{ro} = k_{rowc} \left[\left(\frac{k_{row}}{k_{rowc}} + k_{rw} \right) \left(\frac{k_{rog}}{k_{rowc}} + k_{rg} \right) - k_{rw} - k_{rg} \right] \quad (A48)$$

$$\rho_o = \rho_{or} [1 + C_{op}(P - P_r) - C_{ot}(T - T_r)] \quad (A49)$$

$$\rho_w = \rho_{wr} [1 + C_{wp}(P - P_r) - C_{wt}(T - T_r)] \quad (A50)$$

Here k_{rowc} is the relative permeability of the oil phase at irreducible water saturation; k_{row} is the relative permeability of the oil phase in the oil-water system; k_{rog} is the relative permeability of

the oil phase in the oil-gas system; ρ_{or} and ρ_{wr} are the densities of oil and water at the reference temperature and pressure, respectively, kg/m^3 ; C_{op} and C_{wp} are the compressibility value of oil and water, respectively, $1/\text{MPa}$; C_{ot} and C_{wt} are the coefficients of thermal expansion of oil and water, respectively, $1/^\circ\text{C}$; P_r is the reference pressure, Pa; and T_r is the reference temperature, $^\circ\text{C}$.

The oil phase difference equation is taken as an example to illustrate the linearization process of the equations.

① The first two terms on the left side of the oil phase difference Equation (A44) can be derived as follows:

$$\begin{aligned}
 & \Delta T_o^{l+1} \Delta P_o^{l+1} - \Delta T_o^{l+1} \Delta (\rho_o g D)^{l+1} = \Delta T_o^{l+1} \Phi_o^{l+1} \\
 & = T_{oi+\frac{1}{2},j,k}^{l+1} (\Phi_{oi+1,j,k}^{l+1} - \Phi_{oi,j,k}^{l+1}) + T_{oi-\frac{1}{2},j,k}^{l+1} (\Phi_{oi-1,j,k}^{l+1} - \Phi_{oi,j,k}^{l+1}) + T_{oi,j+\frac{1}{2},k}^{l+1} (\Phi_{oi,j+1,k}^{l+1} - \Phi_{oi,j,k}^{l+1}) \\
 & + T_{oi,j-\frac{1}{2},k}^{l+1} (\Phi_{oi,j-1,k}^{l+1} - \Phi_{oi,j,k}^{l+1}) + T_{oi,j,k+\frac{1}{2}}^{l+1} (\Phi_{oi,j,k+1}^{l+1} - \Phi_{oi,j,k}^{l+1}) + T_{oi,j,k-\frac{1}{2}}^{l+1} (\Phi_{oi,j,k-1}^{l+1} - \Phi_{oi,j,k}^{l+1}) \\
 & = \left(T_{oi+\frac{1}{2},j,k}^l + \delta T_{oi+\frac{1}{2},j,k} \right) \left(\Phi_{oi+1,j,k}^l + \delta \Phi_{oi+1,j,k} - \Phi_{oi,j,k}^l - \delta \Phi_{oi,j,k} \right) + \\
 & \left(T_{oi-\frac{1}{2},j,k}^l + \delta T_{oi-\frac{1}{2},j,k} \right) \left(\Phi_{oi-1,j,k}^l + \delta \Phi_{oi-1,j,k} - \Phi_{oi,j,k}^l - \delta \Phi_{oi,j,k} \right) + \\
 & \left(T_{oi,j+\frac{1}{2},k}^l + \delta T_{oi,j+\frac{1}{2},k} \right) \left(\Phi_{oi,j+1,k}^l + \delta \Phi_{oi,j+1,k} - \Phi_{oi,j,k}^l - \delta \Phi_{oi,j,k} \right) + \\
 & \left(T_{oi,j-\frac{1}{2},k}^l + \delta T_{oi,j-\frac{1}{2},k} \right) \left(\Phi_{oi,j-1,k}^l + \delta \Phi_{oi,j-1,k} - \Phi_{oi,j,k}^l - \delta \Phi_{oi,j,k} \right) + \\
 & \left(T_{oi,j,k+\frac{1}{2}}^l + \delta T_{oi,j,k+\frac{1}{2}} \right) \left(\Phi_{oi,j,k+1}^l + \delta \Phi_{oi,j,k+1} - \Phi_{oi,j,k}^l - \delta \Phi_{oi,j,k} \right) + \\
 & \left(T_{oi,j,k-\frac{1}{2}}^l + \delta T_{oi,j,k-\frac{1}{2}} \right) \left(\Phi_{oi,j,k-1}^l + \delta \Phi_{oi,j,k-1} - \Phi_{oi,j,k}^l - \delta \Phi_{oi,j,k} \right)
 \end{aligned} \tag{A51}$$

The term $(T_{oi\pm\frac{1}{2},j,k} = \frac{(Ak)_{i\pm\frac{1}{2},j,k}}{0.5(\Delta x_i + \Delta x_{i\pm 1})} \left(\frac{k_{ro} \rho_o}{\mu_o} \right)_{i\pm\frac{1}{2},j,k})$ in Equation (A50) can be derived as follows:

$$\begin{aligned}
 \delta T_{oi\pm\frac{1}{2},j,k} & = \frac{(Ak)_{i\pm\frac{1}{2},j,k}}{0.5(\Delta x_i + \Delta x_{i\pm 1})} \delta \left(\frac{k_{ro} \rho_o}{\mu_o} \right)_{i\pm\frac{1}{2},j,k} \\
 & = \frac{(Ak)_{i\pm\frac{1}{2},j,k}}{0.5(\Delta x_i + \Delta x_{i\pm 1})} \left(\frac{\partial}{\partial P_o} \left(\frac{k_{ro} \rho_o}{\mu_o} \right) \delta P_o + \frac{\partial}{\partial T} \left(\frac{k_{ro} \rho_o}{\mu_o} \right) \delta T + \right. \\
 & \quad \left. \frac{\partial}{\partial S_w} \left(\frac{k_{ro} \rho_o}{\mu_o} \right) \delta S_w + \frac{\partial}{\partial S_g} \left(\frac{k_{ro} \rho_o}{\mu_o} \right) \delta S_g \right)_{i\pm\frac{1}{2},j,k} \\
 & = \frac{(Ak)_{i\pm\frac{1}{2},j,k}}{0.5(\Delta x_i + \Delta x_{i\pm 1})} \left(\frac{k_{ro}}{\mu_o} \frac{\partial \rho_o}{\partial P_o} \delta P_o + \frac{k_{ro}}{\mu_o} \frac{\partial \rho_o}{\partial T} \delta T + k_{ro} \rho_o \frac{\partial}{\partial T} \left(\frac{1}{\mu_o} \right) \delta T + \right. \\
 & \quad \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i\pm\frac{1}{2},j,k} \\
 & = \frac{(Ak)_{i\pm\frac{1}{2},j,k}}{0.5(\Delta x_i + \Delta x_{i\pm 1})} \left(\frac{k_{ro} \rho_o C_{op}}{\mu_o} \delta P_o - \frac{k_{ro} \rho_o C_{op}}{\mu_o} \delta T - \frac{k_{ro} \rho_o}{\mu_o^2} \frac{\partial \mu_o}{\partial T} \delta T + \right. \\
 & \quad \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i\pm\frac{1}{2},j,k}
 \end{aligned} \tag{A52}$$

Similarly, terms $\delta T_{oi,j\pm\frac{1}{2},k}$ and $\delta T_{oi,j,k\pm\frac{1}{2}}$ in Equation (A50) can be derived as follows:

$$\begin{aligned}\delta T_{oi,j\pm\frac{1}{2},k} &= \frac{(Ak)_{i,j\pm\frac{1}{2},k}}{0.5(\Delta y_j + \Delta y_{j\pm 1})} \delta \left(\frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j\pm\frac{1}{2},k} \\ &= \frac{(Ak)_{i,j\pm\frac{1}{2},k}}{0.5(\Delta y_j + \Delta y_{j\pm 1})} \left(\frac{\partial}{\partial P_o} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta P_o + \frac{\partial}{\partial T} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta T + \right. \\ &\quad \left. \frac{\partial}{\partial S_w} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta S_w + \frac{\partial}{\partial S_g} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta S_g \right)_{i,j\pm\frac{1}{2},k} \\ &= \frac{(Ak)_{i,j\pm\frac{1}{2},k}}{0.5(\Delta y_j + \Delta y_{j\pm 1})} \left(\frac{k_{ro}}{\mu_o} \frac{\partial \rho_o}{\partial P_o} \delta P_o + \frac{k_{ro}}{\mu_o} \frac{\partial \rho_o}{\partial T} \delta T + k_{ro}\rho_o \frac{\partial}{\partial T} \left(\frac{1}{\mu_o} \right) \delta T + \right. \\ &\quad \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i,j\pm\frac{1}{2},k} \\ &= \frac{(Ak)_{i,j\pm\frac{1}{2},k}}{0.5(\Delta y_j + \Delta y_{j\pm 1})} \left(\frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta P_o - \frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta T - \frac{k_{ro}\rho_o}{\mu_o^2} \frac{\partial \mu_o}{\partial T} \delta T + \right. \\ &\quad \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i,j\pm\frac{1}{2},k}\end{aligned}\quad (A53)$$

$$\begin{aligned}\delta T_{oi,j,k\pm\frac{1}{2}} &= \frac{(Ak)_{i,j,k\pm\frac{1}{2}}}{0.5(\Delta z_k + \Delta z_{k\pm 1})} \delta \left(\frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k\pm\frac{1}{2}} \\ &= \frac{(Ak)_{i,j,k\pm\frac{1}{2}}}{0.5(\Delta z_k + \Delta z_{k\pm 1})} \left(\frac{\partial}{\partial P_o} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta P_o + \frac{\partial}{\partial T} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta T + \right. \\ &\quad \left. \frac{\partial}{\partial S_w} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta S_w + \frac{\partial}{\partial S_g} \left(\frac{k_{ro}\rho_o}{\mu_o} \right) \delta S_g \right)_{i,j,k\pm\frac{1}{2}} \\ &= \frac{(Ak)_{i,j,k\pm\frac{1}{2}}}{0.5(\Delta z_k + \Delta z_{k\pm 1})} \left(\frac{k_{ro}}{\mu_o} \frac{\partial \rho_o}{\partial P_o} \delta P_o + \frac{k_{ro}}{\mu_o} \frac{\partial \rho_o}{\partial T} \delta T + k_{ro}\rho_o \frac{\partial}{\partial T} \left(\frac{1}{\mu_o} \right) \delta T + \right. \\ &\quad \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i,j,k\pm\frac{1}{2}} \\ &= \frac{(Ak)_{i,j,k\pm\frac{1}{2}}}{0.5(\Delta z_k + \Delta z_{k\pm 1})} \left(\frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta P_o - \frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta T - \frac{k_{ro}\rho_o}{\mu_o^2} \frac{\partial \mu_o}{\partial T} \delta T + \right. \\ &\quad \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i,j,k\pm\frac{1}{2}}\end{aligned}\quad (A54)$$

Finally, Equation (A50) can be expressed as:

$$\begin{aligned}\Delta T_o^{l+1} \Delta P_o^{l+1} - \Delta T_o^{l+1} \Delta (\rho_o g D)^{l+1} &= \Delta T_o^{l+1} \Phi_o^{l+1} \\ &= T_{oi+\frac{1}{2},j,k}^l \left(\Phi_{oi+1,j,k}^l - \Phi_{oi,j,k}^l \right) + T_{oi+\frac{1}{2},j,k}^l \left(\delta P_{oi+1,j,k}^l - \delta P_{oi,j,k}^l \right) + \\ &\quad \left(\frac{\partial T_o}{\partial P_o} \delta P_o + \frac{\partial T_o}{\partial T} \delta T + \frac{\partial T_o}{\partial S_w} \delta S_w + \frac{\partial T_o}{\partial S_g} \delta S_g \right)_{i+\frac{1}{2},j,k} \left(\Phi_{oi+1,j,k}^l - \Phi_{oi,j,k}^l \right) + \\ &\quad T_{oi-\frac{1}{2},j,k}^l \left(\Phi_{oi-1,j,k}^l - \Phi_{oi,j,k}^l \right) + T_{oi-\frac{1}{2},j,k}^l \left(\delta P_{oi-1,j,k}^l - \delta P_{oi,j,k}^l \right) + \\ &\quad \left(\frac{\partial T_o}{\partial P_o} \delta P_o + \frac{\partial T_o}{\partial T} \delta T + \frac{\partial T_o}{\partial S_w} \delta S_w + \frac{\partial T_o}{\partial S_g} \delta S_g \right)_{i-\frac{1}{2},j,k} \left(\Phi_{oi-1,j,k}^l - \Phi_{oi,j,k}^l \right) + \\ &\quad T_{oi,j+\frac{1}{2},k}^l \left(\Phi_{oi,j+1,k}^l - \Phi_{oi,j,k}^l \right) + T_{oi,j+\frac{1}{2},k}^l \left(\delta P_{oi,j+1,k}^l - \delta P_{oi,j,k}^l \right) + \\ &\quad \left(\frac{\partial T_o}{\partial P_o} \delta P_o + \frac{\partial T_o}{\partial T} \delta T + \frac{\partial T_o}{\partial S_w} \delta S_w + \frac{\partial T_o}{\partial S_g} \delta S_g \right)_{i,j+\frac{1}{2},k} \left(\Phi_{oi,j+1,k}^l - \Phi_{oi,j,k}^l \right) + \\ &\quad T_{oi,j-\frac{1}{2},k}^l \left(\Phi_{oi,j-1,k}^l - \Phi_{oi,j,k}^l \right) + T_{oi,j-\frac{1}{2},k}^l \left(\delta P_{oi,j-1,k}^l - \delta P_{oi,j,k}^l \right) + \\ &\quad \left(\frac{\partial T_o}{\partial P_o} \delta P_o + \frac{\partial T_o}{\partial T} \delta T + \frac{\partial T_o}{\partial S_w} \delta S_w + \frac{\partial T_o}{\partial S_g} \delta S_g \right)_{i,j-\frac{1}{2},k} \left(\Phi_{oi,j-1,k}^l - \Phi_{oi,j,k}^l \right) + \\ &\quad T_{oi,j,k+\frac{1}{2}}^l \left(\Phi_{oi,j,k+1}^l - \Phi_{oi,j,k}^l \right) + T_{oi,j,k+\frac{1}{2}}^l \left(\delta P_{oi,j,k+1}^l - \delta P_{oi,j,k}^l \right) + \\ &\quad \left(\frac{\partial T_o}{\partial P_o} \delta P_o + \frac{\partial T_o}{\partial T} \delta T + \frac{\partial T_o}{\partial S_w} \delta S_w + \frac{\partial T_o}{\partial S_g} \delta S_g \right)_{i,j,k+\frac{1}{2}} \left(\Phi_{oi,j,k+1}^l - \Phi_{oi,j,k}^l \right) + \\ &\quad T_{oi,j,k-\frac{1}{2}}^l \left(\Phi_{oi,j,k-1}^l - \Phi_{oi,j,k}^l \right) + T_{oi,j,k-\frac{1}{2}}^l \left(\delta P_{oi,j,k-1}^l - \delta P_{oi,j,k}^l \right) + \\ &\quad \left(\frac{\partial T_o}{\partial P_o} \delta P_o + \frac{\partial T_o}{\partial T} \delta T + \frac{\partial T_o}{\partial S_w} \delta S_w + \frac{\partial T_o}{\partial S_g} \delta S_g \right)_{i,j,k-\frac{1}{2}} \left(\Phi_{oi,j,k-1}^l - \Phi_{oi,j,k}^l \right)\end{aligned}\quad (A55)$$

② The source and sink terms of oil phase difference Equation (A44) can be treated as follows:

$$\begin{aligned} M_{oi,j,k}^{l+1} &= \left(\frac{2\pi kh}{\ln \frac{r_e}{r_w} + S - 0.75} \frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k}^{l+1} (P_{oi,j,k}^{l+1} - P_{wfi,j,k}^{l+1}) \\ &= \left(WI \frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k}^{l+1} (P_{oi,j,k}^{l+1} - P_{wfi,j,k}^{l+1}) \end{aligned} \quad (A56)$$

WI can be expressed as:

$$WI = \frac{2\pi hk}{\ln \frac{r_e}{r_w} + S - 0.75} \quad (A57)$$

When the bottom-hole pressure is fixed for oil production, $P_{wfi,j,k}^{l+1}$ is known, and Equation (A55) can be simplified as:

$$\begin{aligned} M_{oi,j,k}^{l+1} &= \left(WI \frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k}^{l+1} (P_{oi,j,k}^{l+1} - P_{wfi,j,k}^{l+1}) \\ &= WI \left[\left(\frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k}^l + \delta \left(\frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k} \right] (P_{oi,j,k}^l + \delta P_{oi,j,k} - P_{wfi,j,k}^l) \\ &= WI \left[\left(\frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k}^l + \left(\frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta P_o - \frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta T - \frac{k_{ro}\rho_o}{\mu_o^2} \frac{\partial \mu_o}{\partial T} \delta T + \right. \right. \\ &\quad \left. \left. \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i,j,k} \right] (P_{oi,j,k}^l + \delta P_{oi,j,k} - P_{wfi,j,k}^l) \\ &= WI \left(\frac{k_{ro}\rho_o}{\mu_o} \right)_{i,j,k}^l (P_{oi,j,k}^l + \delta P_{oi,j,k} - P_{wfi,j,k}^l) + \\ &\quad WI \left(\frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta P_o - \frac{k_{ro}\rho_o C_{op}}{\mu_o} \delta T - \frac{k_{ro}\rho_o}{\mu_o^2} \frac{\partial \mu_o}{\partial T} \delta T + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_w} \delta S_w + \frac{\rho_o}{\mu_o} \frac{\partial k_{ro}}{\partial S_g} \delta S_g \right)_{i,j,k} (P_{oi,j,k}^l - P_{wfi,j,k}^l) \end{aligned} \quad (A58)$$

③ The term on the right side of the oil phase difference Equation (A44) can be derived as follows:

$$\begin{aligned} &\frac{V_B}{\Delta t} ((\rho_o \varphi S_o)_{i,j,k}^{l+1} - (\rho_o \varphi S_o)_{i,j,k}^n) \\ &= \frac{V_B}{\Delta t} ((\rho_o \varphi S_o)_{i,j,k}^l + \delta(\rho_o \varphi S_o)_{i,j,k} - (\rho_o \varphi S_o)_{i,j,k}^n) \\ &= \frac{V_B}{\Delta t} ((\rho_o \varphi S_o)_{i,j,k}^l + (S_o \delta(\rho_o \varphi) + \rho_o \varphi \delta S_o)_{i,j,k} - (\rho_o \varphi S_o)_{i,j,k}^n) \\ &= \frac{V_B}{\Delta t} \left\{ \left[S_o \left(\varphi \frac{\partial \rho_o}{\partial P_o} \delta P_o + \varphi \frac{\partial \rho_o}{\partial T} \delta T + \rho_o \frac{\partial \varphi}{\partial P_o} \delta P_o \right) - \rho_o \varphi \delta(S_w + S_g) \right]_{i,j,k} + \right. \\ &\quad \left. (\rho_o \varphi S_o)_{i,j,k}^l - (\rho_o \varphi S_o)_{i,j,k}^n \right\} \end{aligned} \quad (A59)$$

In summary, Equations (A54), (A57) and (A58) constitute a complete linearization coefficient difference equation of the oil phase, and the variables in the equation are δP_o , δT , δS_w and δS_g .

Similarly, the water and steam phase difference Equation (A45) can be derived as follows:

$$\begin{aligned} &T_{wi+\frac{1}{2},j,k}^l (\Phi_{wi+1,j,k}^l - \Phi_{wi,j,k}^l) + T_{wi+\frac{1}{2},j,k}^l (\delta P_{oi+1,j,k}^l - \delta P_{oi,j,k}^l) - T_{wi+\frac{1}{2},j,k}^l (\delta P_{cowi+1,j,k}^l - \delta P_{cowi,j,k}^l) \\ &+ \left(\frac{\partial T_w}{\partial P_o} \delta P_o + \frac{\partial T_w}{\partial T} \delta T + \frac{\partial T_w}{\partial S_w} \delta S_w \right)_{i+\frac{1}{2},j,k} (\Phi_{wi+1,j,k}^l - \Phi_{wi,j,k}^l) + \\ &T_{wi-\frac{1}{2},j,k}^l (\Phi_{wi-1,j,k}^l - \Phi_{wi,j,k}^l) + T_{wi-\frac{1}{2},j,k}^l (\delta P_{oi-1,j,k}^l - \delta P_{oi,j,k}^l) - T_{wi-\frac{1}{2},j,k}^l (\delta P_{cowi-1,j,k}^l - \delta P_{cowi,j,k}^l) \\ &+ \left(\frac{\partial T_w}{\partial P_o} \delta P_o + \frac{\partial T_w}{\partial T} \delta T + \frac{\partial T_w}{\partial S_w} \delta S_w \right)_{i-\frac{1}{2},j,k} (\Phi_{wi-1,j,k}^l - \Phi_{wi,j,k}^l) + \end{aligned} \quad (A60)$$

$$\begin{aligned}
& T_{wi,j+\frac{1}{2},k}^l \left(\Phi_{wi,j+1,k}^l - \Phi_{wi,j,k}^l \right) + T_{wi,j+\frac{1}{2},k}^l \left(\delta P_{oi,j+1,k}^l - \delta P_{oi,j,k}^l \right) - T_{wi,j+\frac{1}{2},k}^l \left(\delta P_{cowi,j+1,k}^l - \delta P_{cowi,j,k}^l \right) \\
& + \left(\frac{\partial T_w}{\partial P_o} \delta P_o + \frac{\partial T_w}{\partial T} \delta T + \frac{\partial T_w}{\partial S_w} \delta S_w \right)_{i,j+\frac{1}{2},k} \left(\Phi_{wi,j+1,k}^l - \Phi_{wi,j,k}^l \right) + \\
& T_{wi,j-\frac{1}{2},k}^l \left(\Phi_{wi,j-1,k}^l - \Phi_{wi,j,k}^l \right) + T_{wi,j-\frac{1}{2},k}^l \left(\delta P_{oi,j-1,k}^l - \delta P_{oi,j,k}^l \right) - T_{wi,j-\frac{1}{2},k}^l \left(\delta P_{cowi,j-1,k}^l - \delta P_{cowi,j,k}^l \right) \\
& + \left(\frac{\partial T_w}{\partial P_o} \delta P_o + \frac{\partial T_w}{\partial T} \delta T + \frac{\partial T_w}{\partial S_w} \delta S_w \right)_{i,j-\frac{1}{2},k} \left(\Phi_{wi,j-1,k}^l - \Phi_{wi,j,k}^l \right) + \\
& T_{wi,j,k+\frac{1}{2}}^l \left(\Phi_{wi,j,k+1}^l - \Phi_{wi,j,k}^l \right) + T_{wi,j,k+\frac{1}{2}}^l \left(\delta P_{oi,j,k+1}^l - \delta P_{oi,j,k}^l \right) - T_{wi,j,k+\frac{1}{2}}^l \left(\delta P_{cowi,j,k+1}^l - \delta P_{cowi,j,k}^l \right) \\
& + \left(\frac{\partial T_w}{\partial P_o} \delta P_o + \frac{\partial T_w}{\partial T} \delta T + \frac{\partial T_w}{\partial S_w} \delta S_w \right)_{i,j,k+\frac{1}{2}} \left(\Phi_{wi,j,k+1}^l - \Phi_{wi,j,k}^l \right) + \\
& T_{wi,j,k-\frac{1}{2}}^l \left(\Phi_{wi,j,k-1}^l - \Phi_{wi,j,k}^l \right) + T_{wi,j,k-\frac{1}{2}}^l \left(\delta P_{oi,j,k-1}^l - \delta P_{oi,j,k}^l \right) - T_{wi,j,k-\frac{1}{2}}^l \left(\delta P_{cowi,j,k-1}^l - \delta P_{cowi,j,k}^l \right) \\
& + \left(\frac{\partial T_w}{\partial P_o} \delta P_o + \frac{\partial T_w}{\partial T} \delta T + \frac{\partial T_w}{\partial S_w} \delta S_w \right)_{i,j,k-\frac{1}{2}} \left(\Phi_{wi,j,k-1}^l - \Phi_{wi,j,k}^l \right) + \\
& T_{gi+\frac{1}{2},j,k}^l \left(\Phi_{gi+1,j,k}^l - \Phi_{gi,j,k}^l \right) + T_{gi+\frac{1}{2},j,k}^l \left(\delta P_{oi+1,j,k}^l - \delta P_{oi,j,k}^l \right) + T_{gi+\frac{1}{2},j,k}^l \left(\delta P_{cgoi+1,j,k}^l - \delta P_{cgoi,j,k}^l \right) \\
& + \left(\frac{\partial T_g}{\partial T} \delta T + \frac{\partial T_g}{\partial S_g} \delta S_g \right)_{i+\frac{1}{2},j,k} \left(\Phi_{gi+1,j,k}^l - \Phi_{gi,j,k}^l \right) + \\
& T_{gi-\frac{1}{2},j,k}^l \left(\Phi_{gi-1,j,k}^l - \Phi_{gi,j,k}^l \right) + T_{gi-\frac{1}{2},j,k}^l \left(\delta P_{oi-1,j,k}^l - \delta P_{oi,j,k}^l \right) + T_{gi-\frac{1}{2},j,k}^l \left(\delta P_{cgoi-1,j,k}^l - \delta P_{cgoi,j,k}^l \right) \\
& + \left(\frac{\partial T_g}{\partial T} \delta T + \frac{\partial T_g}{\partial S_g} \delta S_g \right)_{i-\frac{1}{2},j,k} \left(\Phi_{gi-1,j,k}^l - \Phi_{gi,j,k}^l \right) + \\
& T_{gi,j+\frac{1}{2},k}^l \left(\Phi_{gi,j+1,k}^l - \Phi_{gi,j,k}^l \right) + T_{gi,j+\frac{1}{2},k}^l \left(\delta P_{oi,j+1,k}^l - \delta P_{oi,j,k}^l \right) + T_{gi,j+\frac{1}{2},k}^l \left(\delta P_{cgoi,j+1,k}^l - \delta P_{cgoi,j,k}^l \right) \\
& + \left(\frac{\partial T_g}{\partial T} \delta T + \frac{\partial T_g}{\partial S_g} \delta S_g \right)_{i,j+\frac{1}{2},k} \left(\Phi_{gi,j+1,k}^l - \Phi_{gi,j,k}^l \right) + \\
& T_{gi,j-\frac{1}{2},k}^l \left(\Phi_{gi,j-1,k}^l - \Phi_{gi,j,k}^l \right) + T_{gi,j-\frac{1}{2},k}^l \left(\delta P_{oi,j-1,k}^l - \delta P_{oi,j,k}^l \right) + T_{gi,j-\frac{1}{2},k}^l \left(\delta P_{cgoi,j-1,k}^l - \delta P_{cgoi,j,k}^l \right) \\
& + \left(\frac{\partial T_g}{\partial T} \delta T + \frac{\partial T_g}{\partial S_g} \delta S_g \right)_{i,j-\frac{1}{2},k} \left(\Phi_{gi,j-1,k}^l - \Phi_{gi,j,k}^l \right) + \\
& T_{gi,j,k+\frac{1}{2}}^l \left(\Phi_{gi,j,k+1}^l - \Phi_{gi,j,k}^l \right) + T_{gi,j,k+\frac{1}{2}}^l \left(\delta P_{oi,j,k+1}^l - \delta P_{oi,j,k}^l \right) + T_{gi,j,k+\frac{1}{2}}^l \left(\delta P_{cgoi,j,k+1}^l - \delta P_{cgoi,j,k}^l \right) \\
& + \left(\frac{\partial T_g}{\partial T} \delta T + \frac{\partial T_g}{\partial S_g} \delta S_g \right)_{i,j,k+\frac{1}{2}} \left(\Phi_{gi,j,k+1}^l - \Phi_{gi,j,k}^l \right) + \\
& T_{gi,j,k-\frac{1}{2}}^l \left(\Phi_{gi,j,k-1}^l - \Phi_{gi,j,k}^l \right) + T_{gi,j,k-\frac{1}{2}}^l \left(\delta P_{oi,j,k-1}^l - \delta P_{oi,j,k}^l \right) + T_{gi,j,k-\frac{1}{2}}^l \left(\delta P_{cgoi,j,k-1}^l - \delta P_{cgoi,j,k}^l \right) \\
& + \left(\frac{\partial T_g}{\partial T} \delta T + \frac{\partial T_g}{\partial S_g} \delta S_g \right)_{i,j,k-\frac{1}{2}} \left(\Phi_{gi,j,k-1}^l - \Phi_{gi,j,k}^l \right) + M_{wi,j,k}^{l+1} + M_{gi,j,k}^{l+1} \\
& = \frac{V_B}{\Delta t} \left((\rho_w \varphi S_w)_{i,j,k}^l + \delta (\rho_w \varphi S_w)_{i,j,k} - (\rho_w \varphi S_w)_{i,j,k}^n \right) + \frac{V_B}{\Delta t} \left((\rho_g \varphi S_g)_{i,j,k}^l + \delta (\rho_g \varphi S_g)_{i,j,k} - (\rho_g \varphi S_g)_{i,j,k}^n \right) \\
& = \frac{V_B}{\Delta t} \left\{ \begin{aligned} & (\rho_w \varphi S_w)_{i,j,k}^l - (\rho_w \varphi S_w)_{i,j,k}^n + \left[\rho_w \varphi \delta S_w + S_w \varphi \frac{\partial \rho_w}{\partial P_o} \delta P_o + S_w \varphi \frac{\partial \rho_w}{\partial T} \delta T + S_w \rho_w \frac{\partial \varphi}{\partial P_o} \delta P_o \right]_{i,j,k} + \\ & (\rho_g \varphi S_g)_{i,j,k}^l - (\rho_g \varphi S_g)_{i,j,k}^n + \left[\rho_g \varphi \delta S_g + S_g \varphi \frac{\partial \rho_g}{\partial T} \delta T + S_g \rho_g \frac{\partial \varphi}{\partial P_o} \delta P_o \right]_{i,j,k} \end{aligned} \right\}
\end{aligned}$$

The linearized energy difference Equation (A46) is:

[illegible]

$$\begin{aligned}
& T_{\text{hgi}-\frac{1}{2},j,k}^l \left(\Phi_{\text{gi}-1,j,k}^l - \Phi_{\text{gi},j,k}^l \right) + T_{\text{hgi}-\frac{1}{2},j,k}^l \left(\delta P_{\text{oi}-1,j,k}^l - \delta P_{\text{oi},j,k}^l \right) + T_{\text{hgi}-\frac{1}{2},j,k}^l \left(\delta P_{\text{cgoi}-1,j,k}^l - \delta P_{\text{cgoi},j,k}^l \right) \\
& + \left(\frac{\partial T_{\text{hg}}}{\partial T} \delta T + \frac{\partial T_{\text{hg}}}{\partial S_g} \delta S_g \right)_{i-\frac{1}{2},j,k} \left(\Phi_{\text{gi}-1,j,k}^l - \Phi_{\text{gi},j,k}^l \right) + \\
& T_{\text{hgi},j+\frac{1}{2},k}^l \left(\Phi_{\text{gi},j+1,k}^l - \Phi_{\text{gi},j,k}^l \right) + T_{\text{hgi},j+\frac{1}{2},k}^l \left(\delta P_{\text{oi},j+1,k}^l - \delta P_{\text{oi},j,k}^l \right) + T_{\text{hgi},j+\frac{1}{2},k}^l \left(\delta P_{\text{cgoi},j+1,k}^l - \delta P_{\text{cgoi},j,k}^l \right) + \\
& \left(\frac{\partial T_{\text{hg}}}{\partial T} \delta T + \frac{\partial T_{\text{hg}}}{\partial S_g} \delta S_g \right)_{i,j+\frac{1}{2},k} \left(\Phi_{\text{gi},j+1,k}^l - \Phi_{\text{gi},j,k}^l \right) + \\
& T_{\text{hgi},j-\frac{1}{2},k}^l \left(\Phi_{\text{gi},j-1,k}^l - \Phi_{\text{gi},j,k}^l \right) + T_{\text{hgi},j-\frac{1}{2},k}^l \left(\delta P_{\text{oi},j-1,k}^l - \delta P_{\text{oi},j,k}^l \right) + T_{\text{hgi},j-\frac{1}{2},k}^l \left(\delta P_{\text{cgoi},j-1,k}^l - \delta P_{\text{cgoi},j,k}^l \right) + \\
& \left(\frac{\partial T_{\text{hg}}}{\partial T} \delta T + \frac{\partial T_{\text{hg}}}{\partial S_g} \delta S_g \right)_{i,j-\frac{1}{2},k} \left(\Phi_{\text{gi},j-1,k}^l - \Phi_{\text{gi},j,k}^l \right) + \\
& T_{\text{hgi},j,k+\frac{1}{2}}^l \left(\Phi_{\text{gi},j,k+1}^l - \Phi_{\text{gi},j,k}^l \right) + T_{\text{hgi},j,k+\frac{1}{2}}^l \left(\delta P_{\text{oi},j,k+1}^l - \delta P_{\text{oi},j,k}^l \right) + T_{\text{hgi},j,k+\frac{1}{2}}^l \left(\delta P_{\text{cgoi},j,k+1}^l - \delta P_{\text{cgoi},j,k}^l \right) + \\
& \left(\frac{\partial T_{\text{hg}}}{\partial T} \delta T + \frac{\partial T_{\text{hg}}}{\partial S_g} \delta S_g \right)_{i,j,k+\frac{1}{2}} \left(\Phi_{\text{gi},j,k+1}^l - \Phi_{\text{gi},j,k}^l \right) + \\
& T_{\text{hgi},j,k-\frac{1}{2}}^l \left(\Phi_{\text{gi},j,k-1}^l - \Phi_{\text{gi},j,k}^l \right) + T_{\text{hgi},j,k-\frac{1}{2}}^l \left(\delta P_{\text{oi},j,k-1}^l - \delta P_{\text{oi},j,k}^l \right) + T_{\text{hgi},j,k-\frac{1}{2}}^l \left(\delta P_{\text{cgoi},j,k-1}^l - \delta P_{\text{cgoi},j,k}^l \right) + \\
& \left(\frac{\partial T_{\text{hg}}}{\partial T} \delta T + \frac{\partial T_{\text{hg}}}{\partial S_g} \delta S_g \right)_{i,j,k-\frac{1}{2}} \left(\Phi_{\text{gi},j,k-1}^l - \Phi_{\text{gi},j,k}^l \right) - Q_{\text{li},j,k}^{l+1} + Q_{\text{hi},j,k}^{l+1} + Q_{\text{welli},j,k}^{l+1} \\
& = \frac{V_B}{\Delta t} \left[\left((1-\varphi) M_R T \right)_{i,j,k}^{l+1} - \left((1-\varphi) M_R T \right)_{i,j,k}^n + \left(\varphi (S_o \rho_o U_o + S_w \rho_w U_w + S_g \rho_g U_g) \right)_{i,j,k}^{l+1} \right. \\
& \quad \left. - \left(\varphi (S_o \rho_o U_o + S_w \rho_w U_w + S_g \rho_g U_g) \right)_{i,j,k}^n \right] \\
& = \frac{V_B}{\Delta t} \left\{ \begin{aligned} & \left[(1-\varphi) M_R T \right]_{i,j,k}^l - \left[(1-\varphi) M_R T \right]_{i,j,k}^n + (1-\varphi) M_R \delta T_{i,j,k} - M_R T \frac{\partial \varphi}{\partial P_o} \delta P_{oi,j,k} + \\ & \left[\varphi (S_o \rho_o U_o + S_w \rho_w U_w + S_g \rho_g U_g) \right]_{i,j,k}^l - \left[\varphi (S_o \rho_o U_o + S_w \rho_w U_w + S_g \rho_g U_g) \right]_{i,j,k}^n + \\ & U_o \left[-\rho_o \varphi \delta (S_w + S_g) + (1 - S_w - S_g) \left(\varphi \frac{\partial \rho_o}{\partial P_o} \delta P_o + \varphi \frac{\partial \rho_o}{\partial T} \delta T + \rho_o \frac{\partial \varphi}{\partial P_o} \delta P_o \right) \right]_{i,j,k} + \\ & U_w \left(\rho_w \varphi \delta S_w + S_w \varphi \frac{\partial \rho_w}{\partial P_o} \delta P_o + S_w \varphi \frac{\partial \rho_w}{\partial T} \delta T + S_w \rho_w \frac{\partial \varphi}{\partial P_o} \delta P_o \right)_{i,j,k} + \\ & U_g \left(\rho_g \varphi \delta S_g + S_g \varphi \frac{\partial \rho_g}{\partial T} \delta T + S_g \rho_g \frac{\partial \varphi}{\partial P_o} \delta P_o \right)_{i,j,k} + \\ & \left(\varphi S_o \rho_o \frac{\partial U_g}{\partial T} \delta T + \varphi S_w \rho_w \frac{\partial U_w}{\partial T} \delta T + \varphi S_g \rho_g \frac{\partial U_g}{\partial T} \delta T \right)_{i,j,k} \end{aligned} \right\}
\end{aligned}$$

④ Equilibrium Equation (A47) can be changed as shown below:

$$\delta T - \frac{\partial f(P)}{\partial P} \delta P = f(P)^n - T^l \quad (\text{A62})$$

After linearizing difference Equations (A44)–(A47), Equations (A44)–(A47) can be simplified as:

$$\begin{bmatrix} a_{oi,j,k}^1 & a_{oi,j,k}^2 & a_{oi,j,k}^3 & a_{oi,j,k}^4 \\ a_{hi,j,k}^1 & a_{hi,j,k}^2 & a_{hi,j,k}^3 & a_{hi,j,k}^4 \\ a_{ri,j,k}^1 & a_{ri,j,k}^2 & a_{ri,j,k}^3 & a_{ri,j,k}^4 \\ a_{si,j,k}^1 & a_{si,j,k}^2 & a_{si,j,k}^3 & a_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi,j,k-1} \\ \delta S_{wi,j,k-1} \\ \delta T_{i,j,k-1} \\ \delta P_{oi,j,k-1} \end{bmatrix} + \begin{bmatrix} b_{oi,j,k}^1 & b_{oi,j,k}^2 & b_{oi,j,k}^3 & b_{oi,j,k}^4 \\ b_{hi,j,k}^1 & b_{hi,j,k}^2 & b_{hi,j,k}^3 & b_{hi,j,k}^4 \\ b_{ri,j,k}^1 & b_{ri,j,k}^2 & b_{ri,j,k}^3 & b_{ri,j,k}^4 \\ b_{si,j,k}^1 & b_{si,j,k}^2 & b_{si,j,k}^3 & b_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi,j-1,k} \\ \delta S_{wi,j-1,k} \\ \delta T_{i,j-1,k} \\ \delta P_{oi,j-1,k} \end{bmatrix} + \quad (\text{A63})$$

$$\begin{aligned}
& \begin{bmatrix} c_{oi,j,k}^1 & c_{oi,j,k}^2 & c_{oi,j,k}^3 & c_{oi,j,k}^4 \\ c_{Hi,j,k}^1 & c_{Hi,j,k}^2 & c_{Hi,j,k}^3 & c_{Hi,j,k}^4 \\ c_{Ri,j,k}^1 & c_{Ri,j,k}^2 & c_{Ri,j,k}^3 & c_{Ri,j,k}^4 \\ c_{si,j,k}^1 & c_{si,j,k}^2 & c_{si,j,k}^3 & c_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi-1,j,k} \\ \delta S_{wi-1,j,k-1} \\ \delta T_{i-1,j,k-1} \\ \delta P_{oi-1,j,k-1} \end{bmatrix} + \begin{bmatrix} d_{oi,j,k}^1 & d_{oi,j,k}^2 & d_{oi,j,k}^3 & d_{oi,j,k}^4 \\ d_{Hi,j,k}^1 & d_{Hi,j,k}^2 & d_{Hi,j,k}^3 & d_{Hi,j,k}^4 \\ d_{Ri,j,k}^1 & d_{Ri,j,k}^2 & d_{Ri,j,k}^3 & d_{Ri,j,k}^4 \\ d_{si,j,k}^1 & d_{si,j,k}^2 & d_{si,j,k}^3 & d_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi,j,k} \\ \delta S_{wi,j,k} \\ \delta T_{i,j,k} \\ \delta P_{oi,j,k} \end{bmatrix} + \\
& \begin{bmatrix} e_{oi,j,k}^1 & e_{oi,j,k}^2 & e_{oi,j,k}^3 & e_{oi,j,k}^4 \\ e_{Hi,j,k}^1 & e_{Hi,j,k}^2 & e_{Hi,j,k}^3 & e_{Hi,j,k}^4 \\ e_{Ri,j,k}^1 & e_{Ri,j,k}^2 & e_{Ri,j,k}^3 & e_{Ri,j,k}^4 \\ e_{si,j,k}^1 & e_{si,j,k}^2 & e_{si,j,k}^3 & e_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi+1,j,k} \\ \delta S_{wi+1,j,k} \\ \delta T_{i+1,j,k} \\ \delta P_{oi+1,j,k} \end{bmatrix} + \begin{bmatrix} f_{oi,j,k}^1 & f_{oi,j,k}^2 & f_{oi,j,k}^3 & f_{oi,j,k}^4 \\ f_{Hi,j,k}^1 & f_{Hi,j,k}^2 & f_{Hi,j,k}^3 & f_{Hi,j,k}^4 \\ f_{Ri,j,k}^1 & f_{Ri,j,k}^2 & f_{Ri,j,k}^3 & f_{Ri,j,k}^4 \\ f_{si,j,k}^1 & f_{si,j,k}^2 & f_{si,j,k}^3 & f_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi,j+1,k} \\ \delta S_{wi,j+1,k} \\ \delta T_{i,j+1,k} \\ \delta P_{oi,j+1,k} \end{bmatrix} + \\
& \begin{bmatrix} g_{oi,j,k}^1 & g_{oi,j,k}^2 & g_{oi,j,k}^3 & g_{oi,j,k}^4 \\ g_{Hi,j,k}^1 & g_{Hi,j,k}^2 & g_{Hi,j,k}^3 & g_{Hi,j,k}^4 \\ g_{Ri,j,k}^1 & g_{Ri,j,k}^2 & g_{Ri,j,k}^3 & g_{Ri,j,k}^4 \\ g_{si,j,k}^1 & g_{si,j,k}^2 & g_{si,j,k}^3 & g_{si,j,k}^4 \end{bmatrix} \begin{bmatrix} \delta S_{gi,j,k+1} \\ \delta S_{wi,j,k+1} \\ \delta T_{i,j,k+1} \\ \delta P_{oi,j,k+1} \end{bmatrix} + \begin{bmatrix} M_{oi,j,k}^{l+1} \\ M_{wi,j,k}^{l+1} + M_{gi,j,k}^{l+1} \\ -Q_{lossi,j,k}^{l+1} + Q_{Hi,j,k}^{l+1} + Q_{welli,j,k}^{l+1} \\ 0 \end{bmatrix} = \begin{bmatrix} h_{oi,j,k} \\ h_{Hi,j,k} \\ h_{Ri,j,k} \\ h_{si,j,k} \end{bmatrix}
\end{aligned}$$

where a, b, c, d, e, f , and g are the coefficients of the corresponding variables. Supposing II is the number of grid points in the x direction, JJ is the number of grid points in the y direction, and KK is the number of grid points in the z direction, there are $II \times JJ \times KK$ grid nodes; thus, the above equations are $4 \times II \times JJ \times KK$ order equations with four unknowns: δP_o , δT , δS_w and δS_g . The equations are simplified as follows:

$$AX = B \quad (A64)$$

where A is a seven-diagonal 4×4 block matrix, X consists of the unknown variables, $X = [\delta P_o, \delta T, \delta S_w, \delta S_g]^T$, and B is a constant term at the right side of the equation.

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