



Article Experimental Study of Crack Propagation in Cracked Concrete

Siyao Wang ^{1,2} and Shaowei Hu ^{1,2,*}

- ¹ School of Water Resources and Hydropower Engineering, Wuhan University, Wuhan 430072, China; whuwangsiyao@126.com
- ² Materials & structural Engineering Department, Nanjing Hydraulic Research Institute, Nanjing 210029, China
- * Correspondence: hushaowei@nhri.cn; Tel.: +86-025-8582-9633

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Abstract: The intersection of cracks has an important role in the key technology of hydraulic fracturing for enhancing the recovery of tight hydrocarbon reservoirs. On the basis of digital image correlation technology, three-point bending tests of concrete beams with an edge crack and a central preset crack were conducted to investigate the propagation of cracks after intersection in concretes. Concrete beams with cracks of different positions, lengths, and approach angles were tested, and results were analyzed. In conclusion, the crack positions, crack lengths, and approach angles significantly influence the crack propagation in naturally cracked concrete. A large distance between the crack tip and central point at the preset transverse crack and crack length indicate a high possibility of the edge crack vertically crossing the preset crack. In particular, the crack restarts from the preset crack tip after intersection when the distance between two cracks is smaller than 30 mm and when the preset crack length is smaller than 40 mm. A large approach angle corresponds to a large carrying capacity of the beam and a high possibility of the crack propagating perpendicularly. An improved criterion of restart cracking after interaction is proposed, and the restart points of all tested beams are predicted and compared with the experimental results. A good agreement is observed, which proves that this criterion is reliable.

Keywords: crack propagation; concrete; preset crack; criterion; interaction

1. Introduction

Hydraulic fracturing is a key technology for enhancing the recovery of tight hydrocarbon reservoirs in petroleum industries [1–3]. Numerous natural fractures exist in shale formations, and these pre-existing natural fractures have an important role in the hydraulic fracturing process because they can influence the process and path of hydraulic fracture propagation. Natural fractures affect the propagation of hydraulic fractures, and the passage of internal driving forces of hydraulic fractures can be greatly influenced by multiple natural fractures. Thus, the interaction of cracks in rock-like materials, such as concrete, should be investigated. The propagation criterion of crack interaction can also guide the study of complex fracture propagation.

A series of criterions were proposed to predict single crack propagation in rock-like materials. Hubbert and Willis [4] proposed the classical model to predict the pore and corresponding fracture pressures using well log and drilling data; their results are the basis for fracture propagation research. Xu and Reinhardt [5,6] proposed the classical double-K fracture criterion to predict crack initiation and propagation in concretes. On the basis of the double-K fracture criterion, crack initiation has been predicted by the initiation fracture toughness, and the stability of the crack propagation has been predicted by the unstable fracture toughness. The maximum tensile stress criterion has been widely

used to predict the appearance of cracks in rock and concrete structures [7–12]. This criterion indicates that cracks will appear when the maximum circumferential stress in the concrete structure exceeds the tensile strength of the material. The fracture toughness criterion has also been widely used to predict the stability of crack propagation in concrete structures [13–15]. The fracture toughness criterion indicates that, if the stress intensity factor (SIF) of the crack tip exceeds the fracture toughness, then the crack will develop and expand. The crack propagation will also enter the unstable propagation stage. However, these criteria can only be used to predict single crack propagation, but multiple crack propagation has been proven to be different from single crack propagation by many studies.

Simple and reasonable multiple crack tests should be conducted to obtain and improve the criterion of multiple crack propagation. In the last century, some efforts have been undertaken for this purpose. Gu [16] conducted an experiment of hydraulic fractures crossing natural fractures at non-orthogonal angles. The test results were analyzed to evaluate the criterion of a crack crossing a natural crack at non-orthogonal angles. Fu [17] presented several laboratory experiments to explore the interaction between hydraulic and pre-existing fractures that are strongly cemented relative to the host material strength but over only a portion of the natural fracture. The results showed that no crossing results are obtained when the strongly cemented region is around 30% of the height of the natural crack. Hassan [18] experimentally investigated the interaction of natural and propagated hydraulic fractures. The results showed that weakly bonded natural fracture surfaces increase the chance of shear slippage occurring and arresting the propagation of hydraulic fractures even at an angle of interaction as high as 90°. Wang [19] experimentally explored the interaction between cemented natural and hydraulic fractures. The experimental results showed that the interface cracks apart and attracts a propagating fracture depending on the frictional strength and the approach angle. The above-mentioned investigations are concentrated on the propagation and criteria of the hydraulic fracture crossing the natural fracture and the interaction of fractures. The general crack propagation after interaction is rarely researched through experiments, and the prediction criterion of crack restart propagation has seldom been studied.

Digital image correlation (DIC) technology has a great advantage in directly observing crack initiation and propagation in solid materials. DIC is an optical measurement to detect the surface displacement and deformation fields of test specimens by analyzing the probabilistic statistical correlation of the light intensity of particles randomly distributed on the specimen surface before and after deformation [20]. The DIC technique has been broadly used to experimentally investigate the crack propagation behavior in the fracture process of quasi-brittle materials at the laboratory scale because of its advantages of full-field measurement, non-contact, and 3D mode [21–23]. The rapid development in image processing techniques and high-resolution digital cameras over the last decade facilitated the utilization of DIC technology in analyzing specimens with multiple growing cracks. Helm [24] presented a series of modifications to the DIC process that allowed the method to automatically analyze specimens with multiple growing cracks. Li [25] researched multiple fatigue crack propagation of reinforced concrete beams strengthened with CFRP under cyclic bending loads. The results of the aforementioned studies indicated that the DIC technique has great advantage in observing crack propagation in concretes with multiple cracks, measuring fracture processes, and providing insight into concrete cracking.

In this study, the DIC technique was used to investigate the crack propagation path in the cracked concrete three-point bending (TPB) beam by measuring the displacement and strain fields of test beam surface. The geometrical parameters of the preset crack including crack position, crack length, and approach angle were analyzed. Furthermore, the influence of the crack position, crack length, and approach angle of the preset crack on the load force (*P*) and crack mouth opening displacement (CMOD) relations and the strain contour of test beams was illustrated. Finally, a criterion of the restart point on the crack upper surface was proposed to predict the propagation path after interaction.

2. Materials and Methods

2.1. Test Beams

The naturally cracked concrete beam tested in the experiment is shown in Figure 1. In this figure, d is the distance from the tip of the initial edge crack to the center of the preset crack on the extension line of initial vertical notch, w is the length of the preset crack, θ is the acute angle measured from the initial fracture propagation direction to the preset crack and a_0 is the notch depth. For all test specimens, the height H is 150 mm, the length L is 800 mm, the span to height ratio S/H is 4, and the thickness B is 100 mm. All test beams were divided into 12 groups and distinguished by the preset crack position, length, and approach angle. Table 1 lists all test beams and the corresponding parameters of the preset crack. All test beams were named as TPB-"X", for the beam without preset crack, X is 1, 2, 3, and 4; for the beam with different distances between the edge crack and the preset crack, X is D10, D30, D50, and D70, respectively; for the beam with different approach angles, X was A30, A45, A60, and A90, respectively.



Figure 1. Schematic view of three-point bending (TPB) beam with edge crack and preset crack.

Test Beam	<i>a</i> ₀ (mm)	<i>d</i> (mm)	w (mm)	θ (°)
TPB-D10	30	10	40	90
TPB-D30	30	30	40	90
TPB-D50	30	50	40	90
TPB-D70	30	70	40	90
TPB-W20	30	30	20	90
TPB-W40	30	30	40	90
TPB-W60	30	30	60	90
TPB-W80	30	30	80	90
TPB-A30	30	30	60	30
TPB-A45	30	30	60	45
TPB-A60	30	30	60	60
TPB-A90	30	30	60	90

Table 1. Dimensions of test beams.

2.2. Materials

All tested concrete beams consist of a standard P.O 42.5 Portland cement of a 28-day standard compressive strength higher than 42.5 MPa, crushed limestones with a maximum diameter of 20 mm, and river sand. The composition proportion of the concrete is given in Table 2. Concrete mixtures were casted into a wooden mold under vibration and demolded after 28 days. All interior surfaces of wooden molds were lubricated with a thin layer of oil before casting. The initial cracks were formed by inserting two steel sheets of 2 mm in thickness during pouring. The steel sheets with two V-shaped tips and the steel sheets with one V-shaped tip were utilized to form the preset and edge cracks, respectively. The measured cube compressive strength, tense strength, Young's modulus, and Poisson's ratio of concrete are $f_c = 46.7$ MPa, $f_t = 3.5$ MPa, E = 33.0 GPa, and $\mu = 0.2$, respectively.

Table 2. Concrete mix proportions.				
Water/Cement Ratio	Cement (kg/m ³)	Water (kg/m ³)	Sand (kg/m ³)	Aggregate (kg/m ³)
0.47	468	219	641	1301

 Table 2. Concrete mix proportions.

Standard TPB fracture tests of concrete beams were conducted to obtain the fracture toughness K_{Ic} and the fracture energy G_f of the concrete. Figure 2 shows the load force—crack mouth opening displacement (*P*-CMOD) curve measured using a clip gauge. The critical loading force can be obtained at the turning point of the *P*-CMOD curve. Moreover, the fracture toughness K_{Ic} and the fracture energy G_f can be obtained on the basis of the test results. The average fracture toughness K_{Ic} and the average fracture energy G_f of the concrete are 0.57 and 110 N/mm, respectively.



Figure 2. Load force - crack mouth opening displacement (P-CMOD) curves of stand TPB tests.

2.3. Experimental Setup

The TPB tests of concrete beams were performed using a mechanical testing machine, as shown in Figure 3. In the test, the concrete beam was placed in the clamp and loaded by a constant displacement rate of 0.05 mm/min right above the initial notch. The crack initiation was monitored and analyzed using acoustic emission technology, and the entire propagation process was monitored and analyzed using the DIC technology. The DIC analysis method of crack propagation is explained in detail in Section 2.4. One clip gauge was placed at the bottom of the notch and fixed by two steel plates to measure the CMOD. The experimental process can be divided into four steps. First, the test beam was placed into the clamp, and the clip gauge was attached on the notch mouth to measure the CMOD. Second, the acoustic emission (AE) sensors were placed on the surface of the test beam to obtain the crack initiation. Third, the DIC equipment was set up to start taking pictures. Finally, the loading force was applied using displacement control with a constant rate of 0.05 mm/min.



Figure 3. Experimental setup.

2.4. DIC Technology

DIC technology was used to obtain the deformation of the surface of the test beam in the experiment, as shown in Figure 3. In this test, digital images were obtained using cameras with a resolution of 2448×2408 pixels, 50 mm fixed focal lenses, and maximum acquisition frequency of 80 Hz. Approximately 0.25 m × 0.25 m area of the mid-span portion was captured by each camera. Moreover, the DIC setup was calibrated using a standard calibration panel with 4 mm and calibration deviation of 0.02 pixel.

In the test, the measuring surface of the test beam was preprocessed by creating an irregular matte black and white speckle pattern to provide adequate contrast, as shown in Figure 4. Moreover, external lighting was directed toward the measuring surface to enhance the image quality. During the entire test process, the images were shot once a second until the failure of the test beam. Meanwhile, the time of the first digital image was recorded and matched on the loading time curve during the loading stage to match the load values with the reference of time in the test.



Figure 4. Matte speckle pattern on the measuring surface of the test beam.

The crack opening displacement (COD) profile along the crack path, which is an important input datum for calculating the cohesive stress distribution in the fracture process zone (FPZ), can be obtained from the displacement fields of specimen surface by DIC technology [26]. As shown in Figure 5, the x-axis and y-axis are along the span and height of the specimen, respectively. Figure 6a shows the displacement contours at the peak load of the standard TPB specimen without central preset crack. The displacement distributions along a line segment (between x = -10 mm and x = 10 mm) of four horizontal cross sections located at y = 3, 45, 55, 70 mm are observed in Figure 6b–e, in which x and y coordinates represent perpendicular and parallel to the crack surface, respectively. The displacement jump in the figures is due to the formation of microcracks; thus, the COD at one y coordinate can be easily obtained by the difference in the displacements between the beginning and end points of the jump. For instance, the value of COD is equal to 25.3 µm at y = 3 mm, 18.4 µm at y = 15 mm, 12.6 µm at y = 25 mm, and 4.5 µm at y = 40 mm. On the basis of this method, a COD is extracted along the y-axis per mm; accordingly, the overall COD profile along the crack path can be determined (Figure 6f).



Figure 5. The crack opening displacements of the propagating crack.



Figure 6. Standard TPB at the peak load. (a) Opening displacement contour. (b) Opening displacement pattern along y = 40 mm. (c) Opening displacement pattern along y = 25 mm. (d) Opening displacement pattern along y = 15 mm. (e) Opening displacement pattern along y = 3 mm. (f) Crack opening displacement (COD) profile along the fracture process zone (FPZ).

The FPZ tip should be defined to determine the crack extension length Δa and investigate the crack reinitiation point. Wu [26] regarded the position of maximum tensile strain as the tip of the FPZ. Chen and Su [27] proposed that COD = 2 µm can be used as the FPZ tip of specimens. In the current study, the location with COD = 3 µm is selected for the identification of the FPZ tip. The length of the cohesive zone of approximately 43 mm is at the peak load.

3. Results and Discussion

3.1. Preset Crack Position

Figure 7 illustrates the *P*-CMOD relations of the test beams with different preset crack positions. Table 3 lists the fracture parameters of the test beam with different preset crack positions. The test results of the beams with the distances of 30 and 50 mm between the preset crack and notch tip are close to each other. Meanwhile, the beam with a distance of 10 mm has a great reduction in carrying capacity. Therefore, the preset crack close to the notch tip accelerates the crack propagation into the unstable stage and induces the reduction in peak load. Figure 8 illustrates the strain contours of the test beams. In the crack path of the beams with different preset crack positions, the crack restart point after interaction is transformed from the preset crack tip to the middle of the preset crack. A long distance between the notch crack tip and the preset crack indicates a weak influence of the preset crack on the crack path.



Figure 7. P-CMOD curves of the beams with different preset crack positions.



Figure 8. Strain contours in the computational domain on the beams with different preset crack positions.

3.2. Preset Crack Length

Figure 9 plots the *P*-CMOD relations of the concrete beams with different preset crack lengths. The results of the beams with the preset crack lengths of 20, 40, 60, and 80 mm have a good agreement with one another on the *P*-CMOD curve, and the peak load is reduced by the increment of the preset crack length. A long preset crack length corresponds to a high critical CMOD. Table 3 shows the fracture parameters of the test beam with different preset crack lengths. Figure 10 shows the strain contours of test beams. The crack restart points after interaction are transformed from the preset crack tip to the middle of the preset crack. A long preset crack length indicates a high possibility of the crack restart point at the middle of the preset crack.



Figure 9. P-CMOD curves of the beams with different preset crack lengths.



Figure 10. Strain contours in the computational domain on the beams with different preset crack lengths.

3.3. Approach Angle

Figure 11 illustrates the *P*-CMOD relations of the concrete beams with different approach angles. The test results of the beams with the approach angles of 30°, 45°, 60°, and 90° indicate that a large approach angle corresponds to a large peak load and a small offset distance from the midpoint of preset crack. Table 3 lists the fracture parameters of test beams with different approach angles. Figure 12 shows the strain contours of the test beams. In the crack path of the beams with different approach angles, the crack restart points after interaction are transformed from the preset crack tip to the middle of the preset crack. A large approach angle results in a high possibility of the crack restart point at the middle of the preset crack.



Figure 11. P-CMOD curves of the beams with different approach angles.



Figure 12. Strain contours in the computational domain on the beams with different approach angles.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Test Beam	E (GPa)	P _{ini} (kN)	CMOD _{ini} (mm)	$CMOD_{ini}$ (mm) P_{max} (kN)	
TPB-D10-2 34.23 6.07 0.11 7.13 0.13 TPB-D10-3 33.17 5.92 0.09 6.72 0.12 Average 33.15 6.08 0.09 6.94 0.12 TPB-D30-1 31.35 6.20 0.05 7.21 0.08 TPB-D30-3 30.19 5.93 0.06 6.83 0.08 Average 31.60 6.00 0.05 7.31 0.09 TPB-D50-1 34.03 5.81 0.07 7.01 0.12 TPB-D50-1 34.03 5.81 0.07 7.01 0.12 TPB-D50-1 35.21 5.82 0.05 8.12 0.08 TPB-D70-1 35.21 5.82 0.05 8.12 0.08 TPB-D70-3 33.15 5.92 0.05 8.14 0.08 TPB-W20-1 32.65 6.11 0.06 8.09 0.07 TPB-W20-3 33.43 6.00 0.06 8.04 0.08 <tr< td=""><td>TPB-D10-1</td><td>32.05</td><td>6.24</td><td>0.08</td><td>6.98</td><td>0.11</td></tr<>	TPB-D10-1	32.05	6.24	0.08	6.98	0.11
TPB-D10-3 33.17 5.92 0.09 6.72 0.12 Average 33.15 6.08 0.09 6.94 0.12 TPB-D30-1 31.35 6.20 0.05 7.21 0.08 TPB-D30-2 33.27 5.88 0.05 7.89 0.10 TPB-D30-3 30.19 5.93 0.06 6.83 0.08 Average 31.60 6.00 0.05 7.31 0.09 TPB-D50-1 34.03 5.81 0.07 7.32 0.11 TPB-D50-2 32.27 5.95 0.07 7.01 0.12 TPB-D70-1 35.21 5.97 0.07 7.45 0.12 Average 32.51 5.97 0.07 7.45 0.12 TPB-D70-1 35.21 5.82 0.08 TPB-W20-3 3.45 6.09 0.07 Average 33.60 5.94 0.05 8.14 0.08 TPB-W20-1 32.65 6.11 0.06 8.09 0	TPB-D10-2	34.23	6.07	0.11	7.13	0.13
Average 33.15 6.08 0.09 6.94 0.12 TPB-D30-1 31.35 6.20 0.05 7.21 0.08 TPB-D30-2 33.27 5.88 0.05 7.89 0.10 TPB-D30-3 30.19 5.93 0.06 6.83 0.08 Average 31.60 6.00 0.05 7.31 0.09 TPB-D50-1 34.03 5.81 0.07 7.01 0.12 TPB-D50-3 31.13 6.16 0.08 8.03 0.12 Average 32.51 5.97 0.07 7.45 0.12 TPB-D70-1 35.21 5.82 0.05 8.12 0.08 TPB-D70-2 32.43 6.09 0.07 745 0.12 TPB-W20-1 32.65 6.11 0.06 8.09 0.07 TPB-W20-2 33.38 5.84 0.06 8.21 0.08 TPB-W40-3 33.25 5.98 0.07 7.31 0.11	TPB-D10-3	33.17	5.92	0.09	6.72	0.12
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Average	33.15	6.08	0.09	6.94	0.12
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-D30-1	31.35	6.20	0.05	7.21	0.08
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Average 31.60 6.00 0.05 7.31 0.09 TPB-D50-1 34.03 5.81 0.07 7.32 0.11 TPB-D50-2 32.37 5.95 0.07 7.01 0.12 TPB-D50-3 31.13 6.16 0.08 8.03 0.12 Average 32.51 5.97 0.07 7.45 0.12 TPB-D70-1 35.21 5.82 0.05 8.12 0.08 TPB-D70-3 33.15 5.92 0.05 8.35 0.07 Average 33.60 5.94 0.05 8.14 0.08 TPB-W20-1 32.65 6.11 0.06 8.09 0.07 TPB-W20-2 33.38 5.84 0.06 8.21 0.08 TPB-W20-3 34.26 6.05 0.07 7.82 0.10 Average 33.43 6.00 0.06 8.04 0.08 TPB-W40-1 35.21 6.08 0.07 7.31 0.11 TPB-W40-1 35.21 6.08 0.07 7.99 0.12 Average 34.02 6.00 0.07 7.41 0.11 TPB-W40-3 33.25 5.98 0.07 6.93 0.10 Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-3 33.15 6.21 0.07 6.32 0.13 TPB-W60-3 33.15 6.21 0.07 6.32 0.13 TPB-W60-3 33.15 6.21 0.07 6.32 0.15	TPB-D30-3	30.19	5.93	0.06	6.83	0.08
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Average 32.51 5.97 0.07 7.45 0.12 TPB-D70-1 35.21 5.82 0.05 8.12 0.08 TPB-D70-2 32.43 6.09 0.06 7.96 0.09 TPB-D70-3 33.15 5.92 0.05 8.35 0.07 Average 33.60 5.94 0.05 8.14 0.08 TPB-W20-1 32.65 6.11 0.06 8.09 0.07 TPB-W20-2 33.38 5.84 0.06 8.21 0.08 TPB-W20-3 34.26 6.05 0.07 7.82 0.10 Average 33.43 6.00 0.06 8.04 0.08 TPB-W40-1 35.21 6.08 0.07 7.31 0.11 TPB-W40-2 33.61 5.95 0.07 7.99 0.12 TPB-W40-3 33.25 5.98 0.07 6.93 0.10 Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-1 30.23 5.93 0.07 6.95 0.13 TPB-W60-2 34.21 6.24 0.09 7.14 0.14 Average 32.53 5.96 0.07 6.32 0.15 TPB-W80-1 30.25 5.96 0.07 6.32 $0.$	TPB-D50-3	31.13	6.16	0.08	8.03	0.12
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TPB-W20-1 32.65 6.11 0.06 8.09 0.07 TPB-W20-2 33.38 5.84 0.06 8.21 0.08 TPB-W20-3 34.26 6.05 0.07 7.82 0.10 Average 33.43 6.00 0.06 8.04 0.08 TPB-W40-1 35.21 6.08 0.07 7.31 0.11 TPB-W40-2 33.61 5.95 0.07 7.99 0.12 TPB-W40-3 33.25 5.98 0.07 6.93 0.10 Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-1 30.23 5.93 0.07 6.95 0.13 TPB-W60-3 33.15 6.21 0.07 6.96 0.12 Average 32.55 5.96 0.07 6.32 0.15 TPB-W80-2 33.59 5.84 0.07 6.31 0.15 TPB-W80-3 31.62 6.11 0.07 6.50 0.15	Average	33.60	5.94	0.05	8.14	0.08
IPS-W20-1 32.65 6.11 0.06 8.09 0.07 TPB-W20-2 33.38 5.84 0.06 8.21 0.08 TPB-W20-3 34.26 6.05 0.07 7.82 0.10 Average 33.43 6.00 0.06 8.04 0.08 TPB-W40-1 35.21 6.08 0.07 7.31 0.11 TPB-W40-2 33.61 5.95 0.07 7.99 0.12 TPB-W40-3 33.25 5.98 0.07 6.93 0.10 Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-1 30.23 5.93 0.07 6.95 0.13 TPB-W60-2 34.21 6.24 0.09 7.14 0.14 TPB-W80-3 33.15 6.21 0.07 6.32 0.15 TPB-W80-3 31.62 6.11 0.07 6.37 0.14 Average 31.82 5.99 0.07 6.50 0.15		22.65		0.00	0.00	0.00
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-W20-1	32.65	6.11	0.06	8.09	0.07
IPB-W20-3 34.26 6.05 0.07 7.82 0.10 Average 33.43 6.00 0.06 8.04 0.08 TPB-W40-1 35.21 6.08 0.07 7.31 0.11 TPB-W40-2 33.61 5.95 0.07 7.99 0.12 TPB-W40-3 33.25 5.98 0.07 6.93 0.10 Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-1 30.23 5.93 0.07 6.95 0.13 TPB-W60-2 34.21 6.24 0.09 7.14 0.14 TPB-W60-3 33.15 6.21 0.07 6.96 0.12 Average 32.53 6.12 0.08 7.02 0.13 TPB-W80-2 33.59 5.84 0.07 6.81 0.15 TPB-W80-2 33.35 5.02 0.08 6.27 0.13 TPB-W80-3 31.62 5.99 0.07 6.50 0.15 TPB-A30-1 33.35 5.02 0.08 6.18 0.11	TPB-W20-2	33.38	5.84	0.06	8.21	0.08
Average33.43 6.00 0.06 8.04 0.08 TPB-W40-1 35.21 6.08 0.07 7.31 0.11 TPB-W40-2 33.61 5.95 0.07 7.99 0.12 TPB-W40-3 33.25 5.98 0.07 6.93 0.10 Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-1 30.23 5.93 0.07 6.95 0.13 TPB-W60-2 34.21 6.24 0.09 7.14 0.14 TPB-W60-3 33.15 6.21 0.07 6.96 0.12 Average 32.53 6.12 0.08 7.02 0.13 TPB-W80-3 30.25 5.96 0.07 6.32 0.15 TPB-W80-1 30.25 5.96 0.07 6.32 0.15 TPB-W80-2 33.59 5.84 0.07 6.37 0.14 Average 31.62 6.11 0.07 6.37 0.14 Average 31.82 5.99 0.07 6.50 0.15 TPB-A30-1 33.35 5.02 0.08 6.27 0.13 TPB-A30-2 31.31 4.97 0.06 5.62 0.11 TPB-A30-3 34.15 5.17 0.08 6.18 0.11 Average 32.94 5.05 0.07 6.36 0.12 TPB-A45-1 34.31 5.99 0.09 6.51 0.13 Average 33.17 6.03 0.08 6.91 0.11 <td>TPB-W20-3</td> <td>34.26</td> <td>6.05</td> <td>0.07</td> <td>7.82</td> <td>0.10</td>	TPB-W20-3	34.26	6.05	0.07	7.82	0.10
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Average	33.43	6.00	0.06	8.04	0.08
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-W40-1	35.21	6.08	0.07	7.31	0.11
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-W40-2	33.61	5.95	0.07	7.99	0.12
Average 34.02 6.00 0.07 7.41 0.11 TPB-W60-1 30.23 5.93 0.07 6.95 0.13 TPB-W60-2 34.21 6.24 0.09 7.14 0.14 TPB-W60-3 33.15 6.21 0.07 6.96 0.12 Average 32.53 6.12 0.08 7.02 0.13 TPB-W80-1 30.25 5.96 0.07 6.32 0.15 TPB-W80-2 33.59 5.84 0.07 6.81 0.15 TPB-W80-3 31.62 6.11 0.07 6.37 0.14 Average 31.82 5.99 0.07 6.50 0.15 TPB-A30-1 33.35 5.02 0.08 6.27 0.13 TPB-A30-1 33.35 5.02 0.08 6.27 0.13 TPB-A30-2 31.31 4.97 0.06 5.62 0.11 TPB-A30-3 34.15 5.17 0.08 6.18 0.11 Average 32.94 5.05 0.07 6.36 0.12 TPB-A45-1 34.31 5.96 0.07 6.98 0.10 TPB-A45-2 35.08 6.13 0.09 7.25 0.11 TPB-A45-3 30.12 5.99 0.09 6.51 0.13 Average 33.17 6.03 0.08 6.91 0.11 TPB-A60-1 28.54 6.13 0.07 7.21 0.11 TPB-A60-3 31.52 6.02 0.06 7.08 $0.$	TPB-W40-3	33.25	5.98	0.07	6.93	0.10
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Average	34.02	6.00	0.07	7.41	0.11
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-W60-1	30.23	5.93	0.07	6.95	0.13
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-W60-2	34.21	6.24	0.09	7.14	0.14
Average 32.53 6.12 0.08 7.02 0.13 TPB-W80-1 30.25 5.96 0.07 6.32 0.15 TPB-W80-2 33.59 5.84 0.07 6.81 0.15 TPB-W80-3 31.62 6.11 0.07 6.37 0.14 Average 31.82 5.99 0.07 6.50 0.15 TPB-A30-1 33.35 5.02 0.08 6.27 0.13 TPB-A30-2 31.31 4.97 0.06 5.62 0.11 TPB-A30-3 34.15 5.17 0.08 6.18 0.11 Average 32.94 5.05 0.07 6.36 0.12 TPB-A45-1 34.31 5.96 0.07 6.98 0.10 TPB-A45-2 35.08 6.13 0.09 7.25 0.11 TPB-A45-3 30.12 5.99 0.09 6.51 0.13 Average 33.17 6.03 0.08 6.91 0.11 TPB-A60-1 28.54 6.13 0.07 7.21 0.11 TPB-A60-3 31.52 6.02 0.06 7.08 0.09 Average 29.95 6.04 0.06 7.11 0.10 TPB-A90-1 30.01 5.97 0.08 7.63 0.09 TPB-A90-2 36.13 6.14 0.09 7.26 0.11 TPB-A90-3 29.55 5.99 0.08 6.65 0.12 Average 31.90 6.03 0.08 7.18 0.11	TPB-W60-3	33.15	6.21	0.07	6.96	0.12
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Average	32.53	6.12	0.08	7.02	0.13
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	TPB-W80-1	30.25	5.96	0.07	6.32	0.15
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	TPB-W80-2	33.59	5.84	0.07	6.81	0.15
Average31.825.990.076.500.15TPB-A30-133.355.020.086.270.13TPB-A30-231.314.970.065.620.11TPB-A30-334.155.170.086.180.11Average32.945.050.076.360.12TPB-A45-134.315.960.076.980.10TPB-A45-235.086.130.097.250.11TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-W80-3	31.62	6.11	0.07	6.37	0.14
TPB-A30-1 33.35 5.02 0.08 6.27 0.13 TPB-A30-2 31.31 4.97 0.06 5.62 0.11 TPB-A30-3 34.15 5.17 0.08 6.18 0.11 Average 32.94 5.05 0.07 6.36 0.12 TPB-A45-1 34.31 5.96 0.07 6.98 0.10 TPB-A45-2 35.08 6.13 0.09 7.25 0.11 TPB-A45-3 30.12 5.99 0.09 6.51 0.13 Average 33.17 6.03 0.08 6.91 0.11 TPB-A60-1 28.54 6.13 0.07 7.21 0.11 TPB-A60-2 29.79 5.98 0.06 7.04 0.09 TPB-A60-3 31.52 6.02 0.06 7.08 0.09 Average 29.95 6.04 0.06 7.11 0.10 TPB-A90-1 30.01 5.97 0.08 7.63 0.09	Average	31.82	5.99	0.07	6.50	0.15
TPB-A30-231.314.970.065.620.11TPB-A30-334.155.170.086.180.11Average32.945.050.076.360.12TPB-A45-134.315.960.076.980.10TPB-A45-235.086.130.097.250.11TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A30-1	33.35	5.02	0.08	6.27	0.13
TPB-A30-334.155.170.086.180.11Average32.945.050.076.360.12TPB-A45-134.315.960.076.980.10TPB-A45-235.086.130.097.250.11TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A30-2	31.31	4.97	0.06	5.62	0.11
Average32.945.050.076.360.12TPB-A45-134.315.960.076.980.10TPB-A45-235.086.130.097.250.11TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A30-3	34.15	5.17	0.08	6.18	0.11
TPB-A45-134.315.960.076.980.10TPB-A45-235.086.130.097.250.11TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	Average	32.94	5.05	0.07	6.36	0.12
TPB-A45-235.086.130.097.250.11TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A45-1	34.31	5.96	0.07	6.98	0.10
TPB-A45-330.125.990.096.510.13Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A45-2	35.08	6.13	0.09	7.25	0.11
Average33.176.030.086.910.11TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A45-3	30.12	5.99	0.09	6.51	0.13
TPB-A60-128.546.130.077.210.11TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	Average	33.17	6.03	0.08	6.91	0.11
TPB-A60-229.795.980.067.040.09TPB-A60-331.526.020.067.080.09Average29.956.040.067.110.10TPB-A90-130.015.970.087.630.09TPB-A90-236.136.140.097.260.11TPB-A90-329.555.990.086.650.12Average31.906.030.087.180.11	TPB-A60-1	28.54	6.13	0.07	7.21	0.11
TPB-A60-3 31.52 6.02 0.06 7.08 0.09 Average 29.95 6.04 0.06 7.11 0.10 TPB-A90-1 30.01 5.97 0.08 7.63 0.09 TPB-A90-2 36.13 6.14 0.09 7.26 0.11 TPB-A90-3 29.55 5.99 0.08 6.65 0.12 Average 31.90 6.03 0.08 7.18 0.11	TPB-A60-2	29.79	5.98	0.06	7.04	0.09
Average 29.95 6.04 0.06 7.11 0.10 TPB-A90-1 30.01 5.97 0.08 7.63 0.09 TPB-A90-2 36.13 6.14 0.09 7.26 0.11 TPB-A90-3 29.55 5.99 0.08 6.65 0.12 Average 31.90 6.03 0.08 7.18 0.11	TPB-A60-3	31.52	6.02	0.06	7.08	0.09
TPB-A90-1 30.01 5.97 0.08 7.63 0.09 TPB-A90-2 36.13 6.14 0.09 7.26 0.11 TPB-A90-3 29.55 5.99 0.08 6.65 0.12 Average 31.90 6.03 0.08 7.18 0.11	Average	29.95	6.04	0.06	7.11	0.10
TPB-A90-2 36.13 6.14 0.09 7.26 0.11 TPB-A90-3 29.55 5.99 0.08 6.65 0.12 Average 31.90 6.03 0.08 7.18 0.11	TPB-A90-1	30.01	5.97	0.08	7.63	0.09
TPB-A90-3 29.55 5.99 0.08 6.65 0.12 Average 31.90 6.03 0.08 7.18 0.11	TPB-A90-2	36.13	6.14	0.09	7.26	0.11
Average 31.90 6.03 0.08 7.18 0.11	TPB-A90-3	29.55	5 99	0.08	6.65	0.12
	Average	31.90	6.03	0.08	7.18	0.11

Table 3. The fracture parameters of test beams.

4. Criterion of Crack Restart Point after Intersection

4.1. Traditional Criterion of Restarting Crack after Crack Intersection

The classical fracture toughness criterion was mainly used to predict the fracture propagation in the concrete. If the equivalent SIF, K_e is greater than the fracture toughness of the concrete, K_{Ic} , then the crack tends to propagate. The equivalent SIF K_e is calculated as [28]

$$K_e = \cos\frac{\alpha}{2} \left(K_{\rm I} \cos^2\frac{\alpha}{2} - \frac{3K_{\rm II}}{2} \sin\alpha \right) \tag{1}$$

where $K_{\rm I}$ and $K_{\rm II}$ represent the SIFs for mode I and II cracks. The calculation of $K_{\rm I}$ and $K_{\rm II}$ is introduced in the next section. α is the fracture propagation angle and is defined in the local polar coordinate system at the fracture tip. In accordance with the extremum problem on circumferential stress σ_{θ} , the angle of crack extension can be calculated as follows [28–30].

$$K_{\rm I}\sin\alpha + K_{\rm II}(3\cos\alpha - 1) = 0 \tag{2}$$

or

$$\alpha = 2\arctan\frac{1}{4}\left(\frac{K_{\rm I}}{K_{\rm II}} \pm \sqrt{\left(\frac{K_{\rm I}}{K_{\rm II}}\right)^2 + 8}\right) \tag{3}$$

The determinations of the K_{I} and K_{II} values of the crack tip are keys to predict the crack propagation after interactions.

4.2. Improved Criterion of Restarting Crack after Crack Intersection

A conventional SIF criterion of crack propagation is mainly considered when the reinitiation point is still at the crack tip after the intersection of cracks. However, the experimental results show that the restart point of the crack is not only at the crack tip but also at the point on the surface of preset crack. The extended Renshaw and Pollard criterion [31] is considered to be the crack propagation criterion, which assumes that the crack can cross the pre-existing natural fracture if the stress near the crack tip on the opposite side of the interfaces, which is obtained from maximum circumferential stress criterion, is adequate to reinitiate a new fracture. Thus, the improved criterion of the crack propagation after interaction is proposed by combining the equivalent SIF criterion and the maximum circumferential stress criterial stress criterion, as follows:

- 1. When $K_e \ge K_{Ic}$ and $\sigma_{max} < f_t$, the crack will restart at the preset crack tip;
- 2. When $K_e < K_{Ic}$ and $\sigma_{max} \ge f_t$, the crack will restart at the point on the surface of the preset crack;
- 3. When $K_e \ge K_{Ic}$ and $\sigma_{max} \ge f_t$, the crack will restart at the crack tip or at the point on the surface of the preset crack or both occur.

where K_e is the equivalent SIF of the preset crack tip; K_{Ic} is the fracture toughness; σ_{max} is the maximum stress on the upper surface of the preset crack; f_t is the tensile strength.

4.3. Determination of the SIF of the Preset Crack Tip

Figure 13 shows the forces of the notched concrete beam with the preset crack in the center, which include the applied mid-span load *P*, support reaction *P*/2, and cohesive stress in the FPZ $\sigma(x)$. The load applied on the half concrete beam is analyzed, as shown in Figure 14a, to simplify the calculation of SIF of the preset crack tip induced by external forces. On basis of the principle of superposition for SIFs [32], SIFs *K*_I, and *K*_{II} can be obtained by adding *K*_{I,1} and *K*_{II,2} and *K*_{II,2}, respectively, as shown in Equation (4).

$$\begin{cases} K_{I} = K_{I,1} + K_{I,2} \\ K_{II} = K_{II,1} + K_{II,2} \end{cases}$$
(4)



Figure 13. External forces applied on the test beam with a preset crack.



Figure 14. Calculation of the stress intensity factor (SIF) of the preset crack tip. (**a**) The load applied on the half concrete beam; (**b**) The force generating stress intensity factor $K_{I,1}$ and $K_{I,2}$; (**c**) The force generating stress intensity factor $K_{I,2}$ and $K_{I,2}$;

As shown in Figure 14b, $K_{I,1}$ and $K_{I,2}$ are SIFs produced by two parts: Part 1, force P_B and moment M_B generated by cohesive stress; Part 2, internal force P_A and moment M_A derived from the force and moment equilibrium principle, as shown in Equations (5) and (6). Figure 14c shows that $K_{II,1}$ and $K_{II,2}$ are SIFs caused by half of the mid-span load P/2 and support reaction P/2.

$$\begin{cases} P_A + P_B = 0\\ M_A + M_B - \frac{1}{2}P_B H - \frac{PS}{4} = 0 \end{cases}$$
(5)

or

$$\begin{cases} P_A = -P_B \\ M_A = -M_B + \frac{PS}{4} + \frac{P_B H}{2} \end{cases}$$
(6)

Therefore, the calculation of the SIFs K_{I} and K_{II} at the crack tip is transformed into calculating $K_{I,1}$ and $K_{I,2}$ and $K_{II,2}$ and $K_{II,2}$, respectively.

In accordance with the calculation in the SIF handbook [33], as shown in Figure 15, $K_{I,1}$ and $K_{II,1}$ can be determined from Equations (7) and (8).

$$\begin{cases} K_{I,1} = \frac{(P/B)}{\sqrt{2h}} f(\eta) \cos \omega(\eta) + \frac{(M/B)}{\sqrt{2h^3}} g(\eta) \sin[\omega(\eta) + \gamma(\eta)] \\ K_{II,1} = \frac{(P/B)}{\sqrt{2h}} f(\eta) \sin \omega(\eta) - \frac{(M/B)}{\sqrt{2h^3}} g(\eta) \cos[\omega(\eta) + \gamma(\eta)] \end{cases}$$
(7)

where $\eta = h_1/h_2$.

$$\begin{cases} P = P_1 - \frac{\eta}{1+\eta} P_3 - \frac{6\eta^2}{(1+\eta)^3} \frac{M_3}{h}, M = M_1 - \left(\frac{\eta}{1+\eta}\right)^3 M_3 \\ f(\eta) = \left(1 + 4\eta + 6\eta^2 + 3\eta^3\right)^{1/2}, g(\eta) = 2\sqrt{3} \left(1 + \eta^3\right)^{1/2} \\ \omega(\eta) = 0.909 - 0.052\eta, \gamma(\eta) = \sin^{-1} \left[\frac{6\eta^2(1+\eta)}{f(\eta)g(\eta)}\right] \end{cases}$$
(8)



Figure 15. Analysis of the SIF of preset crack tip with external forces and bending moments.

The forces acting on the specimens in Figure 14b are introduced into Equations (7) and (8), where $P_1 = P_A = -P_B$, $P_3 = 0$, $M_1 = M_A$, $M_2 = -M_B$, and $M_3 = 0$.

For determining P_B and M_B induced by the cohesive stress, the bilinear softening model of the concrete proposed by Petersson [34] is used to calculate the magnitude of cohesive stress σ transmitted in the FPZ, which can be shown as follows:

$$\sigma(w_i) = \begin{cases} f_t - w_i (f_t - \sigma_s) / w_s & 0 \le w_i < w_s \\ \sigma_s (w_i - w_0) / (w_s - w_0) & w_s \le w_i < w_0 \\ 0 & w_0 \le w_i \end{cases}$$
(9)

When the propagating crack reaches the preset crack, w_i is the COD of the crack at y = i mm (I = 0, 1, ..., d), which can be obtained by the DIC method mentioned in Section 2.4. As shown in Figure 16, σ_s and w_s are the values of stress and displacement at the turning point of the curve, respectively. w_0 is the value of COD when the transmitted stress decreases to 0 and follows Petersson's proposed value [34], as shown as follows,

$$\begin{pmatrix}
\sigma_s = \frac{f_t}{3} \\
w_s = \frac{0.8G_f}{f_t} \\
w_0 = \frac{3.6G_f}{f_t}
\end{cases}$$
(10)

where f_t and G_f are the material characteristics, which are measured from standard TPB specimen without central preset crack on fracture path. As mentioned above, f_t and G_f are equal to 3.5 MPa and 110 N/mm, respectively.



Figure 16. The bilinear softening model of the concrete.

In accordance with the cohesive stress distribution and equivalent principles of force system, resultant force $P_{\rm B}$ and bending moment $M_{\rm B}$ can be obtained as follows:

$$P_B = -\sum_{i=0}^d \sigma(w_i)B\tag{11}$$

$$M_B = -\sum_{i=1}^n \sigma(w_i) \left(i + a_0 - \frac{a_0 + d}{2} \right) B$$
(12)

The boundary collocation method [35] is used to determine the $K_{I,2}$ and $K_{II,2}$ values of the preset crack tip. By choosing *m* points on the boundary outside the crack and combining 2*m* boundary conditions at *m* points into a 2*m*-order linear equation system, the undetermined coefficients X_j and Y_j are solved, and $K_{I,2}$ and $K_{I,2}$ are obtained.

$$\varphi = \frac{PW}{B} \sum_{i=1}^{2m} \left\{ \begin{array}{c} X_j \left(\frac{r_i}{W}\right)^{\frac{j}{2}+1} \left[-\cos\left(\frac{j}{2}-1\right)\theta_i + \frac{\frac{j}{2}+(-1)^j}{\frac{j}{2}+1}\cos\left(\frac{j}{2}+1\right)\theta_i \right] \\ + Y_j \left(\left(\frac{r_i}{W}\right)^{\frac{j}{2}+1}\right) \left[-\sin\left(\frac{j}{2}-1\right)\theta_i + \frac{\frac{j}{2}-(-1)^j}{\frac{j}{2}+1}\sin\left(\frac{j}{2}+1\right)\theta_i \right] \end{array} \right\}$$
(13)

$$\frac{\partial \varphi}{\partial n} = \frac{P}{W} \sum_{j=1}^{2m} \left\{ \begin{array}{l} X_j \left(\frac{r_i}{W}\right)^{\frac{j}{2}} \left[-\cos\left(\frac{j}{2}\theta_i - \omega_i\right) - \frac{j}{2}\cos\left(\left(\frac{j}{2} - 2\right)\theta_i + \omega_i\right) + \left(\frac{j}{2} + (-1)^j\right)\cos\left(\frac{j}{2}\theta_i + \omega_i\right) \right] \\ + Y_j \left(\frac{r_i}{W}\right)^{\frac{j}{2}} \left[-\sin\left(\frac{j}{2}\theta_i - \omega_i\right) - \frac{j}{2}\sin\left(\left(\frac{j}{2} - 2\right)\theta_i + \omega_i\right) + \left(\frac{j}{2} - (-1)^j\right)\sin\left(\frac{j}{2}\theta_i + \omega_i\right) \right] \right\}$$
(14)

$$\begin{array}{l} \varphi(r_i, \theta_i) = \varphi_0(r_i, \theta_i) \\ \frac{\partial \varphi(r_i, \theta_i)}{\partial n} = \frac{\partial \varphi_0(r_i, \theta_i)}{\partial n} \end{array} \quad (i = 1, 2, \cdots, m)$$
(15)

where φ and $\frac{\partial \varphi}{\partial n}$ are stress function and its normal derivative, respectively; $\varphi(r_i, \theta_i)$ and $\frac{\partial \varphi(r_i, \theta_i)}{\partial n}$ are the increment of external surface moment and negative value of tangential component of resultant force of external force from starting point to point (r_i, θ_i) , respectively; and $\varphi_0(r_i, \theta_i)$ and $\frac{\partial \varphi_0(r_i, \theta_i)}{\partial n}$ are the boundary values of $\varphi(r_i, \theta_i)$ and $\frac{\partial \varphi(r_i, \theta_i)}{\partial n}$, respectively. In addition, $r_i = \sqrt{x_i^2 + y_i^2}$, $\theta_i = \text{tg}^{-1} \frac{y_i}{x_i}$, and ω_i is the angle between the outer normal to the boundary and the x-axis. Table 4 lists the boundary conditions, as shown in Figure 17.



Figure 17. The boundary conditions.

Therefore, $K_{I,2}$ and $K_{II,2}$ can be calculated by the following expressions:

$$\begin{cases} K_{\text{I},2} = \lim_{r \to 0} \sqrt{2\pi r} \sigma_{yy}(x,0) = -\sqrt{\frac{2\pi}{W}} \frac{P}{B} X_1 \\ K_{\text{II},2} = \lim_{r \to 0} \sqrt{2\pi r} \sigma_{xy}(x,0) = -\sqrt{\frac{2\pi}{W}} \frac{P}{B} Y_1 \end{cases}$$
(16)

Boundary	$arphi_0$	$\partial \varphi_0 / \partial n$
AB	0	0
BC	$-(y+w/2)\frac{P}{2B}$	0
CD	$-\frac{LP}{4B}$	$-\frac{P}{2B}$
DE	$-(y+w/2)\frac{P}{2B}$	0
EF	$-\frac{SP}{4B}$	0
FA	$-\frac{\dot{SP}}{4B}$	0
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Table 4. The boundary conditions.

4.4. Determination of the Stress on the Preset Crack Upper Surface

The stress on the upper surface of preset crack has a strong relationship with the strain on the upper surface, when the approach angle is close to 90°. The distribution of stress is equal to the distribution of strain on the upper surface when two cracks interact. The distribution of strain can be evaluated by DIC technology. In our experiment, the strain on the upper surface of the preset crack can be divided into three distributions whether the reinitiation point is at the preset crack tip or on the upper surface of the preset crack, as shown in Figure 18.



Figure 18. The distribution of strain on the upper surface of preset crack. (**a**) The distribution of strain before the propagating crack reaches the preset crack (the restart point at the middle point of the preset crack); (**b**) The distribution of strain when two cracks interact (the restart point at the middle point of the preset crack); (**c**) The distribution of strain after two cracks interact (the restart point at the middle point of the preset crack); (**d**) The distribution of strain before the propagating crack reaches the preset crack (the restart point at the tip of the preset crack); (**e**) The distribution of strain before the propagating crack reaches the preset crack (the restart point at the tip of the preset crack); (**e**) The distribution of strain when two cracks interact (the restart point at the tip of the preset crack); (**f**) The distribution of strain after two cracks interact (the restart point at the tip of the preset crack); (**f**) The distribution of strain after two cracks interact (the restart point at the tip of the preset crack).

For the beam with the restart point at the middle point of the preset crack, the transform of the stress distribution on the upper surface are shown in Figure 18a–c. For the beam with the restart point at the tip of the preset crack, the transform of the stress distribution on the upper surface is shown in Figure 18d–f. Figure 18a,d shows no evident maximum point for the distribution of strain before the propagating crack reaches the preset crack. Figure 18b,e shows the maximum point at the middle point of the upper surface of the preset crack (near the propagating crack tip where two cracks interact) when two cracks interact. Figure 18c,f shows the restart point on the upper surface of the preset crack and at the preset crack tip respectively, after two cracks interact.

Therefore, the maximum stress σ_{max} on the upper surface is at the middle point of the upper surface near the propagating crack tip when two cracks interact. In accordance with the maximum circumferential stress criterion, the maximum stress σ_{max} can be obtained by calculating the SIF of the propagating crack tip when two cracks interact. At that time, the stress field around the propagating crack tip can be obtained using Equation (17) [36]. If the maximum principal stress σ_{max} reaches the concrete tensile strength f_t , then a new fracture will initiate on the opposite side of the interface.

$$\begin{cases} \sigma_r = \frac{K_I}{2(2\pi r)^{1/2}} (3 - \cos\theta) \cos\frac{\theta}{2} \\ \sigma_\theta = \frac{K_I}{2(2\pi r)^{1/2}} (1 + \cos\theta) \cos\frac{\theta}{2} \\ \tau_{r\theta} = \frac{K_I}{2(2\pi r)^{1/2}} \sin\theta \cos\frac{\theta}{2} \end{cases}$$
(17)

where K_{I} is the SIF at the propagating crack tip induced by the vertical loading force and can be calculated using Equation (18); (r, θ) are the local polar coordinate systems defined at the crack tip.

$$K_I = K_I(P,a) + K_I^c \tag{18}$$

where $K_I(P, a)$ is the SIF induced by loading force and calculated using Equations (19) and (20). K_I^c is the SIF induced by the cohesion in the crack and can be calculated using the DIC and Equation (23).

$$K_I(P,a) = \frac{3PS\sqrt{a}}{2H^2B}F\left(\frac{a}{H}\right)$$
(19)

$$F\left(\frac{a}{H}\right) = \frac{1.99 - a/H(1 - a/H)\left[2.15 - 3.93a/H + 2.7(a/H)^2\right]}{\left(1 + 2a/H\right)\left(1 - a/H\right)^{3/2}}$$
(20)

For the notched beam subjected to a pair of tensile unit point forces in the crack at a distance *x* from crack mouth as shown in Figure 19, the SIF at the tip of crack is written as follows,

$$K_I = \frac{2}{\sqrt{\pi a}} F(x/a, a/h) \tag{21}$$

where the Green's function F(x/a, a/h) is as follows:

$$F(\frac{x}{a}, \frac{a}{h}) = \frac{3.52(1-\frac{x}{a})}{(1-\frac{a}{h})^{3/2}} - \frac{4.35-5.28\frac{x}{a}}{(1-\frac{a}{h})^{1/2}} + \left\{\frac{1.3-0.3(\frac{x}{a})^{3/2}}{\left[1-(\frac{x}{a})^2\right]^{1/2}} + 0.83 - 1.76\frac{x}{a}\right\} \left[1 - (1-\frac{x}{a})\frac{a}{h}\right]$$
(22)



Figure 19. Beam applied a pair of tensile unit point forces.

Therefore, if the distribution of the loading force along the crack surface is determined, the SIF created by loading force applied in the crack can be calculated with the integral method.

The SIF induced by cohesive force can be calculated by Equations (23) and (24), the continuously distributed force of cohesive force was discretized into N concentrated forces and the SIF induced by cohesive force was transformed to the SIF induced by N concentrated forces as shown in Figure 20. Moreover, the concentrated force P_i was obtained by DIC technology based on the bilinear softening model.

$$K_{I}^{c} = \int_{a_{0}}^{a} \frac{2\sigma(x)}{\sqrt{\pi a}} F_{1}\left(\frac{x}{a}, \frac{a}{D}\right) dx = \sum_{i=1}^{N} \frac{2P_{i}(a-a_{0})}{N\sqrt{\pi a}} F_{1}\left(\frac{x_{i}}{a}, \frac{a}{D}\right)$$
(23)

$$F_1\left(\frac{x_i}{a}, \frac{a}{D}\right) = \frac{3.52(1-x_i/a)}{(1-a/D)^{3/2}} - \frac{4.35-5.28x_i/a}{(1-a/D)^{1/2}} + \left[\frac{1.30-0.30(x_i/a)^{3/2}}{\sqrt{1-(x_i/a)^2}} + 0.83 - 1.76\frac{x_i}{a}\right] \left[1 - \left(1 - \frac{x_i}{a}\right)\frac{a}{D}\right]$$
(24)



Figure 20. Dissociation of the cohesion in propagating fractures.

Notably, the computational formulas of the SIF at the preset crack tip and the maximum stress on the upper surface cannot be obtained when the approach angle is less than 90°. In this case, these formulas are calculated by means of the finite element method. This topic will be discussed and investigated in the follow-up work.

4.5. Comparison of the Predicted and Experimental Results

Table 5 shows the predicted results of the crack restart point on the upper surface of the preset crack after interaction. Only a slight difference between the predicted results of the restart point of the crack and the experimental results is observed. The difference in the results of the restart point can be ignored by considering the randomness and divergence of the concrete properties. The improved criterion has great accuracy in predicting the crack restart point on the upper surface after crack intersection.

Test Beam	Pra (kN)	K_I^c (MPa·m ^{1/2})	$K_{\mathbf{e}}$ (MPa·m ^{1/2})	σ _m (MPa)	Restart Point	
	ie · ·				Experiment	Predicted
TPB-D10-1	6.58	0.43	0.67	3.03	Tip	Tip
TPB-D30-1	6.87	0.48	0.73	3.82	Tip & Middle	Tip & Middle
TPB-D50-1	6.98	0.59	0.48	4.15	Near middle	Middle
TPB-D70-1	7.09	0.62	0.45	4.23	Middle	Middle
TPB-W20-1	6.63	0.56	0.71	3.17	Tip	Tip
TPB-W40-1	6.82	0.50	0.68	3.83	Tip & Middle	Tip & Middle
TPB-W60-1	6.95	0.43	0.43	4.05	Middle	Middle
TPB-W80-1	6.11	0.40	0.41	3.96	Middle	Middle
TPB-A30-1	6.12	-	0.65	3.23	Tip	Tip
TPB-A45-1	6.79	-	0.71	3.45	Tip	Tip
TPB-A60-1	7.05	-	0.67	3.84	Near tip	Tip & Middle
TPB-A90-1	7.11	-	0.42	4.02	Middle	Middle

Table 5. The predicted results and the experimental results of the crack restart point.

5. Conclusions

Crack propagation experiments in cracked concrete beams were presented in this work. The crack interaction behavior was directly observed by using the DIC technology. The experimental results of the test beams with different types of preset cracks were analyzed, and the improved propagation criterion of the crack reinitiation after interaction was proposed. The improved criterion in the crack reinitiation prediction after interaction was accurately evaluated by comparing the predicted and the experimental outcomes. The following conclusions can be drawn according to the experimental results:

- 1. The edge crack of the test beams with different distances between the preset and the edge cracks vertically crosses through the preset crack when the relative distance was sufficiently long. The relative distance of the two cracks greatly influenced the peak load of the *P*-CMOD curve. The peak load was high when the relative distance was long;
- 2. The edge crack of the test beams with different preset crack lengths vertically crosses through the preset crack when such length was sufficiently long. The preset crack length greatly influenced the critical CMOD of the *P*-CMOD curve. The critical CMOD was high when the preset crack length was long;
- 3. The edge crack of the test beams with different approach angles vertically crosses through the preset crack when the approach angle was close to 90°. The approach angle greatly influenced crack propagation path and the carrying capacity. Such a path was close to upright and the peak load was high when the approach angle was large;
- 4. The improved crack initiation criterion for predicting the crack propagation path after interaction was proposed by combining the maximum circumferential stress theory and fracture toughness criterion. The predicted results were compared with the experimental outcomes of the reinitiation after crack interaction. Results indicated that the improved criterion accurately predicted the aforementioned path after crack interaction;
- 5. The DIC technology can effectively observe the crack intersection in concrete by this work. The accuracy crack opening displacement obtained by DIC technology indicated that it had good application prospects in investigating the complex problem of multiple crack propagation processes.

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