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Optimal Configuration of a Gas Expansion Process in a Piston-Type Cylinder with Generalized Convective Heat Transfer Law

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Abstract: Optimal configurations for the working fluid expansion process in a piston-type cylinder with maximum work production are studied by applying finite time thermodynamics. The problem is solved by utilizing the modified Lagrangian. The initial and final volumes, initial internal energy and total time are fixed, and the heat transfer between the working fluid and the external heat bath obeys the generalized convective heat transfer law, which can be transformed into Newton's heat transfer law, the Dulong–Petit heat transfer law and the square convective heat transfer law. The optimal configurations of the expansion process under three different conditions of heat transfer law are provided and compared, respectively. The results show that the heat transfer law has both quantitative and qualitative influences on the optimal configurations of the expansion process.

Keywords: finite time thermodynamics; generalized convective heat transfer law; heated piston-type cylinder; optimal configuration; maximum work production

1. Introduction

Finite time thermodynamics (FTT) [1–13] has been applied to perform performance analyses and optimizations for various thermodynamic cycles and processes, including multi-stream heat exchange system [14], chemical reactors [15–18], electrochemical device [19], biological process [20], Novikov engines [21–23], Agrawal engine [24], Carnot engines [25,26], solar engine [27], internal combustion engine cycles [28–32], thermoelectric devices [33–36], cogeneration plants [37–39], ocean thermal energy conversion plants [40–42], Kalina cycle [43], Rankine cycles [44–46], Brayton cycles [47–49], Maisotsenko cycles [50,51], ratchet engine [52], electron engine [53] and the quantum engine [54]. It is one of the standard problems to determine the optimal configurations (OCs) for thermodynamic processes with the specific optimization objectives in FTT. To solve the OCs problems, Euler–Lagrange formalism and optimal control theory are two important methods. For some optimal control problems, analytic solutions can be derived. While for the other problems without analytic solutions, numerical methods are required to be referred. All thermodynamic parameters can be solved once the OCs are obtained. Therefore, it is more complex and significant to study the problems of OCs.

Many investigations about the OCs of different theoretical and practical engine models have been carried out by applying the Euler–Lagrange formalism or optimal control theory. Rubin [55,56] took account of Newton's heat transfer law (NHTL) $q \propto \Delta(T)$ and took the maximum power production as the optimization objective to investigate the OCs of an endoreversible heat engines (HEs) under different constraint conditions. Ondrechen et al. [57] considered the change of heat-reservoir temperature and investigated the OC of the HE with NHTL. Adopting the similar heat transfer and heat reservoir

models, Chen et al. [58] researched the heat leakage influence on the OC. Refs. [59–62] conducted studies on the optimal piston motions (OPMs) for the four-stroke Otto [59,60] and Diesel [61,62] cycle HEs with NHTL. Watowich et al. [63,64] carried out optimizations for light-driven dissipative engines with NHTL based on the maximum work output (MWP) and minimum entropy generation criteria, and obtained the optimal piston trajectories. Teh et al. [65,66] researched the optimal cycle of adiabatic internal combustion engine (ICE) under the condition of maximum efficiency by considering the heat leakage and chemical reaction loss as major losses of ICEs. Bi et al. [67] optimized the charging and discharging processes of the gas-hydrate with NHTL based on the minimum entropy generation criterion, and determined the OCs of the temperature and gas-hydrate phase change rate.

Band et al. [68,69] studied the optimal motion of a piston-type cylinder with NHTL based on the MWP criterion by using Euler–Lagrange ($E - L$) formalism, and obtained the OC of an ideal gas expansion process (EP), which consists of three phases as follows: (1) an adiabatic phase at the initial time; (2) an intermediate $E - L$ arc; (3) a final instantaneous adiabatic phase. Besides, the OCs of the EP with eight kinds of constraints were analyzed and compared. On the basis of Refs. [68,69], Band et al. [70] extended the obtained results to the external combustion engine in which the heat transfer obeys NHTL. Aizenbud and Band [71] and Salamon et al. [72] took the MWP [71] and the maximum power output [72] as optimization objectives to research the OCs of EP under the condition of NHTL, respectively. Based on the obtained results from Refs. [68,69], Aizenbud et al. [73] further studied the ICE model with NHTL.

Studying the effects of heat transfer laws (HTLs) on the OCs of thermodynamic processes and cycles is one of the most important works of FTT. In fact, heat transfer between the working fluid (WF) and external heat bath (EHB) does not always obey NHTL. There are many different HTLs, such as the linear phenomenological heat transfer law (LPHTL) $q \propto \Delta(T^{-1})$, radiative HTL $q \propto \Delta(T^4)$, Dulong–Petit heat transfer law (DPHTL) $q \propto (\Delta T)^{5/4}$, and the convective-radiative HTL $q \propto \Delta(T) + \Delta(T^4)$. Establishing the universal model and obtaining the universal laws and results are the aim of pursuing FTT, and that is the same for the OC studies. Beside the above simple HTLs, there are three universal HTLs: generalized radiative HTL (GRHTL) $q \propto (T^n - T_{ex}^n)$, generalized convective HTL (GCHTL) $q \propto (T - T_{ex})^n$ and the complex generalized HTL $q \propto (T^n - T_{ex}^n)^m$ which includes GRHTL and GCHTL. The HTL not only significantly affects the OCs of the thermodynamic processes with the specific optimization objectives, but also affects the performances of given thermodynamic processes. Refs. [74–79] took the MWP and the maximum efficiency as optimization objectives to determine the OCs of the endoreversible HEs with different constraints and different HTLs, including the LPHTL [74], radiative HTL [75,79] and GRHTL [76–78]. Yan et al. [80] researched the OC of a HE with variable-temperature heat reservoir and LPHTL based on the MWP criterion. Besides, some studies on the OCs of the HEs with GRHTL [81], GCHTL [82] and complex generalized HTL [83] under condition of variable-temperature heat reservoirs were conducted based on MWP criterion. Burzler and Hoffmann [84] and Burzler [85] adopted a four-stroke Diesel HE model and researched the OPM of the HE with the convective-radiative HTL under the condition of maximum power output. Xia et al. [86] obtained the OPM of an Otto cycle HE with the MWP under the condition of LPHTL. Ge et al. [87] considered both the NHTL and LPHTL, and optimized the minimum entropy generation to obtain the OPM of an Otto cycle. Ma et al. [88] introduced the LPHTL into a light-driven dissipative engine, and obtained the OPM of the engine with the MWP and minimum entropy generation.

On basis of Refs. [68,69], Refs. [89–91] concluded the OC for the EP of WF in a piston-type cylinder with the LPHTL [89], GRHTL [90] and convective-radiative HTL [91]. Moreover, Ref. [90] obtained the first-order approximate analytic solutions of the $E - L$ arcs by adopting Taylor series expansion when n is equal to 2, 3 and 4. Chen et al. [92] further adopted the elimination method to study the intermediate $E - L$ arcs of the EP with the GRHTL, and obtained the analytic solutions when n is equal to 2 and 3. Based on the obtained results in Ref. [89], Refs. [93,94] further investigated the external combustion engine [93] and the ICE [94] models with the LPHTL.

From the above introduction, one can see that, for a piston-type cylinder, lots of authors have studied the OCs under NHTL, LPHTL, radiative HTL, convective-radiative HTL and GRHTL. In Refs. [90,92], the utilized generalized radiative law $q \propto (T^n - T_{ex}^n)$ includes Newtonian $(T - T_{ex})$, radiative $(T^4 - T_{ex}^4)$, linear phenomenological $(T_{ex}^{-1} - T^{-1})$, $(T^2 - T_{ex}^2)$, $(T^3 - T_{ex}^3)$ and other laws. It does not include some special laws, such as $(T - T_{ex})^2$, $(T - T_{ex})^3$, $(T - T_{ex})^{5/4}$. Therefore, in order to fully reveal the effects of HTLs on OCs and enrich FTT theory, the OCs under another generalized law, GCHTL $q \propto (T - T_{ex})^n$ should be studied. Based on Refs. [68,69,89–92], the OCs for the EP of WF in a piston-type cylinder will be studied using the modified Lagrangian in this paper. The initial and final volumes, initial internal energy and total time will be fixed, and the MWP with GCHTL $q \propto (T - T_{ex})^n$ will be obtained. The OCs of the EP under conditions of NHTL ($n = 1$), DPHTL ($n = 5/4$) and square convective heat transfer law (SCHTL) ($n = 2$) will be provided and compared, respectively.

2. Modeling

Figure 1 shows a piston-type cylinder, in which an ideal gas expands to generate work. There are six assumptions: (1) the WF is an ideal gas and the mole number of WF is 1 mol; (2) the ideal gas unevenly absorbs heat from the heat source, and the heat absorption rate is $f(t)$ that is an arbitrarily provided function about the time; (3) the EHB temperature T_{ex} is constant, which the ideal gas can be coupled to; (4) the work generated from this system within a specified time interval can be furthest used by controlling the piston motion; (5) the inertias of the piston and the WF are negligible, and the friction loss due to the movement of the piston is also negligible; (6) the heat transfer obeys the GCHTL [82,95–99]. The heat conductance (U) of the cylinder wall is an important parameter for heat transfer, which is the product of the heat transfer coefficient and the contact area between the WF and cylinder. Nevertheless, this paper simplifies U as a constant [68,69].

Figure 1. Schematic diagram of the model: a mole of ideal gas in a cylinder is pumped by a given heating function $f(t)$ and is coupled to the heat bath.

According to the first law of thermodynamics, for the thermal system shown in Figure 1, one has:

$$\dot{E}(t) = f(t) - \dot{W}(t) - U[(T - T_{ex})\text{Sign}(T - T_{ex})]^n \text{Sign}(T - T_{ex}) \quad (1)$$

where $\dot{E}(t)$ and $\dot{W}(t)$ are the change rate of the WF internal energy and WF expansion power, respectively. Besides, the sign function $\text{Sign}(T - T_{ex})$ is equal to 1 as $T - T_{ex} \geq 0$ and -1 as $T - T_{ex} < 0$.

Within the time interval $(0, t_m)$, the work (W) produced by the system due to the expansion of the heated WF is:

$$W = \int_0^{t_m} p(t) \dot{V}(t) dt \quad (2)$$

where t_m is the expansion time, p is the WF pressure, and V is the WF volume. According to Ref. [68], the process of irreversible efficiency (η) can be written as

$$\eta = W / \{E_p + RT_{ex} \ln[V_m/V(0)]\} \quad (3)$$

where E_p is the WF absorbed energy, and it is equal to $\int_0^{t_m} f(t)dt$. Besides, $RT_{ex} \ln[V_m/V(0)]$ is the maximum energy obtained from the WF that expands from $V(0)$ to V_m under the condition of an isothermal expansion temperature (T_{ex}).

3. Optimal Solutions

Taking the WF as an ideal gas, the relationships of $p(t) = RT(t)/V$ and $E(t) = C_V T(t)$ hold, where R and C_V are the gas constant and specific heat, respectively. The $p(t) = RE(t)/C_V V(t)$ can be obtained by combining above the relationships. Substituting it into Equations (1) and (2) yields:

$$\dot{E}(t) = f(t) - \frac{R}{C_V} E(t) \frac{\dot{V}(t)}{V(t)} - U \{ [E(t)/C_V - T_{ex}] \text{Sign}(T - T_{ex}) \}^n \text{Sign}(T - T_{ex}) \quad (4)$$

$$W = \int_0^{t_m} \frac{R}{C_V} E \frac{\dot{V}(t)}{V(t)} dt \quad (5)$$

The optimal control problem is:

$$\text{maximize } W = \int_0^{t_m} \frac{R}{C_V} E \frac{\dot{V}(t)}{V(t)} dt \quad (6)$$

which is constrained by Equation (4).

For solving the above problem, the modified Lagrangian is established as following:

$$L = \frac{R}{C_V} E(t) \frac{\dot{V}(t)}{V(t)} + \lambda(t) \left\{ \dot{E}(t) - f(t) + \frac{R}{C_V} E(t) \frac{\dot{V}(t)}{V(t)} + U \{ [E(t)/C_V - T_{ex}] \text{Sign}(T - T_{ex}) \}^n \text{Sign}(T - T_{ex}) \right\} \quad (7)$$

where $\lambda(t)$ is the Lagrange multiplier for time. From Equation (7), one has:

$$\frac{\partial L}{\partial E} - \frac{d}{dt} \frac{\partial L}{\partial \dot{E}} = 0, \quad \frac{\partial L}{\partial V} - \frac{d}{dt} \frac{\partial L}{\partial \dot{V}} = 0 \quad (8)$$

The results of Equation (8) are first-order since the relationships between Lagrangian and $\dot{E}(t)$, and between Lagrangian and $\dot{V}(t)$ are linear. Hence, the arbitrary initial state and final state cannot be connected because the solutions have not sufficient free parameters. While the OPM can be composed by the motions contenting the Equation (8) and the motions on the accessible regional boundaries of the control variables, the motions contenting both the Equation (8) and adiabatic jumps must be combined together to derive the OPM because the boundary motions consist of infinitely adiabatic jumps [68].

Combining the modified Lagrangian with Euler-Lagrange equations, i.e., Equation (7) and Equation (8), the following equations can be obtained:

$$\dot{\lambda}(t) = [1 + \lambda(t)] \frac{R}{C_V} \frac{\dot{V}(t)}{V(t)} + nU\lambda(t) \frac{\{ [E(t)/C_V - T_{ex}] \text{Sign}(T - T_{ex}) \}^{n-1}}{C_V} \quad (9)$$

$$[1 + \lambda(t)] \dot{E}(t) + \dot{\lambda}(t) E(t) = 0 \quad (10)$$

From Equations (4) and (9), one has:

$$\dot{E} = f(t) - \frac{E(t)\dot{\lambda}}{1 + \lambda} + U\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^{n-1} \left\{ \frac{E(t)[(n-1)\lambda(t) - 1]}{[1 + \lambda(t)]C_V} + T_{ex} \right\} \quad (11)$$

In the derivation of Equation (11), the following equation is used:

$$[\text{Sign}(T - T_{ex})]^{n+1} = [\text{Sign}(T - T_{ex})]^{n-1} \cdot [\text{Sign}(T - T_{ex})]^2 = [\text{Sign}(T - T_{ex})]^{n-1} \quad (12)$$

Combining Equation (10) with Equation (11) and eliminating $\dot{\lambda}(t)$ yields:

$$\lambda(t) = - \frac{[C_V T_{ex} - E(t)] \{ U\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n - f(t)\text{Sign}(T - T_{ex}) \}}{U[(n-1)E(t) + C_V T_{ex}]\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n + f(t)[E(t) - C_V T_{ex}]\text{Sign}(T - T_{ex})} \quad (13)$$

Introducing the derivative of Equation (13) to the time, there is:

$$\dot{\lambda}(t) = \frac{nU\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n \{ E(t)\dot{f}(t)[C_V T_{ex} - E(t)]\text{Sign}(T - T_{ex}) + \dot{E}(t)\{UC_V T_{ex}\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n + f(t)[nE(t) - C_V T_{ex}]\text{Sign}(T - T_{ex})\} \}}{\{U[(n-1)E(t) + C_V T_{ex}]\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n + f(t)[E(t) - C_V T_{ex}]\text{Sign}(T - T_{ex})\}^2} \quad (14)$$

Substituting $\lambda(t)$ and $\dot{\lambda}(t)$, i.e., Equations (13) and (14), into Equation (11) yields:

$$\dot{E}(t) = \frac{\dot{f}(t)E(t)[C_V T_{ex} - E(t)]\text{Sign}(T - T_{ex})}{f(t)[2C_V T_{ex} - (n+1)E(t)]\text{Sign}(T - T_{ex}) - [2C_V T_{ex} + (n-1)E(t)]U\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n} \quad (15)$$

According to Equation (15), the $E(t)$ with respect to $f(t)$ is solved. Moreover, the $V(t)$ of the WF is obtained by substituting $E(t)$ solved from Equation (15) into Equation (4) and then integrating over time.

$$V(t) = V(0) \left[\frac{E(t)}{E(0)} \right]^{-C_V/R} \times \exp \left\{ \frac{C_V}{R} \int_0^t \frac{f(t) - U\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n \text{Sign}(T - T_{ex})}{E(t)} dt \right\} \quad (16)$$

The optimal process that is determined by Equations (15) and (16) is called the $E - L$ arc.

One can obtain the OPM for the EP in a heated piston-type cylinder, which consists of three phases according to Ref. [68].

The terms $f(t)$ and $U\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^n \text{Sign}(T - T_{ex})$ can be ignored for the adiabatic jump. From Equation (1), one has:

$$E(V) = E(V_i)(V/V_i)^{-R/C_V} \quad (17)$$

If the $V(0)$, $E(0)$, V_m and t_m of the EP are specified, the motion equations of the corresponding phases can be derived.

The initial internal energy $E'(0)$ in the $E - L$ arc is:

$$E'(0) = E(0)[V'(0)/V(0)]^{-R/C_V} \quad (18)$$

where $V'(0)$ is the volume after the initial jump for Phase (1).

Phase (2) is the $E - L$ arc with $V'(0)$ and $E'(0)$, and runs from the $t = 0$ to $t = t_m$. When the heat transfer obeys different HTLs (namely different values of n), different calculation methods will be adopted, and as a result, different $E - L$ arcs will be formed. When n is equal to 1, the analytic solutions of the $E - L$ arc can be determined by combining Equations (15) with (16); while n is not equal to 1, the solution of the $E - L$ arc can only be solved by numerical method because the analytic solutions of $E(t)$ cannot be solved according to Equation (15).

Phase (3) is the final adiabatic phase from $V(t_m)$ to V_m as $t = t_m$

$$E_m = E(t_m)[V_m/V(t_m)]^{-R/C_V} \quad (19)$$

where $V(t_m)$ and $E(t_m)$ are obtained according to $E - L$ arcs at $t = t_m$.

Therefore, any optimal EP from $V(0)$ and $E(0)$ to V_m within the t_m can be derived by singly choosing the $V'(0)$ or $E'(0)$. The OPM for the EP with MWP was transformed into an optimization problem in one dimension.

The objective function within the t_m can be established by integrating the energy conservation equation in Equation (4), i.e.,:

$$W = \int_0^{t_m} f(t)dt + E(0) - E_m - U \int_0^{t_m} \{[E(t)/C_V - T_{ex}] \text{Sign}(T - T_{ex})\}^n \text{Sign}(T - T_{ex})dt \quad (20)$$

There are at least two methods to obtain the optimal $E'(0)$: (1) substituting Equation (20) into the equation $dW/dE'(0) = 0$; and (2) the two method of exhaustion. The analytic solutions of the $E - L$ arc must be derived when the first method is applied. However, the analytic solutions of the $E - L$ arc cannot be derived when n is not equal to 1. Therefore, the method of exhaustion should be used to obtain the optimal $E'(0)$ when n is not equal to 1.

When the heat transfer obeys different HTLs, the $E - L$ arcs can be obtained as follows.

3.1. $E - L$ Arc with $n = 1$

The heat transfer obeys NHTL as n is equal to 1.

Firstly, substituting $n = 1$ into Equation (15) and integrating over time t yields:

$$E(t) = E'(0) \left[\frac{UT_{ex} + f(t)}{UT_{ex} + f(0)} \right]^{1/2} \quad (21)$$

Substituting $E(t)$ from Equation (21) into Equation (16) yields:

$$V(t) = V'(0) \left[\frac{UT_{ex} + f(t)}{UT_{ex} + f(0)} \right]^{-C_V/2R} \exp \left\{ -\frac{Ut}{R} + \frac{C_V}{R} \frac{[UT_{ex} + f(0)]^{1/2}}{E'(0)} \int_0^t [UT_{ex} + f(t)]^{1/2} dt \right\} \quad (22)$$

The $E - L$ arc (phase (2)) is determined by Equations (21) and (22).

Secondly, the optimal $E'(0)$ can be determined according to $dW/dE'(0) = 0$.

Finally, the $E - L$ arcs can be obtained by solving Equations (21) and (22) after the optimal $E'(0)$ is determined.

3.2. $E - L$ Arc with $n = 5/4$

The heat transfer obeys DPHTL [100] as n is equal to $5/4$, i.e., $q \propto (\Delta T)^{5/4}$. The DPHTL is widely applied to heat transfer analysis, especially in the field of forced convection.

Firstly, when $n = 5/4$, Equation (15) becomes:

$$\dot{E}(t) = \frac{\dot{f}(t)E(t)[C_V T_{ex} - E(t)]\text{Sign}(T - T_{ex})}{f(t)[2C_V T_{ex} - 9E(t)/4]\text{Sign}(T - T_{ex}) - U[2C_V T_{ex} + E(t)/4]\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^{5/4}} \quad (23)$$

Substituting $E(t)$ obtained from Equation (23) into Equation (16) yields:

$$V(t) = V'(0) \left[\frac{E(t)}{E'(0)} \right]^{-C_V/R} \exp \left\{ \frac{C_V}{R} \int_0^t \frac{f(t) - U\{[E(t)/C_V - T_{ex}]\text{Sign}(T - T_{ex})\}^{5/4}\text{Sign}(T - T_{ex})}{E(t)} dt \right\} \quad (24)$$

The $E - L$ arc (phase (2)) is determined by Equations (23) and (24).

Secondly, the optimal $E'(0)$ can be determined by applying the method of exhaustion since the analytic solution of the $E - L$ arc cannot be solved.

Finally, after the optimal $E'(0)$ is determined and Equations (23) and (24) are solved, the $E - L$ arcs will be derived.

3.3. $E - L$ Arc with $n = 2$

The heat transfer obeys the SCHTL as n is equal to 2.

Firstly, substituting $n = 2$ into Equation (15) yields:

$$\dot{E}(t) = \frac{\dot{f}(t)E(t)[C_V T_{ex} - E(t)]\text{Sign}(T - T_{ex})}{f(t)[2C_V T_{ex} - 3E(t)]\text{Sign}(T - T_{ex}) - U[2C_V T_{ex} + E(t)][E(t)/C_V - T_{ex}]^2} \quad (25)$$

Substituting $E(t)$ obtained from Equation (25) into Equation (16) yields:

$$V(t) = V'(0) \left[\frac{E(t)}{E'(0)} \right]^{-C_V/R} \exp \left\{ \frac{C_V}{R} \int_0^t \frac{f(t) - U[E(t)/C_V - T_{ex}]^2\text{Sign}(T - T_{ex})}{E(t)} dt \right\} \quad (26)$$

The $E - L$ arc (phase (2)) is determined by Equations (25) and (26).

Secondly, the optimal $E'(0)$ can be determined by applying the method of exhaustion since the analytic solution of the $E - L$ arc cannot be solved.

Finally, the $E - L$ arcs can be obtained by solving Equations (25) and (26) after the optimal $E'(0)$ is determined.

4. Numerical Examples

According to Ref. [68], the parameters $V(0) = 1 \times 10^{-3} \text{ m}^3$, $V_m = 8 \times 10^{-3} \text{ m}^3$, $E(0) = 3780 \text{ J}$, $T_{ex} = 300 \text{ K}$, $C_V = 3R/2$, $t_m = 2 \text{ s}$, $f(t) = a \cdot t \cdot \exp(-t/b)$, $a = 4200 \text{ J/s}^2$ and $b = 1 \text{ s}$ are given in the numerical calculations. Analyzing the effects of the HTLs on the optimal expansion and choosing the U as variable parameter are two essential differences between this paper and Refs. [68,69,89–92].

4.1. Calculation Example for $n = 1$

The MWP, η_W and state variables at switching are listed in Table 1. The $E - t$ and $V - t$ in the $E - L$ arc are shown in Figures 2 and 3, respectively. In Figure 2, the E increases to the peak before decreasing with the increase in the time, and there is a maximum E . It can be concluded from Figure 2 that the temperature, at which the whole $E - L$ arc occurs, should be below the EHB temperature, namely less than $T_{ex} = 300 \text{ K}$. This indicates that the WF is cooled in the initial adiabatic EP, and then is heated by the bath-leaked energy. The energy absorbing from heat producer and the energy leaking into WF are both transformed into work. The energy leaking into WF increases with the augmentation of U , which leads to the increases of MWP and η_W . Moreover, the relationship between the WF internal

energy and U is different in different parts of the $E-L$ arc. The WF internal energy grows with the augmentation of U during the initial and final $E-L$ arcs, then diminishes with the augmentation of U during the intermediate $E-L$ arc. As shown in Figure 3, the WF is compressed slightly in the initial $E-L$ arc, and then monotonically expands until the end of EP. Additionally, with the augmentation of the U , the $V(t_m)/V'(0)$ of WF during the $E-L$ arc increases.

Table 1. Parameters versus U for the case of $n = 1$.

$U(W \cdot K^{-1})$	12.6	14.7	16.8
$V'(0)(10^{-3} \text{ m}^3)$	1.4370	1.4005	1.3702
$E'(0)(J)$	2968.3	3019.7	3064.2
$V(t_m)(10^{-3} \text{ m}^3)$	4.6451	4.8877	5.0944
$E(t_m)(J)$	3385.4	3386.6	3392.2
$E(m)(J)$	2356.2	2438.43	2510.8
$W(m)(J)$	4562.5	4593.6	4622.5
η	0.5942	0.5982	0.6020

Figure 2. Optimal internal energy versus time as $n = 1$.

4.2. Calculation Example for $n = 5/4$

The optimal $E'(0)$ can be obtained by applying the method of exhaustion since the analytic solution of the $E-L$ arc with DPHTL cannot be derived. The following calculation procedure is adopted. When $E'(0)$ is given, the corresponding $V'(0)$ can be obtained by substituting $E'(0)$ into Equation (18). Then, according to Equations (23) and (24), the $E-L$ arc is obtained. In the end, combining Equation (20) and the equations of $E(t)$ and $V(t)$, the corresponding objective function (W) can be obtained. When $E'(0)$ takes the other possible values, the relationship between the W and $E'(0)$ can be derived by repeating the above procedures. Then, the MWP and the corresponding optimal $E'(0)$ are obtained. Finally, the whole $E-L$ arc can be derived after the optimal $E'(0)$ is obtained.

Figure 3. Optimal volume versus time as $n = 1$.

Figure 4 depicts the relationship between the W and $E'(0)$ with different U . It is shown that the curves are in a parabolic shape at the initial parts of the curves, and there is an optimal $E'(0)$ to make the W reach maximum. Then, with the increase in $E'(0)$, one can observe that there exists strong fluctuations in the W . The major reason for the fluctuations is as follows. For such values of $E'(0)$, the denominator of Equation (23) approaches zero during the iterative computation. As a result, the results obtained from Equations (23) and (24) fluctuate strongly. Additionally, some process parameters obtained by using the values of $E'(0)$ are irrational. In a word, the values of $E'(0)$ that make the W fluctuate are irrational. From Figure 4, the optimal $E'(0)$ can be determined.

Figure 4. Work production W versus the values of $E'(0)$ with variable heat conductance U as $n = 5/4$.

Table 2 lists the MWP, η_W and the state variables at switching. Figures 5 and 6 show the characteristics of $E - t$ and $V - t$ in the $E - L$ arc, respectively. In Figure 5, the E increases to the peak before decreasing with the increase in the time, and there is a maximum E . It can be concluded from Figure 5 that the temperature, at which the whole $E - L$ arc occurs, should be below the EHB

temperature, namely, less than $T_{ex} = 300$ K. As a result, the MWP and η_W increase with the increase in U .

Table 2. Parameters versus U for the case of $n = 5/4$.

$U(W \cdot K^{-5/4})$	5.0	5.5	6.0
$V'(0)(10^{-3} \text{ m}^3)$	1.4670	1.4411	1.4189
$E'(0)(J)$	2927.78	2962.71	2993.55
$V(t_m)(10^{-3} \text{ m}^3)$	4.9076	5.0552	5.1856
$E(t_m)(J)$	3317.32	3324.24	3331.11
$E(m)(J)$	2395.02	2447.90	2494.78
$W(J)$	4546.61	4564.50	4581.58
η	0.5921	0.5945	0.5967

Figure 5. Optimal internal energy versus time as $n = 5/4$.

Figure 6. Optimal volume versus time as $n = 5/4$.

4.3. Calculation Example for $n = 2$

The optimal $E'(0)$ can be derived by applying the method of exhaustion since the analytic solution of the $E - L$ arc with SCHTL cannot be derived. Figure 7 depicts the relationship between the objective function (W) and $E'(0)$ with different U . It can be seen from the curves that the W increases to the peak before decreasing with the increase in $E'(0)$, and there are a MWP and its corresponding optimal $E'(0)$.

Figure 7. Work production W versus the values of $E'(0)$ with variable heat conductance U as $n = 2$.

Table 3 lists the MWP, the η_W and the state variables at switching. Figures 8 and 9 show the characteristics of $E - t$ and $V - t$ in the $E - L$ arc, respectively. In Figure 8, the E increases to the peak before decreasing with the increase in the time, and there is a maximum E . It can be concluded from the figure that the temperature, at which the whole $E - L$ arc occurs, should be above the EHB temperature, namely more than $T_{ex} = 300$ K. This indicates that the WF does not absorb heat from the EHB, but releases heat to the EHB during the whole $E - L$ arc. The result is different not only from those with NHTL and DPHTL, but also from those with square $q \propto \Delta(T^2)$, cubic $q \propto \Delta(T^3)$ and radiative $q \propto \Delta(T^4)$ HTLs obtained in Refs. [90,92]. The comparisons among the EPs with different HTLs show that the HTL has both quantitative and qualitative influences on the OCs of the EPs. The energy leaking into the bath increases with the augmentation of U , which leads to the decreases of the MWP and η_W .

Table 3. Parameters versus U for the case of $n = 2$.

$U(W \cdot K^{-2})$	0.1	0.2	0.3
$V'(0)(10^{-3} \text{ m}^3)$	1.0108	1.0134	1.0143
$E'(0)(J)$	3753.00	3746.61	3744.34
$V(t_m)(10^{-3} \text{ m}^3)$	2.1527	2.3598	2.4566
$E(t_m)(J)$	4033.61	3910.56	3861.22
$E(m)(J)$	1681.22	1732.87	1757.48
$W(J)$	4461.83	4450.59	4447.02
η	0.5811	0.5796	0.5792

Figure 8. Optimal internal energy versus time as $n = 2$.

Figure 9. Optimal volume versus time as $n = 2$.

Moreover, Figure 8 shows that the $E(t)$ during the whole $E - L$ arc decreases with the augmentation of U . This result is different from the conditions of NHTL and DPHTL. As shown in Figure 9, the WF is compressed slightly in the initial $E - L$ arc, and then monotonically expands until the end of EP. Additionally, with the augmentation of the U , the $V(t_m)/V'(0)$ of WF during the $E - L$ arc increases.

4.4. Performance Comparisons for Three Special HTLs

In the numerical calculations, the different U are given under the different conditions of n . U is set as 12.6 W/K , $5.5 \text{ W/K}^{5/4}$ and 0.1 W/K^2 when n is equal to 1, $5/4$ and 2, respectively. Figures 10 and 11 show the characteristics of $E - t$ and $V - t$ in the $E - L$ arc with different HTLs, respectively.

Figure 10. Optimal internal energy versus time with different heat transfer laws.

Figure 11. Optimal volume versus time with different heat transfer laws.

In the first, Tables 1–3 list that the differences among the OCs with different HTLs as follows: when the heat transfer obeys the NHTL and DPHTL, with the augmentation of U , the $V'(0)$ decreases, the $E'(0)$, MWP and η_W increase; while the heat transfer obeys SCHTL, with the augmentation of U , the $V'(0)$ increases, and the $E'(0)$, MWP and η_W decrease. Moreover, compared with the NHTL and DPHTL, the MWP and η_W with SCHTL are smaller.

Secondly, Figure 10 shows that the E increases to the peak before decreasing with the increase in time, and there is a maximum E . As shown in Figure 11, the WF is compressed slightly in the initial $E-L$ arc, and then monotonically expands until the end of EP. Additionally, with the augmentation of the U , the $V(t_m)/V'(0)$ of WF during the $E-L$ arc increases.

5. Conclusions

The optimal configurations for the expansion process of working fluid in a piston-type cylinder with maximum work output are studied by applying the modified Lagrangian. The initial and final

volumes, initial internal energy and total time of EP are fixed, and the heat transfer between the working fluid and the external heat bath obeys $q \propto (T - T_{ex})^n$. The optimal configurations under the conditions of $n = 1$, $n = 5/4$ and $n = 2$ are obtained and compared, respectively. The main conclusions are as follows:

- (1) The relationships between the E and time are similar under the conditions of three special heat transfer laws; namely, the E increases to the peak before decreasing with the increase in time, and there is a maximum E . For all of three special heat transfer laws, the working fluid is compressed slightly in the initial $E - L$ arc, and then monotonically expands until the end of expansion process. With the augmentation of the U , $V(t_m)/V'(0)$ of working fluid during the $E - L$ arc increases.
- (2) There are differences among the optimal configurations with three different heat transfer laws. In the cases of $n = 1$ and $n = 5/4$, the temperature at which the whole $E - L$ arc occurs should be below the external heat bath temperature, namely less than $T_{ex} = 300$ K; with the augmentation of U , the $V'(0)$ decreases, and the $E'(0)$, maximum work output and η_W increase. While for $n = 2$, the temperature at which the whole $E - L$ arc occurs should be above the external heat bath temperature, namely more than $T_{ex} = 300$ K. This indicates that the working fluid does not absorb heat from the external heat bath, but releases heat to the external heat bath during the whole $E - L$ arc. With the augmentation of U , the $V'(0)$ increases, and the $E'(0)$, maximum work output and η_W decrease. The results obtained with $n = 2$ are not only different from those with $n = 1$ and $n = 5/4$, but also from those with square $(T^2 - T_{ex}^2)$, cubic $(T^3 - T_{ex}^3)$ and radiative $(T^4 - T_{ex}^4)$ heat transfer laws obtained in the previous studies. Moreover, compared with $n = 1$ and $n = 5/4$, the maximum work output and η_W with $n = 2$ are smaller. It can be concluded that the heat transfer law has both quantitative and qualitative influences on the optimal configurations of the expansion process.
- (3) The generalized convective heat transfer law $q \propto (T - T_{ex})^n$ is introduced into the theoretical model of an ideal gas irreversible expansion process in a piston-type cylinder, so the results obtained are universal and fully reveal the effect of heat transfer laws. The work in this paper can enrich FTT theory.

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Abbreviations

DPHTL	Dulong–Petit heat transfer law
EHB	external heat bath
E-L	Euler–Lagrange
EP	expansion process
FTT	finite time thermodynamics
GCHTL	generalized convective heat transfer law
GRHTL	generalized radiative heat transfer law
HE	heat engine
HTL	heat transfer law
ICE	internal combustion engine
LPHTL	the linear phenomenological heat transfer law
MWP	maximum work output

NHTL	Newton's heat transfer law
OC	optimal configuration
OPM	optimal piston motion
SCHTL	square convective heat transfer law
WF	working fluid

Nomenclature

a	coefficient of the heat absorption rate J/s^2
b	exponent of the heat absorption rate s
C_V	mole specific heat $J/(molK)$
E	internal energy J
f	heat absorption rate W
L	modified Lagrangian
n	heat transfer index
p	pressure Pa
q	heat transfer rate W
R	gas constant $J/(molK)$
$sign$	sign function
T	temperature K
t	time s
U	heat conductance W/K^n
V	volume m^3
W	work output J

Greek

symbol

η	efficiency
λ	Lagrange multiplier

Subscripts

ex	external heat bath
m	final state of expansion process
0	ambient or reference

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