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# Geometry Optimization of Top Metallic Contacts in a Solar Cell Using the Constructal Design Method

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Abstract: Sunlight is a natural resource that can be harnessed by the photovoltaic conversion of sunlight into electricity-utilizing solar cells. The production of most common solar cells consists of a homojunction of a p-type and n-type silicon. The p—n junction is realized by the diffusion of impurities through one surface of the wafer. Silicon wafers have a typical dimension of  $156 \times$ 156 mm<sup>2</sup> and a thickness of 0.15–0.2 mm. Groups of 50–100 solar cells are electrically connected and encapsulated to form a module. The required area for interconnection does not contribute to power generation, and the performance of larger area devices usually suffers from higher resistive losses. In the present work, a theoretical model of the geometric arrangement of the top contact metallic electrodes branched network in a photovoltaic cell is developed. The network structure of the electrodes is obtained from applying the constructal design methodology by the minimization of the overall resistance. As a result, the optimal lengths and geometrical relationships of an electrode branching network with a branching angle are determined. A geometric distribution of the electrode network on the solar cell analyzed by the total resistance of every level of branching is defined. The top metallic contact network presents a tree-shaped geometric arrangement with the main objective of covering a generation area for an enhanced collection of the generated electrical current. The theoretical results obtained are expressed as the total voltage of the arrangement and the lengths of the branched electrode network.

Keywords: solar cell; metallic contact; constructal design; structure

# 1. Introduction

A classical crystalline silicon solar cell consists of a semiconductor p—n junction, finger-like top electrodes, all covered by a metal back electrode, and an anti-reflective coating layer. In such kind of cells, energy conversion occurs in two stages: (i) the absorption of photons that excites electrons from the valence band to the conduction band and (ii) the generation of charge carriers to create electron-hole pairs and, therefore, a photocurrent is generated and collected by metallic contacts [1]. Finger-like top electrodes are used in most silicon solar cells, and for wider electrodes, small series resistance is observed, and collected photocurrent increases. However, wider electrodes increase shadow limiting light transmission through the junction. The task of optimizing grid metallic top electrodes in a solar cell is important to achieve better conversion efficiency. Furthermore, there are



Increasing the collection of electrical energy allows for the design and distribution of the top metallic contact network in a sun-exposed area of the solar cell. This metallic contact network is generally made of silver alloys in silicon solar cells [3,4] and is generally disposed of a H-grid pattern over the top surface of the solar cell.

If the single cell design is used for a solar panel, the current would be too large and the voltage too low for effective operation. Therefore, the surface area is divided into sub-cells that are interconnected. The required area for interconnection does not contribute to power generation, and the performance of larger area devices usually suffers from higher resistive losses. However, methods have been developed to minimize this effect, one of which is the optimization of the cell's contact network. The optimization of the grid design for solar cells has been investigated from different approaches for diverse photovoltaic technologies. Studies about the influence of metal grid patterns on the performance of silicon solar cells of two different patterns, linear and square, have demonstrated both theoretically and experimentally that the power losses are less sensitive to illumination for the square grid than the linear grid; moreover, the power losses of the square grid are also less influenced by metal-grid resistivity [5]. Additionally, the optimization of metal-grid design to improve the solar cell performance by using a multiobjective algorithm shows that the linear grid design is a promising candidate for high-efficiency silicon solar cells compared to circular grid patterns [6]. For thin-film solar panels, it has been reported by Van Deelen et al. [7] that a metallic grid consisting of 20–µm wide lines would improve a solar panel with 11.7% transparent conductive oxide (TCO) only up to a level of 13.8%, though this also indicated how panel configurations can be designed with increased robustness against TCO degradation. It was shown that even with 100-µm wide grid lines, considerable efficiency gains can be achieved over cells that only have TCO as a transparent front contact. Therefore, the geometric optimization of the contact network in solar cells can allow for better coverage and collection of the photogenerated current, however, the metallic grid design requires a specific approach for each photovoltaic technology, subjected to the manufacturing restriction process of current technologies, because any one of them has intrinsic properties that force it to apply specific patterns for the metal grid.

Engineering systems require optimum access to imposed currents, such as heat, fluid flow, and electrical current to reach an acceptable performance, and this is a characteristic that needs to be solved. Some nature-based structures are applied as a solution to the optimal cooling of volumes with a generating heat source, such as the heat transfer processes in the venation of tree leaves [8], applied as high conductive vein paths [9], tree-like cavity structures [10] and a fractal-like microchannel heat sink for high-heat flow applications [11].

Bio-inspired networks applied in highly conducting and transparent micro-scaffold for optoelectronic devices, satisfying requirements dictated by a specific application, have demonstrated that such networks give exceptional performance [12]. The application of an electrode network with a shape inspired by natural structures in a window for harvesting solar energy to generate electrical current has been reported by Han et al. [13]. This network was designed under physical assumptions including a uniform current distribution and minimum overall electrical resistance, defined as the minimization of the voltage drop on a disc power plane. A quasi-fractal configuration, based on a leaf venation type network, was applied experimentally and provided characteristics for the optimal structure of different hierarchical networks in a window.

Space-filling supercapacitor carpets using fractal architecture for energy storage have shown to address the issue of scalability, by using the Peano curve for designing electrodes for an optimal distribution of length within a given surface [14]. The Constructal theory allows for the design of

transport networks with physical currents through the system, subject to local restrictions to satisfy global restrictions. This theory, developed by Bejan [15], explains the relationship between the physical properties and the geometry of the paths where physical currents flow through the system. In a constructal system with imposed currents, i.e., heat, fluids, or nutrients, the flow through the system circulates with the minimum resistance. The geometrical characteristics of the network shape, size, and structure are a result of the optimization of the size of every constructal building element. Applying the constructal methodology, the access between a point and an area or volume is optimized by a transport network, whose geometrical characteristics are reticular or leaf-type structures.

A reticular network is a recurrent shape in constructal design. A high conductivity path for cooling a volume with a heat generation source was developed by Bejan [16], who showed that the path length is defined as a function of the physical properties and geometry of the constructal element and that the final shape of the high conductivity path is reticular. This methodology has been applied in a reticular network of metallic contacts for a rectangular solar cell in which the design of finger-like metallic contacts obeys the constructal principle, as the solar cell contains an electrical volumetric source. The size of every branch of the reticular network of a metallic contact is defined as a function of the solar cell and the metallic contact [17].

Bhakta and Bandyopadhyay [18], using the constructal methodology, generated a network of metallic contacts in a solar cell with an electrical generation term with the main objective of minimizing the power losses. Superficial metallic contacts are distributed to cover more areas in a reticular structure and the minimized maximum voltage drop defines the aspect relation of every rectangular branch of the network. The size of the metallic contacts is a function of the physical parameters, such as the electrical conductivities of the solar cell and the metallic contact material. On the other hand, Constructal theory also allows us to define a leaf-like shaped network in a basic problem of imposed currents. This methodology has been applied to cooling a generating volume with a heat source. An adaptive growth model is defined with an unlimited two-node network where the branches extend towards the system, and a leaf-like structure is shown to improve the global performance of the system [19]. In applications in circuits for a power distribution network, an optimal distribution of paths in a power circuit board with a disc-shaped board has been reported. The conductive paths are defined using a radial arrangement in which the length of the branches and fraction area is defined by the optimum access of the electrical current, minimizing the overall resistances [20]. In this case, the optimal structures reported are mainly tree-like structures.

In the present work, a design of the top metallic contact network in a solar cell is developed using the constructal methodology. The characteristics of the metallic contact network must be such that the spacing between metallic contacts must allow for as much sunlight transmission as possible to the surface of a solar cell, causing minimum shading. We anticipate that the metal fraction is a consequence of the constructal design process, from the main physical variables involved in a solar cell. In the first constructal element, a metallic contact is defined, and the aspect ratio is determined. The successive constructions are defined considering the previous construction, and the geometric characteristics of the first assembly are defined applying the constructal design method recursively.

#### 2. Elemental System

The constructal optimization methodology requires to define an elemental system in which the main geometrical and physical variables are identified. In this case, the elemental system or first element of construction has a finite and fixed volume, which is given as  $Vol = H_0L_0w$ , where w is the thickness of the solar cell in the perpendicular direction to the x - y plane. To define the necessary expressions, that is, to state the overall voltage in the elemental system, the following assumptions are considered. The volume of the solar cell is composed of semiconductor material with a constant value of the electrical conductivity  $\sigma_{sc}$ . At the center of the first constructal element, along the symmetry axis, we consider a metallic contact of width  $D_0$ , length  $L_0$  and a constant value of the electrical conductivity  $\sigma_m$ , which is assumed to be a perfect conductor. The thicknesses of the solar cell and the metallic contact are considered equal, as a first approximation. For simplicity, the metal resistance loss on the back surface and the optical shading loss is not considered. However, the allocation of the metallic material of the contacts in the geometric arrangement is unknown and must be determined as a result of the optimization process.

Here, the solar cell operates under direct current conditions and has a uniform rate of current density per unit length, defined as i''' = i/Vol (A/m<sup>3</sup>). The temperature increments due to the uniform rate i''' are not significant in comparison to the global temperature of the solar cell, hence the solar cell is considered to operate under isothermal conditions with no mechanical or thermal stresses [21]. Additionally, the semiconductor material of the solar cell has a constant source of electrical current per unit volume, as a first approximation, defined as i''' = i/Vol (A/m<sup>3</sup>), and it can also be considered as current density per unit of length. The boundaries of the constructal element are considered without electrical leaks. The elemental system has a known and constant area  $A_0 = H_0L_0$  where the dimensions  $H_0$  and  $L_0$  can vary parametrically. The first constructal element is considered has a slender geometry, that is  $H_0 \ll L_0$ , so that the variations of the voltage in the horizontal direction on the metallic contact are greater than in the vertical direction. Additionally, the electrical current flows through the path of minimum resistance in vertical and horizontal directions. For this we assume that the electrical current source and voltage are two dimensional. The optimal aspect ratio of the constructal elemental system has to be defined by the minimization of the maximum voltage drop.

As a consequence, the admittance of the concurrent network of contacts is enhanced, i.e., an optimal flow of electric current can be obtained [22]. The maximum voltage drop is considered to be at the farthest point of the constructal element, at  $y = H_0/2$  and  $x = L_0$ .

A sketch of the first constructal element with the physical and geometrical variables is depicted in Figure 1.



Figure 1. First constructal element with a metallic contact.

In the process of determining the optimal length of the branches of the metallic contact, with the aid of the aspect ratio, the constant source of electrical current due to the activity of the semiconductor material can be considered.

To determine the optimal aspect ratio of the first constructal element, it is necessary to define the voltage drop in horizontal and vertical directions so that the optimal length of the metallic contact can be defined. These results are obtained under local restrictions, as mentioned before. The area of the constructal element  $A_0$  is considered known and constant [23]. In the vertical direction, the voltage distribution along the semiconductor material is defined by carrying out an electrical balance to define the charge conservation and is given by the following differential equation,

$$\frac{\partial^2 V_0}{\partial y^2} + \frac{i^{\prime\prime\prime\prime}}{\sigma_{sc}} = 0 \tag{1}$$

Consequently, in the horizontal direction, the differential equation of the charge conservation in the metallic contact of length  $L_0$  is given by

$$\sigma_m D_0 \frac{\partial^2 V_0}{\partial x^2} + i^{\prime\prime\prime} H_0 = 0 \tag{2}$$

Equation (2) is analogous to the thermal case for a thin-fin approximation used in constructal theory procedures (Bejan and Lorente, 2008). Both differential equations are subjected to the following boundary conditions:

For the semiconductor material, Equation (1):

$$y = 0 : V_0 = V_0(x); \ y = H_0/2 : \frac{\partial V_0}{\partial y} = 0.$$
 (3)

For the metallic contact, Equation (2):

$$x = 0 : V_0 = V_0(0,0); \quad x = L_0 : \frac{\partial V_0}{\partial x} = 0.$$
 (4)

From Equations (1)–(4), the voltage drop of the constructal element is defined as follows:

$$V_0(x,y) - V_0(0,0) = \frac{i'''H_0}{\sigma_m D_0} \left( L_0 x - \frac{x^2}{2} \right) + \frac{i'''}{2\sigma_{sc}} \left( H_0 y - y^2 \right)$$
(5)

In this manner, the voltage drop at  $y = H_0/2$ ,  $x = L_0$ , is given by;

$$\Delta V_0 = \frac{i'''}{2\sigma_{\rm sc}} \left(\frac{H_0}{2}\right)^2 + \frac{i'''H_0}{\sigma_m D_0} \frac{L_0^2}{2} \tag{6}$$

The above expression can be manipulated algebraically to define the dimensionless overall resistance of the constructal element, defined as;

$$\overline{R}_0 = \Delta V_0 \sigma_s / i''' H_0 L_0. \ \overline{R}_0 = \frac{1}{8} \overline{H}_0 + \frac{1}{2\overline{H}_0} \left(\frac{\overline{\sigma}}{\overline{\Phi}_0}\right).$$
(7)

where  $\overline{H}_0$ ,  $\phi_0$  and  $\overline{\sigma}$ , are the aspect ratios for the solar cell and metallic contact, and a ratio of the electrical conductivities between the semiconductor material and metallic contact, respectively. These dimensionless parameters are defined as follow;

$$\overline{H}_0 = \frac{H_0}{L_0}, \overline{\sigma} = \frac{\sigma_{sc}}{\sigma_m}, \phi_0 = \frac{D_0}{H_0}$$
(8)

To define an optimal aspect ratio of the first constructal element and consequently the length of the metallic contact, we can derive Equation (8) with respect to  $\overline{H}_0$  and therefore, an expression of the aspect ratio can be defined as a function of electrical conductivities and geometric dimensions an is given by;

$$\overline{\mathbf{H}}_{0opt} = 2 \left( \frac{\overline{\sigma}}{\phi_0} \right)^{\frac{1}{2}} \tag{9}$$

where the subscript *opt* (optimal) in the context of the constructal methodology is such that considering the variety of possible configurations of geometric dimensions and physical quantities, Equation (9) define a minimum value of the maximum voltage drop in a slender first constructal element. In successive constructions an aspect ratio can be defined to every ramification of a metallic contact in a top surface of a solar cell.

Substituting Equation (9) in Equation (7), minimum resistance of the system is given by;

$$\overline{R}_{0\,\min} = \frac{1}{2} \left( \frac{\overline{\sigma}}{\phi_0} \right)^{\frac{1}{2}} \tag{10}$$

Combining the previous result with the aspect ratio  $\overline{H}_{0opt}$ , gives a minimum value of the overall resistance as a function of  $H_0^2$ , as reported before in the design of networks for cooling elements with a heat source [16]. This geometric characteristic allows us to define manufacturing parameters to produce the smallest elements with metallic contacts as possible. From Equation (8) and the restriction of area  $A_0$  as constant and known, a dimensionless length ( $\overline{L}_0 = L_0/A_0^{1/2}$ ) of the top contact metallic electrode is given by;

$$\overline{L}_{0 opt} = \frac{1}{\sqrt{2}} \left( \frac{\Phi_0}{\overline{\sigma}} \right)^{\frac{1}{4}} \tag{11}$$

Equation (11) defines the length as a function of the electrical conductivities and width of the electrode. With these parameters an optimal size is obtained, the previous methodology is recursive to successive constructions.

#### 3. First Assembly of Metallic Contacts

A branched top metallic contact network in a solar cell is defined with a second building block. In this second construction element, slender geometry, area, and volume restrictions are considered. In this stage of construction, the lengths  $H_1$  and  $L_1$  are unknown, and the process of optimization applied in the first constructal element is recursive for successive constructions. The same physical assumptions in the solar cell are considered. The metallic contact of this second element with the length  $L_1$  is considered as an electric busbar in which the electrical currents collected by the metallic contacts of length  $L_0$  converge at the contact node.

A branched metallic contact network with an angle  $\alpha$  is analyzed to define the number of metallic contacts and the shape and structure of the network as a first approximation. The applied design methodology is based on the tree-shaped networks used for high conductivity inserts in cooling a volume and for a branched pipe network for fluid flow [24,25]. Constructal design methodology has been applied to a pipe network for fluid flow with a tree-shaped configuration, where the angle between branched pipes is determined by the minimization of the hydraulic resistances of the pipe arrangement [26]. For the design of a branched metallic contacts over the solar cell and, consequently, the proportion of space occupied by the metallic over the solar cell. The branching angle is a design parameter, due to the electric effects that allow the depth between branched electrodes [27].

In a first assembly, a number  $n_1$  of inclined metallic contacts of length  $L_0$  and width  $D_0$  at an angle,  $\alpha$  are located along with the metallic contact of length  $L_1$ . This length is defined in such a way that minimizes the power losses in the branched arrangement. The method applied in the previous section can be recursive, considering the appropriate boundary conditions. A second constructal element, with the bifurcated top metallic contact branches, is depicted in Figure 2.

The electrical current distribution in the vertical direction in the second building constructal element is defined following the same methodology of the previous section; the distribution model of current is similar to Equation (1). In the longitudinal direction, at the center is allocated an insert of a metallic contact of electrical conductivity  $\sigma_m$  and width  $D_1$ .



Figure 2. Bifurcated branched metallic contact in a second constructal element.

The current distribution in the longitudinal direction is given by the following differential equation:

$$\sigma_m D_1 \frac{\partial^2 V_1}{\partial x^2} + i^{\prime\prime\prime} H_1 = 0, \qquad (12)$$

and Equation (12) is subjected to the following boundary conditions:

$$x = 0 : V_1 = V_1(0,0); \ x = L_1 : \frac{\partial V_1}{\partial x} = 2 \frac{D_0}{D_1} \left( \frac{\partial V_0}{\partial x} \right)_{x = L_0 \cos \alpha}.$$
 (13)

At the confluence node at the coordinate  $x = L_1$ , Kirchoff's law is fulfilled with the current collected by two branches of the electrode. The number of inclined metallic contact branches of width  $D_0$ , repeated along the metallic electrode of width  $D_1$ , will be shown later.

With the aid of Equations (11) and (12) horizontal distribution of electric current is defined, from this result and vertical distribution, where the expression is similar to the first constructal element, defined in the previous section; with these two expressions a voltage in a solar cell can be constructed. Substituting the coordinates  $x = L_1$ ,  $y = H_1/2$  and considering that at the maximum length of the metallic contact occurs the maximum voltage drop, we can manipulate algebraically the resulting equation and a dimensionless expression of the resistance of the geometric arrangement of metallic contacts is defined by:

$$\overline{R}_1 = \frac{1}{\overline{H}_1} \left( \frac{\overline{\sigma}}{\phi_1} \right) \left( \frac{1}{2} + 2\overline{A}_1 (1 - \cos \alpha) \right) + \frac{1}{8} \overline{H}_1.$$
(14)

where  $\overline{A}_1 = A_0/A_1$ ,  $\phi_1 = D_1/H_1$  and  $\overline{H}_1 = H_1/L_1$  are a ratio between areas, the portion of the metallic contact material of width  $D_1$  and the aspect ratio of the first assembly of metallic contacts, respectively.

For a value of the angle,  $\alpha = 0$ , the dimensionless overall resistance for a reticular arrangement of metallic contacts, designed by the constructal approach, reported by Morega and Bejan [17], is recovered. The aspect ratio of the constructal element is defined by the following expression;

$$\overline{H}_{1 opt} = 2 \left(\frac{\overline{\sigma}}{\phi_1}\right)^{\frac{1}{2}} \left(1 + 4\overline{A}_1(1 - \cos\alpha)\right)^{\frac{1}{2}}$$
(15)

from this aspect ratio and the definition of the area, the length of the electrode is defined by,

$$\overline{L}_1 = \frac{1}{\sqrt{2}} \left(\frac{\phi_1}{\overline{\sigma}}\right)^{\frac{1}{4}} \left(1 + 4\overline{A}_1(1 - \cos\alpha)\right)^{-\frac{1}{4}}.$$
(16)

Substituting Equation (15) in Equation (14) a first optimization of the voltage drop of the geometrical assembly is defined, as follows;

$$\overline{R}_{1\min} = \frac{1}{2} \left( \frac{\overline{\sigma}}{\phi_1} \right)^{\frac{1}{2}} \left( 1 + 4\overline{A}_1 (1 - \cos \alpha) \right)^{\frac{1}{2}}.$$
(17)

The total dimensionless resistance of the bifurcated arrangement is defined with the aid of Equations (10) and (17), considering the bifurcated metallic network as a series and shunt resistance arrangement. Therefore, we obtain the following expression;

$$\overline{R}_{1 \text{ tot}} = \frac{1}{2} \left( \frac{\overline{\sigma}}{\phi_0} \right)^{\frac{1}{2}} \left\{ \frac{1}{2} + \left( \frac{\phi_0}{\phi_1} \right)^{\frac{1}{2}} \left( 1 + 4\overline{A}_1 (1 - \cos \alpha) \right)^{\frac{1}{2}} \right\}$$
(18)

On the other hand, a ratio of the presence of metallic material over the solar cell, can be defined with the aid of the area ratio and overall voltage of the first assembly. This was reported previously as a *porosity* due to the coverage material by a reticular network of inserts of high thermal conductivity for a thermal case [16]. This area ratio is defined by  $\varphi_1 = A_{p1}/A_1 = constant$ , where  $A_{p1} = D_1L_{1 opt} + n_1D_0L_{0 opt}$  is the area occupied by the metallic material. The allocation of the metallic material over the constructal area of the assembly can be defined by the following expression;

$$\varphi_1 \cong \phi_1 + \frac{\phi_0}{\sin \alpha} \tag{19}$$

An optimal ratio of the portion of the metallic material of the geometric arrangement can be defined by the condition of the minimum total voltage drop and area ratio used in the previous expression. The mentioned ratio can be defined as applying a methodology for branching design in networks for heat transfer reported by Miguel A.F. [28] where the minimum impedance is obtained with the volume as a restriction. By applying the chain rule of the partial derivative to Equation (18) as  $\partial \Delta \overline{V}_{\text{tot}} / \partial \phi_0$  that is defined as;

$$\frac{\partial \Delta \overline{V}_{1 \text{ tot}}}{\partial \phi_0} = \frac{\partial \Delta \overline{V}_{1 \text{ tot}}}{\partial \phi_1} \frac{\partial \phi_1}{\partial \phi_0} + \frac{\partial \Delta \overline{V}_{1 \text{ tot}}}{\partial \phi_0} = 0$$
(20)

where  $\partial \phi_1 / \partial \phi_0$  can be defined from Equation (18) as  $\phi_1 = \varphi_1 - (\phi_0 / \sin \alpha)$ .

Substituting the corresponding derivatives and manipulating algebraically Equation (19), the ratio of the portion of the metallic material for a bifurcated arrangement is less than unity  $\phi_0/\phi_1 \ll 1$  and is defined by geometric parameters as;

$$\frac{\phi_0}{\phi_1} = \left[\frac{\sin\alpha}{\left(1 + 4\overline{A}_1(1 - \cos\alpha)\right)^{\frac{1}{2}}}\right]^{\frac{1}{3}}$$
(21)

An additional characteristic in constructal methodology is the definition of the optimal number of elemental constructions, generally disposed to both sides of the main collector branch. This number is defined as a ratio between the optimal length of the first and the height of the initial building element, i.e.,  $n_1 = 2L_{1 opt}/H_0$ . This feature is reported mainly in the process design of networks of high conductivity material inserts to cooling volumes with a volumetric heat source [17].

The number of branched elements  $n_1$  disposed along with the metallic contact, is defined by the aid of the aspect ratio of the first construction  $\overline{H}_{1 opt}$ , the length  $H_1$ , which can be expressed as  $H_1 \cong 2L_0 \sin \alpha$  and the ratio  $\phi_0/\phi_1$ . The number of branched elements is defined as follow;

$$n_1 = \frac{\phi_1^{3/2}}{\overline{\sigma}} \left[ \frac{\sin \alpha}{\left(1 + 4\overline{A}_1(1 - \cos \alpha)\right)^{\frac{1}{2}}} \right]^{\frac{1}{3}}.$$
(22)

# 4. Second Assembly

The second level of branching of the top metallic contacts in a solar cell is defined by a third building element with a constant area  $A_3$ . A geometrical arrangement of branched metallic contacts with an angle  $\alpha$ , defined in the previous section, is considered. The constructal methodology can be applied recursively, considering the geometrical parameters, to determine the dimensions of the metallic branched networks inclined at an angle  $\beta$ . A sketch of the bifurcated geometrical arrangement is depicted in Figure 3.



Figure 3. A two-level bifurcated branched network of the top contact metallic grid.

At the center of the constructal element, a metallic contact of length  $L_2$  collects the electrical current generated by the solar cell material. Following the methodology presented in Section 3, a dimensionless voltage drop is defined and consequently defines the geometric aspect ratio  $\overline{H}_2$ , the portion of the metallic material ratio  $\phi_1/\phi_2$ , and the number of branched elements that can be arranged along the collector electrode. The geometrical characteristics are defined by the following expression:

$$\overline{H}_2 = 2\sqrt{2} \left(\frac{\overline{\sigma}}{\phi_2}\right)^{\frac{1}{2}} \left[1 + 2\overline{A}_2(1 - \cos\beta) + 4\overline{A}_1\overline{A}_2(1 - \cos\alpha)\right]^{\frac{1}{2}}$$
(23)

where  $\phi_2 = D_2/H_2$  is the allocation of the metallic contact material of width  $D_2$ , and  $\overline{A}_2 = A_1/A_2$  is the area ratio.

Therefore, the length of the metallic contact and the total voltage of the arrangement is defined by

$$\overline{L}_2 = \frac{1}{2^{\frac{3}{4}}} \left(\frac{\phi_2}{\overline{\sigma}}\right)^{\frac{1}{4}} \left[1 + 2\overline{A}_2(1 - \cos\beta) + 4\overline{A}_1\overline{A}_2(1 - \cos\alpha)\right]^{-\frac{1}{4}}$$
(24)

The total resistance of the bifurcated arrangement of metallic contacts is defined considering the previous resistances of the elemental construction and the first configuration, defined by Equation (18). The total resistance is defined as the resistances in series and shunt by the following expression:

$$\overline{R}_{2 \text{ tot}} = \frac{\Delta \overline{V}_{1 \text{ tot}}}{2} + \frac{1}{\sqrt{2}} \left(\frac{\overline{\sigma}}{\Phi_2}\right)^{\frac{1}{2}} \left(1 + 2\overline{A}_2(1 - \cos\beta) + 4\overline{A}_1\overline{A}_2(1 - \cos\alpha)\right)^{\frac{1}{2}}$$
(25)

where  $\phi_0/\phi_1$  is the ratio of the portion of metallic material over the solar cell and is defined by Equation (20).

The corresponding ratio of the portion of the material in the second assembly is a function of the geometrical parameters, and with the geometrical approximation of  $H_1$ , this ratio is defined as a function of the first constructal aspect ratio by

$$\frac{\phi_1}{\phi_2} = \left(\sqrt{2}\sin\beta\right)^{\frac{2}{3}} \left(1 + \frac{2\overline{A}_1\sin\alpha\cos\alpha}{\overline{H}_0}\right)^{\frac{4}{3}} \left(1 + 4\overline{A}_1(1 - \cos\alpha)\right)^{\frac{1}{3}}$$
(26)

The number of branched contacts is defined following the methodology applied in the previous section and is given by

$$n_{2} = \frac{\sin\beta}{\sqrt{2}\left(1 + 2\overline{A}_{2}(1 - \cos\beta) + 4\overline{A}_{1}\overline{A}_{2}(1 - \cos\alpha)\right)^{\frac{1}{2}}} \left(\frac{\phi_{2}}{\overline{\sigma}}\right)^{\frac{1}{2}} \left[\left(\frac{\phi_{1}}{\overline{\sigma}}\right)^{\frac{1}{2}} \frac{1}{2\left(1 + 4\overline{A}_{1}(1 - \cos\alpha)\right)^{\frac{1}{2}}} + \frac{\cot\alpha}{2}\right]$$
(27)

# 5. Results

The geometrical characteristics of a branched metallic network corresponding to different constructions are reported in this section. In the following results, the dimensionless resistance for the constructal elements is plotted for different values of geometrical and physical parameters. The values of the dimensionless parameters are defined with reported data of the metallic contact and typical solar cell characteristic materials [29–31]. The magnitude order of the electrical conductivities ratio  $\bar{\sigma}$  varies from ~ 10<sup>-3</sup> to 10<sup>-6</sup> approximately. In Figure 4, the dimensionless resistance  $\bar{R}_0$  versus the aspect ratio  $\bar{H}_0$  of the first constructal element is shown for a fixed value for the electrical conductivity's ratio  $\bar{\sigma} = 5.33 \times 10^{-5}$ .

The geometrical parameter  $\phi_0$  represents the allocation of metallic material over the solar cell material. For a value of  $\phi_0 = 0.0001$ , the dimensionless resistance of the constructal element decreases monotonically without a minimum value present for an aspect ratio  $\overline{H}_0$ . That is, for a lesser presence of the metallic material, the geometry of the constructal element tends to be a slender geometry ( $\overline{H}_0 \ll 1$ ). A minimum value for the dimensionless resistance is presented for a value of the aspect ratio  $\overline{H}_0 \approx 0.4$ , corresponding to  $\phi_0 = 0.001$ . On the other hand, for a value of  $\phi_0 = 0.1$ , a minimum value of  $\overline{R}_0$  presents at  $\overline{H}_0 \approx 0.02$ .



Figure 4. Dimensionless resistance versus aspect ratio for different values of the geometrical parameter.

For the first assembly of branched metallic contacts, the bifurcated case is initially considered; later, we show results for the number of repeated branched elements along the electrode of width  $D_1$ . The dimensionless resistance is defined by Equation (13) as a function of the geometrical parameter  $\phi_1$ , the aspect ratio of the constructal element  $\overline{H}_1$ , and the angle of branching  $\alpha$ . For the following results, in Figure 5, we fixed values of  $\overline{A}_1 = 0.1$  and  $\overline{\sigma} = 5.33 \times 10^{-5}$ . In Figure 5a, the dimensionless overall resistance  $\overline{R}_1$ , for  $\phi_1 = 0.001$  presents a minimum value at  $\overline{H}_1 \cong 0.5$ , in comparison with  $\phi_1 = 0.1$ , where the minimum value is presented at  $\overline{H}_1 \cong 0.06$ , this is a lesser value of the aspect ratio, i.e., svelteness constructal element. On the other hand, in Figure 5b, for a range of values of the branching angle  $\alpha$ , the dimensionless resistance presents a minimum value at  $\overline{H}_1 \cong 0.064$  for an angle  $\alpha = 10^{\circ}$ ; whereas for an angle of 90°, this corresponds to a reticular arrangement of electrodes, and a minimum value is presented at  $\overline{H}_1 \cong 0.073$ . This means that for a right angle, the aspect ratio of the element is less slender than that corresponding to an angle of 10°. This feature allows the arrangement of electrodes to be smaller in size.



**Figure 5.** Dimensionless resistance  $\overline{R}_1$ , (**a**) for different values of  $\phi_1$  and (**b**) for different branching angle values.

The optimal number of metallic contacts of width  $D_0$  repeated along with the metallic contact of length  $L_1$  can be defined with the aid of Equation (22). For the fixed values of  $\phi_1$  and  $\overline{\sigma}$  used previously in Figure 5, the number of metallic contacts  $n_1$  for an angle  $\alpha = 10^\circ$  is  $n_1 \cong 37$ . For a right-angle arrangement of contacts, corresponding to the highest minimum value of the dimensionless voltage  $\overline{R}_1$  the number of elements is  $n_1 \cong 450$ . The effect of having a large number  $n_1$  of branched metallic contacts is that more of the solar cell area is covered and the resistance of the geometrical arrangement is considered as resistances, in shunt and series, decrease. The influence of the numbers of contacts are shown below.

For the second assembly of bifurcated metallic contacts, the dimensionless resistance of the first construction,  $\overline{R}_2$ , is plotted versus the aspect ratio of the constructal element,  $\overline{H}_2$ , in Figure 6. For fixed values of the area ratios  $\overline{A}_1$  and  $\overline{A}_2$ , the portion of the material in the second constructal element,  $\phi_2$ , and the branching angle  $\alpha$ , the dimensionless voltage drop presents a minimum value of  $\overline{R}_2 \approx 0.016$  at  $\overline{H}_2 \approx 0.065$  for an angle  $\beta = 10^\circ$ , and  $\overline{R}_2 \approx 0.0178$  at  $\overline{H}_2 \approx 0.071$  for the right angle. If the angle of branching increases, the minimum value of the overall resistance presents a value of  $\overline{H}_2$  that is greater than that corresponding to a small value of the angle  $\beta$ . With the aid of Equations (25) and (26), the geometrical ratio of the portion of metallic material for the second assembly is defined with a value less than unity, and the number of branched elements of the second assembly is also defined for fixed values of angle  $\alpha$  and the geometrical and physical parameters mentioned before. The optimal number  $n_2$  for an angle  $\beta = 10^\circ$  is  $n_2 \approx 140$ , and for a right angle of the metallic contacts,  $n_2 \approx 506$ . To show the influence of the optimal number of branched elements of the first and second assembly, the numbers are considered as  $n_1 \approx 37$  and  $n_2 \approx 140$ .



Figure 6. Second assembly dimensionless voltage drop versus aspect ratio.

Considering the overall resistance of every constructal level as resistances in shunt and series and considering the bifurcated arrangement and number of branches  $n_1$  and  $n_2$ , the minimum value of  $\overline{R}_{tot}$  for an aspect ratio of the corresponding constructal element can be determined, as shown in Figure 7. For fixed values of the physical quantities and branching angles defined previously, Figure 7a shows for the first assembly, that the minimum value is presented at  $\overline{H}_1 \cong 0.05$  for both cases. For a bifurcated branched arrangement of contacts, the minimum overall resistance presents a minimum at  $\overline{R}_1$  tot  $\cong 0.017$  in comparison with the  $n_1$  branches, with  $\overline{R}_1$  tot  $\cong 0.011$ .



**Figure 7.** Comparison of the bifurcated and *n* branches, for the (**a**) First assembly and (**b**) Second assembly.

For the second assembly, shown in Figure 7b, the first branching level is considered as resistances in shunt and series, decreases. The influence of the numbers of contacts will be shown in Figure 7b. The minimum value of the overall resistance for the  $n_1$  and  $n_2$ - branching cases is  $\overline{R}_{2 \text{ tot}} \cong 0.016$ for  $\overline{H}_2 \cong 0.06$ , which when compared to the arrangement of the bifurcated contact we obtained  $\overline{R}_{2 \text{ tot}} \cong 0.025$  at the same value of  $\overline{H}_2$ . The value of the overall resistance of the  $n_1$ ,  $n_2$  branches arrangement presents a lower value.

#### 6. Concluding Remarks

In the present work, we developed the fundamentals to determine the length of the metallic contacts network as a function of the physical properties and geometrical features of a common silicon solar cell. Based on the methodology, previously studied for the design of constructal networks for the heat transfer [16] and reticular grid in a solar cell [17], the aspect relation of the constructal elements and allocation of the metallic material between constructal elements were defined. There is a lack of studies on the constructal theory applied to the design of top metallic electrodes in solar cells; therefore, we can improve the harvest of sunlight, and consequently, the generation of electrical energy. For future work, we suggest the incorporation of a thermodynamic model, considering the internal energy and the entropy generation in a solar cell with electrodes. Our model does not consider the electrical characterization parameters like series and shunt resistance, open and short circuit current, and fill factor. Hence, the combination of such parameters with thermal parameters is needed to provide a better understanding and explanation of the solar cell performance.

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### Nomenclature

Α	area	$(m^2)$
D	electrode width	(m)
Н	height of the constructal element	(m)
i	electrical current	(A)
<i>i'''</i>	electrical current per unit of volume	$\left(A/m^3\right)$
L	length of the constructal element	(m)
п	number of branched elements	
V	voltage	(V)
Vol	volume	$(m^3)$
w	thickness of the solar cell	(m)
Greek symbols		
α	angle	(degrees)
β	angle	(degrees)
σ	electrical conductivity	$\left(\Omega^{-1}m^{-1}\right)$
$\varphi$	amount of metallic material	,
Subscripts		
т	metallic	
min	minimum	
SC	solar cell material	
0	elemental construction	
1	first construction	
2	second construction	

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