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# Impact of Price–Quantity Uncertainties and Risk Aversion on Energy Retailer's Pricing and Hedging Behaviors

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**Abstract:** The joint uncertainties of wholesale price and end-user demand quantity often poses huge pricing challenges to energy retailers. However, the literature lacks analysis of such uncertainties' impacts on retailer pricing behaviors and possible hedging behaviors. To study these impacts, this paper proposes four models: a risk-averse or a risk-neutral retailer deciding retail price with or without forward contract. We present closed-form solutions for these four models on optimal retail price, as well as optimal forward position (if allowed). We propose a novel approach of volatility decomposition to describe the relationship between behaviors and different volatility sources. Comparative statics gives detailed analysis of the pricing and hedging behaviors in both uncertainties, as well as their correlation. We obtain profit distributions using Monte Carlo simulations in the context of the California Electricity Market. Results show that the price and quantity uncertainties and their correlation create significant differences in the retailer's behaviors, and the determinants of these differences are different. In addition, forward contract increases expected profit and decreases profit volatility for risk-averse retailers simultaneously. These results could serve as a benchmark for analyses of deregulated, imperfect energy markets coupled with contingent financial markets under both price and quantity uncertainties.

**Keywords:** pricing; price-quantity uncertainties; risk-aversion; risk-neutral; forward contract

## 1. Introduction

Energy industries underwent a deregulation process that broke vertical integration into the wholesale market and the retail market [1]. In the middle of the wholesale and retail markets, the economic decisions of the retailer are critical. One of the most important decisions is setting the price for the end user. When this price is too high, there will be too few end users; when it is too low, there will be profit loss. A proper price will provide correct incentives for end-user energy consumption and ensure retailer profitability, but an improper price is likely to disrupt the energy market and cause crisis. For example, in 2002 in California, the retailers were not allowed to alter retail prices when the wholesale price spiked, and some of them went broke, resulting in end users having disrupted electricity supply [2].

Uncertainties are particularly prevalent in energy industries [3]. They have attracted a great deal of attention from researchers [4–8]. There are mainly two kinds of uncertainties: quantity uncertainty and price uncertainty. Price uncertainty arises in the wholesale market where retailers purchase energy from energy producers. For example, the electricity price could spike to more than $6000 per MWh while the normal range was only around $40 in the east coast of the United States in 1998 [9]. Quantity or demand uncertainty arises in the retail market and is often the cause of price uncertainty. Quantity uncertainty is of great concern due to the difficulty in energy storage because massive storage along the energy supply chain is neither cost-effective nor flexible in response to fast-changing market conditions. Under such complicated uncertainties, it is natural to expect that risk aversion could make a significant difference to a retailer's pricing behavior.

There are numerous studies addressing a retailer's pricing behaviors under uncertainties and risk aversion. Ref. [4] studied the impact of demand uncertainty and risk aversion on the pricing and ordering behaviors of a retailer in a newsvendor model and found that different types of demand uncertainty lead to different pricing behavior. Ref. [10]) studied the impact of yield rate uncertainty on a retailer's optimal pricing and profit, and found that a less random yield rate results in lower optimal price and higher profit. Ref. [11] studied the impact of demand uncertainty on a retailer's optimal pricing and order size, and found that demand variation will lead to a higher optimal price and profit for the additive demand distribution. Although explicit models have analyzed a retailer's pricing behavior under demand quantity uncertainty, little is known about a retailer's pricing behavior facing supply price uncertainty, and even less is known when both supply price uncertainty and demand quantity uncertainty exist. Studying these cases are particularly meaningful under the ongoing energy market deregulation. This paper makes the first analytical attempt to understand a retailer's pricing behavior under the price and quantity uncertainties, as well as the aversion to risk.

Does risk aversion always mean the retailer increases or reduces the retail price? Both choices are reasonable. A risk-averse retailer may reduce the retail price in order to reduce the unit good profit margin, which simultaneously reduces profit uncertainty because the weight (the profit margin) of quantity uncertainty is reduced. On the other hand, the retailer will also choose to increase the retail price to reduce end-user demand, and as a result, reduce the profit uncertainty because the weight (the end-user demand) of price uncertainty is reduced. Price and quantity uncertainties are multiplicative in the retailer's profit, which is why the above two choices are both plausible. Such multiplicative nature makes things complicated.

Pricing is not the only tool against these uncertainties for a risk-averse retailer, hedging is another useful tool that is well studied. Ref. [6] gave a detailed description of energy and electricity financial derivatives, and their applicability in various scenarios. Ref. [12] studied hedging with future contracts in a deregulated energy market, and the superiority of future contracts was demonstrated. Ref. [13] considered supply chain coordination and forward contract design for the electricity market and analyzed the impact of market structure on the supply chain. Ref. [14] studied the optimal strategy for a retailer to choose the quantity of the physical forward contract. They used the lattice Monte Carlo simulation (LMCS) to generate random scenarios and solved them by using mixed integer programming (MIP). In the case of price and quantity uncertainty, due to the insufficiency in energy storage and the short-term inelasticity in energy supply [9], price uncertainty and quantity uncertainty are often correlated, and hedging solutions considering such correlations are generally better than only focusing on price uncertainty. Implicit models often exploited such a correlation by means of long or short forward contracts, and they achieve a partial hedge of the quantity uncertainty and a full hedge of the price uncertainty [15–17]. Ref. [18] demonstrated the hedging strategies against such price and quantity uncertainties in electricity industries. Explicit models like [3,19,20] used a variational approach to construct a financial portfolio to best hedge price and quantity uncertainty. Explicit studies of hedging under both price and quantity uncertainties shall be useful for the current deregulating energy market.

Given the general acceptance of hedging in an energy retailer, we are interested in studying the effect of price and quantity uncertainties, as well as the effect of hedging availability, on the price decision of a risk-averse retailer. We aimed to answer the following questions: Will risk aversion lead to higher or lower price? Will greater price uncertainty and greater quantity uncertainty lead to a higher or a lower price? Will different price-quantity correlations lead to different prices and different forward positions? Will a forward contract in this joint uncertainty case lead to a higher or a lower price? What is the driving force of the retailers' behaviors under these circumstances? These questions would be of great interest to producers, retailers, and consumers along the whole supply chain for the energy industry. This paper addresses these questions by considering the following four models under the joint uncertainty case:

1. A risk-neutral retailer that maximizes expected profit by setting a retail price.
2. A risk-averse retailer that maximizes expected utility by setting a retail price.
3. A risk-neutral retailer that maximizes expected profit by setting a retail price and minimizes profit variance by hedging with forward contracts.
4. A risk-averse retailer that maximizes expected profit while minimizing variance by setting a retail price and hedging with forward contracts.

We obtain analytical solutions for these four models. Through volatility decomposition and the comparative statics, we were able to identify the volatility components in the price–quantity multiplicative uncertainties. We also found out which components dominated and drove the pricing and hedging behaviors under different market conditions. Lastly, we used the results to explain the interacting mechanism between behaviors and dominant volatility components.

We summarize the main contributions of this paper in the following:

1. This work is the first attempt to capture the retailer pricing behaviors under upstream wholesale pricing uncertainty and downstream demand uncertainty.
2. This work provides a framework in formulating and solving the co-optimization of the retail price and forward contract hedging. We obtain analytical optimal solutions for the four models listed above, including unhedged and hedged scenarios.
3. A novel volatility decomposition is proposed to study the multiplied price and quantity uncertainty. It explains the effects of the price uncertainty, quantity uncertainty, and their correlation on the optimal retail price and optimal forward contract.

For the risk aversion effect (by comparing model 1 and 2), we showed that under a big price uncertainty and small quantity uncertainty, a risk-averse retailer will overprice a risk-neutral retailer. On the contrary, under a small price uncertainty and big quantity uncertainty, a risk-averse retailer will underprice a risk-neutral retailer. For the forward contract effect (by comparing model 2 and 4), we show that under a big price uncertainty and small quantity uncertainty, a risk-averse retailer without a forward contract will overprice a risk-averse retailer with a forward contract. On the contrary, under a small price uncertainty and big quantity uncertainty, a risk-averse retailer without a forward contract will underprice a risk-averse retailer with a forward contract. For the uncertainties effect (by comparing all four models), we found that a greater price uncertainty always led to a higher retail price. For the correlation effect on retail price in model 2, when the correlation was positive and price uncertainty was large, a greater quantity uncertainty led to a higher retail price. However, when the correlation was negative, a greater quantity uncertainty led to a lower retail price. For the correlation effect on the forward position, the findings were complicated and are discussed in Section 3. The implications of these findings are also discussed in Section 3.

## 2. Theoretical Framework

In this section, we formulate four models, where we assume all models face demand distribution parametric on retail price $r$ (as specified below), which we believe to be a general assumption in the literature [4,21]. In models 1 and 2, we considered a retailer making a pricing decision, and in models 3 and 4, we considered a retailer making both pricing and hedging decisions. Models 1 and 3 are risk neutral models, thus profit maximization was the goal function. Models 2 and 4 are risk averse models, thus we adopted expected mean-variance utility (as specified below) maximization as the goal function, which is generally used in the literature [3,9]. The properties of the four models are summarized in Table 1. The market structure for models 3 and 4 is illustrated in Figure 1. If we remove the energy derivative market in Figure 1, we obtain the market structure for models 1 and 2. Particularly, model 3 assumed a retailer with two objectives: to maximize profit and to minimize profit variance. In the sense of profit maximization rather than utility maximization, we categorized model 3 as risk-neutral. It is proved in Lemma 1 that the solution to model 3 is a solution to model 4 in the limit of the risk aversion coefficient going to zero, so model 3 was used to demonstrate the effect of risk aversion on a retailer in the presence of a forward market when compared to model 4. Other utility functions like CARA (constant absolute risk aversion) or CVaR (conditional value at risk) could also be used, yet they are beyond the scope of this paper and will not be studied. We do not consider a risk-prone retailer in the energy market because retailers are load-serving entities that aim at energy market operational stability, and strong prudence is required. Therefore, in reality, they are not in a situation to pursue risk. Details of the solution and proofs are provided in the Appendix. We also introduce profit volatility decomposition analysis in this section.

**Table 1.** Model properties.

| Model Properties | Risk-Neutral | Risk-Averse |
|---|---|---|
| Without forward | Model 1: Profit max, no forward | Model 2: Utility max, no forward |
| With forward | Model 3: Profit max, with forward | Model 4: Utility max, with forward |

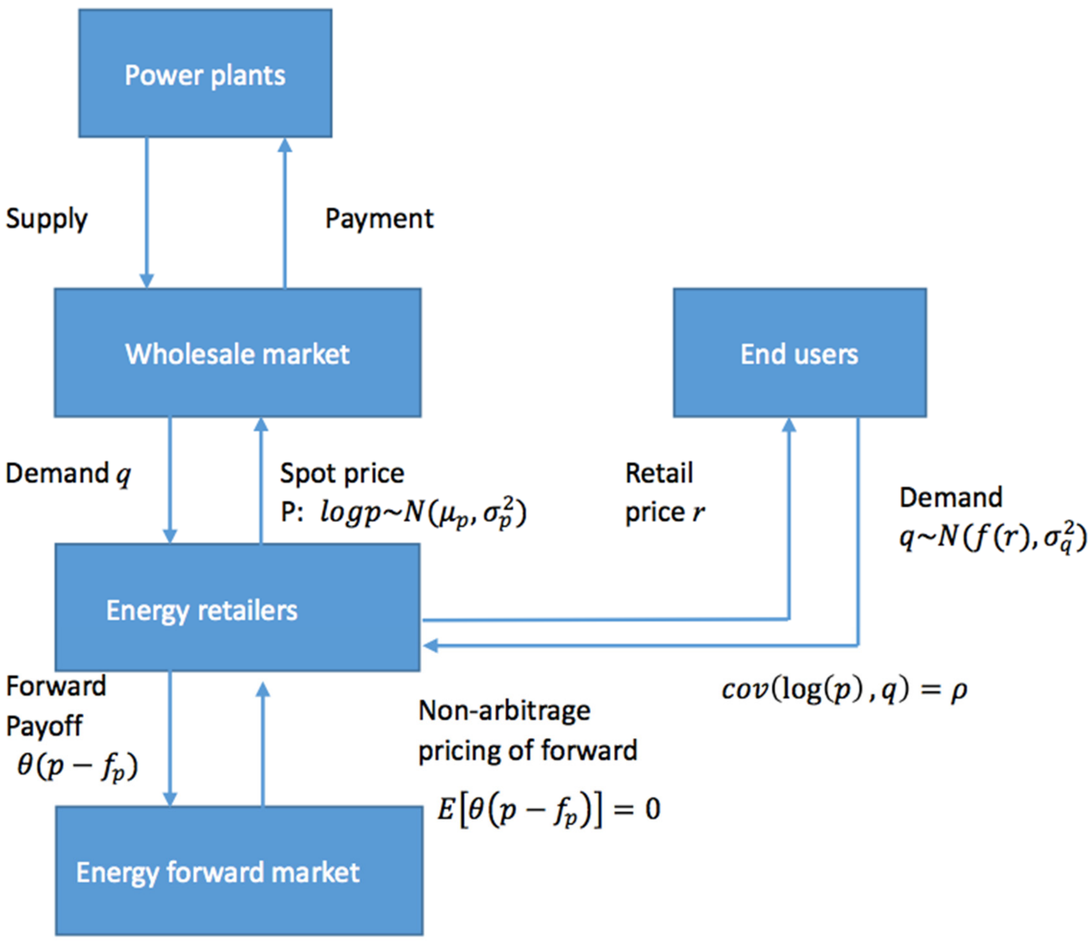


**Figure 1.** The structure of the energy market in models 3 and 4. It involves three markets: wholesale market, retail market, and energy financial market. If we remove the energy forward market, we obtain the market structure for models 1 and 2. We assumed no-arbitrage pricing of forward, i.e., $E\left[\theta\left(p - f_p\right)\right] = 0$.

In all four models, consider a one period setting of {0, 1} in time, where time 0 denotes present and time 1 denotes the future. At time 0, the retailer purchases an energy commodity from an energy producer at a wholesale spot price *p*, which is a random variable (defined below). At this instant, the retail price *r* is set (in models 3 and 4, the forward position $\theta$ is also set). The total consumer demand, *q(r)*, as defined below, is also a random variable parametric on retail price *r*. At time 1, the retailer covers the random consumer demand given the endogenous retail price *r*. In all four models, the retailer's naked position at time 1, $y(p,q,r)$, and the demand function $q(r)$ can be expressed as:

$$\begin{gathered} y(p,q,r) = (r-p) \times q(r) \\ q(r) = a - br + X_q, \text{ where } E\big[X_q\big] = 0,\ var\big(X_q\big) = \sigma_q^2 \end{gathered} \tag{1}$$

Furthermore, where wholesale price *p* and quantity *q* follows the following relation:

$$\begin{gathered} var(\log(p)) = \sigma_p^2, \quad E(\log(p)) = u_p, \quad var(q) = \sigma_q^2, \\ E(q) = u_q = a - br, \quad corr(\log(p), q) = \rho \end{gathered}$$

In models 3 and 4, to hedge the uncertainties in the naked position $y(p,q,r)$, the retailer chooses forward position $\theta$ at time 0 such that its full exposure is:

$$Y(p,q,r,\theta) = y(p,q,r) + \theta\big(p - f_p\big) \tag{2}$$

where $f_p$ is the forward price at time 0.

In models 2 and 4, the retailer maximizes her expected mean variance utility with her risk aversion coefficient $\gamma$:

$$\begin{gathered} \mathrm{E[U(y)]} = E[y] - \tfrac{1}{2}\gamma \times var(y) \\ \text{where } U(y) = y - \tfrac{1}{2}\gamma \times \left(y^2 - E[y]^2\right) \end{gathered} \tag{3}$$

We present the four models one-by-one as follows.

### 2.1. Model 1. Profit Maximization, No Forward

A risk-neutral retailer chooses an optimal retail price r to maximize the expected profit:

$$\begin{gathered} (P1)\max_r J = E[y(p,q,r)] \\ where \quad y(p,q,r) = (r-p) \times q(r) \end{gathered} \tag{4}$$

**Theorem 1.** *(Optimal retail price for a risk-neutral retailer): There exists a unique profit-maximization solution $r_1^*$ for (P1), where:*

$$r_1^* = \frac{a + E[p]b}{2b} \tag{5}$$

### 2.2. Model 2. Utility Maximization, No Forward

A risk-averse retailer chooses the optimal retail price r to maximize their mean-variance utility:

$$\begin{gathered} (P2)\max_r J = E[U(y(p,q,r))] \\ where\ y(p,q,r) = (r-p) \times q(r) \\ U(y) = y - \tfrac{1}{2}\gamma \times \left(y^2 - E[y]^2\right) \end{gathered} \tag{6}$$

Note that $[U(Y)] = E[Y] - \frac{1}{2}\gamma Var(Y)$, which is the mean variance utility

**Theorem 2.** *(Optimal retail price for a risk-averse retailer): There exists a unique utility-maximization solution* $r_2^*$ *for (P2), where:*

$$r_2^* = \frac{a + bE[p] + \gamma \left\{ \begin{array}{c} E[p]var(X_q) + abvar(X_p) \\ +(a + bE[p])cov(X_p, X_q) \\ +cov(X_p X_q, X_q) + bcov(X_p, X_p X_q) \end{array} \right\}}{2b + \gamma\left\{var(X_q) + b^2 var(X_p) + 2bcov(X_p, X_q)\right\}} \tag{7}$$

*where:*

$$X_p = p - E[p],\ X_q = q - E[q]$$

*2.3. Model 3. Profit Maximization with Forward*

A risk-neutral retailer chooses the optimal retail price to maximize their expected profit and chooses an optimal forward position $\theta$ to minimize his profit variance. Note that we assume no-arbitrage in the forward contract, that is, an agent cannot gain any profit using arbitrage in the forward contract [22].

$$\text{s.t. } E[p] = f_p \quad \begin{array}{c} (P3.1)\max_r J_r = E[Y(p, q, r, \theta)] \\ (P3.2)\min_\theta J_\theta = var[Y(p, q, r, \theta)] \\ \\ where\ Y(p, q, r, \theta) = y(p, q, r) + \theta(p - f_p) \end{array} \tag{8}$$

**Theorem 3.** *(Optimal forward and retail price design): There exists a unique variance minimization solution* $\theta_3^*$ *to (P3.2), where:*

$$\theta_3^* = -\frac{cov(y(p, q, r), p)}{var(p)} \tag{9}$$

There exists a unique profit maximization solution $r_3^*$ to $(P3.1)$, where:

$$r_3^* = \frac{a + E[p]b}{2b} \tag{10}$$

*2.4. Model 4. Utility Maximization with Forward*

A risk-averse retailer chooses the optimal retail price r and forward position $\theta$ to maximize his expected utility. Note that we assume no-arbitrage in the forward contract, that is, an agent cannot gain any profit using arbitrage in the forward contract [22].

$$\text{s.t. } E[p] = 0 \quad \begin{array}{c} (P4)\ \max_{r,\theta} J = E[U(Y(p, q, r, \theta))] \\ \\ where\ U(Y) = Y - \frac{1}{2}\gamma \times \left(Y^2 - E[Y]^2\right) \\ Y(p, q, r, \theta) = y(p, q, r) + \theta(p - f_p) \end{array} \tag{11}$$

Note that $[U(Y)] = E[Y] - \frac{1}{2}\gamma Var(Y)$, which is the mean variance utility.

**Theorem 4.** *(Optimal joint forward and retail price design): Given a retail price r, there exists a unique utility maximization forward position* $\theta_4^*$ *for (P4), where:*

$$\theta_4^* = -\frac{cov(y(p, q, r), p)}{var(p)} \tag{12}$$

Given any forward position $\theta$, there exists a unique utility maximization retail price $r_4^*$ for (P4), where:

$$r_4^* = \frac{a + bE[p] + \gamma \left\{ \begin{array}{c} E[p]var\left(X_q\right) + (a-\theta)bvar\left(X_p\right) \\ +(a-\theta+bE[p])cov\left(X_p, X_q\right) \\ +cov\left(X_pX_q, X_q\right) + bcov\left(X_p, X_pX_q\right) \end{array} \right\}}{2b + \gamma\left\{var\left(X_q\right) + b^2var\left(X_p\right) + 2bcov\left(X_p, X_q\right)\right\}} \tag{13}$$

There exists a pair of unique utility maximization solutions $\left(r_4^{**}, \theta_4^{**}\right)$ for model 4, where:

$$r_4^{**} = \frac{A + BC}{1 - BD},\ \theta_4^{**} = \frac{C + AD}{1 - BD} \tag{14}$$

where:

$$\begin{array}{c} A = \dfrac{a + bE[p] + \gamma \left\{ \begin{array}{c} E[p]var\left(X_q\right) + (a)bvar\left(X_p\right) \\ +(a+bE[p])cov\left(X_p, X_q\right) \\ +cov\left(X_pX_q, X_q\right) + bcov\left(X_p, X_pX_q\right) \end{array} \right\}}{2b + \gamma\left\{var\left(X_q\right) + b^2var\left(X_p\right) + 2bcov\left(X_p, X_q\right)\right\}} \\ B = \dfrac{-\gamma\left\{bvar\left(X_p\right) + cov\left(X_p, X_q\right)\right\}}{2b + \gamma\left\{var\left(X_q\right) + b^2var\left(X_p\right) + 2bcov\left(X_p, X_q\right)\right\}} \\ C = \dfrac{avar\left(X_p\right) + cov\left(X_p, X_pX_q\right) + E[p]cov\left(X_p, X_q\right)}{var\left(X_p\right)} \\ D = -b - \dfrac{cov\left(X_p, X_q\right)}{var\left(X_p\right)} \end{array} \tag{15}$$

in which:

$$X_p = p - E[p],\ X_q = q - E[q]$$

The solutions of the four models are presented in the four theorems correspondingly, and some critical properties of the models and the solutions can be obtained for any twice-differentiable distribution $(p, q) \sim f(p, q)$. We present them in the form of a lemma and propositions. Lemma 1 describes the relationship between models, proposition 1 compares the solutions of model 1 and model 2, and propositions 2 and 3 show the conditions when correlation can have an impact on retailer pricing.

**Lemma 1.** *The solution to model 3 is equivalent to the solution to model 4 in the limit of* $\gamma \to 0$.

**Proposition 1.** *Under the condition* $r_1^* > max\left\{E[p], E[p] - 2\rho\sigma_p\sigma_q\right\}$*, the optimal price for a risk-averse retailer without forward,* $r_2^*$*, and the optimal price for a risk-neutral retailer,* $r_1^*$*, have the following relation:*

$$\begin{array}{c} r_2^* < r_1^* \ when \ \frac{var\left(X_p\right)}{var\left(X_q\right)} \to 0 \\ r_2^* > r_1^* \ when \ \frac{var\left(X_q\right)}{var\left(X_p\right)} \to 0 \end{array}$$

**Proposition 2.** *Under the condition* $r_1^* > max\left\{E[p], E[p] - 2\rho\sigma_p\sigma_q\right\}$ *and* $\frac{var\left(X_p\right)}{var\left(X_q\right)} \to 0$, *the optimal price for a risk-averse retailer without forward,* $r_2^*$*, is a decreasing function of* $\sigma_q$*. That is:*

$$\frac{\partial r_2^*}{\partial \sigma_q} < 0 \text{ when } r_1^* > max\left\{E[p], E[p] - 2\rho\sigma_p\sigma_q\right\} \text{ and } \frac{var\left(X_p\right)}{var\left(X_q\right)} \to 0$$

**Proposition 3.** *Under the condition* $r_1^* > max\{E[p], E[p] - 2\rho\sigma_p\sigma_q\}$ *and* $\frac{var(X_q)}{var(X_p)} \to 0$*, the optimal price for a risk-averse retailer without forward,* $r_2^*$*, is an increasing function of* $\sigma_q$ *when* $\rho \to 1$*, and a decreasing function of* $\sigma_q$ *when* $\rho \to -1$*. That is:*

$$\text{with } r_1^* > max\{E[p], E[p] - 2\rho\sigma_p\sigma_q\} \text{ and } \frac{var(X_q)}{var(X_p)} \to 0,$$

$$a. \quad \frac{\partial r_2^*}{\partial \sigma_q} > 0 \text{ when } \rho \to 1.$$
$$b. \quad \frac{\partial r_2^*}{\partial \sigma_q} < 0 \text{ when } \rho \to -1$$

We are now in a position to do a profit volatility decomposition, which is useful in determining the effect of volatility terms on pricing and hedging behaviors.

### 2.5. Profit Volatility Decomposition Analysis

From Figure 2, for model 2, let $\mathrm{p} = \mathrm{X_p} + E[p]$, $q = X_q + E[q]$, and process the variance of profit as follows:

$$\begin{aligned}
\mathrm{var(y)} &= \mathrm{var(rq - pq)} = \mathrm{var}\big(\mathrm{r}(\mathrm{E[q]} + \mathrm{X_q}) - (\mathrm{E[p]} + \mathrm{X_p})(\mathrm{E[q]} + \mathrm{X_q})\big) \\
&= \mathrm{var}\big(\mathrm{rE[q]} + \mathrm{rX_q} - \mathrm{E[p]} \times E[q] - E[p]X_q - E[q]X_p - X_pX_q\big) \\
&= \mathrm{var}\big(\mathrm{rX_q} - E[p]X_q - E[q]X_p - X_pX_q\big) = \mathrm{var}\big((\mathrm{r} - E[p])X_q - E[q]X_p - X_pX_q\big) \\
&= (r - E[p])^2 var(X_q) + (E[q])^2 var(X_p) + var(X_pX_q) \\
&\quad - 2(E[q])(r - E[p])cov(X_p, X_q) - 2(r - E_p)cov(X_pX_q, X_q) \\
&\quad + 2(E[q])cov(X_p, X_pX_q)
\end{aligned} \tag{16}$$

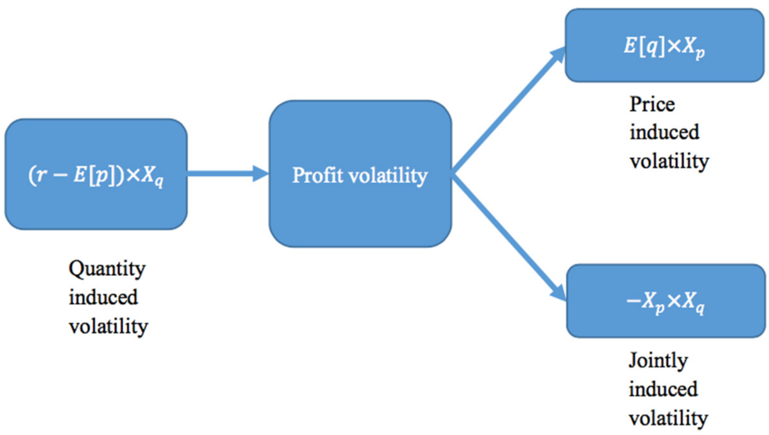


**Figure 2.** Profit volatility decomposition for model 2.

From Figure 3, for model 4, let $\mathrm{p} = \mathrm{X_p} + E[p]$, $q = X_q + E[q]$, and process the variance of profit as follows:

$$\begin{aligned}
\mathrm{var(y)} &= \mathrm{var}\big(\mathrm{rq - pq} + \theta(p - f_p)\big) = \mathrm{var}\big(\mathrm{r}(\mathrm{E[q]} + \mathrm{X_q}) - (\mathrm{E[p]} + \mathrm{X_p})(\mathrm{E[q]} + \mathrm{X_q}) + \theta X_p\big) \\
&= \mathrm{var}\big(\mathrm{rE[q]} + \mathrm{rX_q} - \mathrm{E[p]} \times E[q] - E[p]X_q - (E[q] - \theta)X_p - X_pX_q\big) \\
&= \mathrm{var}\big(\mathrm{rX_q} - E[p]X_q - E[q]X_p - X_pX_q\big) = \mathrm{var}\big((\mathrm{r} - E[p])X_q - (E[q] - \theta)X_p - X_pX_q\big) \\
&= (r - E[p])^2 var(X_q) + (E[q] - \theta)^2 var(X_p) + var(X_pX_q) \\
&\quad - 2(E[q] - \theta)(r - E[p])cov(X_p, X_q) - 2(r - E_p)cov(X_pX_q, X_q) \\
&\quad + 2(E[q] - \theta)cov(X_p, X_pX_q)
\end{aligned} \tag{17}$$

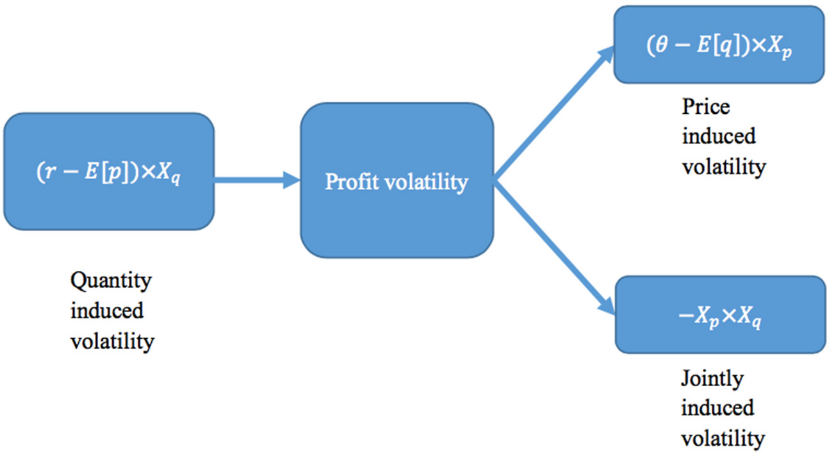


**Figure 3.** Profit volatility decomposition for model 4.

As can be seen in Figure 3, we denote $(r - E[p]) \times X_q$ as the quantity-induced volatility (QIV), where the only volatility comes from quantity. We denote $-E[q]X_p$ or $-(E[q] - \theta)X_p$ as the price-induced volatility (PIV), where the only volatility comes from price. We denote $-X_pX_q$ as the joint induced volatility (JIV). QIV is controlled by retail price r in the form of $|r - E[p]|$, which is the profit margin of each product. When QIV is dominant, the quantity uncertainty is big, and to control for a smaller QIV, the retailer will reduce the profit margin. PIV is controlled by $|E[q]|$ or $|E[q] - \theta|$, which is the naked position of the demand quantity level that is subject to price volatility. When PIV is dominant, the price uncertainty is big, and to control for a smaller PIV, the retailer will reduce the naked position. JIV cannot be controlled by the price or forward position, but the covariance terms containing JIV can have an impact on the final behaviors.

To determine the dominant term of the retailer behavior, the following two-step analysis is implemented:

1. First, for the above three terms (PIV, QIV, JIV), the ones with a low magnitude must not be the dominant terms and thus can be eliminated.
2. Second, check if the behavior variable (e.g., retail price $r$ or forward position $\theta$) and varying parameter (e.g., the quantity volatility $\sigma_q$) are both in the remaining terms. If so, these terms are dominant. If not, the covariance between the behavior variable term and varying parameter term is dominant.

The above analysis can be explained as follows. First, a term is dominant if it is capable of significantly changing the magnitude of profit volatility and cannot be small. Second, a dominant term should have both a behavior variable and varying parameter in it so that the retailer's behavioral response to the varying parameter is reflected in the term.

## 3. Comparative Statics

To apply the above models to stylized examples, we assumed that wholesale price and quantity followed a log-normal normal distribution, that is, $(\log(p), q) \sim N(u_p, u_q, \sigma_p^2, \sigma_q^2, \rho)$, and applied the same parameters of the California Electricity Market found in [9] and given in Table 2.

**Table 2.** Parameters setup with the wholesale log-price mean $u_p$, the demand function intercept a, the demand function slope b, the wholesale log-price and quantity correlation $\rho$, the risk aversion coefficient $\gamma$, the wholesale log-price standard deviation $\sigma_p$, and the quantity standard deviation $\sigma_q$.

| $u_p$ | a | b | $\rho$ | $\gamma$ | $\sigma_p$ | $\sigma_q$ |
|---|---|---|---|---|---|---|
| 3.64 | 119.29 | 0.3382 | 0.7 | 0.0001 | 0.35 | 30 |

To study the effects of price-quantity uncertainties and risk aversion on retailer's pricing and hedging behaviors, we needed to examine the properties of the pricing $r$ (and hedging $\theta$, if allowed) of our four models across the parametric dimension spaces, including quantity uncertainty $\sigma_q$, price uncertainty $\sigma_p$, and price-quantity correlation $\rho$. However, these properties are high-dimensional and complicated, and may be inverted under different conditions, which makes it difficult to fully characterize these properties. To address the difficulty, we conducted comparative statics in the following two subsections. We plotted pricing r (and hedging $\theta$, if allowed) of our four models across quantity uncertainty $\sigma_q$ in Section 3.1, and plotted that across price uncertainty $\sigma_p$ in Section 3.2, since these two parameters change within a year and the changes are identifiable. Moreover, in Section 3.1, we varied $\rho$ and $\sigma_p$ to see how they affected the relations, and in Section 3.2, we only varied $\rho$ to see how they affected the relations, since $\sigma_q$ made little difference in this subsection. The parameters varied according to Table 3. In Table 3, the medium value for $\sigma_p$, $\sigma_q$, and $\rho$ were from Table 2. The low and high value of $\rho$ were taken from [3,23]. The low and high value of $\sigma_q$ were estimated from [24], and the high value of $\sigma_p$ was taken from that during 2002 California electricity crisis [25], the low value assumed a regulated market where wholesale electricity price changed very little over the year.

**Table 3.** Range for the wholesale log-price standard deviation $\sigma_p$, the quantity standard deviation $\sigma_q$, and the wholesale log-price and quantity correlation $\rho$.

| Parametric Range | $\sigma_p$ | $\sigma_q$ | $\rho$ |
|---|---|---|---|
| Low | 0.01 | 10 | −0.7 |
| Medium | 0.35 | 30 | 0 |
| High | 1.2 | 50 | 0.7 |

To characterize the behavior properties in the meantime, we explain these properties in the following subsections using volatility decomposition mentioned in Section 2.5. In the two models that involve risk aversion, retail prices were set to both optimize profit and reduce volatility in the mean-variance goal function. The existence of volatility reduction makes a difference in retail price between the risk averse models and risk neutral models. Such a difference in retail price is referred to as "price hedge," as it results from the variance term in the goal function and serves to reduce volatility in a similar fashion as the forward contract hedge to reduce volatility.

### 3.1. Quantity Uncertainty $\sigma_q$

To study how retailers set the price and forward position in the benchmark setting when only quantity uncertainty varies, we plot Figure 4. Figure 4 shows a graph of retail prices from three models as a function of quantity uncertainty, as well as a graph of forward position from models 2 and 4 as a function of quantity uncertainty. In the left graph, as quantity uncertainty $\sigma_q$ grew, the risk-neutral retail price $r_1^*$ remained the same, but the risk averse price without forward, $r_2^*$, and the risk-averse price with forward, $r_4^{**}$, both decreased with $\sigma_q$. Formally, this is given as: $\frac{\partial r_1^*}{\partial \sigma_q} = 0$, $\frac{\partial r_2^*}{\partial \sigma_q} < 0$, $\frac{\partial r_4^*}{\partial \sigma_q} < 0$. This was due to the "price hedge" that reduced the retail price to reduce the quantity induced volatility (QIV), as shown in Figure 3. The more quantity uncertainty there was, the more "price hedge" that was needed, so the smaller the retail price was. Moreover, $r_2^*$ decreased faster than $r_4^{**}$ with $\sigma_q$, which is formally given as: $\frac{\partial r_2^*}{\partial \sigma_q} < \frac{\partial r_4^*}{\partial \sigma_q} < 0$. This was due to the "forward hedge" that partially hedged the QIV with a price-based forward contract. With QIV partially hedged, the "price hedge" effect was needed less, so $r_4^{**}$ decreased slower than $r_2^*$ with $\sigma_q$. In the right graph, as the quantity uncertainty $\sigma_q$ grew, the variance minimization forward position $\theta_3^*$, and the risk aversion forward position $\theta_4^{**}$, both decreased with $\sigma_q$, which is formally given as: $\frac{\partial \theta_3^*}{\partial \sigma_q} < 0$, $\frac{\partial \theta_4^{**}}{\partial \sigma_q} < 0$. This was due to the forward hedge that partially hedged the QIV with a price-based forward contract. Price uncertainty and quantity uncertainty are correlated, so the retailer will sell a forward contract to hedge QIV. The more quantity uncertainty there was, the more a forward hedge was needed, so the smaller the forward position. Moreover, $\theta_3^*$ decreased faster than $\theta_4^{**}$ with $\sigma_q$, which is formally given as: $\frac{\partial\theta_3^*}{\partial\sigma_q} < \frac{\partial\theta_4^{**}}{\partial\sigma_q} < 0$. This was due to the "price hedge" that hedged the QIV with the retail price. With QIV partially hedged by the retail price, the forward hedge effect was needed less, so $\theta_4^{**}$ decreased slower than $\theta_3^*$.

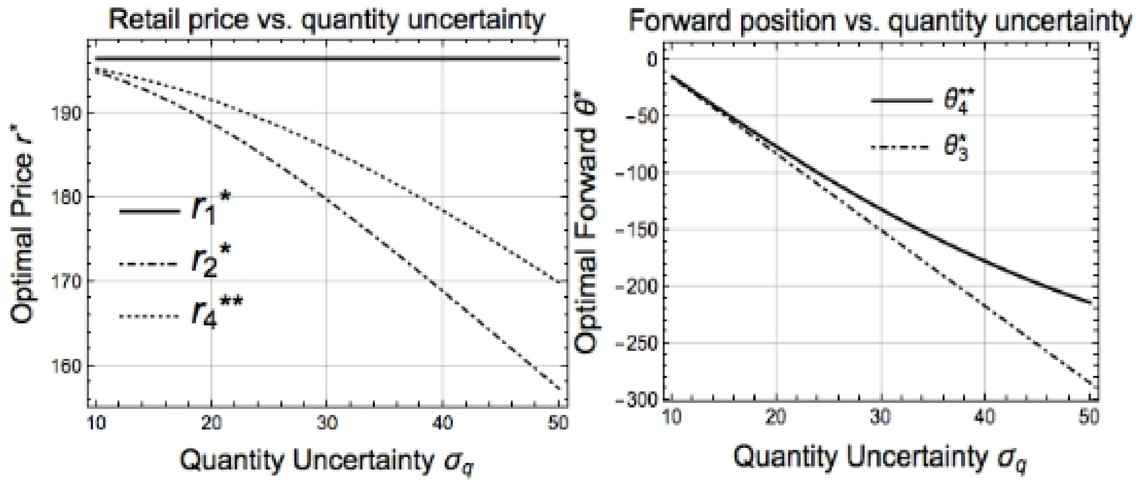


**Figure 4.** Comparison of $r_1^*$, $r_2^*$, and $r_4^{**}$ as from Theorems 1, 2, and 4 as a function of quantity volatility $\sigma_q$ (left panel), and $\theta_4^{**}$ and $\theta_3^*$ from Theorems 4 and 3 as a function of quantity volatility $\sigma_q$ (right panel). The retail price of models 1 and 3 are identical by definition, so we only plot the retail price of model 1. All other parameters were set according to the benchmark setup given in Table 2.

To study how price uncertainty impacted retailers pricing behaviors with quantity uncertainty, we give Figure 5. Figure 5 shows graphs of retail pricing r over the quantity risk axis $\sigma_q$ for three levels of price uncertainty $\sigma_p$. As price uncertainty grew from low ($\sigma_p = 0.01$) to high ($\sigma_p = 1.2$), the risk averse price without forward, $r_2^*$, shifted from below $r_1^*$ to above $r_1^*$, the risk neutral price, which is formally given as: $r_2^*(\sigma_p = 0.01) < r_1^*(\sigma_p = 0.01)$, $r_2^*(\sigma_p = 1.2) > r_1^*(\sigma_p = 1.2)$. This is exactly what propostion 1 reveals. This was due to a regime switching from QIV dominant to PIV dominant. When price uncertainty was small, QIV was dominant, and quantity uncertainty was the major concern for risk-averse retailers. Therefore, "price hedge" mostly happened to reduce QIV, leading to a reduction of the profit margin, and a reduction of the retail price. On the other hand, when price uncertainty was large, PIV was dominant, and price uncertainty became a major concern. Therefore, "price hedge" mostly happened to reduce PIV, leading to a reduction of the expected demand, and an increase of the retail price. Another phenemenon was the risk averse price with a forward, $r_4^{**}$, which remained below $r_1^*$, the risk neutral price, regardless of the price uncertainty, which is formally given as: $r_4^{**}(\sigma_p = 0.01) < r_1^*(\sigma_p = 0.01)$, $r_4^{**}(\sigma_p = 1.2) < r_1^*(\sigma_p = 1.2)$. This was due to the forward hedge eliminating all naked positions, and therefore PIV was always small when forward hedge existed, so QIV remained dominant across different price uncertainties.

To study how price uncertainty and correlation impacted retailers' pricing behaviors with quantity uncertainty, we show Figures 6 and 7. Figure 6 shows graphs of the risk averse price without a forward, $r_2^*$, over the quantity risk axis $\sigma_q$ for three levels of price uncertainty $\sigma_p$ and three levels of price-quantity correlation $\rho$. As price uncertainty grew from low ($\sigma_p = 0.01$) to high ($\sigma_p = 1.2$), $\rho$ grew to cause more of a difference in $r_2^*$. In the top-left panel, when price uncertainty was low, $r_2^*$ almost overlapped in all levels of correlation, and all decreased with $\sigma_q$, which is formally given as:$r_2^*(\sigma_p = 0.01, \rho = 0.7) \approx r_2^*(\sigma_p = 0.01, \rho = 0) \approx r_2^*(\sigma_p = 0.01, \rho = -0.7)$. This is exactly what proposition 2 reveals. When price uncertainty was small ($\sigma_p = 0.01$), QIV was dominant, therefore retailers decreased the profit margin as quantity uncertainty grew. However, in the bottom panel, when price uncertainty was high, $r_2^*$ with high correlation ($\rho = 0.7$) even increased with $\sigma_q$ when $\sigma_q$ was smaller than 30, which is formally given as: $\frac{\partial r_2^*(\sigma_p=1.2,\rho=0.7,\sigma_q<30)}{\partial \sigma_q} > 0$. This is exactly what proposition 3 reveals. When the price uncertainty was large ($\sigma_p = 1.2$) and the quantity uncertainty was small ($\sigma_q < 30$), PIV had the largest uncertainty, JIV the second largest, and QIV the smallest. Then, using our volatility decomposition method, the PIV–JIV covariance was dominant. As quantity uncertainty grew, JIV grew. With a high correlation ($\rho = 0.7$), the "price hedge" of the PIV–JIV covariance decreased PIV, and with low correlation ($\rho = -0.7$), the "price hedge" of PIV–JIV covariance increased PIV. Therefore, we saw $\rho$ cause a difference in the $r_2^* - \sigma_q$ relation when the price uncertainty was large ($\sigma_p = 1.2$).

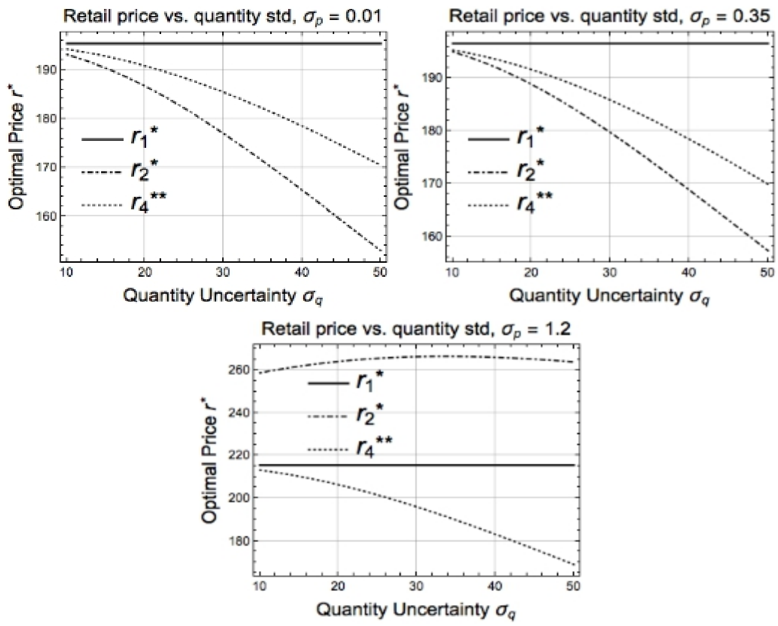


**Figure 5.** Comparison of $r_1^*$, $r_2^*$, and $r_4^{**}$ as from Theorems 1, 2, and 4 as a function of quantity volatility $\sigma_q$ under small (top-left, $\sigma_p = 0.01$ ), medium (top-right, $\sigma_p = 0.35$ ), and large (bottom, $\sigma_p = 1.2$ ) price volatility.

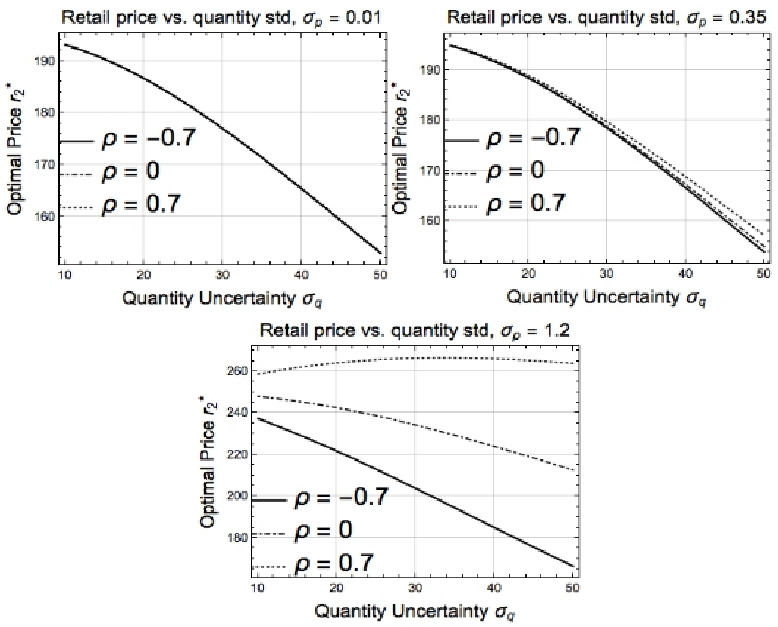


**Figure 6.** Comparison of $r_2^*$ from Theorem 2 as a function of quantity volatility $\sigma_q$ under small (top-left, $\sigma_p = 0.01$ ), medium (top-right, $\sigma_p = 0.35$ ), and large (right, $\sigma_p = 1.2$ ) price volatility. In each plot, the three correlation conditions ($\rho = -0.7, \rho = 0, \rho = 0.7$ ) are also described.

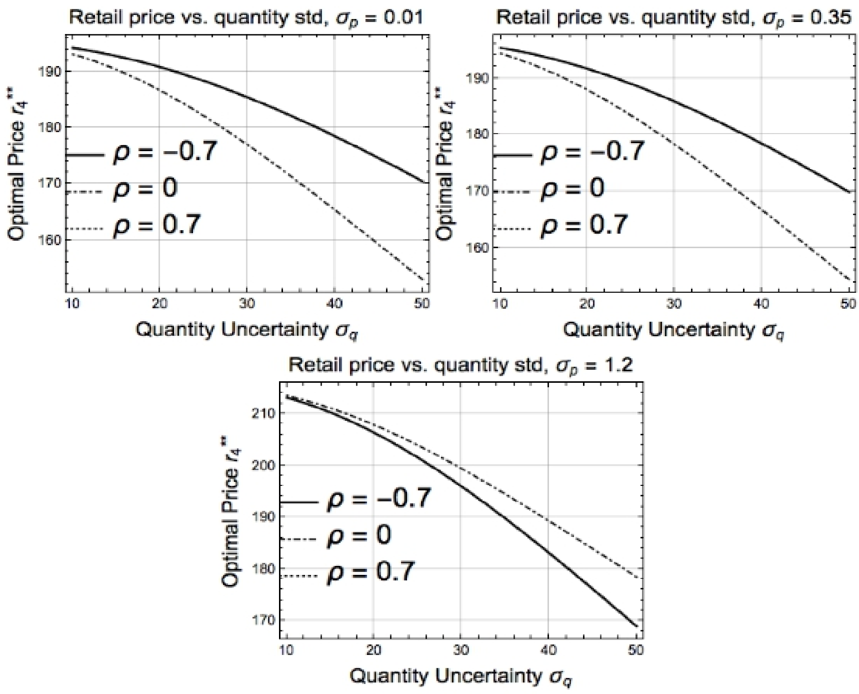


**Figure 7.** Comparison of $r_4^{**}$ from Theorem 4 as a function of quantity volatility $\sigma_q$ under small (top-left, $\sigma_p = 0.01$ ), medium (top-right, $\sigma_p = 0.35$ ), and large (bottom, $\sigma_p = 1.2$ ) price uncertainties. In each plot, the three correlation levels ($\rho = -0.7, \rho = 0, \rho = 0.7$ ) are also described.

Figure 7 shows graphs of the risk-averse price with forward, $r_4^{**}$, over the quantity risk axis $\sigma_q$ for three levels of price uncertainty $\sigma_p$ and three levels of price-quantity correlation ρ. From all three graphs, large- ($\rho = 0.7$) and small- ($\rho = -0.7$) correlated (hereto after named "not uncorrelated") retail prices seemed to overlap, and both were different from the uncorrelated ($\rho = 0$) retail price. This indicates that, in the presence of a forward contract, the effect of price–quantity correlation was not negligible. This phenomenon was due to the partial hedge of quantity uncertainty using a forward contract when the price and quantity uncertainty were not uncorrelated. Such a partial hedge caused the above $r_4^{**} - \sigma_q$ relation difference in the large ($\rho = 0.7$) and small ($\rho = -0.7$) correlation case against the uncorrelated ($\rho = 0$) case. As price uncertainty grew from low ($\sigma_p = 0.01$) to high ($\sigma_p = 1.2$), the uncorrelated retail price $r_4^{**}$ went from below to above the not uncorrelated one, which is formally given as:

$$\begin{gathered} r_4^{**}\Big(\sigma_p = 0.01, \rho = 0.7\Big) = r_4^{**}\Big(\sigma_p = 0.01, \rho = -0.7\Big) > r_4^{**}\Big(\sigma_p = 0.01, \rho = 0\Big) \\ r_4^{**}\Big(\sigma_p = 1.2, \rho = 0.7\Big) = r_4^{**}\Big(\sigma_p = 1.2, \rho = -0.7\Big) < r_4^{**}\Big(\sigma_p = 1.2, \rho = 0\Big) \end{gathered} \tag{18}$$

This was still due to a regime switching from QIV-dominant to QIV-JIV-covariance-dominant. When the price uncertainty was small, QIV was dominant. With large ($\rho = 0.7$) or small ($\rho = -0.7$) correlations, the forward hedge happened to partially hedge the QIV, so the "price hedge" for QIV was needed less compared to the uncorrelation case. Therefore, both the large and small correlated retail price $r_4^{**}$ were higher than the uncorrelated retail price $r_4^{**}$. When the price uncertainty was large, the QIV-JIV covariance was dominant. In all correlation levels, the forward hedge happened to hedge the JIV. Part of the JIV that was not hedged by the forward remained dominant, and the "price hedge" from QIV was needed to hedge JIV. The greater the JIV, the more the "price hedge" was needed to hedge the JIV. The more correlation or anti-correlation that existed, the greater the part of JIV that was hedged by the forward, and the less the "price hedge" was needed, which meant a lower retail price $r_4^{**}$. Therefore, both large and small correlated retail prices $r_4^{**}$ were lower than the uncorrelated retail price $r_4^{**}$.

To study how price uncertainty and correlation impacted retailers' forward hedging behaviors regarding quantity, we show Figure 8. As price uncertainty grew from low ($\sigma_p = 0.01$) to high

($\sigma_p = 1.2$), the range of forward contracts decreased from 10,000 to less than 150. This decrease indicated the regime switching from quantity uncertainty dominance to price uncertainty dominance. In PIV–QIV dominance, price uncertainty was low, so the forward in PIV was mainly used to partially hedge the quantity uncertainty and the ratio of quantity uncertainty to price uncertainty could be as large as 10,000. In PIV–JIV dominance, price uncertainty was large, and the forward in PIV was mainly used to hedge the naked position, which was around 200. Moreover, from the top-left to bottom graph, the slopes of $\theta_4^{**}$ against $\sigma_q$ are inverted for both correlation cases ($\rho = -0.7,\ \rho = 0.7$) when price uncertainty grew from low to high, which is formally given as:

$$\frac{\partial\theta_4^{**}(\sigma_p{=}0.01,\rho{=}0.7)}{\partial\sigma_q} < 0 < \frac{\partial\theta_4^{**}(\sigma_p{=}1.2,\rho{=}0.7)}{\partial\sigma_q}, \frac{\partial\theta_4^{**}(\sigma_p{=}0.01,\rho{=}-0.7)}{\partial\sigma_q} > 0 > \frac{\partial\theta_4^{**}(\sigma_p{=}1.2,\rho{=}-0.7)}{\partial\sigma_q} \tag{19}$$

This was still due to a similar argument of regime switching. In PIV–QIV dominance, where the top-left and the top-right graphs reside, the forward in PIV was mainly used to partially hedge the quantity uncertainty in QIV. Therefore, increasing the quantity uncertainty entailed increasing ($\rho = 0.7$) or decreasing ($\rho = -0.7$) the forward position. In PIV–JIV dominance, where the bottom graph resides, the forward mainly hedged the JIV. Therefore, increasing the quantity uncertainty entailed decreasing ($\rho = 0.7$) or increasing ($\rho = -0.7$) the forward position, which was totally inverted from the PIV–QIV dominance.

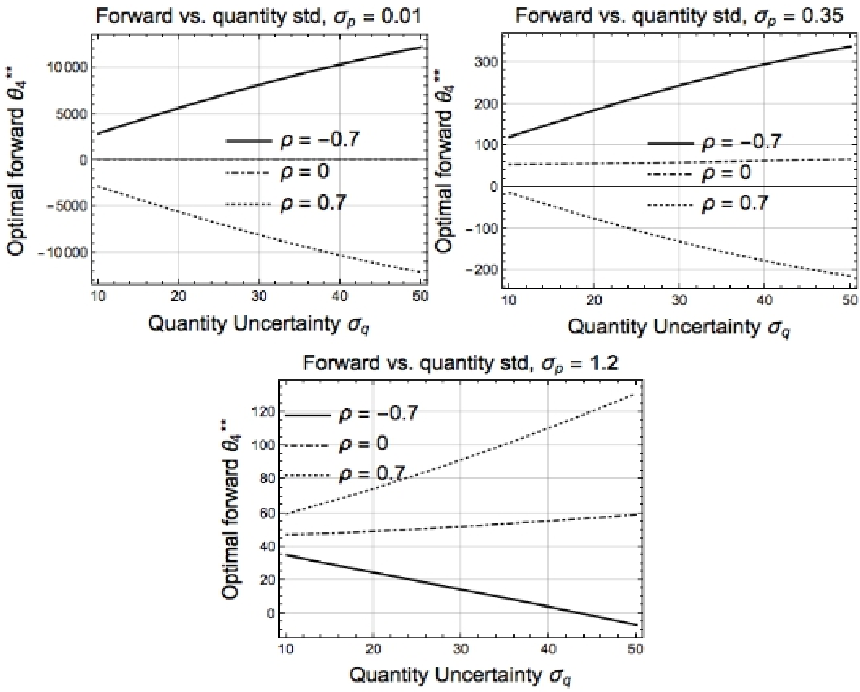


**Figure 8.** Comparison of $\theta_4^{**}$ as from Theorem 4 as a function of quantity volatility $\sigma_q$ under small (top-left, $\sigma_p = 0.01$ ), medium (middle, $\sigma_p = 0.35$ ), and large (right, $\sigma_p = 1.2$ ) price volatility. In each plot, the three correlation conditions ($\rho = -0.7, \rho = 0, = 0.7$ ) are also described.

### 3.2. Price Uncertainty $\sigma_p$

To study how retailers set the price and forward position in the benchmark setting when only price uncertainty varied, we give Figure 9. Figure 9 shows a graph of retail prices from three models as a function of price uncertainty, as well as a graph of the forward position from models 2 and 4 as a function of price uncertainty. In the left graph, as price uncertainty $\sigma_p$ grows, the risk-neutral retail price $r_1^*$, the risk averse price without forward $r_2^*$, and the risk averse price with the forward $r_4^{**}$ all increased with $\sigma_q$, and $r_2^*$ grew the fastest, $r_1^*$ the second fastest, and $r_4^{**}$ the slowest. This is formally given as: $\frac{\partial r_2^*}{\partial\sigma_q} > \frac{\partial r_1^*}{\partial\sigma_q} > \frac{\partial r_4^{**}}{\partial\sigma_q} > 0$. $r_2^*$ grew faster than $r_1^*$ because of the "price hedge" in model 2 that increased the retail price to reduce the price-induced volatility(PIV), as shown in Figure 3. The greater the price uncertainty, the more "price hedge" that was needed, and therefore the greater the retail price. $r_4^{**}$ grew slower than $r_1^*$ because the forward in model 4 hedged all of PIV and part of QIV, and as a result, left QIV dominant. Then, the "price hedge" decreased the retail price to reduce the QIV, and therefore we saw a smaller $r_4^{**}$ compared to $r_1^*$.

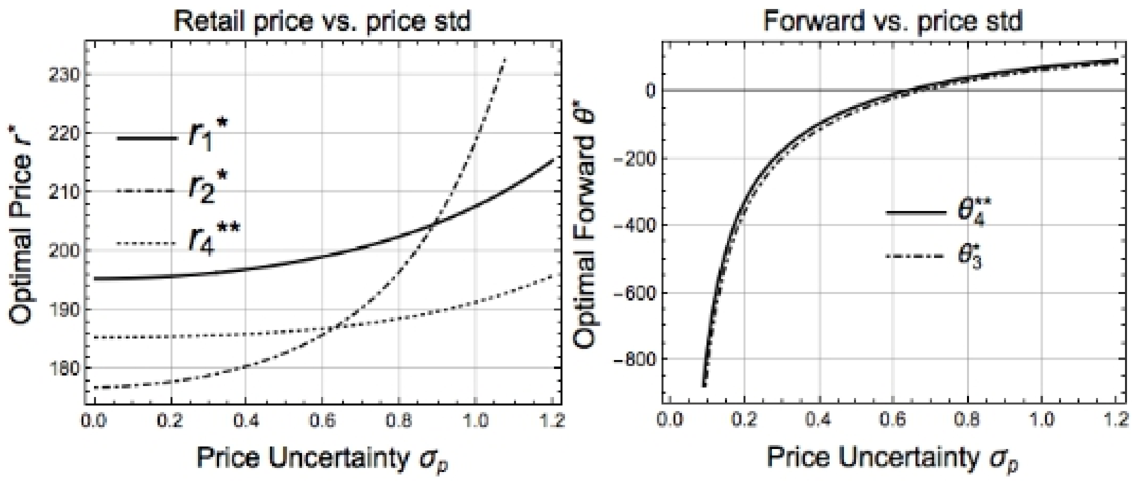


**Figure 9.** Comparison of $r_1^*$, $r_2^*$, and $r_4^{**}$ from Theorems 1, 2, and 4 as a function of price uncertainty $\sigma_p$ (left panel), and $\theta_4^{**}$ and $\theta_3^*$ as from Theorems 4 and 3 as a function of price uncertainty $\sigma_p$ (right panel). The retail price of models 1 and 3 are identical by definition, so we only give the retail price of model 1. All other parameters were set according to the benchmark setup as given in Table 2.

In the right graph, as price uncertainty $\sigma_p$ grew, the variance minimization of the forward position $\theta_3^*$ and the risk aversion of the forward position $\theta_4^{**}$ both increased with $\sigma_q$, which is formally given as: $\frac{\partial\theta_4^{**}}{\partial\sigma_q} > 0$, $\frac{\partial\theta_3^{*}}{\partial\sigma_q} > 0$. This was due to a forward hedge transition from hedging QIV to hedging PIV. Price uncertainty and quantity uncertainty were correlated, and therefore the retailer should sell the forward contract to hedge the QIV. When $\sigma_p$ was small, the greater the price uncertainty, the more hedge each unit of forward contract provided to hedge QIV, and therefore the fewer units of short position forward were needed, which meant an increase in the forward contract position. When $\sigma_p$ was small, PIV was dominant, and therefore the forward position converged to the position of the expected quantity $E[q]$.

To study how correlation affected retailers pricing and hedging behaviors with price uncertainty, we present Figure 10. Figure 10 shows graphs of the risk-averse retail price without a forward, $r_2^*$, the risk-averse retail price with a forward, $r_4^{**}$, and the risk-averse forward position, $\theta_4^{**}$, with respect to the price uncertainty axis $\sigma_p$. In the top-left graph, $r_2^*$ ($\rho = 0.7$) increased the fastest in $\sigma_p$, $r_2^*(\rho = 0)$ was second fastest, and $r_2^*(\rho = -0.7)$ was the slowest, which is formally given as: $\frac{\partial r_2^*(\rho=0.7)}{\partial\sigma_p} > \frac{\partial r_2^*(\rho=0)}{\partial\sigma_p} > \frac{\partial r_2^*(\rho=-0.7)}{\partial\sigma_p}$. This was due to the "price hedge" of the PIV–JIV covariance, as described in Propositions 2 and 3. A large correlation($\rho = 0.7$) led to a positive PIV–JIV covariance such that as $\sigma_p$ increased, the total profit variance increased the fastest, and the "price hedge" increased the fastest as a result. Meanwhile, a small correlation ($\rho = -0.7$) led to a negative PIV–JIV covariance such that the "price hedge" had the slowest increase.

In the top-right graph, $r_4^{**}(\rho = 0.7) = r_4^{**}(\rho = -0.7)$ and both increased slower than $r_4^{**}(\rho = 0)$ in $\sigma_p$. This was due to the forward hedge of JIV. Both large and small correlations made it possible for forward to hedge part of JIV. Then, as $\sigma_p$ increased, JIV increased faster for a medium correlation since no part of JIV could be hedged with a forward. This led to a faster "price hedge" increase as a result.

In the bottom graph, we observe that:

$$\begin{array}{c} \theta_4^{**}(\rho = 0.7) < \theta_4^{**}(\rho = 0) < \theta_4^{**}(\rho = -0.7) \text{ when } \frac{var(X_p)}{var(X_q)} \to 0 \\ \theta_4^{**}(\rho = -0.7) < \theta_4^{**}(\rho = 0) < \theta_4^{**}(\rho = 0.7) \text{ when } \frac{var(X_q)}{var(X_p)} \to 0 \\ \frac{\partial \theta_4^{**}(\rho=0.7)}{\partial \sigma_p} > \frac{\partial \theta_4^{**}(\rho=0)}{\partial \sigma_p} \approx 0 > \frac{\partial \theta_4^{**}(\rho=-0.7)}{\partial \sigma_p} \end{array} \quad (20)$$

This resulted from a regime switching from PIV–QIV dominance to PIV–JIV dominance. When price uncertainty was much smaller than quantity uncertainty, PIV–QIV correlation was dominant. With a large correlation ($\rho = 0.7$), PIV and QIV were negatively correlated. Therefore, an additional short forward position was required to hedge JIV compared to the zero correlation. Likewise, with a small correlation ($\rho = -0.7$), PIV and QIV were correlated; an additional long forward position was required to hedge JIV. When the price uncertainty was much greater than the quantity uncertainty, PIV–JIV correlation was dominant. With a large correlation, PIV and JIV were correlated. Therefore, an additional long forward position was required to hedge JIV compared to the zero-correlation scenario. Likewise, with a small correlation, PIV and JIV were negatively correlated, and an additional short forward position was required to hedge JIV. This result was robust for all three levels of quantity uncertainty.

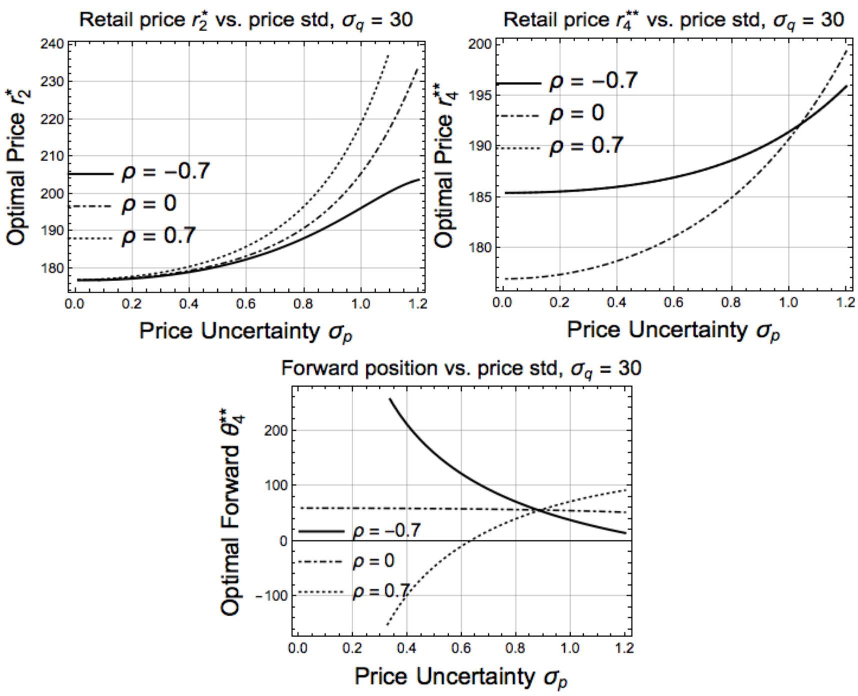


**Figure 10.** Comparison of $r_2^*$ from Theorem 2 and $r_4^{**}$ and $\theta_4^{**}$ from Theorem 4 as a function of price volatility $\sigma_p$ under a small ($\rho = -0.7$), medium ($\rho = 0$), and large ($\rho = 0.7$) correlation.

### *3.3. Profit Distribution Comparison*

To check the effect of the pricing and hedging behavior in the final volatility reduction, Figure 11 plots the distributions of profit under Theorems 1 to 4, and the profit expectation and variance of Theorems 1 to 4 are presented in Table 4. When comparing Theorems 1 (black) and 2 (blue), one can see that price-utility optimization (i.e., Theorem 2) gave a more concentrated profit distribution than profit optimization (i.e., Theorem 1) at a cost of losing some profit. This shows that there was a tradeoff between profit and profit predictability. When comparing Theorems 1 (black) and 3 (red), we found that hedging helped to reduce volatility without affecting the expected profit. However, when comparing Theorems 2 (blue) and 4 (green), we found that the peak of the joint optimization (green) shifted to the right and was lifted up relative to the price utility optimization (i.e., Theorem 2). This meant that hedging not only reduced the volatility but also increased expected profit. This interesting phenomenon requires empirical validation. Finally, comparing Theorems 3 (red) and 4 (green) revealed that joint optimization also helped to concentrate profit distribution at a cost of lower expected profit.

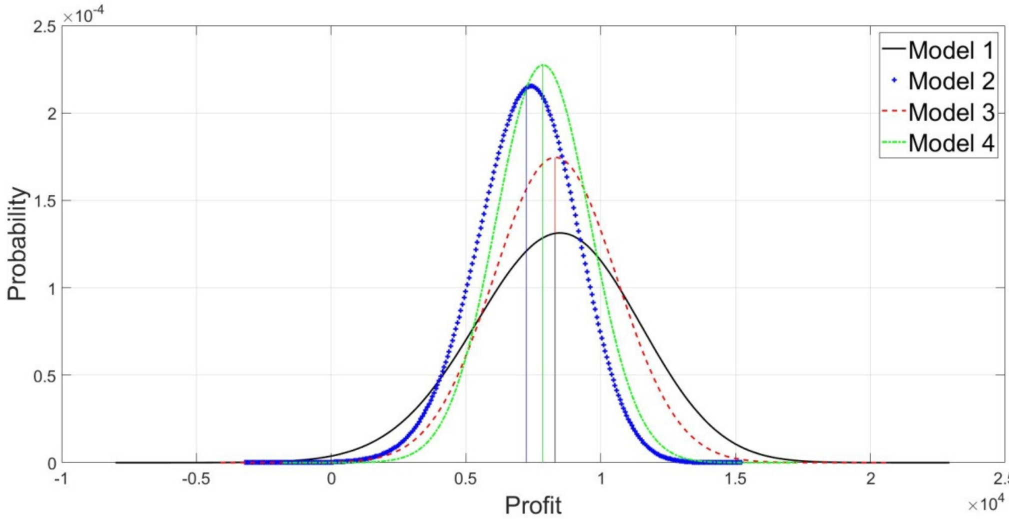


**Figure 11.** Profit distribution under profit maximization with no forward scenario (black line, Theorem 1), utility maximization with no forward (blue +, Theorem 2); profit maximization with forward (red –, Theorem 3); and utility maximization with forward (green —, Theorem 4).

**Table 4.** Profit distribution of the four models.

| Profit Expectation and Deviation | Risk-Neutral | Risk-Averse |
|---|---|---|
| Without forward | 8306.8 ± 3003.8 | 7241.4 ± 1839.3 |
| With forward | 8306.8 ± 2258.4 | 7852.4 ± 1734.2 |

### *3.4. Demand Function Sensitivity Analysis*

Retail electricity demand is known to be insensitive to retail price. Therefore, we would like to study the effect of a smaller demand function slope b on the behaviors of retailers. Specifically, we studied the effect of slope b on the risk-neutral price without a forward, $r_1^*$, risk-averse price without a forward, $r_2^*$, the risk-averse price with a forward, $r_4^{**}$, the risk-neutral forward position $\theta_3^*$, and the risk-averse forward position $\theta_4^{**}$, and we give the relations in Figure 12. From Table 2, b = 0.3382, so we set the demand slope b in the range of [0.01,0.5].

From the Figure 12 left panel, it can be seen that as end-user demand became less sensitive to retail price (as b decreased), the retail price for the four models all increased. This is intuitive since the less sensitive the end users becomes, the less reaction they will have toward a retail price markup. Therefore, the retailer will raise the retail price as the demand slope decreases. From the Figure 12 right panel, it can be seen that as the end-user demand became less sensitive to the retail price (as b decreased), the forward contract first increased then decreased, but the forward contract varied within a much smaller range than the retail price. Therefore, retail price was more sensitive to a demand slope change than the forward position.

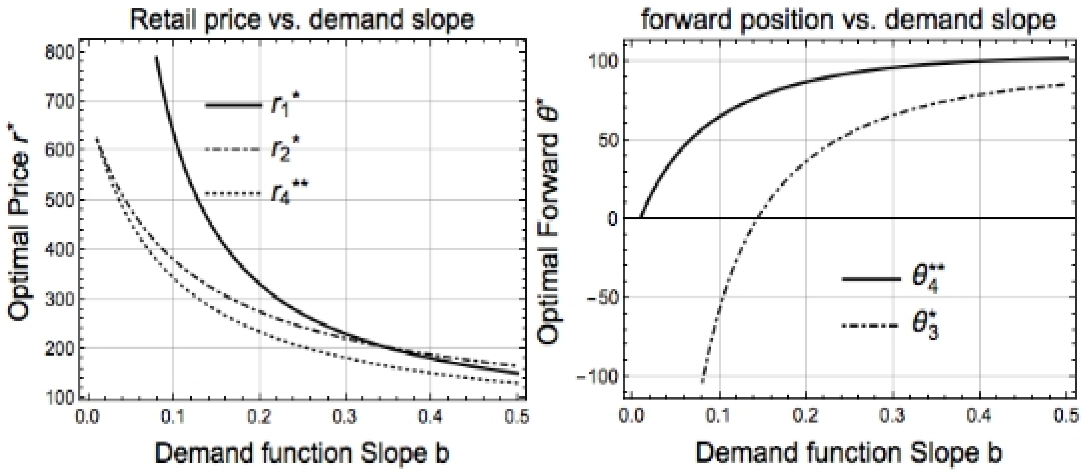


**Figure 12.** Comparison of $r_1^*$, $r_2^*$, and $r_4^{**}$ from Theorems 1, 2, and 4 as a function of the demand slope b (left panel), and $\theta_4^{**}$ and $\theta_3^*$ from Theorems 4 and 3 as a function of the demand slope b (right panel). The retail price of models 1 and 3 are identical by definition, so we only plotted the retail price of model 1. All other parameters were set according to the benchmark setup as in Table 1.

## 4. Conclusions

We addressed the pricing problem of a retailer under price and quantity uncertainties with and without a forward contract. Four different models were proposed: risk-neutral without a forward, risk-averse without a forward, risk-neutral with a forward, and risk-averse with a forward. Closed-form solutions were derived for the four models, and comparative statics were conducted. We found different determinants for a risk-averse retailer's pricing and hedging behaviors under different price and quantity uncertainty cases, as shown in Table 5.

**Table 5.** Optimal pricing and hedging behaviors and their dominant volatility sources.

| Cases | $r_2^*$ vs. $r_1^*$ | | $\partial r_2^*/\partial\sigma_q$ | | $\partial r_4^{**}/\partial\sigma_q$ | | $\partial\theta_4^{**}/\partial\sigma_q$ | | $\partial\theta_4^{**}/\partial\sigma_p$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Relation** | **Dominant** | **Relation** | **Dominant** | **Relation** | **Dominant** | **Relation** | **Dominant** | **Relation** | **Dominant** |
| 1 | < | QIV | − | QIV | − | QIV | − | PIV–QIV | − | PIV–QIV |
| 2 | < | QIV | − | QIV | − | QIV | + | PIV–QIV | + | PIV–QIV |
| 3 | > | PIV | + | PIV–JIV | − | QIV | + | PIV–JIV | − | PIV–JIV |
| 4 | > | PIV | − | PIV–JIV | − | QIV | − | PIV–JIV | + | PIV–JIV |

Notes: "−" means less than 0, "+" means greater than 0, and PIV–JIV means the covariance between PIV and JIV, PIV–QIV means the covariance between PIV and QIV. Case 1: $\frac{var(X_p)}{var(X_q)} \to 0, \rho \to 1$; Case 2: $\frac{var(X_p)}{var(X_q)} \to 0, \rho \to -1$; Case 3: $\frac{var(X_q)}{var(X_p)} \to 0,\ \rho \to 1$; Case 4: $\frac{var(X_q)}{var(X_p)} \to 0,\ \rho \to -1$.

We show that, for a retailer under price and quantity risk,

(a) Under a large price uncertainty and small quantity uncertainty, a risk-averse retailer overpriced a risk-neutral retailer. On the contrary, under a small price uncertainty and large quantity uncertainty, a risk-averse retailer underpriced a risk-neutral retailer. This finding supplements the finding that a risk-averse retailer only underprices a risk-neutral retailer in the sole presence of quantity uncertainty [4].

(b) Correlation created a great difference for a risk-averse retailer pricing behavior without a forward only under a large price uncertainty, but there was very little difference in risk-averse retailer pricing behavior with a forward regardless of whether the price uncertainty was large or small.

(c) Correlation created a great difference for a risk-averse retailer hedging behavior, no matter whether the price uncertainty was large or small.

(d) A risk-averse retailer with a forward contract had a larger expected profit and smaller profit volatility compared to a risk-averse retailer without a forward contract.

This work expands the understanding of a retailer's decision-making in the presence of both price and quantity uncertainties, and such a presence is prevalent in an energy market. We show that a retailer's pricing behaviors may be affected by the price and quantity uncertainty and their correlation. With the increasing penetration of renewables in the energy market that might cause price and quantity to be negatively correlated [23,26], and the fast development of the energy finance market, we will observe new retailer pricing behaviors as predicted by our models. Regulators and policy-makers could gain insights from the factors that determine retailer behaviors and provide corresponding incentives to stabilize energy prices. One possible example is to control the correlation using the strategic feed-in of renewables such that the retail price might stay in a desired range.

This work has a number of potential extensions. First, this analysis framework can be applied to the study of a traditional newsvendor problem, where the financial forward contract is replaced by a physical forward contract. Second, we can relax the monopolistic setting to a more general oligopolistic setting and study the interactions of retailers. Third, these results call for empirical validation in real energy markets. The results will improve our understanding of the energy markets.

Finally, yet importantly, our models only consider a simple forward contract with a non-arbitrage constraint. One interesting potential future work is to allow multiple forwards and options in a market that could be replicated into any continuous payoff structure. This would further improve hedging quality and provide better profit profiles.

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## Nomenclature

| | |
|---|---|
| $r$ | Retail price in the retail market |
| $p$ | Wholesale price in the wholesale market |
| $q$ | Retail demand by consumers |
| $\theta$ | Forward position by the retailer |
| $a$ | Intercept of the demand function |
| $b$ | Slope of the demand function |
| $u_p$ | Expectation of the logarithm of wholesale price |
| $u_q$ | Expectation of end-user demand |
| $\sigma_p$ | Standard deviation of the logarithm of wholesale price |
| $\sigma_q$ | Standard deviation of end-user demand |
| $\rho$ | Price–quantity correlation |
| $U$ | Retailer's utility |
| $PIV$ | Price-induced volatility |
| $QIV$ | Quantity-induced volatility |
| $JIV$ | Jointly induced volatility |

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