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The Effect of Price-Dependent Demand on the Sustainable Electrical Energy Supply Chain

Ivan Darma Wangsa , Tao Ming Yang and Hui Ming Wee *

Department of Industrial and Systems Engineering, Chung Yuan Christian University, Taoyuan 32023, Taiwan; ivan_darma@yahoo.com (I.D.W.); taoming1129@gmail.com (T.M.Y.)

* Correspondence: weehm@cycu.edu.tw; Tel.: +886-3-2654409

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Abstract: In order to identify the optimal structure of an electricity power network under the main assumption of a price dependent demand of electrical energy, we presented an optimization model that aims at analyzing the effect of price-dependent demand on the sustainable electrical supply chain system (SESCS). The system included a power generation system, transmission and distribution substations, and many customers. The electrical energy was generated and transmitted through multiple substations to our customers, and the demand for electricity by the customers is dependent on the price of electricity. In the study, we considered the transmission and the distribution costs which depend on the capacities of power generation, transmission rates and distances between stations. We utilized the inventory theory to develop our model and proposed a procedure to derive an optimal solution for this problem. Finally, numerical examples and sensitivity analysis are provided to illustrate our study and consolidate managerial insights.

Keywords: sustainable electrical energy supply chain; inventory theory; transmission and distribution costs; price-dependent demand

1. Introduction

In recent years, the consumption of electricity has increased rapidly. Total electricity consumption in the U.S. in 2015 was more than 13 times greater than in 1950 [1]. According to the electricity consumption for residential, commercial, transportation and industrial sectors has been increasing since 2015. At the end of 2018, an upsurge in the residential electricity use is predicted to occur at a rate of 100 million kWh/day, due mainly to the need for air-conditioning in homes due to increasing climate change.

The variations in electricity price can influence the demand for electricity. The prices of electricity are set to reduce the variations in demand. Therefore, it is usually highest in the summer when total demand is high (at peak hours). In general, prices are usually higher for residential and commercial customers because it costs more to distribute electricity to them. The electricity price for industrial use is usually cheaper due to government regulation and the economics of scales [1]. With the projected consumption growth and the variations in the price of electricity, our research is to provide insight to government and practitioner in their energy policy.

Banbury [2] was the first to study the electricity supply chain. A literature survey on electricity market models has been done by [3]. The model to predict electricity consumption has been developed by [4]. The electricity prices and expansion planning of power capacity has been optimized by using a stochastic programming by [5]. Genoese and Genoese [6] developed a model to measure of energy storage by applying the agent-based simulation. Pavković et al. [7] investigated the impact of high-speed wind energy for optimizing electrical energy storage systems. A mathematical model to find the break-even point of increasing electrical power capacity and electricity supply uncertainty has

been presented by [8]. Fossati et al. [9] proposed a genetic algorithm to minimize cost and optimize energy in micro-grid systems. Luo et al. [10] classified the electrical energy storage technologies into six forms of stored energy. The first study using a simple inventory model to analyze the electrical supply chain was conducted by [11,12]. Taylor et al. [13] used a two-stage game theory model to study price and capacity competition in electricity market. Wu et al. [14] derived a model to study the conventional generation with intermittent supply. Ouedraogo [15] developed an electricity supply-demand model for the African power systems. Wangsa and Wee [16] developed the electrical supply chain using inventory theory and considering the blackout cost.

Goyal [17] focused on the integrated vendor-buyer inventory under a constant demand rate. Later, other researchers, such as [18–23] continued his work. The integrated model considering lead time and demand uncertainties have been developed by [24–27].

In reality, pricing is a marketing strategy to control the buying behavior of customers. Pricing mechanisms, such as promotions, can be employed to induce customers to buy more products. However, increasing price may result in reduced demand and consumption; this is also true in the consumption of electricity.

The purpose of our study is to analyze the effect of price-dependent demand on the sustainable electrical supply chain system (SESCS). The power is transmitted through distribution networks to multiple customers whose demands are influenced by the price of electricity. We propose a procedure to find an optimal solution. The remainder of the paper is organized as follows: in Section 2, we describe the analogy and problem of SESCO. In Sections 3 and 4, we present the development of the mathematical model. A numerical example and sensitivity analysis are discussed in Sections 5 and 6, respectively. Section 7 concludes the work by providing insights, managerial implications and directions for future research.

2. Analogies between Sustainable Electrical Supply Chain System (SESCS) and Supply Chain Inventory System (SCIS)

In this section, we explain the problem description of SESCO with price-dependent demand. In this study, it is shown that determining the capacity of SESCO has the same process or analogy as determining the order quantity in the Supply Chain Inventory System (SCIS). The SCIS involves a vendor-buyer coordination and freight forwarding [27]. The buyer sells items to the end-customers and orders items to the vendor. The vendor produces the items and sends in batch to the buyer. The buyer will then sell the items to the customers.

Similarly, in SESCO, electricity generated in a power generation will be transmitted through a transmission line and distribution substation to the customers whose demands are influenced by the price of electricity. The electricity demand is represented as $D(p)$ kWh per year. The power generation produces the electricity in a batch size of $Q_t g n m$ kWh where m is power generation's factor, n is transmission factor's impact on the transmission substation and g is distribution factor's effect on the distribution substation (integer). The finite power supply rate is P kWh per year, [$P > D(p)$] and a fixed setup cost of \$ S . The electricity energy of $Q_t g n$ kWh is supplied by the power generator to the transmission substation, then $Q_t g$ kWh of electricity is supplied to the distribution substation and Q_t kWh of electricity is consumed by the customers.

In order to maximize the profit of SESCO, we consider the sales revenue, production cost, setup cost of the power generation, ordering cost of customers and transmission/distribution costs of substations. The transmission and distribution costs are function of the power plant, the transmission substation and the distribution substation with maximum capacities of W_p^x , W_t^x , W_d^x , respectively.

The analogies between SESCO and SCIS are depicted in Figure 1.

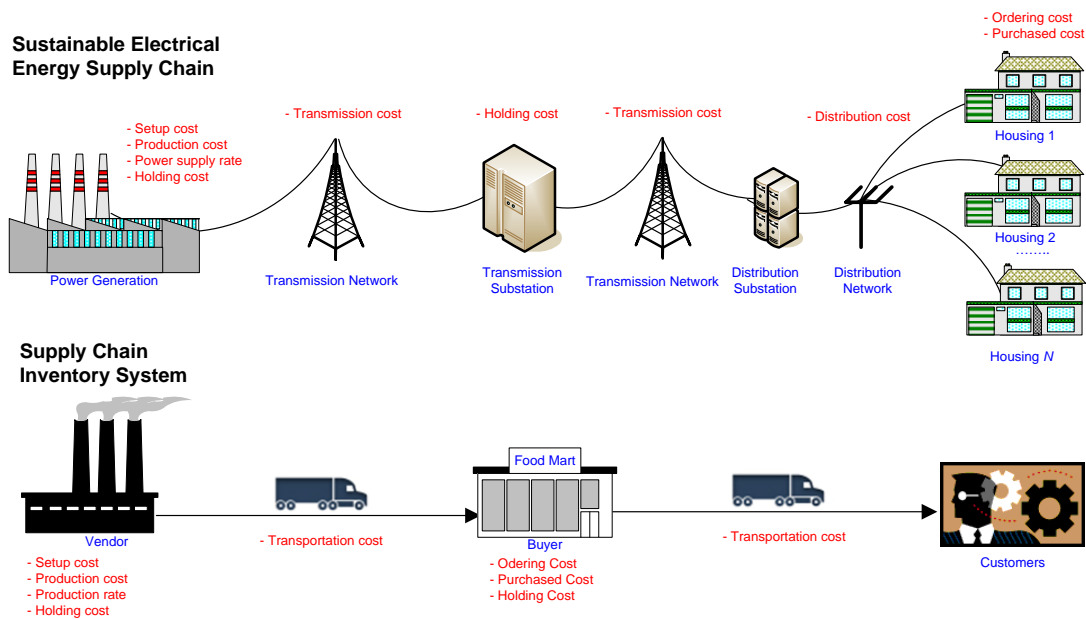


Figure 1. The analogies between Sustainable Electrical Supply Chain System (SESCS) and Supply Chain Inventory System (SCIS).

3. Assumptions

We utilized some notations in the development of the mathematical model (see Abbreviations) and utilized the following assumptions in our model:

1. The model consists of power generation, transmission and distribution substations as well as multiple customers.
2. Power supply blackouts do not occur.
3. The demand rate of the customers depends on the selling price.
4. We included the four types of demand rate functions:

$$D_i(p) = \begin{cases} \text{Increasing linear} & \beta + \gamma p \\ \text{Increasing quadratic} & \beta + \gamma p^2 \\ \text{Increasing multiplicative} & \beta + \gamma^p \\ \text{Decreasing multiplicative} & \beta - \gamma^p \end{cases}$$

where, $\beta > 0$ is a scaling factor, and $\gamma > 1$ is a price elasticity coefficient.

5. The finite power supply rate P is higher than the demand rate of the customers, $[P > D(p)]$.
6. The power consumption is the sum of the electricity consumed by customers from i th to N th, $(Q = \sum_{i=1}^N Q_i)$. The electricity consumption for customer i th should be in proportion to his or her electrical demand. There are $[Q_i = D_i(p)Q/D(p)]$.
7. The electrical energy (Qt) in kWh is the percentage of electricity used (Q) within a particular unit of time (t) by customers.
8. The amount of electricity energy: at generation is $Qtgnm$ kWh; at the transmission substation is $Qtgn$ kWh, at the distribution substation is Qtg kWh, and the customers consumed energy is Qt kWh.
9. The maximum capacity of power generation must be greater than at the transmission and distribution substations $(W_p^x > W_t^x > W_d^x)$ in KVA.
10. The power generation process will include the transmission and distribution costs.

4. Mathematical Model

In this section, we explain how we first derived the total cost function with regard to customers, the distribution and transmission substations, and the total profit function of power generation.

4.1. The Customer's Ordering Cost

Ordering cost is defined as the customer's total cost per unit time:

$$TC_c(Q) = \frac{A \cdot D(p)}{Q \cdot t} \quad (1)$$

4.2. The Distribution Cost

We utilized the same methodology as [16,27–29] to determine the distribution cost, which is incurred by the distribution substation. The distribution cost for partial load F based on the adjusted inverse function can be calculated using the equation below:

$$F = C_t \left(\frac{W_d^x}{W_d^y} \right) \quad (2)$$

where α is the coefficient of the adjusted inverse function (0–1); its function is to increase the transmission rate per kVA per mile as the W_d^y increases. The distribution cost per kVA per mile is as follows:

$$F_z = \alpha F + (1 - \alpha)C_t \quad (3)$$

By combining Equation (2) into Equation (3) and simplifying, we can determine the unit rate:

$$F_z = C_t + \alpha C_t \left(\frac{W_d^x - W_d^y}{W_d^y} \right) \quad (4)$$

The estimated total cost for the distribution substation as a function of demand, correction factor and distance with adjusted inverse yields is shown in the equation below:

$$F_d = \left\{ C_t + \alpha C_t \left(\frac{W_d^x - W_d^y}{W_d^y} \right) \right\} D(p) \cdot d_d \cdot \Delta \quad (5)$$

The actual power supply capacity is presented as ($W_d^y = Qtg\Delta$). Therefore, the distribution cost can be expressed in the following equation:

$$TC_d(Q, g) = \frac{D(p)}{Qtg} \alpha C_t W_d^x d_d + D(p) d_d \Delta (1 - \alpha) C_t \quad (6)$$

4.3. Total Cost Incurred by the Transmission Substation

This total cost consists of the energy holding cost and transmission cost. We calculated the average of these two elements using the equation below:

$$\bar{I}_t = \frac{Egn}{2} \quad (7)$$

where, $E = Qt$, the energy holding cost at the transmission substation is represented below:

$$r_t p \bar{I}_t = r_t p \left(\frac{Qtgn}{2} \right) \quad (8)$$

We used the same methodology to determine the percentage of the transmission cost of distribution (Equation (6)), which can be calculated using the equation below:

$$F_t(Q, g, n) = \frac{D(p)}{Qtgn} \alpha C_t W_t^x d_t + D(p) d_t \Delta (1 - \alpha) C_t \quad (9)$$

Thus, the total cost incurred at the transmission substation is the sum of Equations (8) and (9). Therefore, we calculated the total cost of the transmission substation using the following equation:

$$TC_t(Q, g, n) = \frac{D(p)}{Qtgn} (\alpha C_t W_t^x d_t) + r_t p \left(\frac{Qtgn}{2} \right) + D(p) d_t \Delta (1 - \alpha) C_t \quad (10)$$

4.4. Total Profit of Power Generation

The total profit of power generation can be presented by the expression:

Total profit = sales revenue – production cost – setup cost – holding cost – transmission cost.

The electricity sales revenue and production cost are given in the following equation:

$$D(p) \cdot (p - v) \quad (11)$$

Power generation creates electrical energy in $(Qtgnm)$ kWh in one production run. Therefore, we determine the setup cost for power generation using the equation below:

$$\frac{D(p)}{Qtgnm} S \quad (12)$$

The electrical energy, $E = Qt$, the production energy $(Egnm)$, and the transmission station receive the electricity in m times of (Egn) . The average energy holding for the generation of power can be evaluated as follows:

$$\bar{I}_p = \frac{\left[Egnm \left(\frac{Egn}{P} + (m-1) \frac{E}{D(p)} \right) - \frac{m^2 (Egn)^2}{2P} \right] - \left[\frac{E}{D(p)} (1 + 2 + \dots + (m-1)) E \right]}{\frac{Egnm}{D(p)}} \quad (13)$$

Equation (13) can be rewritten as:

$$\bar{I}_p = \frac{Egn}{2} \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \quad (14)$$

Hence, the power generation's energy holding cost is determined via the following equation:

$$r_p v \frac{Qtgn}{2} \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \quad (15)$$

Similar to the previous transmission cost, the production energy $(Qtgnm)$ can be calculated via the capacity of power generation (W_p^x) , and network from the power generation to the transmission substation (d_p) . Thus, the transmission cost can be calculated as follows:

$$F_p(Q, g, n, m) = \frac{D(p)}{Qtgnm} \alpha C_t W_p^x d_p + D(p) d_p \Delta (1 - \alpha) C_t \quad (16)$$

Hence, the total profit for power generation can be expressed by:

$$\begin{aligned} \Pi_p(Q, g, n, m) = & D(p) \cdot [p - v - d_p \Delta(1 - \alpha) C_t] - \frac{D(p)}{Qtgnm} \left(S + \alpha C_t W_p^x d_p \right) \\ & - r_p v \frac{Qtgn}{2} \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \end{aligned} \quad (17)$$

The joint total profit is the total profit of power generation and the total cost incurred by the customers, the distribution substation, and the transmission substation, which is illustrated below:

$$\text{Max } \Pi(Q, g, n, m) = \Pi_p(Q, g, n, m) - TC_t(Q, g, n) - TC_d(Q, g) - TC_c(Q) \quad (18)$$

$$\begin{aligned} \text{Max } \Pi(Q, g, n, m) = & D(p) \cdot [p - v - (d_p + d_t + d_d) \Delta(1 - \alpha) C_t] \\ & - \frac{D(p)}{Qt} \left[A + \frac{\alpha C_t W_d^x d_d}{g} + \frac{\alpha C_t W_t^x d_t}{gn} + \frac{(S + \alpha C_t W_p^x d_p)}{gnm} \right] \\ & - \frac{Qtgn}{2} \left\{ r_t p + r_p v \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \right\} \end{aligned} \quad (19)$$

The following is a simplified version of the equation above:

$$\bar{y}_1 = \alpha C_t W_d^x d_d \quad (20)$$

$$\bar{y}_2 = \alpha C_t W_t^x d_t \quad (21)$$

$$\bar{y}_3 = S + \alpha C_t W_p^x d_p \quad (22)$$

$$\bar{V} = [p - v - (d_p + d_t + d_d) \Delta(1 - \alpha) C_t] \quad (23)$$

Therefore, Equation (19) can be rewritten as follows:

$$\begin{aligned} \text{Max } \Pi(Q, g, n, m) = & D(p) \cdot \bar{V} - \frac{D(p)}{Qt} \left(\frac{A \cdot gnm + \bar{y}_1 \cdot nm + \bar{y}_2 \cdot m + \bar{y}_3}{gnm} \right) \\ & - \frac{Qtgn}{2} \left\{ r_t p + r_p v \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \right\} \end{aligned} \quad (24)$$

We examined the effect of (g, n, m) on $\Pi(Q, g, n, m)$ on fixed $\Pi(Q)$ by employing the first and the second partial derivatives of Equation (24) with respect to (g, n, m) :

$$\frac{\partial \Pi(Q, g, n, m)}{\partial g} = \frac{D(p)}{Qt} \left(\frac{\bar{y}_1}{g^2} + \frac{\bar{y}_2}{g^2 n} + \frac{\bar{y}_3}{g^2 nm} \right) - \frac{Qtgn}{2} \left\{ r_t p + r_p v \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \right\} \quad (25)$$

$$\frac{\partial^2 \Pi(Q, g, n, m)}{\partial g^2} = \frac{-2D(p)}{Qt} \left(\frac{\bar{y}_1}{g^3} + \frac{\bar{y}_2}{g^3 n} + \frac{\bar{y}_3}{g^3 nm} \right) < 0 \quad (26)$$

$$\frac{\partial \Pi(Q, g, n, m)}{\partial n} = \frac{D(p)}{Qt} \left(\frac{\bar{y}_2}{gn^2} + \frac{\bar{y}_3}{gn^2 m} \right) - \frac{Qtg}{2} \left\{ r_t p + r_p v \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \right\} \quad (27)$$

$$\frac{\partial^2 \Pi(Q, g, n, m)}{\partial n^2} = \frac{-2D(p)}{Qt} \left(\frac{\bar{y}_2}{gn^3} + \frac{\bar{y}_3}{gn^3 m} \right) < 0 \quad (28)$$

$$\frac{\partial \Pi(Q, g, n, m)}{\partial m} = \frac{D(p)}{Qt} \left(\frac{\bar{y}_3}{gnm^2} \right) - \frac{Qtgn}{2} r_p v \left(1 - \frac{D(p)}{P} \right) \quad (29)$$

and:

$$\frac{\partial^2 \Pi(Q, g, n, m)}{\partial m^2} = \frac{-2D(p)}{Qt} \left(\frac{\bar{y}_3}{gnm^3} \right) < 0 \quad (30)$$

For the fixed-integers (g, n, m) , the function of $\Pi(Q, g, n, m)$ is a concave function of (Q) . Hence, the maximum value of $\Pi(Q, g, n, m)$ is located at point (Q^*) , which satisfies $\frac{\partial \Pi}{\partial Q} = 0$. To calculate

the optimal solution for fixed-integers (g, n, m) , we utilized the partial derivatives with respect to Q , as shown in the following equation:

$$\frac{\partial \Pi(Q, g, n, m)}{\partial Q} = \frac{D(p)}{Q^2 t} \left(\frac{A \cdot gnm + \bar{y}_1 \cdot nm + \bar{y}_2 \cdot m + \bar{y}_3}{gnm} \right) - \frac{tgn}{2} \left\{ r_t p + r_p v \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \right\} \quad (31)$$

In the equation below, we set Equation (31) equal to zero and solved for, Q .

$$Q^* = \frac{1}{t \cdot g \cdot n} \sqrt{\frac{2D(p) \cdot (A \cdot gnm + \bar{y}_1 \cdot nm + \bar{y}_2 \cdot m + \bar{y}_3)}{m \cdot \left\{ r_t p + r_p v \left[m \left(1 - \frac{D(p)}{P} \right) - 1 + \frac{2D(p)}{P} \right] \right\}}} \quad (32)$$

4.5. Procedure

In order to obtain the optimal solution values for the proposed model, we established the procedure below:

- Step 1
 - a. We set $m = 1$.
 - b. We set $n = 1$.
 - c. We set $g = 1$.
- Step 2 We calculated the optimal Q^* by using Equation (32).
- Step 3 Next, we calculated the actual electrical power capacities.
 - a. Distribution Substation
 - (a.1) We determined the actual capacity of the distribution substation via $(W_d^y = Q^* t g \Delta)$. We checked, if $(W_d^y \leq W_d^x)$ was satisfied. Then we revised the power consumption (Step a.2). Otherwise, $(W_d^y > W_d^x)$, we went on to Step (b.1).
 - (a.2) We revised the power consumption $(Q^* = \frac{W_d^x}{t g \Delta})$ and followed Step (b.1).
 - b. Transmission Substation
 - (b.1) To obtain the actual capacity of the transmission substation, we utilized $(W_t^y = Q^* t g n \Delta)$. We then checked if $(W_t^y \leq W_t^x)$ was satisfied and revised the power consumption (Step b.2). If it was not satisfied, we followed Step (c.1).
 - (b.2) We revised the power consumption $(Q^* = \frac{W_t^x}{t g n \Delta})$ and proceeded to Step (c.1).
 - c. Power Generation
 - (c.1) We determined that the actual capacity of the transmission substation could be calculated via $(W_p^y = Q^* t g n m \Delta)$. We checked, if $(W_p^y \leq W_p^x)$ was satisfied, revised the power consumption (Step c.2) and proceeded to Step (4).
 - (c.2) We revised the power consumption $(Q^* = \frac{W_p^x}{t g n m \Delta})$ and moved on to Step (4).
- Step 4 We computed $\Pi(Q, g, n, m)$ using Equation (24).
- Step 5 We set $g = g + 1$ and repeated Steps (2 to 4).
- Step 6 If $\Pi(Q_g^*, g, n_g, m_g) \geq \Pi(Q_{g-1}^*, g-1, n_{g-1}, m_{g-1})$ then we went to Step (5). If it was equal, we moved on to Step (7).
- Step 7 We set $n = n + 1$ and repeated Steps (1c to 6).
- Step 8 If $\Pi(Q_n^*, g_n^*, n, m_n) \geq \Pi(Q_{n-1}^*, g_{n-1}^*, n-1, m_{n-1})$ we went on to Step (7). If they were equal, we moved to Step (9).

Step 9 Step 9 We set $m = m + 1$ and repeated Steps (1b to 8).

Step 10 If $\Pi(Q_m^*, g_m^*, n_m^*, m) \geq \Pi(Q_{m-1}^*, g_{m-1}^*, n_{m-1}^*, m - 1)$, we moved on to Step (9), otherwise we went to Step (11).

Step 11 We set $\Pi(Q^*, g^*, n^*, m^*) = \Pi(Q_{m-1}^*, g_{m-1}^*, n_{m-1}^*, m - 1)$, then (Q^*, g^*, n^*, m^*) were the optimal solution.

5. Numerical Example

In this example, we utilized artificial data to demonstrate the application of the model. The data values are given in Tables 1–3.

Table 1. Customer Data.

Parameter	Customer			
	1	2	3	4
Types of Demand Function	Increasing Linear	Increasing Quadratic	Increasing Multiplicative	Decreasing Multiplicative
Demand Function	$\beta + \gamma p$	$\beta + \gamma p^2$	$\beta + \gamma p$	$\beta - \gamma p$
Scaling Factor	30,000	30,000	30,000	30,000
Price Elasticity Coefficient	5	5	5	5
Avg. Time of Electrical Consumption (hours)	24	24	24	24
Ordering Cost (\$)	20	20	20	20
Price of Electricity (\$/kWh)	1.20	1.20	1.20	1.20
Holding Cost Rate (%/year)	0.20	0.20	0.20	0.20
Power Supply Loss Factor	0.1125	0.1125	0.1125	0.1125
Power Factor Correction (kVA/kWh)	1.25	1.25	1.25	1.25

Table 2. Power Generation, Transmission Substation and Distribution Substation Data.

Parameters	Power Generation	Transmission Substation	Distribution Substation
Power Supply Rate (kWh/year)	650,000	-	-
Production Cost (\$/kWh)	0.85	-	-
Setup Cost (\$)	5600	-	-
Holding Cost Rate of the Power Generation (%/year)	0.20	-	-
Transmission and Distribution Rates (\$/kVA/mile)	0.000455	-	-
Maximum Capacity of Power Generation (kVA)	500,000	-	-
Maximum Capacity of Transmission Substation (kVA)	-	350,000	-
Maximum Capacity of Distribution Substation (kVA)	-	-	10,500

Table 3. The Network Data for Power Generation, and Transmission and Distribution Substations.

From—To	Transmission Substation	Distribution Substation	Customers
Power Generation (miles)	80	-	-
Transmission Substation (miles)	-	10	-
Distribution Substation (miles)	-	-	3

Using the mathematical model developed in previous section, we solved for the parameters given above. Table 4 shows the details of the procedure for obtaining the optimal solution. The optimal values are as follows: the customer power consumption, $Q^* = 350$ kW or 350,000 Watt; the electrical power factors of the distribution, the transmission and the power generation are 1, 2 and 7 times, respectively. Thus, the total profit is \$17,712.07/year.

Table 4. Details of the Procedures for this Example.

m	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$W_d^y^*$	Q^* Revisited	$W_t^y^*$	$W_p^y^*$	$\Pi(.)$
1	10	1	30,006.0	30,007.2	30,006.9	29,993.1	351.42	10,542.59	350.00	105,000.00	105,000.00	13,481.57
2	6	1	30,006.0	30,007.2	30,006.9	29,993.1	341.57	10,247.24	341.57	61,483.45	122,966.91	16,204.24
3	4	1	30,006.0	30,007.2	30,006.9	29,993.1	365.55	10,966.51	350.00	42,000.00	126,000.00	17,100.27
4	3	1	30,006.0	30,007.2	30,006.9	29,993.1	381.12	11,433.53	350.00	31,500.00	126,000.00	17,444.63
5	3	1	30,006.0	30,007.2	30,006.9	29,993.1	315.33	9459.89	315.33	28,379.68	141,898.39	17,621.25
6	3	1	30,006.0	30,007.2	30,006.9	29,993.1	270.01	8100.23	270.01	24,300.69	145,804.12	17,621.42
7	2	1	30,006.0	30,007.2	30,006.9	29,993.1	352.40	10,571.91	350.00	21,000.00	147,000.00	17,712.07 ←
8	2	1	30,006.0	30,007.2	30,006.9	29,993.1	314.42	9432.73	314.42	18,865.46	150,923.72	17,631.05

* the local solution; ←the optimal solution.

Table 5. The Results.

Decision Variables	Values
Demand of Customer 1	30,006.00 kWh/year
Demand of Customer 2	30,007.20 kWh/year
Demand of Customer 3	30,006.90 kWh/year
Demand of Customer 4	29,993.10 kWh/year
Electrical power consumption of Customer 1	87.51 kW
Electrical power consumption of Customer 2	87.51 kW
Electrical power consumption of Customer 3	87.51 kW
Electrical power consumption of Customer 4	87.47 kW
Electrical power Generation	7 times
Electrical power transmission factor	2 times
Electrical power distribution factor	1 times
Energy transmitted by Power Generation	117,600.00 kWh
Energy transmitted by Transmission Substation	16,800.00 kWh
Energy transmitted by Distribution Substation	8400.00 kWh
Energy consumed by Customer 1	2100.19 kWh
Energy consumed by Customer 2	2100.27 kWh
Energy consumed by Customer 3	2100.25 kWh
Energy consumed by Customer 4	2099.29 kWh
Actual capacity of Power Generation	147,000.00 kVA
Actual capacity of Transmission Substation	21,000.00 kVA
Actual capacity of Distribution Substation	10,571.91 kVA

In our solution (Table 5), we have $m = 7$ batches. It means the electricity produce by the power generator has a total power of 117,600.00 kWh. But not all the electricity (117,600.00 kWh) generated is transmitted at once, but in 7 times with 16,800.00 kWh each. Since the generator has a device to minimize the holding cost (Equation (15)) of the electricity, for each batch, the transmission substation receives 16,800.00 kWh of electricity; it then transmits in two batches to the distribution station at 8400 kWh each. This is done to minimize the transmission cost (Equation (9)) and distribution cost (Equation (6)).

Based on those results, the electrical power consumption of customers 1, 2, 3 and 4 are 87,507.87 Watt; 87,511.37 Watt; 87,510.49 Watt; and 87,470.26 Watt, respectively. The demand of customers 1, 2, 3 and 4 are 30,006 kWh/year; 30,007.2 kWh/year; 30,006.9 kWh/year; and 29,993.1 kWh/year, respectively. The results are shown in Table 5.

6. Discussion

In this section, we discuss the effect of changes in the parameters as well as summarize the results of the sensitivity analysis.

6.1. Price Elasticity Coefficient (γ)

In the equation discussed above, we set the price elasticity coefficient (γ) to 5. Here, we discuss the effect of changing this coefficient from 5 to 50. Table 6 and Figure 2 show that if the price elasticity increases, then the total profit increases by an average of 0.02%. Table 6 shows an increase in the demand for customers 1, 2 and 3 and contrasts these results with customer 4's demand. We set the demand function for customer 4 as decreasing. In many practical situations, the price elasticity can be a result of advertising that allows marketers to achieve higher profits.

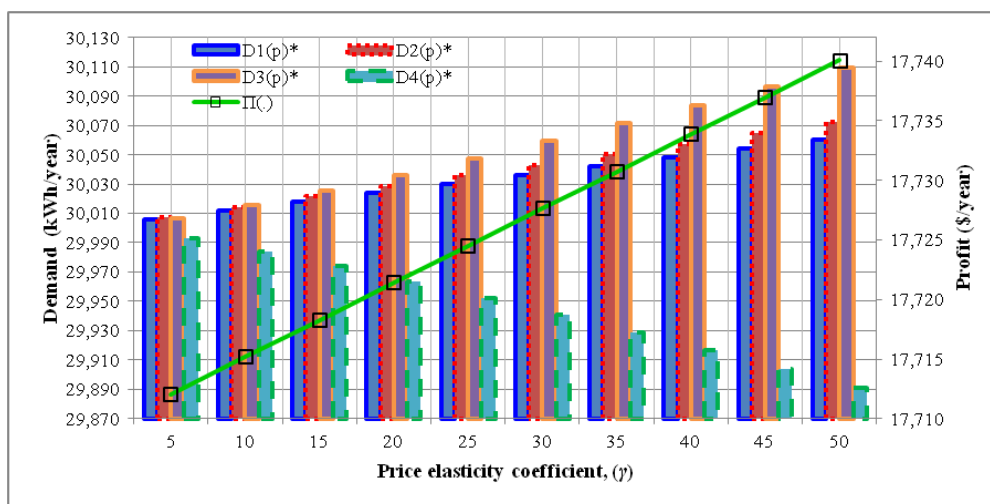


Figure 2. The Effect of Changes in the Price Elasticity Coefficient on Profit and Demand.

Table 6. Effect of Changes in the Price Elasticity Coefficient Parameter.

Price Elasticity Coefficient, (γ)	m^*	n^*	g^*	$D_1(p)^*$	$D_1(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
5	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	17,712.07
10	7	2	1	30,012.00	30,014.40	30,015.85	29,984.15	350.00	17,715.19
15	7	2	1	30,018.00	30,021.60	30,025.78	29,974.22	350.00	17,718.30
20	7	2	1	30,024.00	30,028.80	30,036.41	29,963.59	350.00	17,721.41
25	7	2	1	30,030.00	30,036.00	30,047.59	29,952.41	350.00	17,724.52
30	7	2	1	30,036.00	30,043.20	30,059.23	29,940.77	350.00	17,727.64

Table 6. Cont.

Price Elasticity Coefficient, (γ)	m^*	n^*	g^*	$D_1(p)^*$	$D_1(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
35	7	2	1	30,042.00	30,050.40	30,071.27	29,928.73	350.00	17,730.75
40	7	2	1	30,048.00	30,057.60	30,083.65	29,916.35	350.00	17,733.86
45	7	2	1	30,054.00	30,064.80	30,096.35	29,903.65	350.00	17,736.97
50	7	2	1	30,060.00	30,072.00	30,109.34	29,890.66	350.00	17,740.08

* the local solution.

6.2. Scaling Factor (β)

Table 7 and Figure 3 present the impact of the scaling factor of demand (β). The results are similar to the price elasticity coefficient on the optimal solutions. They show that when the scaling factor increases, consumer demand, power consumption, and total profit also increase. The parameters $D_1(p)$, $D_2(p)$, $D_3(p)$, $D_4(p)$, Q and Π increase (6.66% to 15%), (6.66% to 14.9%), (6.66% to 14.9%), (6.7% to 15%), (−8.2% to 8%), and (10.6% to 30%), respectively.

Table 7. Effect of Changes in the Scaling Factor Parameter.

Scaling Factor, (β)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
15,000	5	2	1	15,006.00	15,007.20	15,006.90	14,993.10	323.34	4639.38
16,000	5	2	1	16,006.00	16,007.20	16,006.90	15,993.10	334.55	5432.56
18,000	5	2	1	18,006.00	18,007.20	18,006.90	17,993.10	350.00	7061.27
20,000	6	2	1	20,006.00	20,007.20	20,006.90	19,993.10	321.44	8722.39
23,000	6	2	1	23,006.00	23,007.20	23,006.90	22,993.10	346.87	11,339.64
26,000	6	2	1	26,006.00	26,007.20	26,006.90	25,993.10	350.00	14,011.84
28,000	7	2	1	28,006.00	28,007.20	28,006.90	27,993.10	338.85	15,835.38
30,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	17,712.07
32,000	7	2	1	32,006.00	32,007.20	32,006.90	31,993.10	350.00	19,598.27
35,000	8	2	1	35,006.00	35,007.20	35,006.90	34,993.10	343.99	22,448.29

* the local solution.

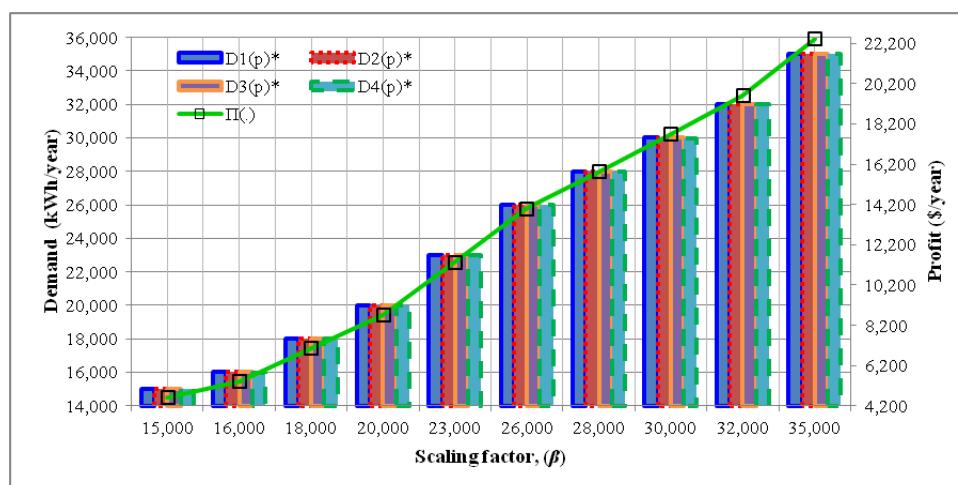


Figure 3. The Effect of Changes in the Scaling Factor on Profit and Demand.

6.3. Price of Electricity (p)

We examined the effects of changes in the price of electricity (p) starting from \$1.16/kWh to \$1.25/kWh. It must be noted that we kept the production cost at ($v = \$0.85/\text{kWh}$). Table 8 and Figure 4

show that the results were not affected by changes in the customer demand function. The results show that the price of electricity is also not affected by optimal power consumption. If the price of electricity was to increase and the production cost was to remain unchanged, then the total profit would increase from 5.27% to 9.12%.

Table 8. Effect of Changes in the Price of Electricity Parameter.

Price of Electricity, (p)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
1.16	7	2	1	30,005.80	30,006.73	30,006.47	29,993.53	350.00	12,978.61
1.17	7	2	1	30,005.85	30,006.84	30,006.57	29,993.43	350.00	14,161.97
1.18	7	2	1	30,005.90	30,006.96	30,006.68	29,993.32	350.00	15,345.34
1.19	7	2	1	30,005.95	30,007.08	30,006.79	29,993.21	350.00	16,528.70
1.20	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	17,712.07
1.21	7	2	1	30,006.05	30,007.32	30,007.01	29,992.99	350.00	18,895.45
1.22	7	2	1	30,006.10	30,007.44	30,007.12	29,992.88	350.00	20,078.83
1.23	7	2	1	30,006.15	30,007.56	30,007.24	29,992.76	350.00	21,262.21
1.24	7	2	1	30,006.20	30,007.69	30,007.36	29,992.64	350.00	22,445.59
1.25	7	2	1	30,006.25	30,007.81	30,007.48	29,992.52	350.00	23,628.98

* the local solution.

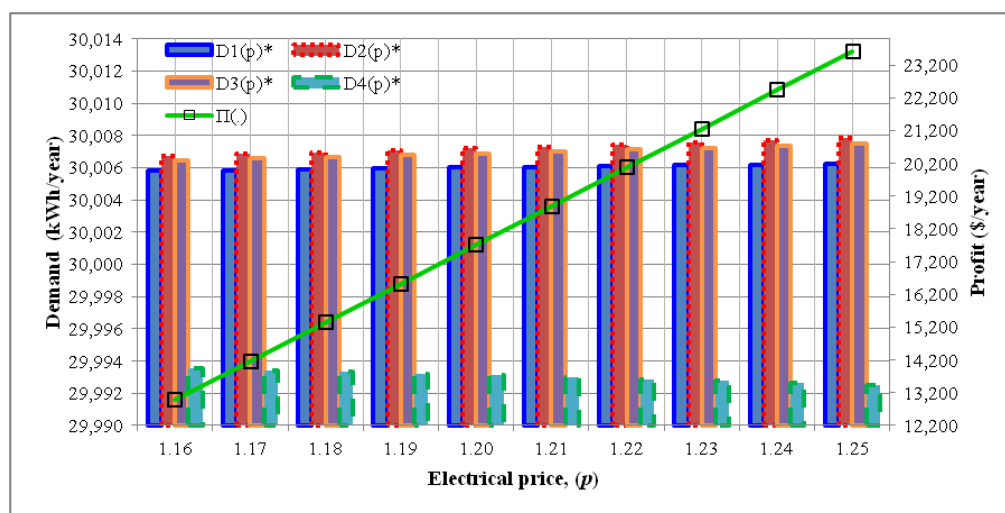


Figure 4. The Effect of Changes in the Price of Electricity on Profit and Demand.

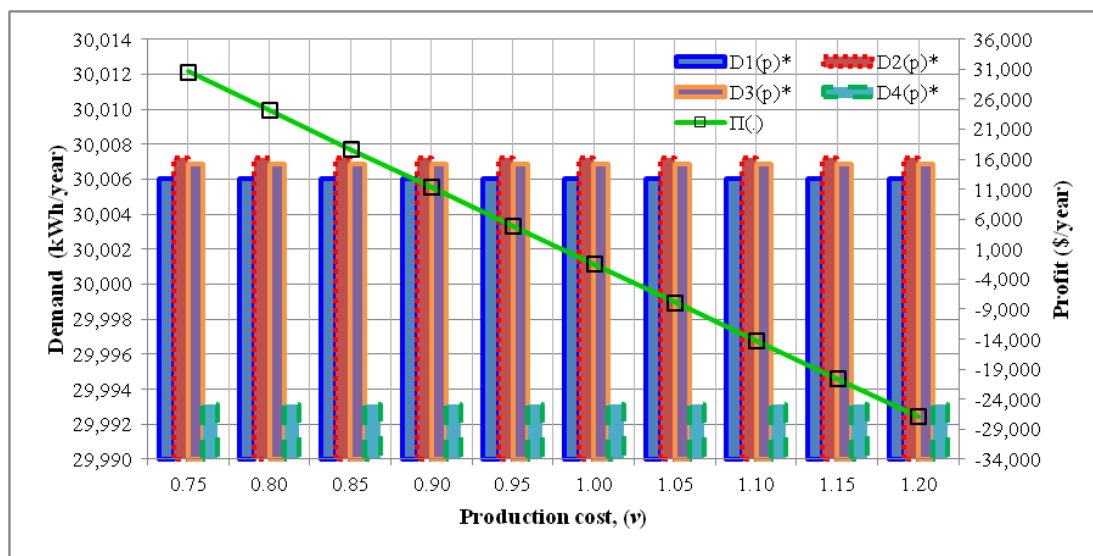
6.4. Production Cost (v)

Next, we examined the effects of changes in the production cost (v). In this case, we set the price of electricity to \$1.20/kWh and production costs from \$0.75/kWh to \$1.10/kWh. We assumed that the production cost would not be more than the price of electricity ($v < p$). Table 9 and Figure 5 show that the customer demand function is similar to the price of electricity in that it is not affected by changes in production cost. In contrast, the results showed that the production cost is affected by the optimal power consumption, which decreases at an average of -0.36% . If the production cost increases and the price of electricity remains unchanged, then the total profit will decrease by -21.03% to -423.85% . Therefore, the profit resulting from the production cost is different from the profit resulting from the price of electricity.

Table 9. Effect of Changes in the Production Cost Parameter.

Production Cost, (v)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(\cdot)$
0.75	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	30,566.30
0.80	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	24,139.19
0.85	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	17,712.07
0.90	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	344.56	11,287.31
0.95	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	337.22	4871.34
1.00	6	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	−1500.19
1.05	6	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	−7858.81
1.10	4	3	1	30,006.00	30,007.20	30,006.90	29,993.10	349.16	−14,215.97
1.15	4	3	1	30,006.00	30,007.20	30,006.90	29,993.10	343.69	−20,544.70
1.20	4	3	1	30,006.00	30,007.20	30,006.90	29,993.10	338.46	−26,868.36

* the local solution.

**Figure 5.** The Effect of Changes in the Production Cost on Profit and Demand.

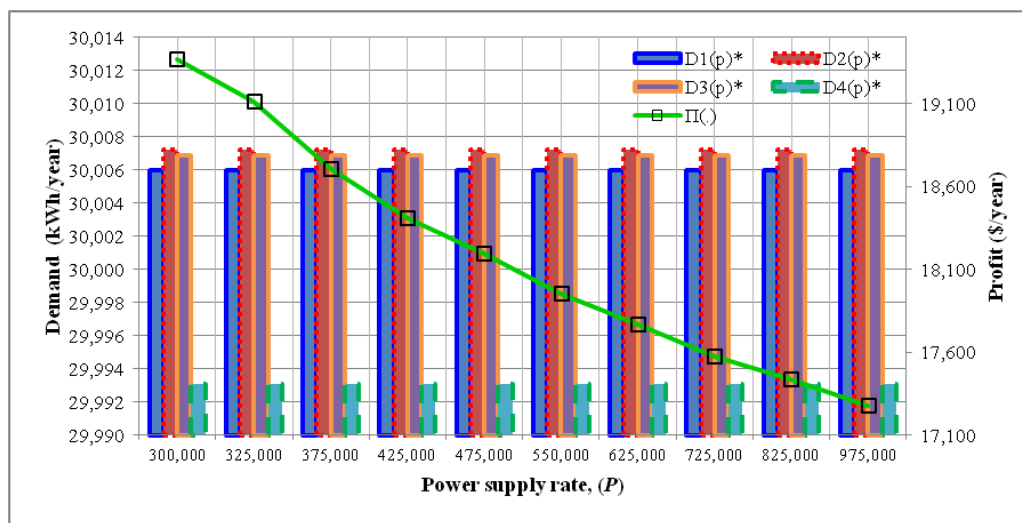
6.5. Power Supply Rate (P)

In addition, we examined the effects of changes in the power supply rate (P), assuming that it is higher than customer demand ($P > D(p)$). For example, we investigated changes in the power supply rate starting from 300,000 kWh/year, 325,000 kWh/year to 975,000 kWh/year. We found that if the power's supply rate rises (the demand of the customers will be unchanged) the impact on total profit will decrease (average -0.41%), which is caused by the fact that if the holding cost of electricity increases, the power supply rate will also increase. However, if the power supply rate and customer demand of customer increase simultaneously, profitability increases. Further, we also analyze with some combination of parameters. The results are shown in Table 10 and Figure 6.

Table 10. Effect of Changes in the Power Supply Rate Parameter.

Power Supply Rate, (P)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
300,000	8	2	1	30,006.00	30,007.20	30,006.90	29,993.10	346.58	19,369.73
325,000	8	2	1	30,006.00	30,007.20	30,006.90	29,993.10	341.38	19,110.62
375,000	8	2	1	30,006.00	30,007.20	30,006.90	29,993.10	333.52	18,703.94
425,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	18,410.00
475,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	18,197.76
550,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	17,951.76
625,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	350.00	17,764.81
725,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	349.83	17,575.70
825,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	347.20	17,433.05
975,000	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	344.33	17,275.19

* the local solution.

**Figure 6.** The Effect of Changes in the Power Supply Rate on Profit and Demand.

6.6. Effect of Changes on Combination of Parameters

6.6.1. Price Elasticity Coefficient (γ), Scaling Factor (β), and Power Supply Rate (P)

In this analysis, we examined the effects of changes on the price elasticity coefficient (γ), scaling factor (β) and power supply rate (P). Table 11 and Figure 7 show that if these variables increase simultaneously, then the total profit will increase significantly at an average of 7.57%.

Table 11. Effect of Changes on Power Supply Rate, Scaling Factor, and Price Elasticity Coefficient Parameters.

Power Supply Rate, (P)	Scaling Factor, (β)	Price Elasticity Coefficient, (γ)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
300,000	15,000	5	6	2	1	15,006.00	15,007.20	15,006.90	14,993.10	285.91	5095.24
325,000	16,000	10	6	2	1	16,012.00	16,014.40	16,015.85	15,984.15	295.00	5866.16
375,000	18,000	15	6	2	1	18,018.00	18,021.60	18,025.78	17,974.22	312.37	7447.45
425,000	20,000	20	6	2	1	20,024.00	20,028.80	20,036.41	19,963.59	328.85	9076.85
475,000	23,000	25	6	2	1	23,030.00	23,036.00	23,047.59	22,952.41	350.00	11,648.80
550,000	26,000	30	7	2	1	26,036.00	26,043.20	26,059.23	25,940.77	328.75	14,197.13
625,000	28,000	35	7	2	1	28,042.00	28,050.40	28,071.27	27,928.73	339.88	15,901.58
725,000	30,000	40	7	2	1	30,048.00	30,057.60	30,083.65	29,916.35	349.98	17,597.38
825,000	32,000	45	7	2	1	32,054.00	32,064.80	32,096.35	31,903.65	350.00	19,324.64
975,000	35,000	50	5	3	1	35,060.00	35,072.00	35,109.34	34,890.66	336.51	21,953.34

* the local solution.

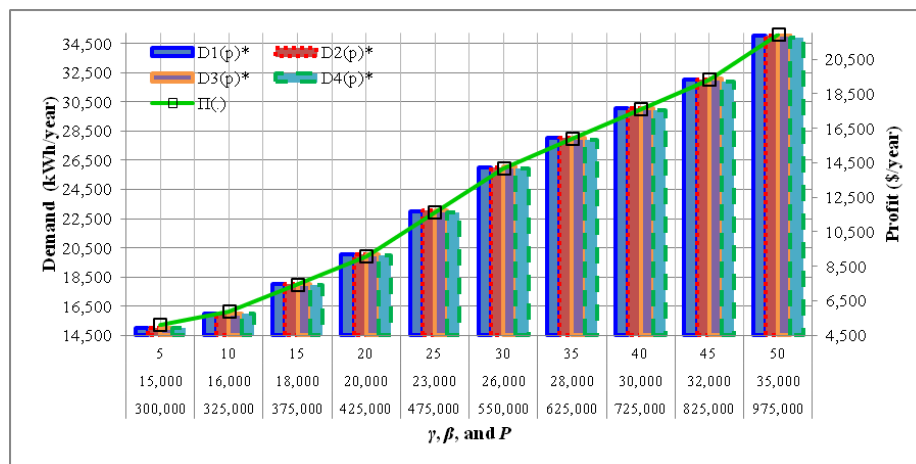


Figure 7. The Effect of Simultaneous Changes in the Power Supply Rate, Scaling Factor and Price Elasticity Coefficient on Profit and Demand.

6.6.2. Price of Electricity (p) and Production Cost (v)

Last, we examined the effects of changes in the price of electricity (p) and production cost (v). Table 12 and Figure 8 show that if these variables increase simultaneously, this will cause a significant decrease (−20.30% to −1,633.38%) in total profits. The decrease of total profit is dependent on a stepwise decline in the price of electricity and production costs.

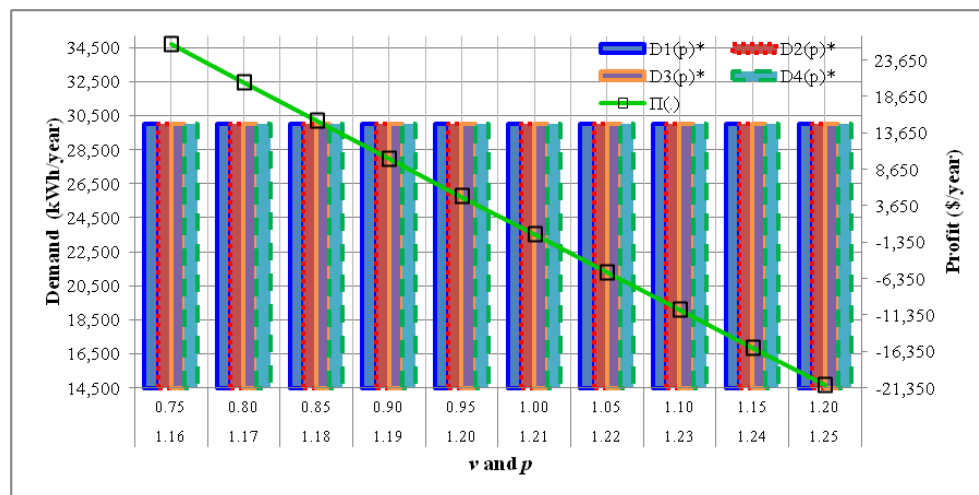


Figure 8. The Effect of Simultaneous Changes in the Price of Electricity and Production Cost on Profit and Demand.

Table 12. Effect of Changes in the Price of Electricity and Production Cost Parameters.

Electrical Price, (p)	Production Cost, (v)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(\cdot)$
1.16	0.75	7	2	1	30,005.80	30,006.73	30,006.47	29,993.53	350.00	25,832.77
1.17	0.80	7	2	1	30,005.85	30,006.84	30,006.57	29,993.43	350.00	20,589.06
1.18	0.85	7	2	1	30,005.90	30,006.96	30,006.68	29,993.32	350.00	15,345.34
1.19	0.90	7	2	1	30,005.95	30,007.08	30,006.79	29,993.21	344.86	10,103.69
1.20	0.95	7	2	1	30,006.00	30,007.20	30,006.90	29,993.10	337.22	4871.34
1.21	1.00	6	2	1	30,006.05	30,007.32	30,007.01	29,992.99	350.00	−316.84
1.22	1.05	6	2	1	30,006.10	30,007.44	30,007.12	29,992.88	350.00	−5492.13

Table 12. Cont.

Electrical Price, (p)	Production Cost, (v)	m^*	n^*	g^*	$D_1(p)^*$	$D_2(p)^*$	$D_3(p)^*$	$D_4(p)^*$	Q^*	$\Pi(.)$
1.23	1.10	6	2	1	30,006.15	30,007.56	30,007.24	29,992.76	350.00	−10,667.44
1.24	1.15	6	2	1	30,006.20	30,007.69	30,007.36	29,992.64	350.00	−15,842.75
1.25	1.20	4	3	1	30,006.25	30,007.81	30,007.48	29,992.52	336.53	−20,989.26

* the local solution.

7. Conclusions, Managerial Implications and Directions for Future Research

In this paper, we propose a mathematical model that assumes price-dependent customer demands. We define and examine four types of customer demands: namely demand may increase linearly, quadratically, multiplicatively or decreasing multiplicatively. The model developed is based on the inventory theory where we examine how the optimal decision variables and the total profits are affected by the price elasticity coefficient (γ), scaling factor (β), price of electricity (p), production cost (v) and the power supply rate (P) parameters.

The sensitivity analysis illustrates that the price elasticity coefficient (γ), scaling factor (β) and price of electricity (p) are significant in maximizing the profit. When the forementioned parameters increase, the consumer demands also increase. When other parameters remain the same, it is obvious that the profit will decrease when the production cost (v) and power supply rate (P) increase. Based on our results, we provide insights to the production and marketing managers in designing a profitable and sustainable electrical energy supply chain.

For future study, researchers may wish to extend the proposed model to include mark-up pricing or subsidized electricity pricing. Scholars may also consider incorporating discount pricing strategies as well as the effect of carbon emissions. Moreover, researchers may wish to look into a more complex type of electrical supply chain, such as a multi-transmission and distribution substation.

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Abbreviations

Index

i the i th customer, $i = 1, 2, \dots, N$

Decision Variables

Q The customer's power consumption, ($Q = \sum_{i=1}^N Q_i$) in kW.

g The electrical power distribution factor's effect on the distribution substation (integer).

n The electrical power transmission factor's impact on the transmission substation (integer).

m The electrical power generation's factor (integer).

Parameters

$D(p)$ Customers' average electrical demand rate, [$D(p) = \sum_{i=1}^N D_i(p)$] in kWh per year.

P The power supply rate, [$P > D(p)$] in kWh per year.

t Customer's average electricity consumption in hour(s).

E Customers' average energy consumption, ($E = Qt$) in kWh.

A Cost per order in dollars.

S Cost per setup of power generation in dollars.

p Price of electricity in dollars per kWh.

v	Production cost, ($v < p$) in dollars per kWh.
r_t	Annual percent of holding cost rate per unit time of the transmission substation.
r_p	Annual percent of holding cost rate per unit time of power generation.
Δ	Power factor correction in kVA per kWh.
C_t	The per mile transmission and distribution rates in dollars per kVA.
d_d	The per mile transportation network from the distribution substation to the customers in miles.
d_t	The per mile transportation network from the transmission substation to the distribution substation in miles.
d_p	The per mile transportation network from power generation to the transmission substation in miles.
W_d^x	Maximum capacity of the distribution substation in KVA.
W_t^x	Maximum capacity of the transmission substation in KVA.
W_p^x	Maximum capacity of power generation ($W_p^y > W_t^y > W_d^y$) in KVA.
W_d^y	Actual capacity of the distribution substation in KVA.
W_t^y	Actual capacity of the transmission substation in KVA.
W_p^y	Actual capacity of the power generation capacity in KVA.
α	The power supply loss factor, $0 \leq \alpha \leq 1$.
β	The scaling factor, $\beta > 0$.
γ	The price elasticity coefficient, $\gamma > 1$.

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