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Power Density Analysis and Optimization of SMPMSM Based on FEM, DE Algorithm and Response Surface Methodology

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Received: 9 August 2019; Accepted: 19 September 2019; Published: 24 September 2019



Abstract: Surface-mounted permanent magnet synchronous motors (SMPMSM) with high power density and good speed regulation are widely used in industrial applications. In order to further improve its power density, this paper studied the relationship between the thickness of the stator yoke, the thickness of the rotor yoke, the relative magnet span of the motor and the motor power density using the finite element simulation method. On this basis, a response surface model between the three parameters and the power density of the motor was established. Based on this model and a differential evolution algorithm, the motor was optimized and the power density was improved; finally, the optimization results were verified using the finite element simulation method. In addition, the optimization results showed that, when other structure parameters remain unchanged, there is an optimal combination of parameters that can maximize the motor's power density, including the thickness of the stator yoke, the thickness of the rotor yoke and relative magnet span of the motor.

Keywords: response surface model; permanent magnet synchronous motor; power density; differential evolution algorithm

1. Introduction

Surface-mounted permanent magnet synchronous motors with high power density [1] and good speed regulation are widely used in many fields, but at present, in some special applications, such as aerospace [2], pump drive [3], robot applications [4], etc., higher power density of the motor is required due to the limitation of motor weight, dimensions or the requirements for driving performance.

At present, the main method of motor design is to calculate the main dimensions of the motor based on the formulas in the design manual [5], and the theoretical basis of this method is to simplify the magnetic field in the motor into magnetic circuit for calculation. However, the magnetic circuit model simplified by the magnetic field cannot accurately reflect the distribution of magnetic field in the motor, so the motor design results obtained based on the formula method generally cannot achieve the optimal combination of dimensions to achieve full utilization of electromagnetic materials, which is unfavorable to improving the power density of the motor. Therefore, the design results obtained by the formula method need to be optimized to improve the motor's performance.

In order to address the motor optimization issue, some researchers adopted the combination of a lumped parameter model and optimization algorithm to optimize the motor and used the finite element method to verify the optimization results [6,7], which took less time. However, some researchers believe the lumped parameter model is unreliable [8], so they used the finite element model during optimization; for example, D A. Lowther [9] classified the motor variables into control variables and

structural variables, and calculated and obtained the optimal control parameters by the use of an additional control strategy emulator during optimization to ensure that the optimal solution for control variables can also be found during optimization.

Regarding the optimization problems of SMPMSM, researchers have made a lot of achievements: A Sarikhani [10] optimized the cost and torque pulsation of SMPMSM by combining a motor physical model with the genetic-particle swarm algorithm. R Ilka [11] studied the volume and efficiency issues of SMPMSM, and according to the k-means algorithm, he classified the Pareto-front obtained from the NSGA-II algorithm to assist designers in choosing the best solution. B. Xiaohua [12] derived the expression formula between SMPMSM cogging torque and permanent magnet size on the basis of energy method and Fourier decomposition method, and further optimized the permanent magnet size to reduce the motor cogging torque. L Xiaoyu [13] grouped the parameters affecting motor performance, and he used the golden section method, the PSO and the NSGA-II algorithm to optimize the specific parameters in each group, which improved motor output performance while reducing its manufacturing cost. However, presently, the researchers' optimization on SMPMSM mainly focuses on the cost, torque pulsation, cogging torque and efficiency, while the research on motor power density is less.

In this paper, the main goal is to improve motor power density. First, the relationship between the three key structural parameters of SMPMSM and power density was studied, and then the motor structural parameters were optimized on this basis. Second, the response surface analysis method was adopted during optimization to construct the response surface model between motor structural parameters and power density, and the optimal size combination was found on the basis of the differential evolution (DE) algorithm and the response surface model to achieve an improvement in motor power density. Finally, the finite element simulation method was used to verify the optimized motor power density.

2. Effect of Structure Dimensions on Power Density

2.1. Structure Parameters of the Motor

The structure diagram of the surface-mounted permanent magnet synchronous motor is as shown in Figure 1. H_1 is the thickness of the stator yoke of the motor, H_2 is the thickness of the rotor yoke of the motor, and α is relative magnet span. Changing H_1 and H_2 will affect the internal reluctance of the motor, and changing relative magnet span α , the internal magnetomotive force of the motor will change. Therefore, the internal magnetic field will change by changing the above three parameters and the output performance and power density of the motor will be affected.

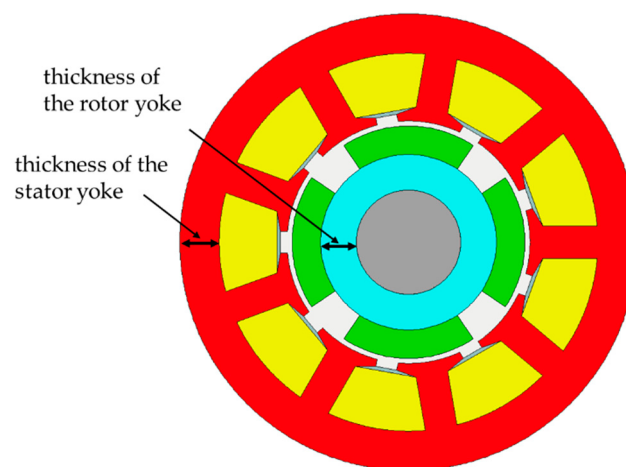


Figure 1. Structure diagram of the surface-mounted permanent magnet synchronous motor.

$$\alpha = \frac{\beta_1}{\beta_2} \quad (1)$$

The parameters of the motor model designed based on the formula method [14,15] are shown in Table 1:

Table 1. Parameters of the motor.

Abbreviation	Parameter	Value
H_1 (mm)	thickness of the stator yoke	5.5
H_2 (mm)	thickness of the stator yoke	5.0
α	relative magnet span	0.75
N_s	slot number	9
N_p	pole number	4
D_1 (mm)	stator outer diameter	64
D_2 (mm)	stator bore diameter	34
L_m (mm)	stack length	72
L_a (mm)	air gap length	0.7
H_m (mm)	magnet thickness	4
N_p	pole number	4
N_s	slot number	9
W_T (mm)	tooth width	4.5
P	pitch	2
N_L	winding layer	2
N_T	turns	22

2.2. 2D Finite Element Analysis

In order to obtain the impact of H_1 , H_2 and α change on motor power density, the Motor-CAD (version 12.1.7) software was used in this paper to conduct the simulation analysis on motor. The motor mesh subdivision is shown in Figure 2: the air gap part with the most complex magnetic field change was divided into four layers for calculation in order to obtain more accurate simulation results. During the simulation, the effective value of motor phase current was kept unchanged (15 A). The specific results obtained by finite element simulation would be analyzed and discussed in the following sections.

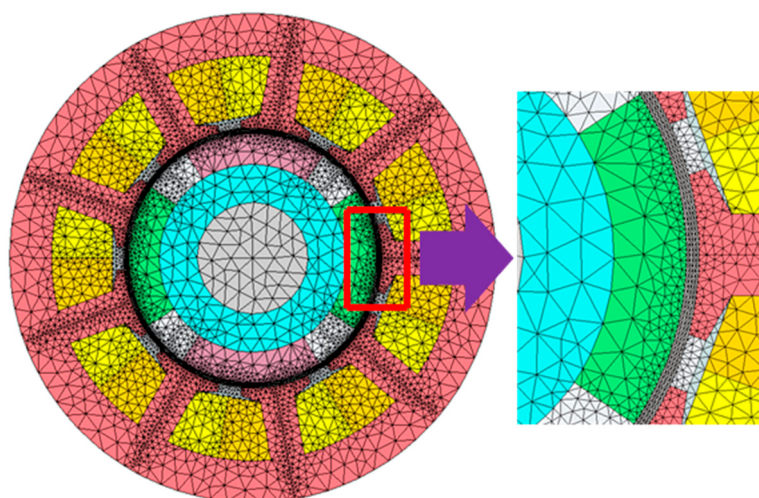


Figure 2. The mesh of motor model.

2.3. Analysis of the Thickness of the Stator Yoke

At present, most motor design theories generally consider the air-gap flux of the brushless permanent magnet motor as its working flux. As a matter of fact, the coil is wound on the teeth of the

stator core of the motor and the magnetic force lines in the stator, through the medium of teeth, form the magnetic flux, which gets cut and cross-linked with the current coil of the motor, so the tooth flux cut cross-linked by the current coil of the brushless motor is the effective working flux of the brushless permanent magnet motor, and according to Equation (1), the change of the average magnetic density on the motor teeth can directly reflect the changes of the effective working flux of the motor [16,17].

$$\phi = BS \quad (2)$$

where ϕ_T is magnetic flux, B is magnetic density and S is cross-sectional area.

As shown in Figure 3a, the motor power density first increases and then decreases with the increase of the thickness of stator yoke while keeping other structure dimensions (H_2 and α) unchanged. This is because when the thickness of the stator yoke is relatively small, there is magnetic saturation at the stator yoke, which causes large total magnetic resistance in the motor, and at this time, the effective flux of the motor is small, the output power is low and the power density is low. Therefore, as the thickness of the stator yoke increases, the total magnetic resistance inside the motor decreases and the average magnetic density of the stator teeth increases (see Figure 3b), so at this time, the effective flux increases, the output power increases and the power density increases; as the thickness of the stator yoke further increases, the magnetic saturation at the stator yoke disappears, and the stator yoke is no longer the main source of magnetic resistance in the motor. Afterwards, the increase of magnetic density at the motor teeth gradually slows down and its values also get stable gradually (see Figure 3b), and at the same time the output power of the motor will show a similar trend. However, as the thickness of the stator yoke increases, the motor gets heavier and heavier, so the power density of the motor starts to decrease gradually.

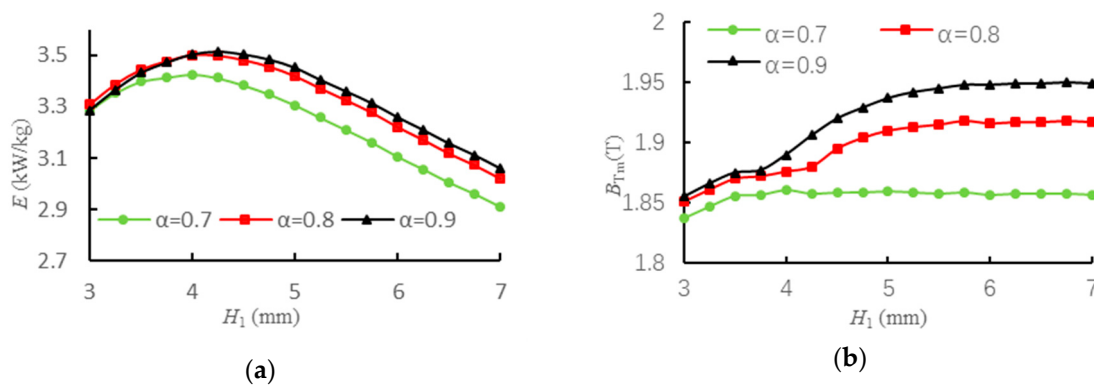


Figure 3. (a) Changes of the power density with the thickness of the stator yoke; (b) Changes of the magnetic density at the motor teeth with the thickness of the stator yoke.

2.4. Analysis of the Thickness of the Rotor Yoke

As shown in Figure 4a, the power density of the motor first rises and then declines with the increase of the thickness of the rotor yoke while keeping other structure dimensions (H_1 and α) unchanged. This is because, similar to the thickness of the stator yoke, the changes of the thickness of the rotor yoke will also have an impact on the total magnetic resistance of the motor. With the increase of the thickness of the rotor yoke, the total magnetic resistance gradually decreases and the magnetic density at the motor teeth increases, so at this time, the output power and power density also gradually increase. When the thickness of the rotor yoke increases to a certain extent, the magnetic density at the motor teeth and the output power gradually stabilizes (see Figure 4b), but at this time the weight of the motor is still gradually increasing with the increase of the thickness of the rotor yoke, so the power density of the motor begins to decline.

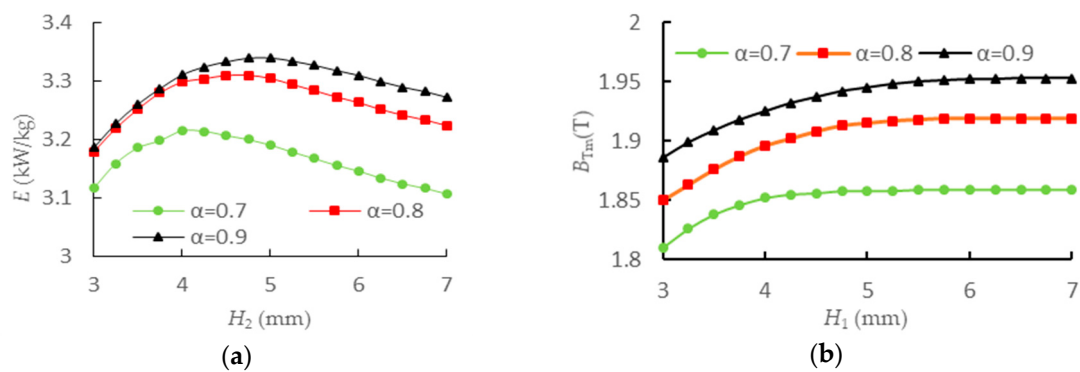


Figure 4. (a) Changes of power density with the thickness of the rotor yoke; (b) Changes of the magnetic density at the motor teeth with the thickness of the rotor yoke.

2.5. Analysis of the Polar Arc Coefficient

As shown in Figure 5a, the motor power density also shows a trend of first rising and then falling with the increase of the polar arc coefficient while keeping other structure dimensions (H_1 and H_2) unchanged. This is because when the polar arc coefficient is low, the magnetic density at the motor teeth is low and the effective magnetic flux of the motor is small, so the output power of the motor is small and the power density is low. As the polar arc coefficient increases, the magnetic density at the motor teeth gradually increases (as shown in Figure 5b), so at this time, the output power and power density of the motor also increase; however, if relative magnet span increases further, magnetic saturation phenomenon will occur inside the motor, as shown in Figure 5b, then the magnetic density at the motor teeth will gradually get stable and the output power of the motor also presents a similar trend as the increase of the polar arc coefficient will increase the amount of permanent magnet and thus increase the weight of the motor, so the power density of the motor starts to decline gradually.

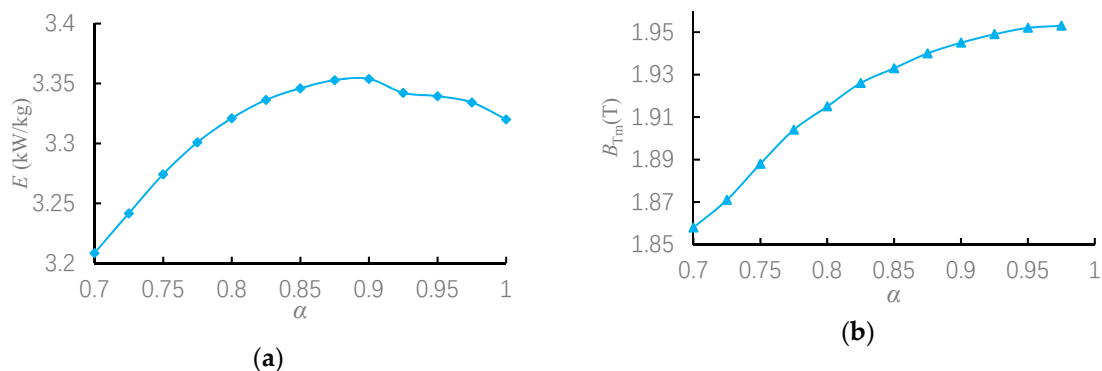


Figure 5. (a) Changes of the power density with the polar arc coefficient; (b) Changes of the magnetic density at the motor teeth with the polar arc coefficient.

3. Motor Optimization

3.1. Construction of Response Surface Model

The response surface analysis is an experiment-based optimization method, which can express the response of the system in the form of mathematical model through specific experimental design and analysis of experimental results [18], and then through analyzing the mathematical model, optimization in the design can be achieved. In this paper, the power density of the motor is the “response” in the response surface model and the thickness of the stator yoke, the thickness of the rotor yoke and relative magnet span are three factors in the response surface model.

In this paper, the experiment was designed with the Box–Behnken method, combined with factor design and incomplete aggregation design, is a three-level experimental design method for the response surface model, with each factor taking three levels. This method can be used to estimate the polynomial of interaction with fewer experiments, so it has higher efficiency and specific experimental designs [19,20]; results are shown in Table 2. The construction method of the response surface model can be performed by referring to [21,22].

Table 2. Table of experimental designs.

Number	H_1 (mm)	H_2 (mm)	α	Power Density (kW/kg)
1	5	3	0.9	3.304
2	3	3	0.8	3.322
3	5	7	0.9	3.375
4	7	5	0.7	2.91
5	5	5	0.8	3.417
6	5	3	0.7	3.243
7	5	7	0.7	3.217
8	3	5	0.7	3.283
9	7	5	0.9	3.06
10	3	5	0.9	3.285
11	7	7	0.8	2.955
12	3	7	0.8	3.202
13	7	3	0.8	2.892

Based on the above experimental results, the response surface model can be obtained as follows:

$$E = 0.039457 + 0.30716 \cdot H_1 + 0.062291 \cdot H_2 + 6.17416 \cdot \alpha + 0.011465 \cdot H_1 \cdot H_2 + 0.18588 \cdot H_1 \cdot \alpha + 0.12164 \cdot H_2 \cdot \alpha - 0.059289 \cdot H_1^2 - 0.021771 \cdot H_2^2 - 4.52988 \cdot \alpha^2 \quad (3)$$

3.2. Motor Power Density Optimization Based on Differential Evolution Algorithm

DE algorithm [23] is a heuristic parallel algorithm based on population difference. Compared with other heuristic algorithms, DE algorithm has the advantages of lesser control parameters, fast convergence, good robustness and it has been widely used in [24] various engineering fields in recent years.

The main control parameters of DE are population size (NP), scaling factor (F) and crossover probability (CR); when the population size is large, the premature convergence of the algorithm may be small, but the whole optimization process requires a large amount of computation and a long time; when the population size is small, the information carried by the population is not rich enough, which is not conducive to whole optimization process. When CR is small, the amount of information exchange between individuals in the population is small and the algorithm is stable, but the rate of convergence is slow; when CR value is large, there is a large amount of information exchange between individuals in the population and the algorithm converges quickly, but it is easy to fall into local optimum and premature phenomenon takes place. The scaling factor F mainly affects the global optimization capability of the algorithm: when F is small, the algorithm has better local search ability and increasing F can enhance the global search ability of the algorithm and avoid prematurity, but the convergence rate will slow down. Therefore, it is very important for the algorithm to reasonably select the control parameters. In this paper's optimization process, the control parameters of the algorithm are set as $NP = 100$, $F = 0.85$, $CR = 0.7$.

For the optimization process involved in this paper, the variation ranges of the motor dimensions are those of the three variables involved in the construction of the response surface model and the other dimensions of the motor remain unchanged during the optimization process. In addition, during

the optimization of power density, the rated power of the motor should be guaranteed to meet the requirements of the performance. Therefore, this optimization problem can be described as:

$$\begin{cases} E = \min f(H_1, H_2, \alpha) \\ 3 \text{ mm} \leq H_1 \leq 7 \text{ mm} \\ 3 \text{ mm} \leq H_2 \leq 7 \text{ mm} \\ 0.7 \leq \alpha \leq 0.9 \\ P_{out} \geq P_{rated} \end{cases} \quad (4)$$

As shown in Table 3, the power density of the motor after optimization is 3.453 kW/kg, which is 5.4% higher than the initial design result.

Table 3. Motor data comparison before and after optimization.

Motor Model	H_1 (mm)	H_2 (mm)	α	Power Factor	Efficiency	Power Density (kW/kg)
Before optimization	5.5	5.0	0.75	0.948	94.32%	3.275
After optimization	4.4	4.9	0.84	0.954	94.12%	3.453

4. Simulation Verification

In order to verify the accuracy of the optimization results, the finite element simulation method was adopted to calculate the power density of the motor after initial design and optimization, respectively. After optimization, the power density of the motor was 3.506 kW/kg (the error was 1.5% compared with the optimization result 3.453 kW/kg), a 7.1% increase over the initial design 3.275 kW/kg. In addition, as shown in Figure 6a,b, before optimization, the magnetic flux density at the stator yoke and rotor yoke of the motor was relatively low, with a maximum value of about 1.5 T, resulting to low power density of the motor due to insufficient utilization of the performance of magnetic permeability materials resulted. After optimization, the flux density at the yoke of the motor increases with the maximum value about 1.85 T, which fully made use of magnetic permeability materials and there is basically no magnetic saturation phenomenon inside the motor, as well the power density of the motor was effectively improved.

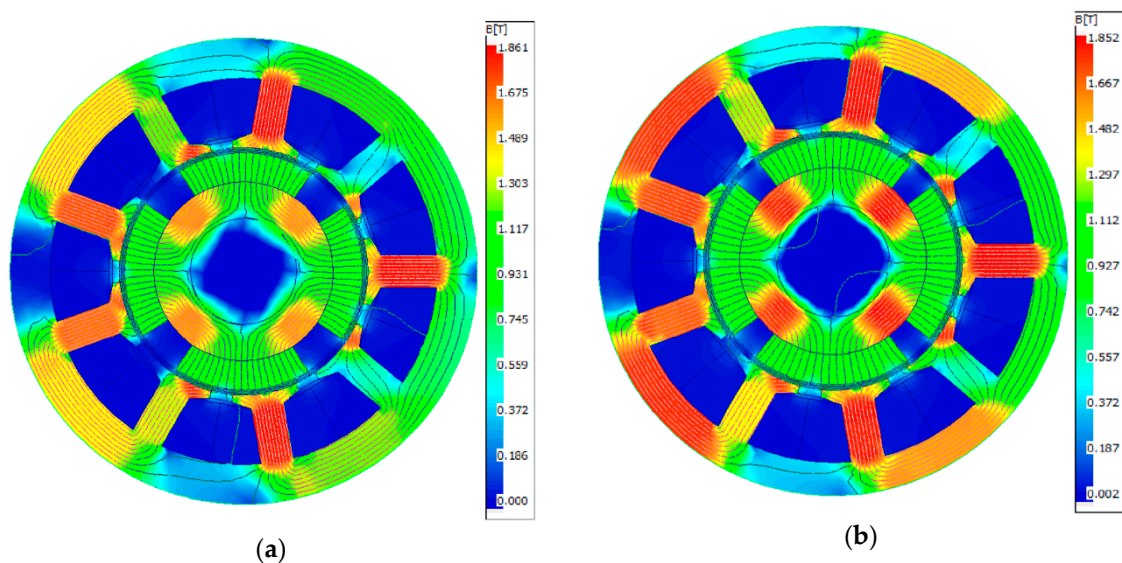


Figure 6. (a) Cloud diagram of motor magnetic flux density before optimization; (b) Cloud diagram of motor magnetic flux density after optimization.

5. Conclusions

In this paper, the relationship between the three main structural parameters of surface-mounted permanent magnet motor and motor power density was studied by the finite element simulation method; it was found that with the increase of stator magnet yoke thickness, rotor magnet yoke thickness and pole-arc coefficient, the motor power density showed an initial increase, which was followed by a decrease.

In order to find the best combination of the three structural parameters of motor to improve the motor power density, the response surface analysis method was used in this paper to construct a response surface model between the motor structural parameters and the power density, and then the motor was optimized based on the DE algorithm to achieve that the motor power density was increased by 7.1%. The optimization results showed that the optimization method proposed in this paper was simple and effective for the motor optimization problem, therefore, the researchers can optimize the motor structural parameters in accordance with this method to improve various performance of the motor (such as torque ripple, cogging torque, losses, temperature rise and noise).

Author Contributions: Software, J.H.; validation, Y.W.; formal analysis, J.H., and W.W.; resources, S.S.; data curation, Y.W.; writing—original draft preparation, W.W.; writing—review and editing, J.H.; supervision, Y.Y.; project administration, S.S.

Funding: Grant number 6141B06260405.

Conflicts of Interest: The authors declare no conflict of interest.

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