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A Dynamic Stochastic Hybrid Model to Represent Significant Wave Height and Wave Period for Marine Energy Representation

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Abstract: This paper presents a methodology to represent ocean wave power generation based on real data observation for significant wave height (SWH or H_s) and wave period (WP or T). This technique is based on a hybrid model, which considers Fourier series and stochastic differential equations, allowing a continuous time representation of the random changes in the parameters associated with wave power generation (H_s and T). The methodology is explained, including estimation methods and a validation procedure. The data series generated by the models were used to create simulated wave power output applying a transformed matrix and a theoretical model. The results validate the utilization of this technique, when the objective is to obtain a robust dynamic representation of a random process, oriented to linear studies.

Keywords: continuous stochastic modeling; wave power; short-term forecast

1. Introduction

The inclusion of wave energy in electrical power systems (EPS) is a current research topic at global level, due to its energy potential—the prospective world production has been theoretically estimated at more than 100,000 TWh per year [1]—and the increasing interest to include renewable energy supplies [2]. For example, North American, European and Asian countries have installed wave energy conversion facilities [3], e.g. Ireland with the ocean energy buoy [4]; the United Kingdom with Pelamis, Ceto I and Ceto II, Oyster and Limpet [5,6]; the United States with PowerBuoy [7]; and Denmark with Wave dragon and wave star [6,8]. Additionally, in African and Oceanian regions, some studies related to implementation and integration of wave energy conversion are being developed [9]. Even though there is increased interest in this energy source, there still exists geographic zones with significant potential, which have not been intervened, for example, the case of Chile in South America [10], which has access to the sea in its whole continental territory.

In all those implemented examples and in future projects, it is necessary to study before a real implementation, the effect of integrating wave energy into the grid, because this type of power induces random perturbations that can trigger damage effects in the system at operational level. Due to the

inherent random nature of wave motion, stochastic modeling tools are widely used in this topic [11,12]. In fact, numerous studies focused on the integration and operation of machinery that uses the sea as the primary source consider spectrum analysis [13], statistical modeling [14], such as time series and regressions [15], together with heuristic methods, such as neural networks, and numerical models [16], such as the Navier stokes equations, to generate hybrid models [17]. With these models, it is possible to characterize the dynamics that induce the penetration of wave power in the power systems, useful for operational studies, and to predict future behaviors, necessary for planning studies.

In the literature, it is possible to find different ways to address the characterization of wave power. Some studies, such as the work published by Wu et al. [18], seek representations for parameters on which the power generation and the energy flow depend. Particularly, it is of special interest to represent the significant wave height (SWH or H_s) and the wave period (WP or T), whose actual measurement is easy to obtain given current technologies [19]. With this information, it is possible to perform statistical studies that allow obtaining information of the power that is generated under the respective conditions of H_s and T , and thus obtaining a representation for this type of generation.

Other studies, such as those conducted by Hatalis et al. [20] and Reikard and colleagues [16,21], directly model the output of power generated by the sea, submitting the available H_s and T data to a transformation, previous to the analysis and statistical modeling.

In particular, this paper proposes a dynamic stochastic hybrid method based on stochastic differential equations (SDE) and Fourier analysis to obtain a continuous model of H_s and T . The theory of linear waves [13] is used due to the characteristics of the sample data and because these models are oriented to be applied in small signal stability analysis, where only linear components are needed. Both models are used to generate series of H_s and T , which are considered as input in two preset models that deliver the generated wave power. The procedure order is chosen with the aim to avoid nonlinear problems of the dynamics associated with the output power.

The power models considered correspond to a theoretical model, which represents wave power in deep water [22], and a power transformation matrix [16], associated to the floating oscillating water column device. This unit operates in environmental conditions (water depth, wave heights and wave period) in which the buoys that extract the actual H_s and T measurements are located. These data have a time resolution of 3 h and are used for the estimation of the parameters associated to the H_s and T hybrid models.

The advantage of applying SDE in the representation of wave power generation can be seen in the applications to continuous time analysis, such as the case of stability analysis. As SDE capture the random dynamic behavior, they are suitable to represent the effect of a perturbation induced on the grid by, for example, the installation of new wave generation parks. Another advantage of this stochastic hybrid method is that it allows a random dynamic representation of power, which also permits simulations to be performed in any time interval, regardless of the original interval of the data sampling. This is advantageous in scenarios where complete missing data or when shorter intervals than the observed are needed, such as the case of stability studies. In addition, the importance of developing a model to represent the electrical power that can be extracted from the Chilean coast contributes directly to the integration of this technology into Chilean energy base, which remains untapped and can be a real alternative to respond to the constant growth of electric consumption in the country.

The paper is organized as follows: Section 2 presents the formulation of the problem and the procedure used to estimate and validate the model proposed for the characterization of H_s and T . In Section 3, the proposed methodology is applied, presenting systematically each stage to follow, from the presentation of the data to the validation of the model. In Section 4, the results of the models are presented including simulations and error measures. In addition, the series generated by the models are submitted to a transformation to obtain a series of power generation, in which the results are discussed. Section 5 presents the conclusions of the article.

2. Problem Formulation

This paper proposes robust dynamic stochastic models of H_s and T , which allow representing wave power. These models are oriented to linear studies of inclusion, such as small signal stability analysis, where the linearized equation of the complete system are investigated with the objective to maintain the stability at a generation level. Thus, the models to be presented consider simplified components, i.e., they consider linear and stationary elements, despite the non-stationarity and non-linearity of the ocean behavior.

2.1. Theoretical Model

The proposed models characterize the variation of H_s and T over time considering real data observation for the parametric adjustment and validation. The methodology consider two parts: a first analysis based on Fourier series, which is used to describe the seasonal behavior of waves in deep waters [23], where the theory of linear waves can be applied [13]; and a second part corresponding to the representation of the randomness inherent to the oceans, which is characterized by the Ornstein-Uhlenbeck (O-U) process. Mathematically, this is expressed in Equation (1) for H_s and in Equation (2) for T .

$$H_t = \alpha + \sum_{i=1}^n \gamma_i \sin(2\pi f_i t) + \delta_i \cos(2\pi f_i t) + U_t \quad (1)$$

$$\ln(T_t) = \beta + \sum_{i=1}^n \zeta_i \sin(2\pi f_i t) + \kappa_i \cos(2\pi f_i t) + U_t \quad (2)$$

where H_t is the H_s at the instant t , T_t is the T at the instant t , f_i is frequency associated to seasonal changes, U_t O-U process represents the noise, and α , γ_i , δ_i , β , ζ_i and κ_i are parameters to be estimated.

2.2. Numerical Model

To compute numerical values, according to our theoretical model, Equations (1) and (2) can be written in their differential form using Ito's formula [24], and are expressed in Equations (3) and (4), respectively.

$$dH_t = \left(\sum_{i=1}^n ((-\gamma_i 2\pi f_i - \eta \delta_i) \sin(2\pi f_i t) + (\delta_i 2\pi f_i - \eta \gamma_i) \cos(2\pi f_i t)) + \eta H_t - \eta \alpha \right) dt + \nu dW_t \quad (3)$$

$$dT_t = T_t \left(\sum_{i=1}^n (-\zeta_i 2\pi f_i \sin(2\pi f_i t) + \kappa_i 2\pi f_i \cos(2\pi f_i t)) + \eta (\ln T_t - \beta) - \sum_{i=1}^n \zeta_i \cos(2\pi f_i t) + \kappa_i \sin(2\pi f_i t) \right) + \frac{1}{2} \nu^2 dt + \nu T_t dW_t \quad (4)$$

The first stage of the proposed procedure consists of decomposing the data sample into its frequency spectrum f_i , using the periodogram, which corresponds to an approximation of the spectral density of the data series. With those frequencies, it is achievable to establish an equation composed of a sum of sine and cosine, linear in the parameters α , γ_i and δ_i in Equation (1), and in the coefficients β , ζ_i and κ_i in Equation (2), allowing the use of linear regression techniques for parametric estimation as least square. Only the significant frequencies must be considered, which correspond to those frequencies with a non-zero associated parameter in the sinusoidal function.

The second stage consists in associating SDE to the residuals of the Fourier adjustment, which is the Ornstein-Uhlenbeck process. In general, SDE can be used to simulate the continuous behavior of a variable, which is why in engineering topics it is common to find applications to power systems for

short-term studies and real time studies [25–27], such as the case of small signal stability [28]. However, as an application to the wave generation area, it is not a highly exploited resource [29].

The SDE can be considered as a generalized case of an ordinary differential equation, where the variable under study, state or dependent presents a random behavior in its evolution. One of the most common SDEs used to describe physical phenomena is the so-called Ornstein-Uhlenbeck (O-U) process, with Equation (5).

$$dU_t = \eta U_t dt + \nu dW_t \quad (5)$$

where U_t corresponds to the O-U process, which represents the state variable or dependent, $\eta < 0$ and $\nu > 0$ are parameters and W_t is the Brownian motion or Wiener process. Besides, given the initial condition for the process U_t , it follows a normal distribution, i.e., $(U_t | U_0 = u_0) \sim N(u_0 e^{\eta t}, \frac{\nu^2 (e^{2\eta t} - 1)}{2\eta})$.

2.3. Model Validation

To adjust the parameter of the O-U process and validate it, the residuals left by the Fourier analysis are used as data. To validate the results once the model is adjusted, the method called uniform residuals analysis [30] is used, which in simple words determines if the actual observations of a certain phenomenon are the result of a process with the characteristics of the proposed model.

With the parameters of the models of H_s and T estimated and validated, the SDE of the models are simulated. The data generated allow a representation of power generation, transforming the data with physical and empirical models.

2.4. Summary

To summarize, the methodology is described in the following stages:

- Have a data source of H_s and T .
- Apply the Fourier analysis to the data sample, i.e., estimate the significant frequencies by means of the periodogram and the amplitudes associated to those frequencies, using the least square estimation technique.
- Adjust the residuals of the previous estimation to an O-U process, using them as observation for the parametric adjustment, which is made by quasi-maximum likelihood technique [31] for the drift parameter η , and quadratic variation for the diffusion coefficient ν .
- Validate the resulting model with the techniques of uniform residuals [30].
- Simulate data series of H_s and T , using the Milstein scheme for the corresponding SDE [24].
- Transform the simulated series into power generation information by means of the physic model for generated power in deep water [22] and the power matrix of the floating oscillating water column device [16].

3. Implementation to the Chilean Case

This section presents the systematic application of the modeling methodology. The implementation allowed the generation of future data series of H_s and T , which provide information on the generation power through the transformation of these series of data.

3.1. Data Sources and Characteristics

The series of H_s and T correspond to H_s measurements in meters and T in seconds, with a resolution of 3 h, which were generated by two buoys located on the coast of Valparaiso (see Figure 1, red marks) belonging to the *Servicio Hidrográfico y Oceanográfico de la Armada de Chile* (SHOA). The dataset was used with the objective of illustrating the application of the methodology. There are no better samples of data for the Chilean coast. Table 1 summarizes the specifications of this information:

Figure 1. Buoys location.

Table 1. Data source.

Bouy	Location	Time Resolution	Available Period	Measures
Triaxys (Sample 1)	32°59' S, 71°49' W	3 h	7 February 2002 to 5 March 2002	Significant wave height (meters) Wave period (seconds)
Watchkeeper (Sample 2)	32°58' S, 71°39' W	3 h	30 August 2009 to 10 October 2010	Significant wave height (meters) Wave period (seconds)

3.2. Parameter Adjustment

According to the methodology presented in the Section 2, the frequencies of Equations (1) and (2) must be estimated by means of the periodogram, for each sample of data. The significant frequencies (Hz) estimated are shown in Tables A1 and A2 in the Appendix A. Then, with that information, it was possible to estimate the coefficients α , γ_i and δ_i in Equation (1), and the coefficients β , ζ_i and κ_i in Equation (2) for each sample of data, using the least Square technique. The estimated values for each parameter of each model are summarized in Tables A3 and A4 in the Appendix A.

The residuals left by this adjustment were fitted by the O-U process in Equation (5), hybridizing the methods of Fourier analysis and SDE. These residues corresponded to the observations of the continuous stochastic process O-U, which was used to estimate the parameters of the proposed process as a model.

For the parametric estimation of η , the quasi-maximum likelihood technique [31] was used. Under this method, the $\tilde{\eta}$ estimator is given by Equation (6).

$$\tilde{\eta}_n = \frac{1}{\Delta} \ln \frac{\sum_{i=1}^n U_{(i-1)\Delta} U_{i\Delta}}{\sum_{i=1}^n U_{(i-1)\Delta}^2} \quad (6)$$

where $U_{i\Delta}$, $i = 1, \dots, n$ are equidistant observations of the O-U process and Δ is the time interval between observations.

It was not possible to estimate parameter ν using this method, and in this case the estimation was done by means of the quadratic variation of the process. The estimator was obtained by equating the theoretical expression of the quadratic variation with its discretization. For the O-U process, the estimator $\tilde{\nu}$ is given by Equation (7).

$$\tilde{\nu} = \sqrt{\frac{1}{t} \sum_{i=1}^n (U_{i\Delta} - U_{(i-1)\Delta})^2} \quad (7)$$

where t corresponds to the total observation time interval.

The values of the parameters delivered by Equations (6) and (7) for each model and each respective sample are presented in Tables 2 and 3.

Table 2. Estimated parameters of O-U process for H_s series.

Sample 1		Sample 2	
Parameter	Value	Parameter	Value
η	−22.0653	η	−4.900198
ν	1.83798	ν	2.083687

Table 3. Estimated parameters of O-U process for T series.

Sample 1		Sample 2	
Parameter	Value	Parameter	Value
η	−36.58495	η	−13.05809
ν	1.369082	ν	1.406986

3.3. Validation Procedure

Once the parameters were estimated, the model Was validated. The method used for the present work corresponded to the analysis of uniform residuals [30]. This technique determined whether the data being adjusted could actually be described by the proposed process. The result of this analysis was graphically observed by means of a quantile-quantile plot, which compared the experimental distribution of the data with the theoretical distribution of the stochastic process. If the identity line were obtained, then it could be concluded that the model satisfactorily adjusts the observed data. Figure 2a–d shows the quantile-quantile plot of this analysis for each sample data and each model.

The quantile-quantile plots in Figure 2a,c correspond to the settings for Sample 1, which consisted of 248 observations. It was observed that the points in the graphics followed approximately the identity line, thus it was affirmed that there is no reason to doubt these adjustments. For the plot in Figure 2b,d, corresponding to Sample 2, which consisted of a total of 3207 observations, the line was not precisely the identity line. The deviation of the curve suggests that the proposed process failed to describe certain characteristics of the data, removing the precision of the model. The difference between the results of Samples 1 and 2 might be due to the fact that they were very different in size: Sample 2 consisted of many data. Tthe nonlinear and non-stationary behaviors of the oceans might be very strong and do not allow the correct fit of the proposed hybrid model. This was not observed in Sample 1, which corresponded to a much shorter observation interval in which it is possible that nonlinear and non-stationary effects were not fully captured.

Figure 2. Quantile-Quantile plots of uniform residuals of H_s and T models. The dashed red line shows the identity function.

4. Results

Once the model was validated, the projections of present and future values of H_s and T must be done to apply the respective transformations to the series and obtain simulations of generated power.

4.1. Simulated Series of H_s and T

To generate the trajectories made by Equations (1) and (2), the numerical scheme of Milstein was used, corresponding to an approximation up to the second-order of the Ito–Taylor expansion of the SDEs under study in Equations (3) and (4) [24].

Equations (8) and (9) express the simulation schemes for the H_s and T models, respectively.

$$H_{t_{j+1}} = H_{t_j} + \left(\sum_{i=1}^n ((-\gamma_i 2\pi f_i - \eta \delta_i) \sin(2\pi f_i t_j) + (\delta_i 2\pi f_i - \eta \gamma_i) \cos(2\pi f_i t_j)) \right. \\ \left. + \eta H_{t_j} - \eta \alpha \right) (t_{j+1} - t_j) + \nu (W_{t_{j+1}} - W_{t_j}) \quad (8)$$

$$T_{t_{j+1}} = T_{t_j} + T_{t_j} \left(\sum_{i=1}^n -\zeta_i 2\pi f_i \sin(2\pi f_i t_j) + \kappa_i 2\pi f_i \cos(2\pi f_i t_j) + \eta (\ln T_{t_j} - \beta) \right. \\ \left. - \sum_{i=1}^n \zeta_i \cos(2\pi f_i t_j) + \kappa_i \sin(2\pi f_i t_j) \right) + \frac{1}{2} \nu (t_{j+1} - t_j) + \nu T_{t_j} (W_{t_{j+1}} - W_{t_j}) \\ + \frac{\nu^2}{2} T_{t_j} ((W_{t_{j+1}} - W_{t_j})^2 - (t_{j+1} - t_j)) \quad (9)$$

where $W_{t_{j+1}} - W_{t_j}$ corresponds to the increments of the Brownian motion, which is simulated by means of the relation $W_{t_{j+1}} - W_{t_j} = \sqrt{(t_{j+1} - t_j)} N_j$, where $N_j \sim N(0, 1)$.

Both equations were simulated with a time interval $(t_{j+1} - t_j) = 0.12$ h, allowing the generation of more refined series of H_s and T than the originals. In total, 500 simulations were performed for

both equations, as plotted in Figure 3a–d. By way of comparison, the average of the 500 simulated trajectories is plotted in black together with the graph of the real data trajectory in red. The blue line in the graphs indicates to the left side the 80% of data used for the parametric adjustments and to the right side the 20% of data used for comparing the performance of the model at the moment of forecasting future values. This allowed measuring the prediction errors of the settings in the next stage.

Figure 3. Simulated trajectories of H_s and T .

Figure 3a, corresponding to the shortest sample of H_s data, shows a good behavior describing present and future values: it was possible to capture the variations of slope and the randomness present in these variations. Something similar could be thought for the second longest H_s data sample (Figure 3b); nevertheless, at first sight, it was not observed due to the large amount of data plotted in the figure.

In the case of T , Figure 3c, corresponding to the shorter sample, shows that the variation—understood as amplitude around an imaginary moving average—of the model was much lower than that of the real data; this effect can be seen more clearly in Figure 3d. However, the ability of the model to follow the trend of the wave period stands out.

4.2. Hybrid Model Performance

The models were subject to a validation process and the errors of the results given by the models were measured by the mean percentage error (MPE).

Table 4 summarizes the values obtained for the MPE of the H_s and T models for both data samples.

Table 4. MPE values.

Sample	MPE Fit	MPE Forecast
H_s Sample 1	−0.956104%	−8.851932%
H_s Sample 2	−3.619523%	−3.368609%
T Sample 1	−1.078481%	−3.439984%
T Sample 2	−2.008694%	−6.136858%

Table 4 shows the adjustment error, corresponding to the comparison of the 500 trajectories generated with the 80% of total data used for the parametric adjustment. On the other hand, the forecast error corresponds to the comparison with the 20% of total data used for the respective part of the average of the 500 trajectories.

The adjustment errors of H_s and T models of both data samples were acceptably low: less than 3% for the adjustment and less than 9% for the future forecast. However, these models use linear wave theory, and it is expected that the error of forecasting increases due to the non-linearities associated to the physical measurements, but, as the paper seeks a robust model that can be useful in small signal analysis, the measurements errors are in an acceptable range.

4.3. Simulated Power Generation

4.3.1. Physical Power

The interest variable of power generation was obtained by means of two methods. The first corresponded to a transformation of the data generated in the previous section through the physical relationship for the deep-water wave-power level [22], expressed in Equation (10), where it takes as an input of H_s (SWH) and T (WP) the series generated by the proposed models.

$$J = \frac{\rho g^2 H_s^2 T}{32\pi} \quad (10)$$

where $\rho = 1025 \text{ kg/m}^3$ is the mass density of sea water and $g = 9.8 \text{ m/s}^2$ gravity acceleration.

The series for the power generated under Equation (10) for both samples of data are presented graphically in Figure 4a,b, respectively.

As shown in Figure 4a, a good agreement of the present data was observed, following the slope changes correctly. For future projection, although the change in trend was well described, the random variation was greater in the real data than in the simulated average trajectory.

On the other hand, Figure 4b shows a remarkable decrease in the description of variability by the model, in both present and future values. Regarding the characterization of trend change, a good performance was observed.

Figure 4. Plots of theoretical power.

Graphically, a decrease of the variance should be noted. This was due to the fact that the simulated observed trajectory corresponded to an average of 500 simulations and not to a single trajectory.

4.3.2. Floating Oscillating Water Column

The second method to obtain a series representing the power generation corresponded to a transformation of the data generated for H_s and T through the power matrix of the floating oscillating water column device (OWC) [16], which was obtained experimentally together with the Pierson–Moskowitz spectrum analysis. The selection of this device was based on the fact that the operating ranges of the machine matched the data provided by the buoys. The power matrix for this device is presented in Table 5.

Table 5. Matrix conversion for the floating oscillating water column.

H_s (meters)	T (seconds)												
	6	7	8	9	10	11	12	13	14	15	16	17	18
1.0	8	17	27	42	56	59	52	44	40	38	40	38	30
1.5	17	39	61	96	126	132	117	99	89	87	89	85	66
2.0	30	69	108	170	224	235	208	177	159	154	159	151	118
2.5	47	108	169	266	350	368	324	276	249	241	248	236	185
3	68	155	244	383	504	530	467	398	358	347	357	340	266
3.5	93	212	332	521	686	721	636	542	487	472	486	463	362
4	121	276	433	680	896	942	831	708	636	616	634	605	473
4.5	154	350	548	861	1130	1190	1050	896	805	780	803	765	599
5	190	432	677	1060	1400	1470	1300	1110	994	963	991	945	739
5.5		523	819	1290	1690	1780	1570	1340	1200	1170	1200	1140	894
6		622	975	1530	2020	2120	1870	1590	1430	1390	1430	1360	1860
6.5		730	1140	1800	2370	2490	2190	1870	1680	1630	1670	1600	1250
7		847	1330	2080	2750	2880	2540	2170	1950	1890	1940	1850	1450

By means of a bilinear interpolation process, it was possible to obtain the intermediate values in the power matrix. The results of the transformation of the data series can be seen graphically in Figure 5a,b.

Figure 5. Plots of OWC power.

In Figure 5a,b, the black path corresponds to the power trajectory generated by the average of the 500 simulated H_s trajectories. In red, the power generated by the actual data is plotted and the blue vertical line represents the set point to the left, and the forecast to the right.

The graphical results are analogous to those observed in Figure 4a,b.

5. Conclusions

The traditional techniques that are used to capture randomness in EPS consider deterministic models mainly. The results or estimations are validated with Monte Carlo-like simulations. In our case, we use stochastic differential equations to make sure we capture the randomness that is self-sustained in time and inherent to marine resources. Here, the validation process for the obtained parameters is based on the use of the residual method for these particular differential equations. This should be regarded as very important because it allows having a robust method to assess quality in the estimated parameters.

The objective of this work is to propose a stochastic dynamical model, which is based on Fourier analysis, applicable to linear wave theory, and stochastic differential equations. With this approach, it is intended to represent the seasonal changes in the waves and their stochastic characteristics. The proposed models were applied to the characterization of two fundamental variables in the wave energy generation: significant wave height (SWH or H_s) and wave period (WP or T). Since we had real observations of these two variables, they were used to adjust the parameters involved in the models by means of statistical techniques to obtain a characterization of the particular region in which the models were applied.

Wave power was obtained using the data series generated by the proposed models in two different procedures: a theoretical equation valid for deep water and the power matrix of the floating oscillating wave converter (OWC), which was chosen given its conditions of operation, applicable to the region where the real measures of H_s and T were obtained. The order followed in the procedure to generate series of power data facilitated the use of linear models, since it avoided directly modeling the power, which has a non-linear behavior.

The paper includes the systematic explanation of the methodology described above, explaining the methods of parametric estimation, validation of models, simulation of present and future values, and adjustment of error measures, which is done by means of the mean percent error.

The results show the good performance of the proposed hybrid models for H_s and T when adjusting present values in time windows in the range of one month, with an error of 1%. For larger samples, the results show a failure in the precision of the method since it could not capture nonlinear and non-stationary components of the oceans, presenting an error close to 3.6%. The same

is true for projections of future values, increasing the error to 8%. It can be concluded that a linear approximation of the ocean behavior is useful and valid to be applied to inclusion studies of this energy source in the short term, such as in the case of small signal analysis for power systems, where the equations are reduced to the linear case. Within this same context, an additional advantage of the proposed methodology, based on techniques in continuous time, is to generate data series with temporal resolutions finer than the series of data delivered by the measurement devices.

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Appendix A

Estimated parameter of the Fourier series.

Table A1. Significant frequencies of H_s series in Hertz.

Sample 1				Sample 2			
Frequency	Value	Frequency	Value	Frequency	Value	Frequency	Value
f_1	0.004	f_1	0.00031	f_{17}	0.00679	f_{33}	0.01543
f_2	0.008	f_2	0.00062	f_{18}	0.0071	f_{34}	0.01605
f_3	0.02	f_3	0.00093	f_{19}	0.00741	f_{35}	0.01636
f_4	0.04	f_4	0.00123	f_{20}	0.00833	f_{36}	0.01698
f_5	0.044	f_5	0.00154	f_{21}	0.00926	f_{37}	0.01821
f_6	0.016	f_6	0.00185	f_{22}	0.01019	f_{38}	0.01883
f_7	0.028	f_7	0.00278	f_{23}	0.01049	f_{39}	0.01944
f_8	0.056	f_8	0.0034	f_{24}	0.0108	f_{40}	0.02037
f_9	0.012	f_9	0.00401	f_{25}	0.01142	f_{41}	0.0213
f_{10}	0.085	f_{10}	0.00432	f_{26}	0.01173	f_{42}	0.0216
f_{11}	0.035	f_{11}	0.00463	f_{27}	0.01265	f_{43}	0.02222
f_{12}	0.08	f_{12}	0.00494	f_{28}	0.01296	f_{44}	0.02346
f_{13}	0.095	f_{13}	0.00525	f_{29}	0.01327	f_{45}	0.02469
		f_{14}	0.00586	f_{30}	0.01358	f_{46}	0.025
		f_{15}	0.00617	f_{31}	0.01389	f_{47}	0.02654
		f_{16}	0.00648	f_{32}	0.01512	f_{48}	0.02901

Table A2. Significant frequencies of T series in Hertz.

Sample 1		Sample 2					
Frequency	Value	Frequency	Value	Frequency	Value	Frequency	Value
f_1	0.02	f_1	0.00031	f_9	0.00926	f_{17}	0.00401
f_2	0.024	f_2	0.00062	f_{10}	0.01235	f_{18}	0.00432
f_3	0.028	f_3	0.00123	f_{11}	0.01543	f_{19}	0.00494
f_4	0.036	f_4	0.00154	f_{12}	0.01574	f_{20}	0.00802
f_5	0.148	f_5	0.00216	f_{13}	0.01698	f_{21}	0.01142
f_6	0.04	f_6	0.00617	f_{14}	0.02284	f_{22}	0.01451
		f_7	0.00648	f_{15}	0.00247	f_{23}	0.01728
		f_8	0.00741	f_{16}	0.00309	f_{24}	0.01111

Table A3. Estimated parameter (Param.) of H_s series.

Sample 1		Sample 2							
Param.	Value	Param.	Value	Param.	Value	Param.	Value	Param.	Value
α	2.16444961	α	2.39881082	δ_{15}	0.10190936	γ_{28}	−0.03842381	γ_{40}	0.10726438
γ_1	−0.52041385	δ_1	−0.19222480	γ_{16}	0.03398044	δ_{28}	−0.05910388	γ_{41}	0.04956495
γ_2	0.41558115	γ_2	−0.15419630	δ_{16}	−0.16064538	δ_{29}	0.10063493	γ_{42}	−0.02771636
γ_3	0.09459525	δ_2	0.03681891	γ_{17}	0.16748966	γ_{30}	0.16845335	δ_{42}	−0.04672657
δ_3	−0.25239546	δ_3	0.12849686	δ_{18}	−0.08177618	δ_{30}	−0.09055345	γ_{43}	−0.02215749
γ_4	−0.25232424	δ_4	0.17815537	γ_{19}	−0.03679207	γ_{31}	0.03616671	γ_{44}	−0.04518168
δ_4	−0.08251500	γ_5	−0.05721734	δ_{20}	−0.10214604	γ_{32}	−0.05036617	δ_{44}	0.10847667
γ_5	0.24262507	δ_6	0.18201542	γ_{21}	0.13634887	δ_{32}	0.11979486	γ_{45}	0.04394374
γ_6	0.14223323	δ_7	−0.08131101	δ_{21}	0.02400411	γ_{33}	−0.07533975	δ_{45}	−0.06698280
γ_7	0.09393246	γ_8	−0.04664260	γ_{22}	0.19341073	δ_{33}	0.08307758	γ_{46}	−0.08760114
δ_7	0.12739113	γ_9	−0.04585039	δ_{22}	0.02624639	γ_{34}	−0.07814249	γ_{47}	0.02297228
γ_8	0.05406861	δ_9	−0.03364261	δ_{23}	−0.21046561	δ_{34}	0.13582857	δ_{47}	0.07782056
δ_8	−0.11663150	γ_{10}	−0.04234483	γ_{24}	−0.08737163	δ_{35}	0.05468955	δ_{48}	−0.05274041
δ_9	−0.20366855	γ_{11}	0.10815016	δ_{24}	−0.17233841	γ_{36}	0.03178428		
γ_{10}	−0.13781228	γ_{12}	0.06563947	δ_{25}	−0.03774840	δ_{36}	0.10403597		
δ_{11}	−0.13920062	δ_{12}	−0.16001306	γ_{26}	−0.07277297	δ_{37}	−0.05799956		
γ_{12}	0.10484563	γ_{13}	0.03034168	δ_{26}	−0.03582647	γ_{38}	−0.05561688		
δ_{12}	−0.07390363	δ_{13}	−0.15443755	γ_{27}	−0.16194854	δ_{38}	0.06869097		
γ_{13}	0.10370307	δ_{14}	0.13252216	δ_{27}	0.03378120	γ_{39}	−0.02730011		

Table A4. Estimated parameter (Param.) of T series.

Sample 1		Sample 2							
Param.	Value	Param.	Value	Param.	Value	Param.	Value	Param.	Value
β	2.54443881	β	2.51146578	ζ_6	0.01326335	ζ_{13}	0.01474221	ζ_{19}	−0.01603285
ζ_1	−0.10642786	κ_1	−0.09307068	κ_6	−0.04702249	κ_{13}	0.01431980	κ_{19}	−0.02947341
κ_1	−0.03693054	ζ_2	0.01816989	κ_7	−0.02284808	κ_{14}	0.01474160	ζ_{20}	0.01269677
κ_2	0.05297084	ζ_3	−0.01925755	ζ_8	0.01640484	ζ_{15}	0.03424544	ζ_{21}	−0.01593799
κ_3	0.03609110	κ_3	0.04059361	κ_8	−0.02593874	κ_{15}	−0.01646934	κ_{21}	−0.01850130
ζ_4	0.03558274	ζ_4	−0.02864632	ζ_9	0.03548206	κ_{16}	−0.01797831	κ_{22}	0.02033713
κ_5	0.04967259	κ_4	0.03466398	κ_{10}	−0.02844656	κ_{17}	−0.02317474	κ_{23}	−0.02872125
ζ_6	−0.04178142	ζ_5	0.02753775	κ_{11}	0.03153106	ζ_{18}	−0.02400249	κ_{24}	0.04060761
κ_6	−0.05086355	κ_5	−0.02510704	κ_{12}	−0.01274014	κ_{18}	−0.02580914		

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