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Optimal Economic Dispatch for Integrated Power and Heating Systems Considering Transmission Losses

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Received: 4 June 2019; Accepted: 26 June 2019; Published: 28 June 2019

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Abstract: To address the problem of the supply–demand imbalance caused by network transmission losses in integrated power and heating systems (IPHS), this paper presents an optimal economic dispatch strategy to minimize system operation cost and realize coordination and optimization between power and heat. Firstly, an innovative economic dispatch model considering transmission losses is developed, where both power and heat transmission losses models are established with good precision together. In addition, the coordination equation is derived from the formulated nonlinear, multi-constrained coupling optimization problem, where the coordination relationship of units' outputs is clearly analyzed in an analytic way. Then, a double-λ-iteration algorithm is proposed, which can not only effectively solve the nonlinear coupling optimization problem but also decrease computation burden with faster convergence rate. Finally, simulations performed on five case studies illustrate the satisfying performance of the presented strategy.

Keywords: integrated power and heating systems; network transmission losses; economic dispatch; nonlinear coupling; double-λ-iteration algorithm

1. Introduction

As a core part of integrated energy systems [1–3], IPHS refer to the organic coordination and optimization of electric energy and heat energy in the process of planning, construction and operation, where production, transmission, distribution, conversion, storage and consumption have been performed holistically. With coupling elements such as combined heat and power (CHP) [4–6] units being integrated, the originally independent power system and heating system can be coupled closely in IPHS. By this way, IPHS can not only achieve diversified energy supply and efficient energy conversion but also ensure energy supply to be sustainable and reliable.

Technologies related to IPHS have been highly valued by researchers and technicians in the global energy field [7–10]. At present, relevant research has mainly focused on promoting the application of distributed energies and cogeneration technologies, increasing the use proportion of renewable energy resources, and advancing collaborative optimization among multiple energy resources, etc. The work studied in this paper will focus on the economic dispatch of IPHS, which aims at minimizing the total system operation cost for power and heat generation of multiple units, while meeting users' actual energy demands and scheduling units' optimal outputs.

The economic dispatch problem (EDP) is one of the most important optimization problems of power system operation, which aims at minimizing the total operation cost while satisfying both system-level and unit-level constraints [11]. The EDP can be typically formulated as a constrained optimization problem, and existing solving algorithms for economic dispatch can be classified into two categories: analytic optimization algorithms such as iteration method [12], Newton's method [13], linear programming method [14], and heuristic optimization algorithms such as genetic algorithm [15], particle swarm optimization [16], and bee colony optimization [17]. It can be noted that EDP of power systems considering power transmission losses has been studied well [11], and there have been many methods to handle the EDP. However, the EDP of power systems is only the study of the single power optimization dispatch, which cannot realize collaborative optimization and economic distribution among multiple energy resources, and it cannot meet the developing trend of the future power grid to integrated energy systems with diversified energy supply and consumption.

Extending this issue to IPHS, domestic and foreign scholars have done some noticeable research. The authors in [18] have proposed a classical CHP economic dispatch model. In addition, the authors in [19] have developed a day-ahead economic dispatch model for regional IPHS to comprehensively consider the wind curtailment cost, electric vehicle dispatch cost and so on. The authors in [20] have presented a holistic optimization dispatching method to minimize the operation cost of the integrated community energy system. However, none of them have considered transmission losses. With power transmission losses, the authors in [21] have designed a line-up competition algorithm and the authors in [22] have proposed an optimization technique based on time-varying acceleration particle swarm optimization to handle CHP multi-objective optimization problems, but neither of them have considered heat transmission losses and network transmission constraints. Defining power and heat losses coefficients, the authors in [23] have presented a distributed neurodynamic-based approach to solve the EDP of integrated energy systems, but neither power nor heat transmission losses model have been established, which means that both of them have been simply considered as constants to a certain extent.

At present, the EDP of IPHS considering network transmission losses is a relatively novel but complex problem, and there have hardly been good model combinations to estimate both power and heat transmission losses at the same time. In other words, the EDP discussed in most of the existing literature is based on the ideal conditions without network transmission losses. Although the goal of economic dispatch has been achieved through reasonable allocations, the optimal solutions are accompanied by many problems due to the existences of transmission losses. On one hand, due to neglecting power transmission losses, it is easy to cause supply–demand imbalance [21,24] and suboptimal solutions [25,26], which cannot meet users' actual load demands well. On the other hand, due to neglecting heat transmission losses, it is easy to cause insufficient heat supply, which will damage users' experiences and even bring serious economic losses [7].

To address the above problems, this paper has studied the optimal economic dispatch of IPHS with network transmission losses, and our major contributions of this paper can be given as follows:

- A novel economic dispatch model is developed for IPHS with network transmission losses, where both power and heat transmission losses are considered with good precision together, and network transmission constraints are extra considered for the practical application significance. In addition, supply–demand equality constraints and output inequality constraints are routinely considered.
- By constructing the systemic Lagrangian function, the coordination equation is derived from the formulated nonlinear, multi-constrained coupling optimization problem, where the coordination relationship of units' outputs is clearly analyzed, and the optimal solutions are illustrated in an analytic way considering output inequality constraints.
- A double-λ-iteration algorithm is proposed to effectively solve this innovative EDP, which can not only decrease computation burden but also protect the privacy of power-heat subsystems to a large extent. More importantly, it can provide optimal analytic solutions with faster convergence rate than heuristic optimization algorithms.
- The total cost of power and heat generation is minimized while ensuring the supply–demand balance, and all of the units' outputs are optimized to relieve the transmission line and pipeline congestions. Moreover, simulations performed on five case studies illustrate the satisfying performance of the presented approach.

The rest of this paper is organized as follows: Section 2 introduces some basics on IPHS. Section 3 develops a novel economic dispatch model with network transmission losses. Section 4 analyzes the coordination relationship of units' outputs. Section 5 proposes a double-λ-iteration algorithm. Section 6 shows the simulation results of the presented method, and conclusions and perspectives for future works are given in Section 7.

2. Integrated Power and Heating Systems

Traditional power systems and heating systems simply optimize for the single energy form of electric energy or heat energy, which cannot achieve complementary advantages and collaborative benefits between energy resources. Relying on advanced communication and control technologies, IPHS can realize mutual coordinations among electric energy, heat energy, energy storage and load through the optimization dispatch, and it can also construct a cost-effective, eco-friendly and flexible way with the integration of production, supply and consumption.

Generalized IPHS involve the production, transmission, distribution, consumption and other aspects of electric energy and heat energy, so it is very complex to carry out research on its whole. At present, scholars mainly start from narrow IPHS. By developing a novel economic dispatch model with network transmission losses, this paper further reduces the imbalance between supply-side and demand-side caused by transmission losses, while meeting users' energy demands and minimizing the enterprises' production cost. The structure diagram of the IPHS studied in this paper is shown in Figure 1, which can be mainly used in industrial parks, intelligent factories, intelligent buildings, etc.

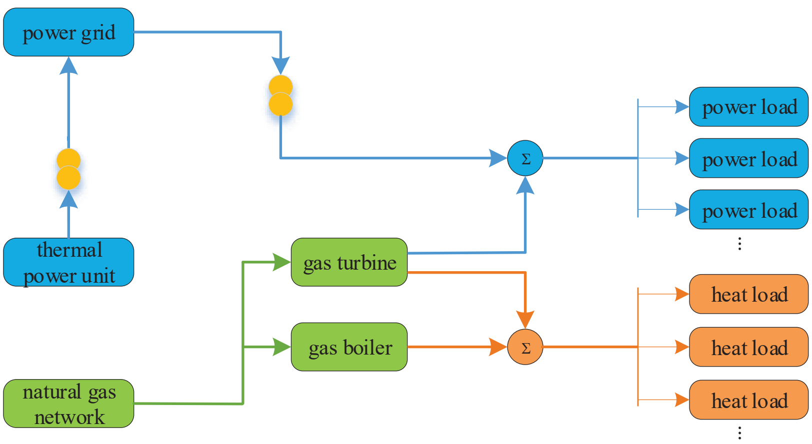


Figure 1. The structure diagram of the IPHS studied in this paper.

Notice that there is no consideration of energy storage units in Figure 1; the reasons can be attributed to two aspects. First, energy storage units are mainly used to stabilize the fluctuations of intermittent renewable energy resources, which should be considered as a whole rather than as a separate part if necessary. Second, for the EDP of IPHS studied in this paper, it can be classified into the single-period economic dispatch field such as [12,22,23,25], which aims at guiding the supply-side to develop an optimal generation scheme and dispatching different units to arrange a reasonable output plan. However, for ensuring the balance constraint between initial state of charge (SOC) and final SOC in an integral dispatch cycle, the single-period economic dispatch is not applicable.

Remark 1. *For modeling electric energy storage, the nonlinear constraints illustrated in [27] can be linearized to obtain the linear constraints with good precision. In addition, heat energy storage can be modeled in similar ways [28,29]. More importantly, most commercial battery systems can be simplified and assumed to be linearized* *models in application, so these linearized ways are effective to a large extent. To be noted, all of this literature discussed above are based on the multi-period economic dispatch field.*

3. Economic Dispatch Model

In this section, the problem formulation of the EDP is presented, which aims at minimizing the total system operation cost for power and heat generation of multiple units to provide the desired amount of power and heat within the units' capabilities.

The objective function can be given by

$$\min\ F_T = F_P + F_C + F_H, \tag{1}$$

where F_T is the total system operation cost, F_P is the total operation cost for power-only units, F_C is the total operation cost for CHP units, and F_H is the total operation cost for heat-only units.

The operation cost for power-only units is usually approximated by a quadratic function as [12]

$$F_P = \sum_{i=1}^{N_p} f_i(P_i) = \sum_{i=1}^{N_p} (\alpha_i + \beta_i P_i + \gamma_i P_i^2), \tag{2}$$

where $f_i(P_i)$ and P_i are the operation cost function and output power associated with the ith power-only unit, respectively, α_i, β_i and $\gamma_i > 0$ are the the operation cost parameters.

The operation cost for CHP units is usually approximated by a quadratic function as [18]

$$F_C = \sum_{j=1}^{N_c} f_j(O_j, H_j) = \sum_{j=1}^{N_c} (\alpha_j + \beta_j O_j + \gamma_j O_j^2 + \delta_j H_j + \theta_j H_j^2 + \varepsilon_j O_j H_j), \tag{3}$$

where $f_j(O_j, H_j)$, O_j and H_j are the operation cost function, output power and output heat associated with the jth CHP unit, respectively, α_j, β_j, $\gamma_j > 0$, δ_j, $\theta_j > 0$ and ε_j are the operation cost parameters.

The operation cost for heat-only units is usually approximated by a quadratic function as [21]

$$F_H = \sum_{k=1}^{N_h} f_k(T_k) = \sum_{k=1}^{N_h} (\alpha_k + \beta_k T_k + \gamma_k T_k^2), \tag{4}$$

where $f_k(T_k)$ and T_k are the operation cost function and output heat associated with the kth heat-only unit, respectively; α_k, β_k and $\gamma_k > 0$ are the operation cost parameters.

The EDP is subject to several operational constraints. Firstly, the power supply–demand equality constraint is given by

$$\Delta P = \sum_{i=1}^{N_p} P_i + \sum_{j=1}^{N_c} O_j - P_D - P_L = 0, \tag{5}$$

where ΔP is the system power mismatch, P_D is the system power load demand, P_L is the power transmission losses that can be expressed by [11]

$$P_L = \sum_{i=1}^{N_p} \sum_{m=1}^{N_p} P_i B_{im} P_m + 2 \sum_{i=1}^{N_p} \sum_{j=1}^{N_c} P_i B_{ij} O_j + \sum_{j=1}^{N_c} \sum_{n=1}^{N_c} O_j B_{jn} O_n, \tag{6}$$

where B_{ij} is the entry of the losses coefficient matrix $\boldsymbol{B}$ on the ith row and the jth column. $B_{ij} = B_{ji}$ can be calculated according to the transmission line parameters and the average daily operating status of the power system [24].

To be noted, the EDP of power systems considering power transmission losses can be regarded as a basic topic in this field [11]. Differing from this topic, this paper extends the issue to IPHS including both power and heat transmission losses. In addition, we use $\boldsymbol{B}$ matrix losses formula, for it can give a sufficiently accurate estimation of the total power transmission losses in the offline mode with a small amount of computation.

Secondly, the heat supply–demand equality constraint is given by

$$\Delta H = \sum_{k=1}^{N_h} T_k + \sum_{j=1}^{N_c} H_j - H_D - H_L = 0, \tag{7}$$

where ΔH is the system heat mismatch, H_D is the system heat load demand, and H_L is the heat transmission losses that can be expressed by [30]

$$H_L = \sum_{g=1}^{n} 2\pi \frac{t_{sw,f} - t_{av,g}}{R_h} l_g, \tag{8}$$

where n is total segments of the heat medium flowing through the pipeline, l_g is the length of the heat medium flowing through each segment of the pipeline, $t_{sw,f}$ is the supply–water temperature in the heating network node f, $t_{av,g}$ is the mean temperature of the medium around the heating network pipeline g, and R_h is the total thermal resistance of pipeline per kilometer from the heat medium to the surrounding medium.

Remark 2. *The power transmission losses P_L considered in Formula (6) is nonlinear with the output power, which will cause the equality constraint (5) to not be a simple linear equality constraint, and the output power and power transmission losses cannot be obtained simultaneously, so how to handle this nonlinear constraint is one of our major challenges. Although the heat transmission losses H_L considered in Formula (8) is linear with the supply–return–water temperature difference, the supply–water temperature and the mass flow are variable and coupled in Formula (16), so that the equality constraint (7) is not also a simple linear equality constraint, so how to handle this nonlinear constraint is our another major challenge.*

Then, the output capacity constraint of power-only units is given by

$$P_i^{min} \le P_i \le P_i^{max}, \tag{9}$$

where P_i^{min} and P_i^{max} are the lower bound and upper bound of the output power associated with the ith power-only unit.

In addition, the output capacity constraint of heat-only units is given by

$$T_k^{min} \le T_k \le T_k^{max}, \tag{10}$$

where T_k^{min} and T_k^{max} are the lower bound and upper bound of the output heat associated with the kth heat-only unit.

Moreover, the heat-power feasible operation region of CHP units is given by [21]

$$\begin{cases} O_j^{min}(H_j) \le O_j \le O_j^{max}(H_j), \\ H_j^{min}(O_j) \le H_j \le H_j^{max}(O_j), \end{cases} \tag{11}$$

where $O_j^{min}(H_j)$, $O_j^{max}(H_j)$, $H_j^{min}(O_j)$ and $H_j^{max}(O_j)$ constitute the linear inequalities that define the feasible operation region of the jth CHP unit. The linear inequalities can be expressed by

$$b_{mj} O_j + c_{mj} H_j \ge d_{mj}, \ m = 1, 2, 3, \tag{12}$$

where b_{mj}, c_{mj} and d_{mj} are the coefficients of the linear inequalities associated with the jth CHP unit, and the heat-power feasible operation region of CHP units is depicted in Figure 2.

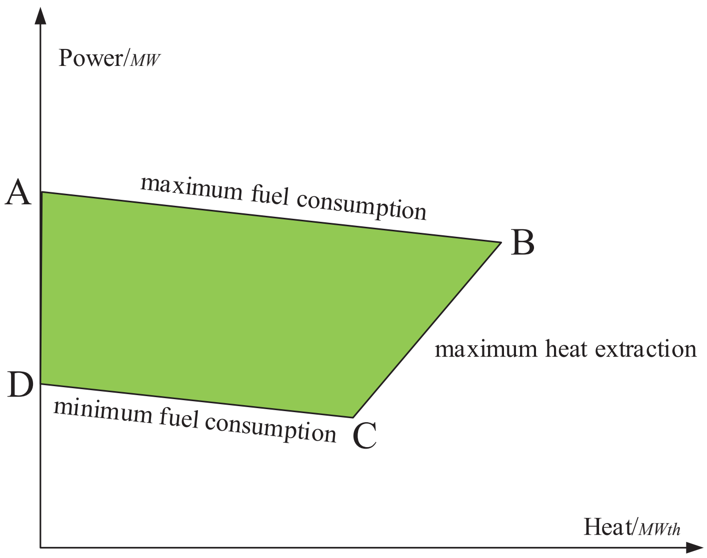


Figure 2. The heat-power feasible operation region of CHP units.

In addition, the transmission capacity constraint of power network lines is given by [13]

$$P_{l,e}^{min} \leq P_{l,e} \leq P_{l,e}^{max}, \tag{13}$$

where $P_{l,e}$ is the transmission power of the power network line e, and $P_{l,e}^{min}$ and $P_{l,e}^{max}$ are the lower bound and upper bound of the transmission power associated with the power network line e.

Finally, the transmission capacity constraints of heating network pipelines are given by [7]

$$t_{sw,f}^{min} \leq t_{sw,f} \leq t_{sw,f}^{max}, \tag{14}$$

where $t_{sw,f}^{min}$ and $t_{sw,f}^{max}$ are the lower bound and upper bound of the supply–water temperature in the heating network node f:

$$m_g^{min} \leq m_g \leq m_g^{max}, \tag{15}$$

where m_g is the mass flow in the heating network transmission pipeline g, m_g^{min} and m_g^{max} are the lower bound and upper bound of the mass flow associated with the heating network transmission pipeline g.

$$q_f = cm_g(t_{sw,f} - t_{rw,f}), \tag{16}$$

where q_f is the transmission heat in the heating network node f, $t_{rw,f}$ is the return–water temperature in the heating network node f, and c is the specific heat capacity of the heat medium.

Remark 3. *Based on the optimization dispatch models in [21,22], this paper develops both power and heat transmission losses models at the same time. In addition, network transmission constraints are extra considered for their practical application significance. Compared with the EDP of power systems considering power transmission losses, there are some solving difficulties in the studied EDP of IPHS. For one thing there exist power-heat couplings both in the objective function and in the constraint conditions, which will cause output power and output heat to be mutually influential when solving the optimal solutions. For another, there exist more nonlinear constraints than the EDP of power systems considering power transmission losses, which will make the EDP studied in this paper more complex to solve. As a result of these, this formulated EDP in this paper is evidently different from the EDP of power systems and it cannot be directly solved.*

4. Output Coordination Relationship

In this section, the coordination relationship of units' outputs is analyzed, which will motivate us to propose the double-λ-iteration algorithm for solving this EDP effectively in the next section.

4.1. Analytic Solutions without Transmission Losses and Inequality Constraints

A core concept of analytic solutions for economic dispatch is the incremental cost (IC), which is the difference in costs as a result of adding/subtracting one unit of power or heat. Mathematically speaking, IC is the derivative of the operation cost function with respect to the output power or output heat. When transmission losses and inequality constraints are neglected, the Lagrangian function based on the developed economic dispatch model is given by

$$L = F_T - \lambda_p \Delta P - \lambda_h \Delta H, \tag{17}$$

where λ_p and λ_h are Lagrangian multipliers associated with the power supply–demand equality constraint and the heat supply–demand equality constraint, respectively.

Furthermore, we can obtain the first-order Karush–Kuhn–Tucker (KKT) optimality conditions [31], which can be expressed by

$$\begin{cases} \frac{\partial L}{\partial P_i} = \frac{\partial f_i(P_i)}{\partial P_i} - \lambda_p = 0, \\ \frac{\partial L}{\partial O_j} = \frac{\partial f_j(O_j,H_j)}{\partial O_j} - \lambda_p = 0, \\ \frac{\partial L}{\partial H_j} = \frac{\partial f_j(O_j,H_j)}{\partial H_j} - \lambda_h = 0, \\ \frac{\partial L}{\partial T_k} = \frac{\partial f_k(T_k)}{\partial T_k} - \lambda_h = 0, \\ \frac{\partial L}{\partial \lambda_p} = \sum_{i=1}^{N_p} P_i + \sum_{j=1}^{N_c} O_j - P_D = 0, \\ \frac{\partial L}{\partial \lambda_h} = \sum_{k=1}^{N_h} T_k + \sum_{j=1}^{N_c} H_j - H_D = 0. \end{cases} \tag{18}$$

Based on the coordination Equation (18), we can obtain

$$\begin{cases} \frac{\partial f_i(P_i)}{\partial P_i} = \frac{\partial f_j(O_j,H_j)}{\partial O_j} = \lambda_p, \\ \frac{\partial f_k(T_k)}{\partial T_k} = \frac{\partial f_j(O_j,H_j)}{\partial H_j} = \lambda_h. \end{cases} \tag{19}$$

Therefore, it can be known that the necessary conditions for the existence of a minimum-cost operating point are that all ICs of the output power for power-only units and CHP units must be equal to λ_p. Meanwhile, all ICs of the output heat for heat-only units and CHP units must be equal to λ_h.

Furthermore, the optimal Lagrangian multipliers denoted by λ_p^* and λ_h^* can be expressed by

$$\lambda_p^* = \frac{P_D + \sum_{i=1}^{N_p} \frac{\beta_i}{2\gamma_i} + \sum_{j=1}^{N_c} \frac{\beta_j + \varepsilon_j H_j}{2\gamma_j}}{\sum_{i=1}^{N_p} \frac{1}{2\gamma_i} + \sum_{j=1}^{N_c} \frac{1}{2\gamma_j}}, \tag{20}$$

$$\lambda_h^* = \frac{H_D + \sum_{k=1}^{N_h} \frac{\beta_k}{2\gamma_k} + \sum_{j=1}^{N_c} \frac{\delta_j + \varepsilon_j O_j}{2\theta_j}}{\sum_{k=1}^{N_h} \frac{1}{2\gamma_k} + \sum_{j=1}^{N_c} \frac{1}{2\theta_j}}. \tag{21}$$

Consequently, the optimal outputs denoted by P_i^*, O_j^*, H_j^* and T_k^* can be calculated by the coordination equation.

4.2. Analytic Solutions with Transmission Losses and Inequality Constraints

When transmission losses and inequality constraints (9)–(11) are considered, the necessary conditions for the existence of a minimum-cost operating point may be expanded slightly as

$$\begin{cases} (\beta_i + 2\gamma_i P_i) pf_{p_i} \geq \lambda_p,\ P_i = P_i^{min}, \\ (\beta_i + 2\gamma_i P_i) pf_{p_i} = \lambda_p,\ P_i^{min} < P_i < P_i^{max}, \\ (\beta_i + 2\gamma_i P_i) pf_{p_i} \leq \lambda_p,\ P_i = P_i^{max}, \end{cases} \tag{22}$$

$$\begin{cases} (\beta_j + 2\gamma_j O_j + \varepsilon_j H_j) pf_{p_j} \geq \lambda_p,\ O_j = O_j^{min}(H_j), \\ (\beta_j + 2\gamma_j O_j + \varepsilon_j H_j) pf_{p_j} = \lambda_p,\ O_j^{min}(H_j) < O_j < O_j^{max}(H_j), \\ (\beta_j + 2\gamma_j O_j + \varepsilon_j H_j) pf_{p_j} \leq \lambda_p,\ O_j = O_j^{max}(H_j), \end{cases} \tag{23}$$

$$\begin{cases} (\delta_j + 2\theta_j H_j + \varepsilon_j O_j) pf_{h_j} \geq \lambda_h,\ H_j = H_j^{min}(O_j), \\ (\delta_j + 2\theta_j H_j + \varepsilon_j O_j) pf_{h_j} = \lambda_h,\ H_j^{min}(O_j) < H_j < H_j^{max}(O_j), \\ (\delta_j + 2\theta_j H_j + \varepsilon_j O_j) pf_{h_j} \leq \lambda_h,\ H_j = H_j^{max}(O_j), \end{cases} \tag{24}$$

$$\begin{cases} (\beta_k + 2\gamma_k T_k) pf_{h_k} \geq \lambda_h,\ T_k = T_k^{min}, \\ (\beta_k + 2\gamma_k T_k) pf_{h_k} = \lambda_h,\ T_k^{min} < T_k < T_k^{max}, \\ (\beta_k + 2\gamma_k T_k) pf_{h_k} \leq \lambda_h,\ T_k = T_k^{max}, \end{cases} \tag{25}$$

where pf_{p_i} and pf_{p_j} are penalty factors of the power transmission losses associated with the ith power-only unit and the jth CHP unit, respectively, which can be expressed by

$$pf_{p_i} = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} = \frac{1}{1 - 2(\sum\limits_{m=1}^{N_p} B_{im} P_m + \sum\limits_{j=1}^{N_c} B_{ij} O_j)}, \tag{26}$$

$$pf_{p_j} = \frac{1}{1 - \frac{\partial P_L}{\partial O_j}} = \frac{1}{1 - 2(\sum\limits_{n=1}^{N_c} B_{jn} O_n + \sum\limits_{i=1}^{N_p} B_{ij} P_i)}, \tag{27}$$

where pf_{h_j} and pf_{h_k} are penalty factors of the heat transmission losses associated with the jth CHP unit and kth heat-only unit, respectively, which can be expressed by

$$pf_{h_j} = \frac{1}{1 - \frac{\partial H_L}{\partial H_j}} = \frac{1}{1 - \frac{2\pi l_g}{c m_g R_h}}, \tag{28}$$

$$pf_{h_k} = \frac{1}{1 - \frac{\partial H_L}{\partial T_k}} = \frac{1}{1 - \frac{2\pi l_g}{c m_g R_h}}. \tag{29}$$

Let Ω_p denote the set of power-only units for which the optimal $P_i = P_i^{min}$ or $P_i = P_i^{max}$. The optimality condition (22) can be rewritten as

$$\lambda_p = (\beta_i + 2\gamma_i P_i) pf_{p_i},\ \forall i \notin \Omega_p. \tag{30}$$

Let Ω_{co} denote the set of CHP units for which the optimal $O_j = O_j^{min}(H_j)$ or $O_j = O_j^{max}(H_j)$. The optimality condition (23) can be rewritten as

$$\lambda_p = (\beta_j + 2\gamma_j O_j + \varepsilon_j H_j) pf_{p_j},\ \forall j \notin \Omega_{co}. \tag{31}$$

Let Ω_{ch} denote the set of CHP units for which the optimal $H_j = H_j^{min}(O_j)$ or $H_j = H_j^{max}(O_j)$. The optimality condition (24) can be rewritten as

$$\lambda_h = (\delta_j + 2\theta_j H_j + \varepsilon_j O_j) p f_{h_j}, \ \forall j \notin \Omega_{ch}. \tag{32}$$

Let Ω_h denote the set of heat-only units for which the optimal $T_k = T_k^{min}$ or $T_k = T_k^{max}$. The optimality condition (25) can be rewritten as

$$\lambda_h = (\beta_k + 2\gamma_k T_k) p f_{h_k}, \ \forall k \notin \Omega_h. \tag{33}$$

Furthermore, we can obtain the optimal Lagrangian multipliers with transmission losses and inequality constraints, which can be expressed by

$$\lambda_p^* = \frac{P_D + P_L - \sum\limits_{i \in \Omega_p} P_i - \sum\limits_{j \in \Omega_{co}} O_j + \sum\limits_{i \notin \Omega_p} \frac{\beta_i}{2\gamma_i} + \sum\limits_{j \notin \Omega_{co}} \frac{\beta_j + \varepsilon_j H_j}{2\gamma_j}}{\sum\limits_{i \notin \Omega_p} \frac{1}{2\gamma_i p f_{p_i}} + \sum\limits_{j \notin \Omega_{co}} \frac{1}{2\gamma_j p f_{p_j}}}, \tag{34}$$

$$\lambda_h^* = \frac{H_D + H_L - \sum\limits_{k \in \Omega_h} T_k - \sum\limits_{j \in \Omega_{ch}} H_j + \sum\limits_{k \notin \Omega_h} \frac{\beta_k}{2\gamma_k} + \sum\limits_{j \notin \Omega_{ch}} \frac{\delta_j + \varepsilon_j O_j}{2\theta_j}}{\sum\limits_{k \notin \Omega_h} \frac{1}{2\gamma_k p f_{h_k}} + \sum\limits_{j \notin \Omega_{ch}} \frac{1}{2\theta_j p f_{h_j}}}. \tag{35}$$

Therefore, we can further calculate the optimal outputs considering transmission losses and inequality constraints by the constrained optimality conditions (30)–(33), respectively.

Based on the above analysis, the optimal Lagrangian multipliers expressed by (34) and (35) cannot be obtained directly—the reason is that there are many unknown variables to calculate previously, such as network transmission losses, penalty factors and so on, so that the optimal solutions of this EDP cannot be solved directly. Thus, the following method is designed to deal with the intractable EDP in the next section.

5. Double-λ-Iteration Algorithm

In this section, a double-λ-iteration algorithm is presented, which can be divided into λ_p-iteration of the power subsystem and λ_h-iteration of the heating subsystem according to the output coordination relationship and system variable types.

Initialization: Assume the iteration performed at discrete time instants is denoted by s. $P_i(0)$, $O_j(0)$, $H_j(0)$, $T_k(0)$, $\lambda_p(0)$, $\lambda_h(0)$ and $t_{sw,f}(0)$ can be set any fixed admissible value. Network transmission constraints could be handled by using the following ways. Firstly, the transmission capacity constraint of power network lines (13) is considered as follows:

$$\begin{cases} P_i'(s) = P_i(s), \ P_{l,e}^{min} \leq P_i(s) < P_{l,e}^{max}, \\ P_i'(s) = P_i^{max} = P_{l,e}^{max}, \ P_i(s) \geq P_{l,e}^{max}, \end{cases} \tag{36}$$

$$\begin{cases} O_j'(s) = O_j(s), \ P_{l,e}^{min} \leq O_j(s) < P_{l,e}^{max}, \\ O_j'(s) = O_j^{max}(H_j) = P_{l,e}^{max}, \ O_j(s) \geq P_{l,e}^{max}. \end{cases} \tag{37}$$

Then, the constraint of supply–water temperatures (14) is considered as follows:

$$\begin{cases} t_{sw,f}'(s) = t_{sw,f}^{min}, \ t_{sw,f}(s) \leq t_{sw,f}^{min}, \\ t_{sw,f}'(s) = t_{sw,f}(s), \ t_{sw,f}^{min} < t_{sw,f}(s) < t_{sw,f}^{max}, \\ t_{sw,f}'(s) = t_{sw,f}^{max}, \ t_{sw,f}(s) \geq t_{sw,f}^{max}. \end{cases} \tag{38}$$

In addition, the mass flow is calculated and its constraint (15) is considered as follows:

$$m_g(s) = \frac{q_f(s)}{c(t'_{sw,f}(s) - t_{rw,f}(s))}, \tag{39}$$

$$\begin{cases} m'_g(s) = m_g^{min},\ m_g(s) \le m_g^{min}, \\ m'_g(s) = m_g(s),\ m_g^{min} < m_g(s) < m_g^{max}, \\ m'_g(s) = m_g^{max},\ m_g(s) \ge m_g^{max}. \end{cases} \tag{40}$$

Furthermore, the transmission heat can be updated as follows:

$$\begin{cases} q'_f(s) = cm'_g(s)(t'_{sw,f}(s) - t_{rw,f}(s)), \\ T'_k(s) = q'_f(s),\ T_k(s) \ge q'_f(s), \\ H'_j(s) = q'_f(s),\ H_j(s) \ge q'_f(s). \end{cases} \tag{41}$$

It can be noted that the supply–water temperature and the mass flow are optimized in the process of scheduling, which have to meet transmission constraints of the heating network. In other words, the supply–water temperature and the mass flow will be updated and determined whether they meet above transmission constraints in each iteration.

Main algorithm: $P_L(s)$, $H_L(s)$, $pf_{p_i}(s)$, $pf_{p_j}(s)$, $pf_{h_j}(s)$, $pf_{h_k}(s)$ can be calculated by using their own formulas. Moreover, system Lagrangian multipliers can be updated by using (34) and (35), respectively, so that units' outputs and supply–water temperatures can be updated as follows:

$$P_i(s+1) = \begin{cases} \frac{1}{2pf_{p_i}(s)\gamma_i}\lambda_p(s+1) - \frac{\beta_i}{2\gamma_i}, & \forall i \notin \Omega_p, \\ P_i^{min}\ or\ P_i^{max}, & \forall i \in \Omega_p, \end{cases} \tag{42}$$

$$O_j(s+1) = \begin{cases} \frac{1}{2pf_{p_j}(s)\gamma_j}\lambda_p(s+1) - \frac{\varepsilon_j}{2\gamma_j}H'_j(s) - \frac{\beta_j}{2\gamma_j}, & \forall j \notin \Omega_{co}, \\ O_j^{min}(H'_j(s))\ or\ O_j^{max}(H'_j(s)), & \forall j \in \Omega_{co}, \end{cases} \tag{43}$$

$$H_j(s+1) = \begin{cases} \frac{1}{2pf_{h_j}(s)\theta_j}\lambda_h(s+1) - \frac{\varepsilon_j}{2\theta_j}O_j(s+1) - \frac{\delta_j}{2\theta_j}, & \forall j \notin \Omega_{ch}, \\ H_j^{min}(O_j(s+1))\ or\ H_j^{max}(O_j(s+1)), & \forall j \in \Omega_{ch}, \end{cases} \tag{44}$$

$$T_k(s+1) = \begin{cases} \frac{1}{2pf_{h_k}(s)\gamma_k}\lambda_h(s+1) - \frac{\beta_k}{2\gamma_k}, & \forall k \notin \Omega_h, \\ T_k^{min}\ or\ T_k^{max}, & \forall k \in \Omega_h, \end{cases} \tag{45}$$

$$t_{sw,f}(s+1) = \frac{1}{cm_g(s)}q_f(s+1) + t_{rw,f}(s). \tag{46}$$

Convergence: The system power mismatch and system heat mismatch are updated by using (5) and (7) respectively, then the convergence condition can be given by

$$\xi = \max \begin{cases} \mid \Delta P(s+1) \mid \\ \mid \Delta H(s+1) \mid \end{cases} \le \mu, \tag{47}$$

where ξ is the maximum absolute value between the system power mismatch and the system heat mismatch, and $\mu > 0$ is the convergence factor that can be regarded as an extremely small positive constant.

In summary, the EDP of IPHS is solved by the proposed double-λ-iteration algorithm, where the original optimization problem can be divided into λ_p-iteration of the power subsystem and λ_h-iteration of the heating subsystem. Thereinto, CHP units working as the bond between subsystems can implement double-λ-iteration to achieve bidirectional information alternations and coordinated resources' allocations. Through reduplicative iterations, the optimal solutions can not be obtained until satisfying the convergence condition.

Remark 4. *By adopting the double-λ-iteration algorithm, the power subsystem has no use for providing privacy information such as operation cost parameters, etc. to the heating subsystem and vice versa, so that the computation burden can be decreased and the privacy of power-heat subsystems can be protected to a large extent, which mean a lot more importance to generation enterprises in the practical power industries. In addition, the presented method can be regarded as an analytic optimization algorithm, which can provide clear analytic solutions with faster convergence rate than heuristic optimization algorithms.*

6. Simulation Results

In this section, the proposed double-λ-iteration algorithm is applied to this EDP on the 10-unit IPHS as shown in Figure 3, where power-only units $G_{p1} - G_{p4}$ correspond to nodes 1–4, CHP units $G_{c1} - G_{c2}$ correspond to nodes 5–6, heat-only units $G_{h1} - G_{h2}$ correspond to nodes 7–8, and power load unit and heat load unit correspond to nodes 9–10, respectively. In addition, the blue solid lines correspond to power network lines, and the red solid lines correspond to heating network pipelines.

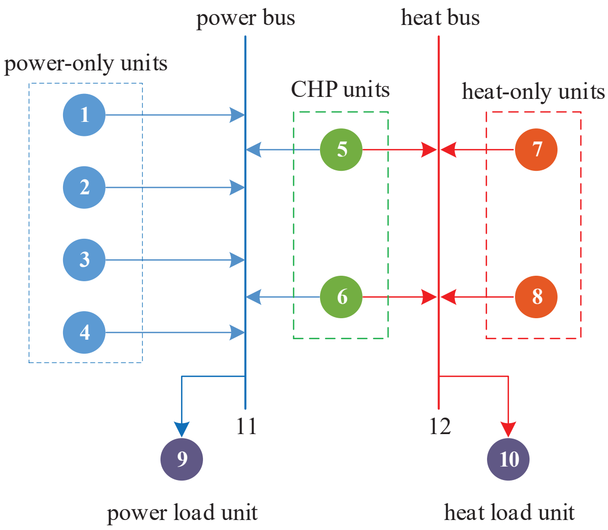


Figure 3. The structure schematic diagram of the 10-unit IPHS.

Based on the basic parameters given in Appendix A, simulations performed on five case studies demonstrate the effectiveness of the proposed algorithm with output inequality constraints, network transmission constraints, load fluctuation demand and unit commitment capability. The simulation results are shown in Tables 1 and 2.

Table 1. The optimal outputs of different units.

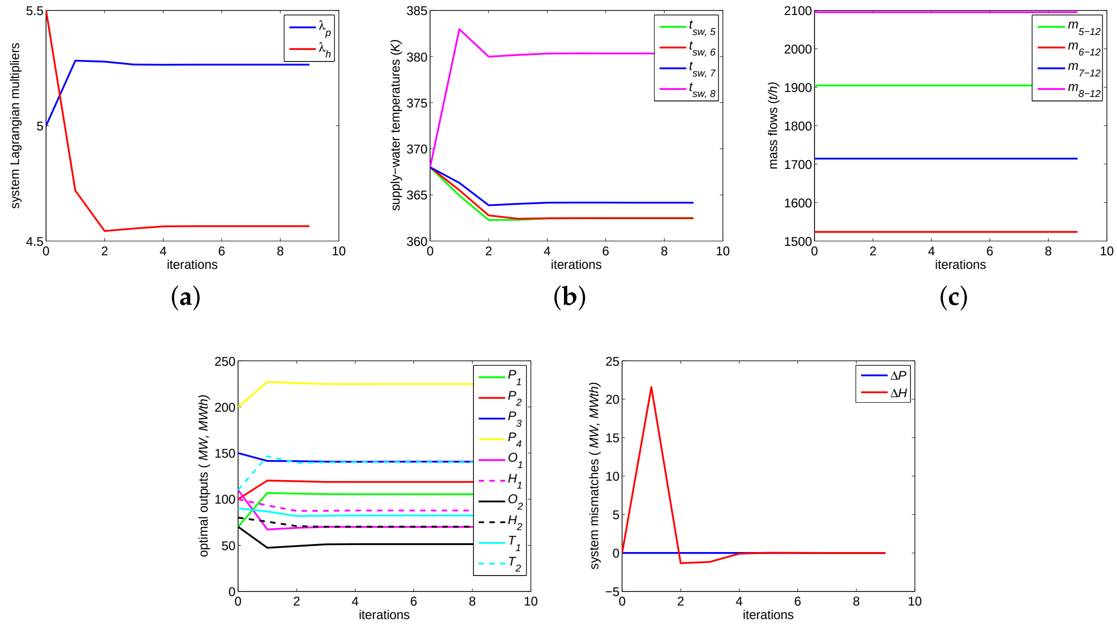
Case	P_1	P_2	P_3	P_4	O_1	H_1	O_2	H_2	T_1	T_2
1	105.3540	118.6603	140.5492	224.7903	69.7815	87.6679	51.2016	70.1857	82.3175	140.1510
2	100.0000	119.9328	141.7102	226.5014	70.4617	87.6043	51.7260	70.0128	82.4121	140.2929
3	100.0000	122.2493	143.7622	220.0000	71.6620	87.4872	52.6314	69.7137	82.5750	140.5400
4	100.0000	134.3102	154.6392	220.0000	76.7202	91.9576	55.5212	72.5051	87.6725	148.1873
5	100.0000	122.2495	143.7623	220.0000	71.6617	87.4884	52.6315	69.7136	82.5766	140.5424

Table 2. The optimal Lagrangian multipliers and minimum total system operation cost.

Case	λ_p^*	λ_h^*	$F_T^*[\times 10^3\$]$
1	5.2648	4.5640	7.1477
2	5.2865	4.5674	7.1480
3	5.3252	4.5733	7.1484
4	5.5344	4.7568	7.4046
5	5.3252	4.5734	7.1484

6.1. Case Study 1: Without Output Inequality Constraints

In this case study, output inequality constraints are not considered. The initial power load demand and heat load demand are 700 MW and 380 MWth, initial system Lagrangian multipliers are $\lambda_P(0) = 5.0$ and $\lambda_h(0) = 5.5$, and the initial supply–water temperature can be set $t_{sw,f}(0) = 368$ K, respectively. After a few iterations, system Lagrangian multipliers and units' outputs tend to be stable gradually. In addition, the network transmission losses reach P_L = 10.3370 MW and H_L = 0.3225 MWth, and two supply–demand equality constraints are satisfied including transmission losses finally. The simulation waveforms are depicted in Figure 4, and it can be noted that the power-only unit G_{p1} is not fulfilling the constraint on its maximum output power P_1^{max} = 100 MW.



(**a**) (**b**) (**c**) (**d**) (**e**)

Figure 4. Case study 1: Without output inequality constraints. (**a**) system Lagrangian multipliers; (**b**) supply-water temperatures; (**c**) mass flows; (**d**) optimal outputs; (**e**) system mismatches.

6.2. Case Study 2: With Output Inequality Constraints

This case study is based on IPHS of the case study 1 considering output inequality constraints. The maximum output power constraint $P_1^{max} = 100$ MW is forced on the G_{p1} to better visualize the behavior of the presented method in Figure 5. In this case study, the G_{p1} is not exceeding its maximum output power and the other power-only units and CHP units have to increase their output power with respect to the previous case to supply for the saturation of the G_{p1}. The network transmission losses reach $P_L = 10.3321$ MW and $H_L = 0.3225$ MWth, and the system converges to new optimal solutions in a few iterations including output inequality constraints.

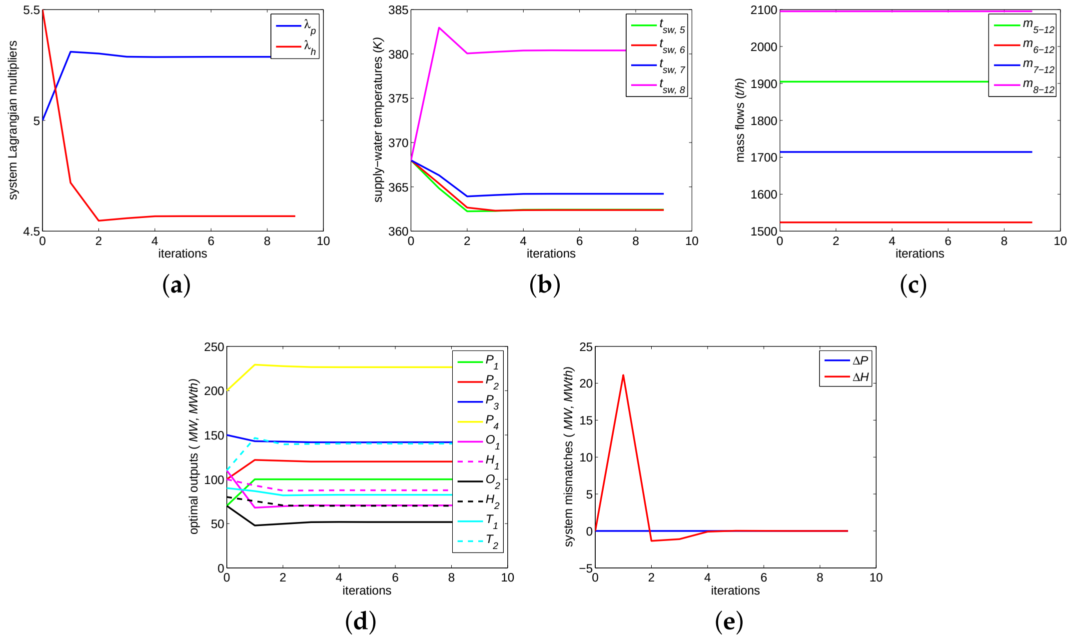


Figure 5. Case study 2: With output inequality constraints. (**a**) system Lagrangian multipliers; (**b**) supply-water temperatures; (**c**) mass flows; (**d**) optimal outputs; (**e**) system mismatches.

6.3. Case Study 3: With Network Transmission Constraints

This case study is based on IPHS of the case study 2 considering network transmission constraints. The bound constraint of the supply–water temperature is better visualized the behavior of the proposed algorithm in Figure 6, and the G_{p4} is not exceeding the maximum transmission capacity of the power network line $P_{l,4-11}^{max} = 220$ MW so that the other power-only units and CHP units have to increase their output power with respect to the case study 2 to supply for the saturation of the G_{p4}. The network transmission losses reach $P_L = 10.3050$ MW and $H_L = 0.3161$ MWth, and the system converges to new optimal solutions in a few iterations including network transmission constraints.

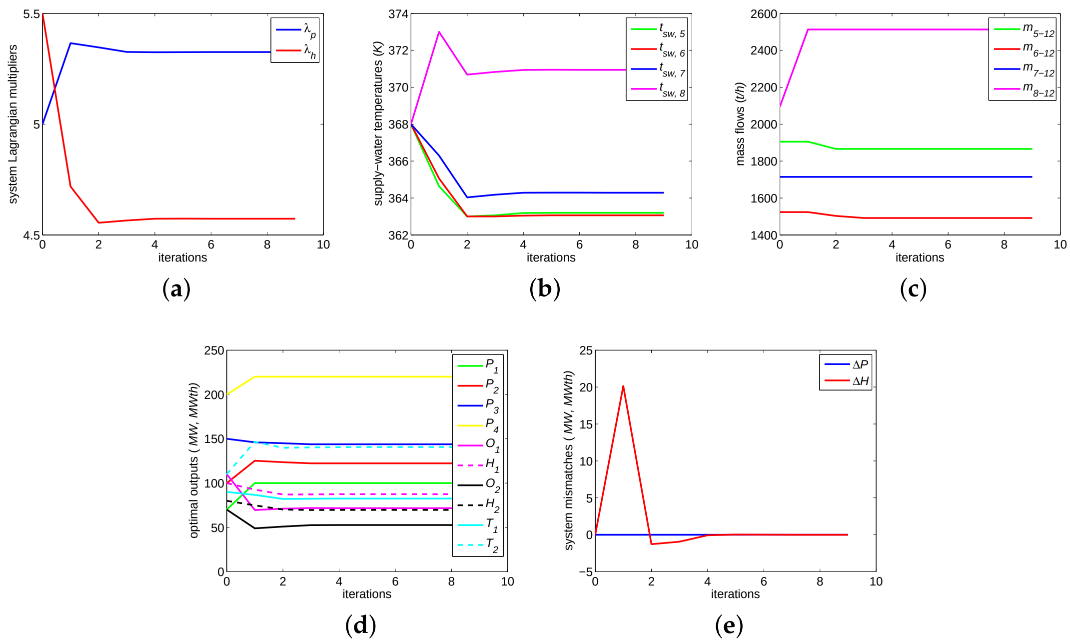


Figure 6. Case study 3: With network transmission constraints. (**a**) system Lagrangian multipliers; (**b**) supply-water temperatures; (**c**) mass flows; (**d**) optimal outputs; (**e**) system mismatches.

6.4. Case Study 4: With Load Fluctuation Demand

This case study is based on IPHS of the case study 3 considering load fluctuation demand. Two load fluctuations are considered: when $s = 9$, the system power load demand and heat load demand are increased by 50 MW and 30 MWth, respectively; then, when $s = 15$, the system power load demand and heat load demand are decreased by 20 MW and 10 MWth, respectively. After a few iterations, two supply–demand equality constraints are satisfied including transmission losses, and the system converges to new optimal solutions finally considering load fluctuation demand. The simulation waveforms are depicted in Figure 7.

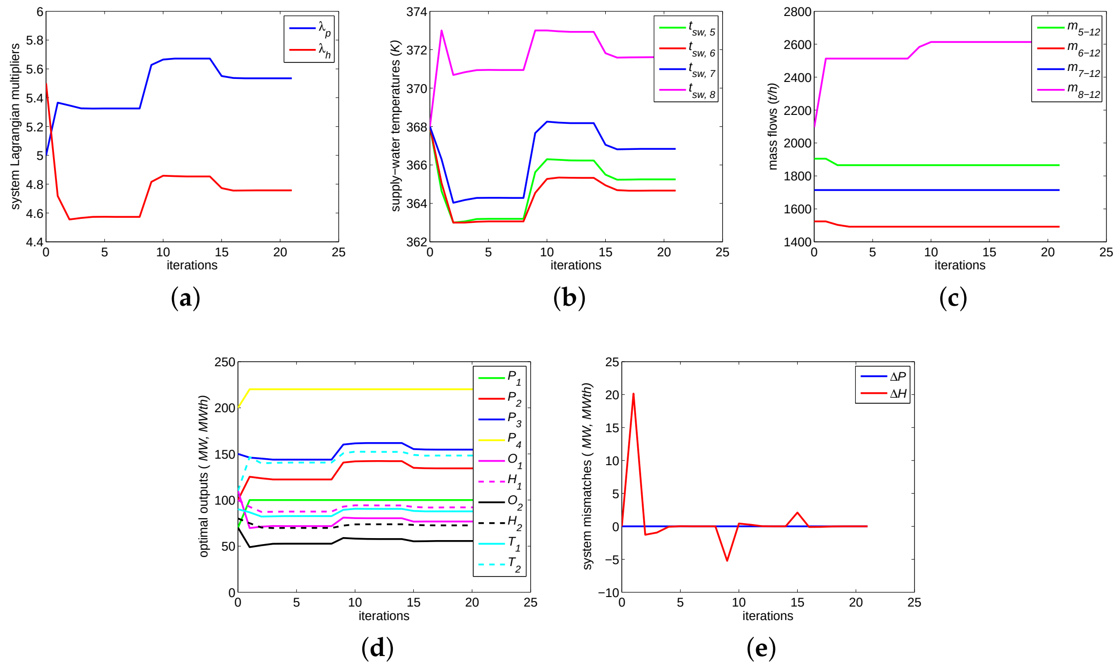


Figure 7. Case study 4: With load fluctuation demand. (**a**) system Lagrangian multipliers; (**b**) supply-water temperatures; (**c**) mass flows; (**d**) optimal outputs; (**e**) system mismatches.

6.5. Case Study 5: With Unit Commitment Capability

This case study is also based on IPHS of the case study 3 considering unit commitment capability. Disconnection and reconnection of the power-only unit G_{p1} are considered during the simulation as shown in Figure 8. When $s = 9$, the G_{p1} is removed from the system, the system detects the disconnection of this unit and perceives the power mismatch and also calculates new solutions under the new condition. Obviously, the remaining units have to provide more output power to compensate for the amount of power previously generated by the disconnected unit; then, when $s = 16$, the G_{p1} is reconnected and the system properly responds to this new condition. The system detects the presence of an additional unit and reaches the same solutions prior to disconnection as shown in Figure 8.

Remark 5. *It should be noted that the simulations performed on five case studies are run in Matlab R2010a, which can only retain results to the fourth decimal place, so it means that there are rounding errors in simulation results. Thus, the convergence factor μ is considered to partly reflect the error margin of simulation results in this paper, which is also the convergence condition of the presented approach.*

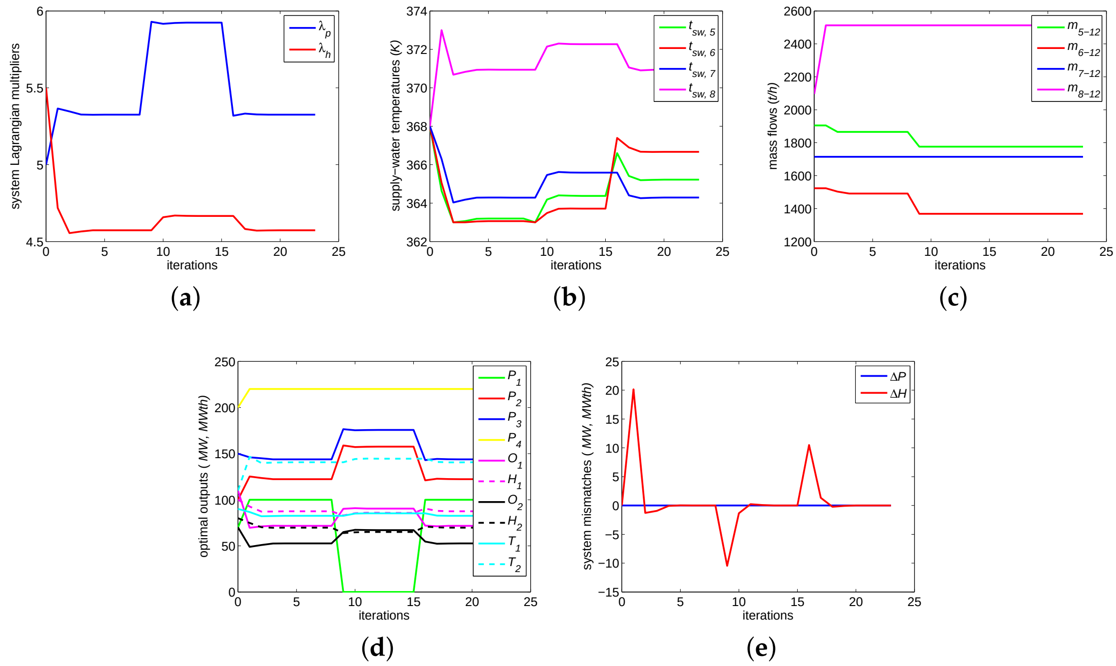


Figure 8. Case study 5: With unit commitment capability. (**a**) system Lagrangian multipliers; (**b**) supply-water temperatures; (**c**) mass flows; (**d**) optimal outputs; (**e**) system mismatches.

7. Conclusions

This paper has proposed a novel economic dispatch model to investigate the problem of the supply–demand imbalance caused by network transmission losses in IPHS, where both power and heat transmission losses models have been established with good precision at the same time. Based on the optimization dispatch model, the coordination relationship of units' outputs has been analyzed, and the optimal solutions have been illustrated in an analytic way. The optimization target has been realized by developing a double-λ-iteration algorithm with faster convergence rate, where all of units' outputs are optimized to relieve the transmission line and pipeline congestions, while ensuring the supply–demand balance including transmission losses. Simulations performed on five case studies have been run in Matlab R2010a (MathWorks, Natick, MA, USA), and the results have shown that the presented method can effectively solve this innovative EDP in fewer iterations than heuristic optimization algorithms—the reason can be attributed to the fact that there provides a better and faster updating direction for the optimal solutions. Furthermore, the proposed approach has provided the satisfying performance under consideration of output inequality constraints, network transmission constraints, load fluctuation demand and unit commitment capability.

It can be noted that only electric energy and heat energy have been considered in this paper; our proposed method is also regarded as a centralized method that has some disadvantages inherent compared with distributed methods. Driven by this content, future works will focus on distributed optimal energy management for integrated energy systems considering power–heat–gas network transmission losses, intermittent renewable energy resources, multiple energy storage units, etc.

Author Contributions: All of the authors have contributed to this work. The original ideas were provided by B.H. and C.Z.; C.Z. performed the case studies and wrote the manuscript with guidance from B.H.; B.H., Q.S. and R.H. reviewed the manuscript and provided valuable suggestions.

Funding: This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant 61573094, and in part by the NSFC under Grant 61603085.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

F_T	total operation cost for IPHS
F_P	total operation cost for power-only units
F_C	total operation cost for CHP units
F_H	total operation cost for heat-only units
N_p	total number for power-only units
N_c	total number for CHP units
N_h	total number for heat-only units
$f_i(P_i)$	operation cost function for the ith power-only unit
$f_j(O_j, H_j)$	operation cost function for the jth CHP unit
$f_k(T_k)$	operation cost function for the kth heat-only unit
α_i, β_i, γ_i	operation cost parameters for the ith power-only unit
α_j, β_j, γ_j, δ_j, θ_j, ε_j	operation cost parameters for the jth CHP unit
α_k, β_k, γ_k	operation cost parameters for the kth heat-only unit
P_i	output power for the ith power-only unit
O_j	output power for the jth CHP unit
H_j	output heat for the jth CHP unit
T_k	output heat for the kth heat-only unit
ΔP	system power mismatch
ΔH	system heat mismatch
P_D	system power load demand
H_D	system heat load demand
P_L	power transmission losses
H_L	heat transmission losses
B_{ij}	element of the losses coefficient matrix $\boldsymbol{B}$
n	total segments of the heat medium flowing through the pipeline
l_g	length of the heat medium flowing through the gth pipeline
R_h	total thermal resistance of pipeline per kilometer from heat medium to surrounding medium
$t_{sw,f}$	supply–water temperature in the fth heating network node
$t_{rw,f}$	return-water temperature in the fth heating network node
$t_{av,g}$	mean temperature of the medium around the gth heating network pipeline
P_i^{min}	lower bound of the output power for the ith power-only unit
P_i^{max}	upper bound of the output power for the ith power-only unit
T_k^{min}	lower bound of the output heat for the kth heat-only unit
T_k^{max}	upper bound of the output heat for the kth heat-only unit
$O_j^{min}(H_j)$	lower bound of the output power for the jth CHP unit
$O_j^{max}(H_j)$	upper bound of the output power for the jth CHP unit
$H_j^{min}(O_j)$	lower bound of the output heat for the jth CHP unit
$H_j^{max}(O_j)$	upper bound of the output heat for the jth CHP unit
$P_{l,e}$	transmission power for eth power network line
$P_{l,e}^{min}$	lower bound of the transmission power for the eth power network line
$P_{l,e}^{max}$	upper bound of the transmission power for the eth power network line
$t_{sw,f}^{min}$	lower bound of the supply–water temperature in the fth heating network node
$t_{sw,f}^{max}$	upper bound of the supply–water temperature in the fth heating network node
m_g	mass flow in the gth heating network pipeline
m_g^{min}	lower bound of the mass flow in the gth heating network pipeline
m_g^{max}	upper bound of the mass flow in the gth heating network pipeline
q_f	transmission heat in the fth heating network node
c	specific heat capacity of the heat medium
L	Lagrangian function
λ_p	Lagrangian multiplier of the output power
λ_h	Lagrangian multiplier of the output heat

pf_{p_i} penalty factor of power transmission losses for the *i*th power-only unit
pf_{p_j} penalty factor of power transmission losses for the *j*th CHP unit
pf_{h_j} penalty factor of heat transmission losses for the *j*th CHP unit
pf_{h_k} penalty factor of heat transmission losses for the *k*th heat-only unit
μ convergence factor

Appendix A

The relevant parameters of IPHS are given in Tables A1–A7.

Table A1. The operation cost function parameters and output limit parameters of power-only units.

Unit	α_i	β_i	γ_i	P_i^{min} [MW]	P_i^{max} [MW]
G_{p1}	25	3.0	0.010	10	100
G_{p2}	40	3.2	0.008	25	170
G_{p3}	75	2.6	0.009	30	200
G_{p4}	100	2.4	0.006	40	300

Table A2. The operation cost function parameters of CHP units.

Unit	α_j	β_j	γ_j	δ_j	θ_j	ε_j
G_{c1}	1250	2.2	0.016	1.2	0.016	0.008
G_{c2}	680	1.2	0.024	0.4	0.022	0.021

Table A3. The operation cost function parameters and output limit parameters of heat-only units.

Unit	α_k	β_k	γ_k	T_k^{min} [MWth]	T_k^{max} [MWth]
G_{h1}	650	1.6	0.018	0	1695
G_{h2}	520	1.2	0.012	0	1250

Table A4. The heat-power feasible operation region parameters of CHP units.

Unit	FOR (H [MWth], O [MW])
G_{c1}	$A_1(0,187),\ B_1(153,132),\ C_1(121,42),\ D_1(0,63)$
G_{c2}	$A_2(0,94),\ B_2(122,68),\ C_2(106,22),\ D_2(0,36)$

Table A5. The capacity limit parameters of power network lines.

Line	$P_{l,e}^{min}$[MW]	$P_{l,e}^{max}$[MW]	*Line*	$P_{l,e}^{min}$[MW]	$P_{l,e}^{max}$[MW]
1–11	0	130	2–11	0	180
3–11	0	210	4–11	0	220
5–11	0	120	6–11	0	100

Table A6. The parameters of heating network nodes and pipelines.

Pipeline	l_g [km]	m_g^{min} [t/h]	m_g^{max} [t/h]	R_h [mK/W]	Node	$t_{sw,f}^{min}$ [K]	$t_{sw,f}^{max}$ [K]
5–12	2.8	0	2700	20	5	363	373
6–12	2.5	0	2700	20	6	363	373
7–12	3.0	0	2700	20	7	363	373
8–12	2.6	0	2700	20	8	363	373

Table A7. The initial output parameters of different units.

Unit	Initial Output	Unit	Initial Output
G_{p1}	$P_1(0) = 70$	G_{c1}	$O_1(0) = 110, H_1(0) = 100$
G_{p2}	$P_2(0) = 100$	G_{c2}	$O_2(0) = 70, H_2(0) = 80$
G_{p3}	$P_3(0) = 150$	G_{h1}	$T_1(0) = 90$
G_{p4}	$P_4(0) = 200$	G_{h2}	$T_2(0) = 110$

The mean temperature of the medium around the heating network pipeline and the return–water temperature in the heating network node can be set $t_{av,g} = 273$ K and $t_{rw,f} = 323$ K respectively, and the specific heat capacity of the heat medium can be given by $c = 4.2$ kJ/(kgK). In addition, the convergence factor can be set $\mu = 0.0001$, and the losses coefficient matrix $\boldsymbol{B}$ can be given by

$$\boldsymbol{B} = \begin{bmatrix} 49 & 14 & 15 & 15 & 20 & 25 \\ 14 & 45 & 16 & 20 & 18 & 19 \\ 15 & 16 & 39 & 10 & 12 & 15 \\ 15 & 20 & 10 & 40 & 14 & 11 \\ 20 & 18 & 12 & 14 & 35 & 17 \\ 25 & 19 & 15 & 11 & 17 & 39 \end{bmatrix} \times 10^{-6}.$$

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