An Improved Mixed Integer Linear Programming Approach Based on Symmetry Diminishing for Unit Commitment of Hybrid Power System

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Abstract: In this paper, the mixed integer linear programming (MILP) for solving unit commitment (UC) problems in a hybrid power system containing thermal, hydro, and wind power have been studied. To promote its efficiency, an improved MILP approach has been proposed, while the symmetric problem in MILP formulas has been solved by reforming hierarchical constraints. Experiments on different scales have been conducted to demonstrate the effectiveness of the proposed approach. The results indicate a dramatic efficiency promotion compared to other popular MILP approaches in large scale power systems. Additionally, the proposed approach has been applied in UC problems of the hybrid power system. Two indexes, fluctuation degree and output degree, have been proposed to investigate the performance of renewable energy sources (RES). Several experiments are also implemented and the results show that the integration of pumped hydroelectric energy storage (PHES) can decrease the output of thermal units, as well as balance wind power fluctuation according to the load demand.

Keywords: unit commitment; mixed integer linear programming (MILP); symmetry diminishing; improved hierarchical constraints; PHES; renewable energies; RES

1. Introduction

The unit commitment (UC) optimization is a large-scale, non-convex, and mixed-integer linear optimization problem that is hard to solve. As a kind of branch-and-cut based algorithm, MILP is one of the effective methods for solving the UC problems in large-scale systems [1]. Numerous studies have devoted to either improve the MILP algorithms or enlarge the application of MILP algorithms in various applications. Among these, the symmetry phenomena in UC problems and MILP formulas have attracted great attention from researches. In the UC problem, symmetric situation occurs frequently. Especially in large scale system, scheduling identical generators originates an algorithmic issue known as the symmetric problem. The symmetric problem emerges in the MILP formula if the variables can be substituted without changing the structure of mixed integer linear problem. Plenty of studies have proved that the symmetry may have great influence on the performance of MILP, while this problem will produce redundant calculation for exploring equivalent search regions unnecessarily [2].

The methods for solving a symmetric problem can be categorized into two main methodologies: Reformulation methods and removal methods [1]. In reformulation methods, isomorphism pruning [3]

and orbital branching [4] are often used to reduce symmetry. However, these methods require rewriting formulations that are not easy tasks and this could lead to a risk in which the dimension of the solving problem will increase. In removal methods, dynamically and statically constraints are used for symmetry diminishing. The dynamically one reduces symmetry during the tree searching process and an additional software called off-the-shelf need to be implemented. The other one is much more simple and exploits symmetry only by adding hierarchies in the selection process and no special software is needed. Therefore, this study static removal strategy is taken to set priorities to the generators status variables.

Actually, adding hierarchical constraints into MILP can be supported by many related efforts and articles. Yokoyama et al. [5] utilized hierarchical relationships between design and operation variables to search the K-best solutions. The results demonstrated that their work is superior to the conventional method with the solution optimality and computation efficiency are improved a lot. Lima et al. [6] made a comparison between the case with and without symmetry breaking constraints. The results of their study state that symmetry breaking constraints performance well in UC MILP models without having a significant impact on the size of the model. Alemany et al. [1] presented a way to reduce the computational burden of the Branch and Cut algorithm without changing the structure of the problem. In these works, the methodology proposed by Alemany et al. [1] lead to a considerable reduction of the search for the branch-and-cut enumeration tree without complex constraints, which is much more superior to other works. Unfortunately, with the system scale increasing, the advantage of their approach is becoming weak. Therefore, this paper presents an improved symmetry breaking approach, which is derived from Reference [1] to diminish symmetry in an efficient way. The UC MILP model comes from [7,8] is taken in this paper without considering ramping up and down constraints as these constraints may prevent the utilization of symmetry breaking constraints [6].

In addition, renewable energy sources (RES), regarded as clean and economical energy sources, are likely to be connected to power systems [9,10] for distributed utilization; the study of UC for power systems containing RES has become a popular topic in research and applications [11]. Thus, in this paper, a hybrid power system is implemented with thermal, hydro, and wind to investigate the UC problems of hybrid systems. The hybrid system is shown in Figure 1.
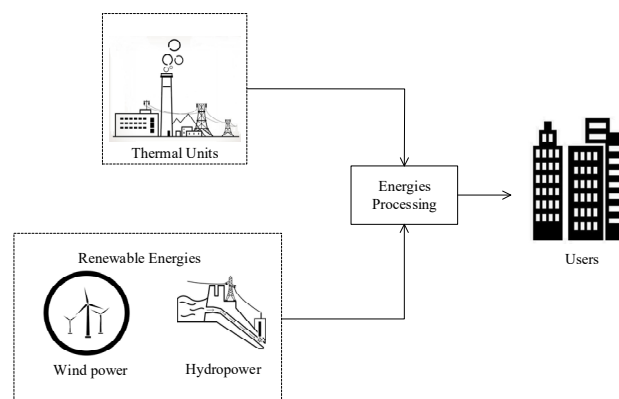

Figure 1. The hybrid power system.

In a hybrid power system, the energy storage system (ESS) is one of the most critical components. The ESS is an indispensable part of a hybrid power system, because the use of an ESS can not only improve power quality, but also ensure the stable and reliable operation of the whole system [12,13]. Recently, pumped hydroelectric energy storage (PHES) is demonstrated to be one of the most effective ESSs for the hybrid system, showing striking advantages on capacity, cost and sufficiency [11,14]. Therefore, this study uses PHES as an ESS to decrease load fluctuation, which was caused by RES integration. Furthermore, in order to discuss the performance of PHES in detail, two indexes are presented: Firstly, an index called fluctuation degree is introduced in this paper to assess load

fluctuation with and without PHES integration. What is more, considering the performance of PHES is closely related to two factors: System scale and capacity of storage; a new index called output degree is proposed to evaluate the influence of these factors on PHES output.

From the above discussion, we were motivated to present an improved MILP model for UC of hybrid power systems integrating RES and investigate the impact of different factors on PHES in detail. The main contributions of this article are: (1) An improved MILP model based on hierarchical constraints has been proposed and applied in different scales of UC problems; (2) an index called fluctuation degree is presented to evaluate the impact of RES integration; and (3) an index called output degree has been proposed to investigate PHES output. The framework of this paper is presented in Figure 2.
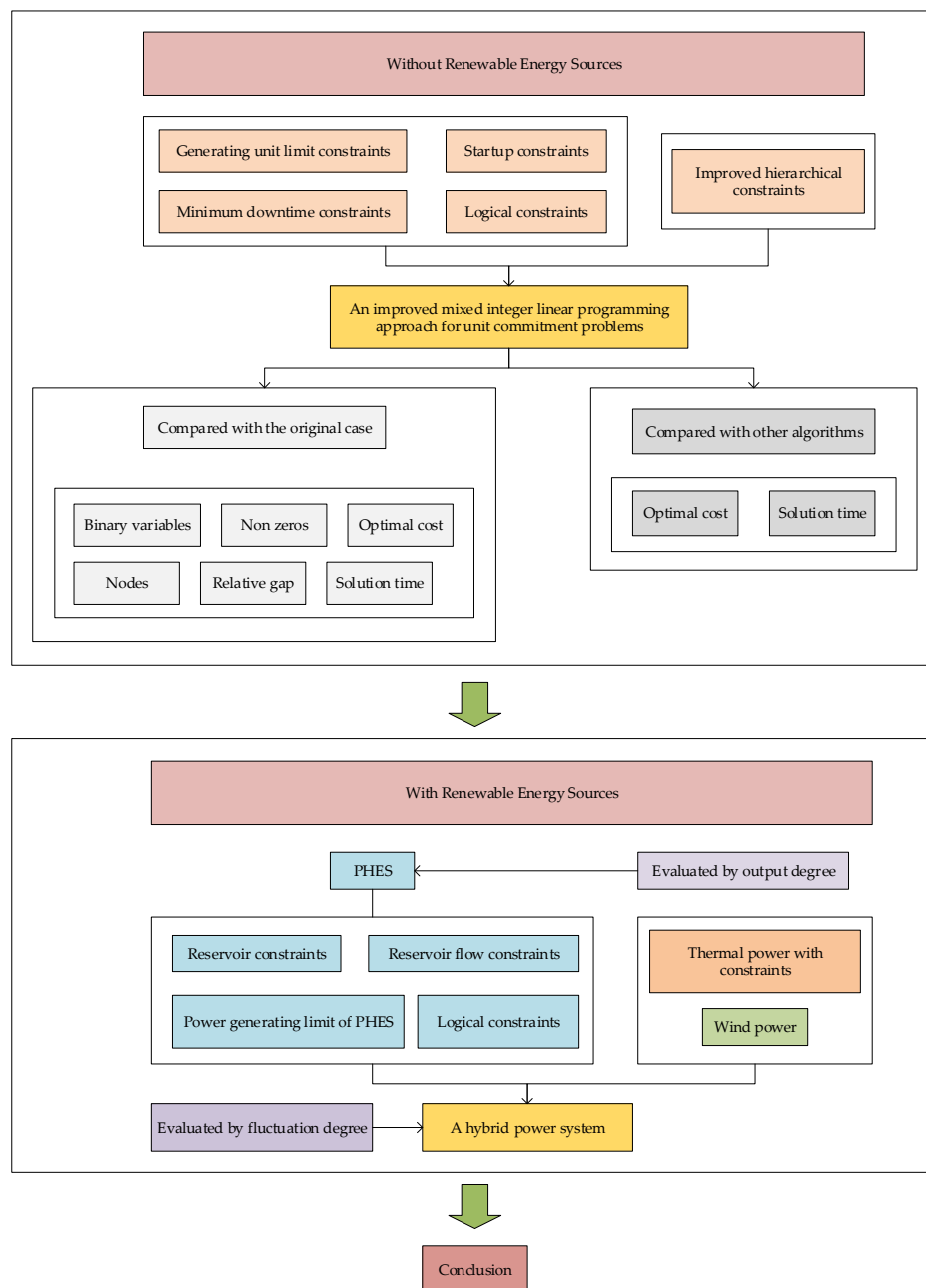

Figure 2. The framework of the improved approach.

The rest part of the paper is organized as follows: Section 2 gives the MILP formulations for UC problems of hybrid systems are introduced. Section 3 describes the proposed MILP approach

with symmetry breaking constraints. Section 4 validates the proposed MILP model in comparative experiments, while the impact of RES in different system scales and the impact of two factors of PHES are evaluated. The conclusions are drawn in Section 5.

2. Formulations for UC Problem

The hybrid system, which consists of thermal units, PHES and wind power is studied in this paper. To minimize operational cost is commonly the objective target. The unit commitment problem can be formulated as [7]:

$$\min f = \sum_{t=1}^{T} \sum_{i=1}^{Gen} [Cp_{i,t} + Cu_{i,t}] \tag{1}$$

where:

$$Cp_{i,t} = a_i onoff_{i,t} + b_i p_i + c_i p_i^2 \tag{2}$$

$$Cu_{i,t} = z_{i,k,1} HST_i + z_{i,k,2} CST_i \tag{3}$$

Subject to:

$$Load_t = \sum_{t=1}^{T} \sum_{i=1}^{Gen} (p_{i,t} + Ph_t + Pw_t) \tag{4}$$

$$\sum_{t=1}^{T} \sum_{i=1}^{Gen} onoff_{i,t} P_{i\max} + Ph_t + Pw_t - Load_t \geq reserve_t \tag{5}$$

$$Cu_{i,t}, p_{i,t}, Ph_t, Pw_t \in \prod_{i,t} \tag{6}$$

where $\prod_{i,t}$ represents the set of feasible production quantities for generator j in time period i. The function $Load_i$ gives the demand at time t, while $reserve_t$ gives the spinning reserve requirement. The function $Cp_{i,t}$ that describes the cost of generation is generally assumed a quadratic function. Typically, it is modeled as a piecewise linear function. Perspective cuts, which is known as a linear approximation, are given in References [15,16] in detail. a, b and c are the parameters of generating. Function $Cu_{i,t}$ is the cost of startup. $p_{i,t}$, Ph_t and Pw_t give the thermal, hydro and wind power output in the time period, respectively. In addition, $onoff_{i,t}$ is the generators status variable. $z_{i,t,1}$ and $z_{i,t,2}$ are the status variables of hot and cold startup, respectively. This paper is focused on the set of constraints in Equation (6).

2.1. Constraints for Thermal Units

The thermal constraints are including generation limit constraint, cold/hot startup constraint, minimum down time constraint, and logical constraint [7]. The first one limits the thermal unit outputs. The next one chooses the cold or hot startup mode. The third one could control the unit's downtime as generators are not allowed to re-start immediately and the last constraint can ensure the logical of variables in UC problems. These constraints can be one or more in the following (7)–(10).

(1) Generating Unit Limit Constraints

$$P_{i\min} \cdot onoff_{i,t} \leq p_{i,t} \leq P_{i\max} \cdot onoff_{i,t} \tag{7}$$

(2) Startup Constraints

$$\begin{cases} z_{i,t,1} + z_{i,t,2} \leq y_{i,t,1} \\ z_{i,t,2} \geq y_{i,t,1} - \sum_{k=DT+1}^{DT+Tcold+1} onoff_{i,t-k} \end{cases} \tag{8}$$

(3) Minimum Downtime Constraints

$$\sum_{k=\max(t-DT+1,1)}^{t} y_{i,k,2} \leq 1 - onoff_{i,t}, \ t \in [L_i + 1, T] \tag{9}$$

$y_{i,t,1}$ and $y_{i,t,2}$ represent the status of startup and shutdown, respectively. DT is the minimum down time of generators and $Tcold$ is the minimum cold time. L_i is the continuous down time of unit i at t_0. $L_i = \max\{0, \min[T, (1 - onoff_{i,0})(DT_i + T_{i,0})]\}$, in this equation, $onoff_{i,0}$ is the original state of unit i; $T_{i,0}$ is the continuous up (positive value)/down (negative value) time before t_0.

(4) Logical Constraint

$$onoff_{i,t} - onoff_{i,t-1} = y_{i,t,1} - y_{i,t,2} \tag{10}$$

2.2. Constraints for PHES

PHES undertakes the task of regulating the peak value and filling valley to save energy in power systems [11,17]. In this part, up/down reservoir constraints, up/down reservoir flow constraints, logical constraints, and power generation of PHES constraints are been taken from Reference [18]. The first one is use to constrain the capacity of up and down reservoirs. The second one presents the flow limitation of up and down reservoirs. The formulation of the third one is the same as Equation (10) to ensure variables' logical and the power generation of PHES is controlled by the last one.

(1) Reservoir Constraints

$$\begin{cases} V_{\min}^u \leq V_t^u \leq V_{\max}^u \\ V_{\min}^d \leq V_t^d \leq V_{\max}^d \end{cases} \tag{11}$$

(2) Reservoir Flow Constraints

When $Ph_t > 0$, PHES is working on generating mode. As the capacity of up reservoir decreasing, the capacity of down reservoir will increase. Δt, which represents a time interval, equals 1 h in this paper.

$$\begin{cases} -u_{2t} \cdot M <= V_{t+1}^u - V_t^u + \Delta t Ph_t / \eta_1 <= u_{2t} \cdot M \\ -u_{2t} \cdot M <= V_{t+1}^d - V_t^d - \Delta t Ph_t / \eta_1 <= u_{2t} \cdot M \end{cases} \tag{12}$$

On the contrary, when $Ph_t < 0$, PHES is working on pumping. The rising capacity of up reservoir comes the decreasing capacity of the down reservoir.

$$\begin{cases} -u_{1t} \cdot M <= V_{t+1}^u - V_t^u + \Delta t Ph_t \cdot \eta_2 <= u_{1t} \cdot M \\ -u_{1t} \cdot M <= V_{t+1}^d - V_t^d - \Delta t Ph_t \cdot \eta_2 <= u_{1t} \cdot M \end{cases} \tag{13}$$

In addition, η_1 and η_2 are fixed constants, which describe the efficiency of generating and pumping, respectively, while ignoring the change of water-head in PHES.

(3) Logical Constraints

$$u_{1t} + u_{2t} = 1 \tag{14}$$

(4) Power Generating limits of PHES

$$Ph_{t\min} \leq Ph_t \leq Ph_{t\max} \tag{15}$$

Where

$$\begin{cases} Ph_{t\min} = \max\left[-PP_{\max}, -V_t^d / (\Delta t \eta_2)\right] \\ Ph_{t\max} = \min[PG_{\max}, (V_t^u / \Delta t) \cdot \eta_1] \end{cases} \tag{16}$$

2.3. Wind Power Generation

The outputs of wind turbines are different even though they install in the same place and with the same rated power. Therefore, through a comprehensive literature review, Equation (17), which comes from Reference [19] is adopted in this paper to generate wind power for a given wind speed input.

$$Pw_t = \begin{cases} 0 & \text{for } v_t < v_i \text{ and } v_t > v_o \\ p_r\left(\frac{v_t - v_i}{v_r - v_i}\right) & \text{for } v_i \leq v_t \leq v_r \\ p_r & \text{for } v_r \leq v_t \leq v_o \end{cases} \tag{17}$$

where p_r is the rated power. v_t is the wind speed (in m/sec), and v_i, v_0 and v_r are the cut-in, cut-out and rated wind speeds, respectively.

3. New Formulation based on System Breaking

3.1. Symmetry in Mixed Integer Linear Programming

The Mixed integer linear programming (MILP) problem can be defined as:

$$\min\{cx + hy : Ax + Gy \geq b, x \in Z^n, y \in \Re^p\} \tag{18}$$

where A and G are $m \times n$ matrices, c and h are $n - vectors$, and b is $m - vector$. x, y are variables, where x is the integer variable and y is the continuous variable.

The set of all feasible solutions is Q. From Q, it is obvious that the MILP problem that describes in Equation (18) may induce multiple equivalent solutions, each of them consisting a symmetry group G. Further details can be found in Reference [2]. Mathematically, the symmetry group G of the problem proposed in Equation (19) can be regarded as the set of all permutations π of the n variables mapping Q on itself and mapping each feasible solution on a feasible solution within the same objective value:

$$G = \left\{\pi \in \prod^n \middle| \forall x \in Q : \pi(x) \in Q \wedge c^T x = c^T \pi(x)\right\} \tag{19}$$

If the equivalence of these sub-problems is uncertain, this may lead to solving unnecessary problems which will make easy problem become complex because of redundant calculations. Therefore, the main challenge of symmetry breaking problem is to discern a subset of a symmetry group in order to relieve the computational burden.

The static symmetry method is effective by constructing hierarchical constraints and adding them to the initial formulations. These methods ensure some of the symmetric solutions breaking, while at least one optimal solution keeping. Generally, the static symmetry can divide into two options: One is to compute a subgroup G_{LP} of G; another is to impose hierarchical decisions. However, another option is to utilize the knowledge of the model. In Reference [20], Sherali H.D. and Smith J.C. proposed an alternative to this knowledge-based method. The authors impose hierarchical decision to decrease the symmetry effect on their model.

In UC formulations, the symmetry problems are driven by the identical units' status variables in the system. In a given scheduling pattern with identical units, there are several identical patterns can be found by permuting the state variables among the identical generator. Therefore, this paper proposed hierarchical decisions, which were suggested by Reference [20].

To describe the symmetry breaking theory in UC problem, the example in Reference [1] would be interesting. In this example, a set Q containing 9 feasible solutions can be represented as follows:

$$
Q = \begin{cases}
\{[0,1,0,1]\} \mapsto f_1 = 186 \\
\{[1,0,1,0]\} \mapsto f_2 = 186 \\
\{[0,1,1,0]\} \mapsto f_3 = 196 \\
\{[1,0,0,1]\} \mapsto f_4 = 196 \\
\{[0,1,1,1]\} \mapsto f_5 = 204 \\
\{[1,0,1,1]\} \mapsto f_6 = 204 \\
\{[1,1,0,1]\} \mapsto f_7 = 204 \\
\{[1,1,1,0]\} \mapsto f_8 = 204 \\
\{[1,1,1,1]\} \mapsto f_9 = 212
\end{cases}
$$

1 and 0 here are the values of $onoff_{i,t}$, they are represented as vectors $[onoff_{1,1}, onoff_{2,1}, onoff_{1,2}, onoff_{2,2}]$, and the corresponding objective value is f_i. It obviously that some permutation of the solutions have the equivalent f.

3.2. Improvements on Hierarchical Constraints

The author in Reference [1] added the hierarchical constraints to MILP for symmetry diminishing, the inequality is described in the following:

$$
onoff_{1,t} \geq onoff_{2,t} \tag{20}
$$

After solving UC MILP with Equation (20), the set Q can be reduced into four feasible solutions:

$$
Q = \begin{cases}
\{[1,0,1,0]\} \mapsto f_1 = 186 \\
\{[1,0,1,1]\} \mapsto f_2 = 204 \\
\{[1,1,1,0]\} \mapsto f_3 = 204 \\
\{[1,1,1,1]\} \mapsto f_4 = 212
\end{cases}
$$

Even though there are two solutions of 204, the set Q decreased a lot that verifies the efficient performance of hierarchical constraints. Equation (20) ensures the order of the binary vectors to avoid redundant calculations.

However, as the scale of power systems increasing, the performance of Equation (20) is not working well with symmetry problems aggrandizing. Therefore, this paper proposed a slight improvement on the Equation (20) without considering hierarchies one by one. The improved hierarchical constraints, which adjust the order of symmetry diminishing, can improve the performance of MILP in the large scale of the systems. The proposed constraints are represented as follows:

$$
onoff_{1,t} \geq onoff_{3,t} \geq \ldots \geq onoff_{2(n-1),t} n \in B_{Gen} \tag{21}
$$

$$
onoff_{2,t} \geq onoff_{4,t} \geq \ldots \geq onoff_{2n,t} n \in B_{Gen} \tag{22}
$$

Generally, the improved hierarchical algorithm outlines of this paper can be written in the following steps:

1. Identify the identical generating units from the total group G.
2. Form a subset B_{Gen} of G.
3. Activate the units from the highest order to meet the load and spinning reserves.
4. Turn on units in lower order if needed.
5. Keep all the lower units deactivate if the higher one is not activated.
6. Solve for economic dispatch in UC.

4. Results of Experiments

To be honestly evaluate the performance of improved constraints, UC problems with different scales are studied in the following experiments. In addition, two indexes have been used to evaluate the performance of RES.

4.1. Efficiency Test

In order to test the computational efficiency of the proposed approach, a popular power system with 10 thermal units are selected. Based on the 10-unit system, the cases with 20, 40, 60, 80, and 100-units were established by duplication. Both the basic data of the thermal units and 24 h load are from Reference [21]. The spinning reserve is set to 10% load demand in this and the latter experiments.

4.1.1. Computational Results with Improved Constraints

The model is implemented in MATLAB using Gurobi as the solver, with default gap set to 0.05%. Each case is solved considering the basic approach (BA) [1] as well as the improved approach (IA) proposed in this paper. Additionally, in order to do a fair comparison between two approaches, different gap values 0.1%, 0.5%, and 1% have been chosen to verify their results. The simulation results are exhibited in Table 1.

Table 1. Comparison of performance between different hierarchical constraints.

Index	Approach	Units			
		40	**60**	**80**	**100**
B Variables	BA	6720	10,080	13,440	16,800
	IA	6720	10,080	13,440	16,800
Non zeros	BA	49,136	73,944	98,752	123,560
	IA	48,656	73,464	98,272	123,080
Optimal Cost	BA	2,242,974	3,361,385	4,482,150	5,600,467
	IA	2,242,596	3,360,419	4,480,326	5,598,415
Nodes	BA	19	283	1320	1971
	IA	1369	1402	1325	356
Relative gap	BA	1.0×10^{-5}	1.2×10^{-5}	2.0×10^{-5}	5.0×10^{-5}
	IA	2.0×10^{-6}	2.5×10^{-5}	1.8×10^{-5}	4.3×10^{-5}
Solution time (gap: default)	BA	6.56	16	40.11	126.67
	IA	15.58	34.88	26.45	26.66
Solution time (gap: 0.1%)	BA	6.02	11.44	19.02	22.41
	IA	5.73	8.58	13.05	19.13
Solution time (gap: 0.5%)	BA	2.05	4.3	9.06	13.3
	IA	2.08	4.02	5.72	14.92
Solution time (gap: 1%)	BA	1.3	4.06	7.8	11.06
	IA	1.17	2.25	3.75	7.75

Table 1 presents that IA can get a better optimal value with less time cost than BA, while the relative gap is narrow in most cases.

Furthermore, these two approaches will be compared in large scale system with default gap set to 1%. Optimal cost and solution time are as the indexes to evaluate the performance of proposed constraints. The simulation results are shown in Table 2.

Table 2 describes that in large scale systems, from the index of optimal cost and solution time, IA performance better in most cases. In the 600, 700, and 1000-units cases, the solution times of IA are more than BA. However, if the gap set to 2%, the solution times of IA in these cases are 816.13 s, 873.5 s and 1222.14 s, respectively, which are less than the solution times of BA (1193.52 s, 1249.78 s and 1446.11s). Similarly, the optimal costs of IA are more than BA in 200, 600, 700, 800, and 900-units cases. If the gap in these cases is set to 2%, the advantages of IA will be validated. Therefore, Table 3 presents the results of simulation with the gap set to 2%.

Table 2. Comparison of performance in large scale (gap: 1%).

Scale	Optimal Cost		Solution Time	
	BA	IA	BA	IA
100	5,636,172	5,634,435	25	9
200	11,274,242	11,293,907	118	45
300	16,925,682	16,922,865	212	109
400	22,552,186	22,550,605	259	250
500	28,205,128	28,177,326	575	337
600	33,794,489	33,856,539	1621	683
700	39,466,484	39,497,405	674	686
800	45,063,024	45,167,260	1946	867
900	50,734,953	50,792,242	2684	1416
1000	56,347,579	55,916,000	4505	5137

Table 3. Comparison of performance in large scale (gap: 2%).

Scale	Optimal Cost		Solution Time	
	BA	IA	BA	IA
100	56,909,362	56,494,104	16.2	12
200	11,392,386	11,343,173	46.47	49.97
300	16,923,744	16,864,586	356	94.91
400	22,668,351	22,613,340	322	243.28
500	28,289,153	28,352,109	588.72	171.39
600	33,924,776	33,868,078	619.02	516.3
700	39,867,856	39,580,050	553.39	679.35
800	45,420,972	45,246,954	1193.52	816.13
900	50,942,836	50,873,907	1249.78	873.5
1000	56,770,346	56,569,896	1446.11	1222.14

The results demonstrate that IA uses less time to obtain better optimal cost with an appropriate gap value.

4.1.2. Comparison with Other MILP Approaches

In order to validate the performance of the proposed approach, some popular MILP algorithms are adopted for comparison, including EPL (Extended Priority List) [21], DPLR (Adaptive Lagrangian Relaxation) [22], LSLR (Local Search Lagrangian Relaxation) [23], and SHCMIP [24]. The results are presented in Tables 4 and 5, respectively.

Table 4. Comparison of optimal cost between different algorithms ($).

Scale	EPL [21]	DPLR [22]	LSLR [23]	SHCMIP [24]	IA
10	563,977	564,049	564,970	563,978	563,938
20	1,124,369	1,128,098	1,125,064	1,123,342	1,123,299
40	2,246,508	2,256,195	2,242,968	2,243,079	2,242,595
60	3,366,210	3,384,293	3,361,244	3,361,766	3,360,339
80	4,489,322	4,512,391	4,482,403	4,482,103	4,480,327
100	5,608,440	5,640,488	5,600,457	5,601,954	5,598,290

Table 5. Comparison of computing time between different algorithms (s).

Scale	EPL [21]	DPLR [22]	LSLR [23]	SHCMIP [24]	IA
10	0.72	108	2.80	0.96	0.34
20	2.97	299	5.40	4.59	1.34
40	11.9	1200	13.5	7.54	16.91
60	23.0	3199	25.8	41.4	18.20
80	44.4	8447	39.7	69.6	26.77
100	64.5	12,437	61.9	83.7	52.33

4.2. Solutions of UC Problems for Hybrid Power System

In this subsection, the MILP formulations for UC problems of hybrid power systems containing thermal, hydro and wind power have been evaluated. The PHES data is cited from Reference [18] and the wind data is referred to from Reference [19].

4.2.1. UC results with Different Hybrid System Compositions

In this subsection, the improved MILP approach will be applied in UC problems of hybrid power systems with different scales from 100-unit to 1000-unit, with default gap set to 1%. A new index called fluctuation degrees is defined to illustrate load fluctuation with RES integration.

$$I_{\text{fluct}} = \frac{1}{T} \sum_{t=1}^{T} \left(net\_load_t - avg_t \right)^2 \tag{23}$$

where *net_load* is the net load and *avg* is the mean value of net load. *net_load* and *avg* are defined as follows:

$$net\_load_t = \begin{cases} Load_t & \text{for thermal system} \\ Load_t - Pw_t & \text{for thermal} - \text{wind system} \\ Load_t - Pw_t - Ph_t & \text{for thermal} - \text{wind} - \text{PHES system} \end{cases} \tag{24}$$

$$avg_t = \frac{1}{T} \sum_{t=1}^{T} net\_load_t \tag{25}$$

The key specifications of wind turbine have been exhibited in Table 6, and the optimal cost and the fluctuation index between two situations have been described in Table 7. The results demonstrate that: (1) The operation cost of thermal units steadily decreases as the integration of RES; and (2) the integration of PHES could restrain the load fluctuation efficiently.

Table 6. Key specifications of the wind turbine.

Rated Power	Rated Wind Speed	Cut in Wind Speed	Cut off Wind Speed
240 MW	5 m/s	3.5 m/s	25 m/s

In conclusion, the combination of PHES in the hybrid system can not only decrease the power output of thermal units, but can also balance the load fluctuation with RES integrating. The fluctuation of the thermal unit output becomes narrow which would be beneficial for reducing operational frequency of startup/shutdown and smoothing power variation, contributing to a decrease in operational cost.

Table 7. Comparison of the improved MILP of UC problems results in different hybrid systems.

Scale	Cost ($)		I_{fluct}	
	Thermal-Wind	**Thermal-Wind-PHES**	**Thermal-Wind**	**Thermal-Wind-PHES**
100	4,991,126	4,932,701	5,869,100	52,365,00
200	10,021,498	9,894,946	23,478,000	20,464,000
300	15,022,335	14,853,533	52,822,344	46,329,000
400	20,049,056	19,800,901	93,906,389	82,330,000
500	25,018,330	24,733,338	146,730,000	128,160,000
600	30,040,031	29,732,068	211,289,375	183,430,000
700	34,915,518	34,632,363	287,588,316	248,640,000
800	39,840,756	39,559,527	375,630,000	328,080,000
900	45,027,102	44,363,237	475,401,094	406,490,000
1000	50,128,293	49,533,136	8,924,857,149	8,798,565,181

4.2.2. The Factors that Affect the Performance of PHES

In this part, the factors which may have great influence on the performance of PHES will be discussed in detail, with default gap set to 0.05%. An index called output degree is proposed for evaluating.

$$I_{output} = \frac{1}{Num} \sum_{i=1}^{Num} ang - output_i^2 \tag{26}$$

where *avg* is the average value of the outputs in one mode. $output_i$ represents the output of PHES, which could be positive or negative value depends on its mode. *Num* is the number of outputs in one mode. The definition of them are:

$$avg = \frac{1}{Num} \sum_{i=1}^{Num} output_i \tag{27}$$

$$output_t = \begin{cases} generating\_power & \text{for generating mode} \\ pumping\_power & \text{for pumping mode} \end{cases} \tag{28}$$

$$Num = \begin{cases} \text{the total of positive number, for generating mode} \\ \text{the total of negative number, for pumping mode} \end{cases} \tag{29}$$

To describe the variation tendency of PHES, Figures 3 and 4 are taken to describe the data of output degree. The simulation results are exhibited in Figures 3 and 4.
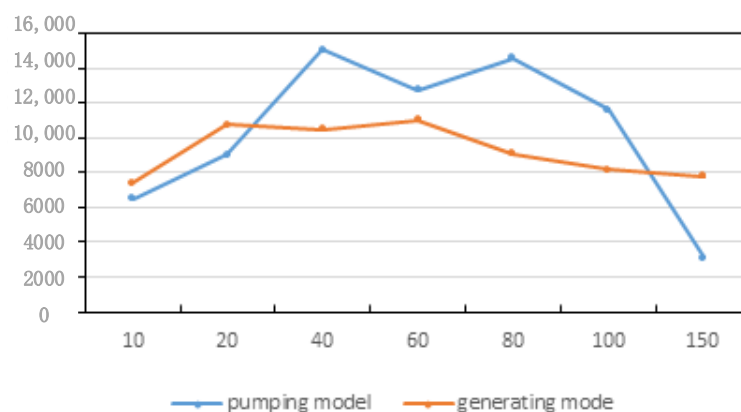
Figure 3. Comparison of PHES in different scale of systems.

From Figure 1, it is obvious that the fluctuation of pumping power is great. After 80-unit case, the blue line decreases dramatically. In Figure 2, the fluctuation of generating power is fierce and it is becoming smooth from 450 MW. The results of these figures demonstrate that the output of PHES

cannot keep increasing without any limitation. These results describes that capacity of PHES and the scale of power systems are the main factors that have a great influence on the performance of PHES. Therefore, a suitable capacity and scale can balance the fluctuation caused by RES better, while it can avoid unnecessary waste of resources at most.
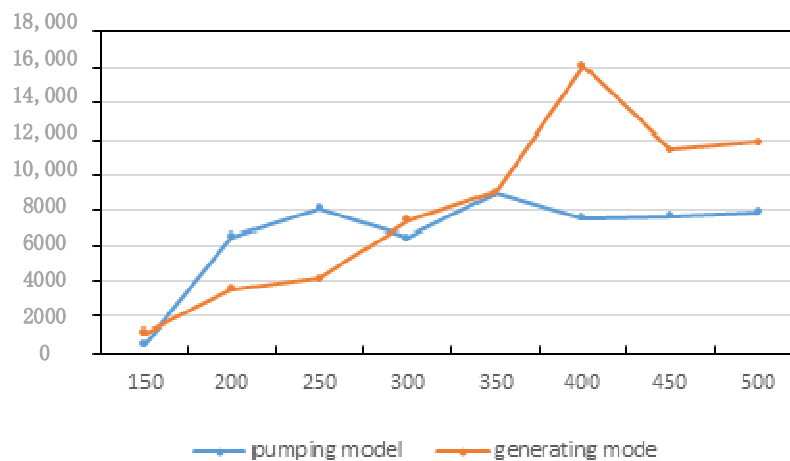

Figure 4. Comparison of PHES output with different capacities.

5. Conclusions

Symmetric problem appears in MILP formula if this method is applied to solve the UC model of a power system in which there are identical generators existing. In this paper, an improved MILP approach has been proposed for efficiency promotion by solving the symmetric problem with the method of reforming the hierarchical constraints. To validate the effectiveness of the proposed approach, it has been applied and compared to traditional methods in solving UC problem of power systems with different scales. Then, this approach was applied to investigate the UC problem of a hybrid power system containing thermal, hydro and wind power. The influence of RES integration, as well as impact of PHES has been studied.

Based on experimental results, the following conclusions could be drawn.

(1) The proposed approach was suitable for the large-scale systems of UC as it can obtain a better objective value with less time cost in most cases. To compare with other algorithms, the proposed approach can still maintain its advantages on objective values and time cost.
(2) The integration of RES can decrease the thermal outputs, and furthermore, decrease the operational cost.
(3) As one of the most effective ESSs, PHES can reduce the load fluctuation which caused by RES integration, as well as ensure the stable and reliable operation of the whole system. The performance of PHES is closely related to two factors: The scale of system and the capacity of it. A suitable PHES can avoid unnecessary waste of resources in the real world.
(4) Considering that the change of water-head is ignored in PHES for implementing an ideal model, more studies on UC problem with realistic PHES features would be interesting in future research.

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Nomenclature

T	Scheduling horizon
Gen	Number of generation units
$Cp_{i,t}$	Generating cost (\$)
a_i, b_i, c_i	Parameters of generating
$onoff_{i,t}$	Binary state variable. 1 meaning on and 0 off
p_i	Thermal power variable (MW)
$Cu_{i,t}$	Startup cost (\$)
$z_{i,k,1}, z_{i,k,2}$	Hot/cold state variable
HST_i, CST_i	Hot/cold startup cost (\$)
$Load_t$	Hourly system demand (MW)
Ph_t	Hydropower variable (MW)
Pw_t	Wind power variable (MW)
$P_{i,\max}, P_{i,\min}$	Maximum/Minimum output of thermal units (MW)
$reserve_t$	Spinning reserve (MW)
$y_{i,t,1}, y_{i,t,2}$	Startup/Shutdown state variable
DT	Minimum down time (h)
$Tcold$	Minimum cold time (h)
V_t^u, V_t^d	Up/down reservoir capacity
$V_{\min}^u, V_{\max}^u$	Minimum/Maximum capacity of up reservoir
$V_{\min}^d, V_{\max}^d$	Minimum/Maximum capacity of down reservoir
u_{1t}, u_{2t}	Pumping/Generating mode state variable
η_1, η_2	Efficiency of generating/pumping
$Ph_{t\min}, Ph_{t\max}$	Minimum/Maximum output of pumped-storage plant (MW)
$PP_{\max}, PG_{\max}$	Maximum pumping/generating power (MW)
p_r	Rate power of wind (MW)
v_t	Wind speed variable (m/sec)
v_i, v_r	Cut-in/Cut/off variable (m/sec)

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