

Article

An Optimal Source-Load Coordinated Restoration Method Considering Double Uncertainty

Ping Jiang and Qiwei Chen *

School of Electrical Engineering, Southeast University, Nanjing 210096, China; jping@seu.edu.cn

* Correspondence: chenqiwei0529@163.com; Tel.: +86-139-5193-9388

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Abstract: The security of power system restoration is severely affected by uncertain factors, especially the start-up time of generating unit and the amount of load pick-up. Solving the optimization restoration problem is challenging since it needs to determine different priorities in which units and loads are restored with the consideration of double uncertainty. Therefore, an optimal source-load coordinated restoration method that is based on information gap decision theory (IGDT) is proposed. Firstly, the time-domain restoration characteristics of black-start unit (BSU), non-black-start unit (NBSU), and load are described with analysis of double uncertainty. On this basis, a coupled multi-objective optimization model is built with double uncertainty, in which source-load coordinated restoration is realized. Then, IGDT is adopted to convert the uncertainty optimization model to a certain one with robustness, which tolerates the most uncertainty and still meets the desired requirement. Finally, the optimization model is solved by non-dominated genetic algorithm II (NSGA-II). The effectiveness and robustness of the proposed method is further illustrated through a case study based on the IEEE 39-bus system.

Keywords: power system restoration; load restoration; uncertainty multi-objective optimization; information gap decision theory

1. Introduction

With the continuous development of power system, the operation of the system has been more complex than ever. Due to some occasional and inevitable factors, blackouts have become a serious threat to the power system [1,2]. Besides, the security of power system has been severely affected by uncertain factors. Therefore, the investigation on source-load coordinated restoration method considering double uncertainty plays supporting role in restoration decision, which is beneficial to ensure the stability and rapidity of power system in restoration process.

The procedure of power system restoration can be divided into three stages: black-start, reconfiguration of the network and load restoration [3,4]. There is no obvious sequence as three stages are interleaved with each other. It is necessary to restore part of the important loads to balance the output of the restarted units during the black-start stage, which ensures the stable operation of the restarted units and achieves the minimum steady output as soon as possible after connecting to the grid [5,6].

In recent years, theoretical investigations on unit and load restoration have attracted researchers' interests. To maximize the generating power output, the start-up of generating units need to be optimized [7]. Sun W. et al. applied mixed-integer linear programming (MILP) algorithm to identify the optimal start-up sequence of NBSUs [8]. An optimal restoration approach based on the Wide Area Measurement System (WAMS) was proposed by Nourizadeh S. et al. [9]. Gu X. et al. proposed an extended black-start restoration strategy, in view of the fact that some current black-start resources with large rated power can provide enough cranking power for more than one non-black-start unit

(NBSU) simultaneously [10]. The feasibility of selecting microgrids as black start power was analyzed by Wu Z. et al. It adopts the Dijkstra algorithm to search for the extended black start paths [11]. Lin Z. et al. put forward a novel method using the entropy weight-based decision-making theory to evaluate and optimize the sequence of NBSUs [12]. In terms of load restoration, a DC optimal load shed recovery with transmission switching model was proposed to increase the operating capacity, when considering load demands as continuous variables [13]. In [14], a mixed-integer model that was considering discrete load pickup was proposed. An optimization model was formulated to maximize load pick-up for a given substation by Qu H. et al. [15]. Gholami A. et al. proposed a two-stage hierarchical approach to load restoration problem, which determines the optimal amount and location of load pick-up [16]. In [17], synchrophasors were used for the prediction of restorable size of load pick-up during the load restoration. Furthermore, a multi-objective optimization method coordinating unit restarting with load restoration was employed into the extended black-start by Wang D. et al. [18]. The method shows that unit and load can be effectively and coordinately restored at the black-start stage.

The aforementioned methods can yield the global optimum given that sufficient modeling details and appropriate estimations are available. However, the double uncertainty, including the start-up time of generating unit and the amount of load pick-up, severely affects the performance of restoration process, which would further reduce the applicability of classical methods. Start-up time of generating unit is closely related to the characteristics of the thermal system, which can be represented as shutting down time of the unit [19]. Besides, the amount of load pick-up is fuzzy uncertain, owing to the influence of precipitation, temperature, cold pick-up and discrete access [20]. The distribution characteristics and membership function of uncertainty are hard to obtain, result in the limitation of fuzzy random method. Nevertheless, new ideas are provided by information gap decision theory (IGDT), which can tolerate the most uncertainty and still provides the desired performance [21]. The IGDT model neither need distribution characteristics nor membership function of uncertainties. This method has been widely adopted in purchasing strategy in electricity market [22] and optimal scheduling [23].

On this basis, an optimal source-load coordinated restoration method based on IGDT is proposed in this paper, which meets the restoration requirements in the fluctuation range of double uncertainty. The time-domain restoration characteristics of black-start unit (BSU), NBSU and load are described with analysis of double uncertainty. Based on that, two objectives are considered and coupled with each other, in which the generating power output of restarted units and the weighted amount of load pick-up both are maximized. Then IGDT is adopted to convert the uncertainty multi-objective optimization model to a certain one with robustness and non-dominated genetic algorithm II (NSGA-II) is applied to solve it. More specifically, this study makes the following contributions that:

- (1) A coupled multi-objective optimization model is built, which takes account of the double uncertainty in restoration process and realizes source-load coordinated restoration.
- (2) IGDT is adopted to power system restoration in order to convert the uncertainty multi-objective optimization model to a certain one with robustness. The requirement of distribution characteristics and membership function of uncertainties are avoided. Moreover, the optimal solution tolerates the most uncertainty and still meets the desired requirement.

The rest of the paper is arranged as follows: Section 2 presents the double uncertainty analysis. Section 3 presents the proposed source-load coordinated restoration model considering double uncertainty. Section 4 presents the solution to the model based on IGDT and NSGA-II. A case study is shown in Section 5. The conclusions are drawn in the last section.

2. Analysis of Double Uncertainty

In the process of source-load coordinated restoration, the double uncertainty including the start-up time of generating unit and the amount of load pick-up severely affect restoration time, generating

power output, and stability. Therefore, a study of double uncertainty is conducted in this section. The time-domain restoration characteristics of BSU, NBSU, and load are described with analysis of double uncertainty, which lays the foundation for later modeling.

2.1. Uncertainty of Start-Up Time of Generating Unit

First of all, a brief analysis of BSU i is carried out. The time-domain restoration characteristic is shown in Figure 1.

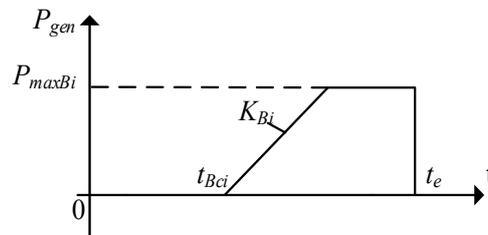


Figure 1. The time-domain restoration characteristic of BSU.

where K_{Bi} is the ramping rate of BSU i ; t_e is the entire restoration time for system; and, t_{Bci} is the time when BSU i begins to ramp up. BSU is not involved in optimization, owing to only one BSU in a system after blackouts. Therefore, suppose $t_{Bci} = 0$ can reduce complexity in later modeling. The generating power output function $P_{BSUi}(t)$ of BSU i can be expressed as:

$$P_{BSUi}(t) = \begin{cases} K_{Bi}t & 0 < t < \frac{P_{maxBi}}{K_{Bi}} \\ P_{maxBi} & \frac{P_{maxBi}}{K_{Bi}} < t \end{cases} \quad (1)$$

where P_{maxBi} is the maximum generating power output of BSU i .

In contrast, a brief analysis of NBSU j affected by uncertainty is carried out. The time-domain restoration characteristic is shown in Figure 2.

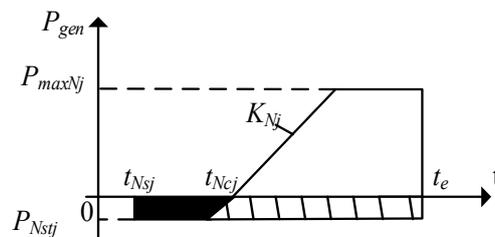


Figure 2. The time-domain restoration characteristic of NBSU.

NBSU j is being cranked, but not paralleled with system from t_{Nsj} to t_{Ncj} . After t_{Ncj} , NBSU j begins to ramp up. Obviously, the length of time when NBSU j is being cranked but not paralleled with system ($t_{Nsj} \sim t_{Ncj}$) is affected by the temperature and pressure of the thermal system, which is positively related to the shutting down time of the NBSU j ($0 \sim t_{Nsj}$). Thus, the start-up time of NBSU j is uncertain and $\overline{t_{Ncj}}$ is used to describe the uncertainty time when NBSU j begins to ramp up. K_{Bi} is ramping rate of NBSU j ; P_{maxNj} is the maximum generating power output of NBSU j ; P_{Nstj} is the

cranking power requirement of NBSU j . The generating power output function $P_{NBSUj}(t)$ of NBSU j can be expressed as:

$$P_{NBSUj}(t) = \begin{cases} 0 & 0 < t < t_{Nsj} \\ -P_{Nstj} & t_{Nsj} < t < \overline{t_{Ncj}} \\ K_{Nj}(t - \overline{t_{Ncj}}) & \overline{t_{Ncj}} < t < \frac{P_{\max Nj}}{K_{Nj}} + \overline{t_{Ncj}} \\ P_{\max Nj} & \frac{P_{\max Nj}}{K_{Nj}} + \overline{t_{Ncj}} < t \end{cases} \quad (2)$$

Although the distribution characteristics and membership function of $\overline{t_{Ncj}}$ are hard to obtain, the fluctuation range of the time from t_{Nsj} to $\overline{t_{Ncj}}$ is available by dispatcher. Therefore, $\overline{t_{Ncj}}$ can be expressed as:

$$\begin{cases} \overline{t_{su j}} = \overline{t_{Ncj}} - t_{Nsj} \\ \overline{t_{su j}} \in U(a_1, t_{Esuj}) \\ U(a_1, t_{Esuj}) = \{\overline{t_{su j}} : |(\overline{t_{su j}} - t_{Esuj}) / t_{Esuj}| \leq a_1\} \end{cases} \quad (3)$$

where $\overline{t_{su j}}$ is also uncertain, which represents the length of time when NBSU j is being cranked. t_{Esuj} is predicted length of that time based on experience and historical data, and a_1 is the uncertainty radius of $\overline{t_{su j}}$. In other words, values of $\overline{t_{su j}}$ whose deviation from t_{Esuj} is nowhere greater than $a_1 t_{Esuj}$.

2.2. Uncertainty of Amount of Load Pick-Up

The time-domain restoration characteristic about feeder k of load l is analyzed and shown in Figure 3.

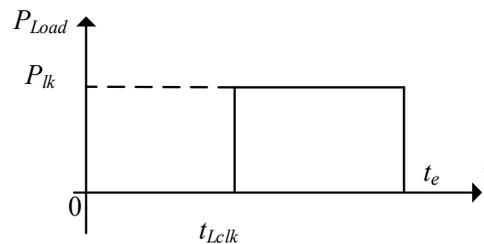


Figure 3. The time-domain restoration characteristic of load.

The time-domain restoration characteristic of load is simplified as a step function. Owing to the influence of precipitation, temperature, cold pick-up, and discrete access, P_{lk} is uncertain, which represents the amount of load pick-up in feeder k of load l . Therefore, $\overline{P_{lk}}$ is used to describe the uncertainty amount of load pick-up in feeder k of load l . t_{Lclk} is the time when feeder k of load l is restored. The function of load pick-up $P_{Loadlk}(t)$ can be expressed as:

$$P_{Loadlk}(t) = \begin{cases} 0 & 0 < t < t_{Lclk} \\ \overline{P_{lk}} & t_{Lclk} < t \end{cases} \quad (4)$$

Similarly, in order to accurately express the uncertainty amount of load pick-up for later modeling, $\overline{P_{lk}}$ can be expressed as:

$$\begin{cases} \overline{P_{lk}} \in U(a_2, P_{Elk}) \\ U(a_2, P_{Elk}) = \{\overline{P_{lk}} : |(\overline{P_{lk}} - P_{Elk}) / P_{Elk}| \leq a_2\} \end{cases} \quad (5)$$

where P_{Elk} is predicted amount of load pick-up in feeder k of load l ; a_2 is the uncertainty radius of $\overline{P_{lk}}$.

3. Source-Load Coordinated Restoration Model Considering Double Uncertainty

The main task of source-load coordinated restoration is to optimize the restoration sequence of NBSUs and loads. The interval between two units begin to restart is set as a time-step, in which important loads are coordinately restored to absorb excessive active power that is generated. The source-load coordinated restoration model is formulated as a multi-objective optimization problem; the optimization model, including objective function and constraints, are described in detail in this section.

3.1. Objective Function

3.1.1. Objective of Unit Restoration

The maximum generating power output of restarted units can be expressed as:

$$f_1 = \max \left(\sum_{i=1}^n \int_0^{t_e} P_{BSUi}(t) + \sum_{j=1}^m \int_0^{t_e} P_{NBSUj}(t) \right) \quad (6)$$

where n is the number of BSUs in the system; m is the number of NBSUs in the system.

It is not easy to solve the objective function containing the integral process. Therefore, the calculation of objective function is converted to that of the maximum area that is encircled by time-domain characteristics curve of units and X axis in this paper. The computational efficiency is obviously improved, which only needs simple algebraic operations. As mentioned above, the time when BSUs begin to ramp is the same as that when the system starts to restore. The first part in Equation (6) can be ignored, as it represents the generating output of BSUs, which is constant.

The second part in Equation (6) can be expressed as the area of white trapezoid in Figure 2. The following equation represents the generating power output of NBSU j .

$$\frac{1}{2} P_{NBSUj}(t) \left[(t - \overline{t_{Ncj}}) + (t - \frac{P_{NBSUj}(t)}{K_{Nj}} - \overline{t_{Ncj}}) \right] - (t - t_{Nsj}) P_{Nstj} \quad (7)$$

where t_{Nsj} is a decision variable to be solved. The maximum generating output of all the NBSUs in system can be expressed as:

$$f_1 = \max \sum_{j=1}^m P_{NBSUj}(t) \cdot (t - \overline{t_{su_j}} - 2t_{Nsj} - \frac{P_{NBSUj}(t)}{2K_{Nj}}) - \sum_{j=1}^m (t - t_{Nsj}) P_{Nstj} \quad (8)$$

3.1.2. Objective of Load Restoration

While the NBSUs are being restored, the important load should be taken into account to balance active power and accelerate restoration process. The maximum weighted amount of load pick-up can be expressed as:

$$f_2 = \max \sum_{l=1}^{N_k} \sum_{k=1}^{N_p} \omega_{lk} P_{Loadlk}(t) \quad (9)$$

where N_k is the number of load nodes in the system; N_p is the number of feeders belonging to load l . ω_{lk} is the weight of load in feeder k of load l . The weight is determined by Analytic Hierarchy Process with the type of load by Wang D. et al. [18], or even set according to the need and preference of the dispatcher.

In summary, two objectives are coupled with each other through time variable t . The objective function of the model is as follow:

$$\text{Max}(f_1, f_2) \quad (10)$$

3.2. Constraints

3.2.1. Cranking Power Constraint

The sum of cranking power for NBSUs to be restarted and the active power of loads pick-up must be less than the available generating power output provided by restarted units. Therefore, the cranking power constraint to NBSU j is as follows:

$$\sum_{i=1}^n P_{BSU_i}(t) + \sum_{j=1}^m P_{NBSU_j}(t) - \sum_{l=1}^{N_k} \sum_{k=1}^{N_p} P_{Loadlk}(t) - \sum_{j=1}^m (1 - u_j(t)) \cdot u_j(t + \Delta t) P_{Nstj} \geq 0 \quad (11)$$

There are four parts in Equation (11). The first and second parts represent generating power output provided by restored BSUs and NBSUs, respectively. The third part is the amount of load pick-up and the last part is the cranking power for NBSU j to be restarted. $u_j(t)$ indicates the state of NBSU j at time t . Only when $u_j(t) = 0$ and $u_j(t + \Delta t) = 1$, NBSU j begins to ramp up. NBSU should keep operating state once restarted. Assuming that NBSU j no longer shuts down after restarted, it can be expressed as follow:

$$u_j(t) < u_j(t + \Delta t) \quad (12)$$

3.2.2. Maximum Power Constraint to Load Pick-Up at One Time

The maximum power of load to be restored at one time must meet the requirements of the system stability; otherwise, the excessive voltage drop and frequency deviation may lead to failure of system restoration. The maximum power constraint to load pick-up at one time is given by:

$$\overline{P}_{lk} \leq \Delta f_{\max} \left(\sum_{i=1}^n \frac{P_{BSU_{ni}}}{f_{di}} + \sum_{j=1}^m \frac{u_j(t) P_{NBSU_{nj}}}{f_{dj}} \right) \quad (13)$$

where $P_{BSU_{ni}}$ and $P_{NBSU_{nj}}$ are the active rated power of BSU i and NBSU j , respectively; Δf_{\max} is the permitted frequency drop; f_{di} and f_{dj} are the frequency response rate of BSU i and NBSU j , which can be obtained from reference [24].

3.2.3. Reactive Power Constraint

The reactive power generated by restoration paths must be less than the possible maximum reactive power that is absorbed by BSUs; otherwise, the excessive reactive power may lead to sustained overvoltage and transformer over excitation. The reactive power constraint is given, as follows:

$$\sum_{p=1}^{N_p} Q_p < \sum_{i=1}^n Q_{Bi\max} \quad (14)$$

where N_p is the number of transmission lines in the restoration paths; Q_p is the charging reactive power of the transmission line p , $Q_{Bi\max}$ is the maximum reactive power absorbed by BSU i .

Besides, the charging reactive power of restoration paths may also lead to the self-excitation of BSU. In order to avoid this situation, the self-excitation constraint to BSU is expressed as:

$$\sum_{p=1}^{N_p} Q_p < \sum_{i=1}^n K_{Qi} S_{Bi} \quad (15)$$

where K_{Qi} and S_{Bi} are the short circuit ratio and the rated power of BSU i , respectively.

3.2.4. System Operating Constraint

The system operating constraint includes voltage and reactive power output of units.

$$\begin{cases} Q_{Gi\min} \leq Q_{Gi} \leq Q_{Gi\max} \\ Q_{Gj\min} \leq Q_{Gj} \leq Q_{Gj\max} \\ V_{i\min} \leq V_i \leq V_{i\max} \\ V_{j\min} \leq V_j \leq V_{j\max} \\ V_{l\min} \leq V_l \leq V_{l\max} \end{cases} \quad (16)$$

where Q_{Gi} and Q_{Gj} are the reactive power output of BSU i and NBSU j , respectively. $Q_{Gi\min}$ and $Q_{Gj\min}$ are the minimum reactive power output of BSU i and NBSU j , respectively. $Q_{Gi\max}$ and $Q_{Gj\max}$ are the maximum reactive power output of BSU i and NBSU j , respectively. V_i , V_j and V_l are voltage amplitude of BSU i , NBSU j and load l , respectively. $V_{i\max}$, $V_{j\max}$ and $V_{l\max}$ are the permitted voltage upper limits of BSU i , NBSU j and load l , respectively. $V_{i\min}$, $V_{j\min}$ and $V_{l\min}$ are the permitted voltage lower limits of BSU i , NBSU j and load l , respectively.

3.2.5. Maximum Critical Hot-Start Time Constraint

To make the unit restarted within the maximum critical time, the maximum critical hot-start limitation of NBSU must be longer than the time when the NBSU begins to ramp up. Therefore, the maximum critical hot-start constraint to NBSU is given by:

$$0 < t_{Nsj} < t_{CH} \quad (17)$$

where t_{CH} is the maximum critical hot-start limitation of NBSU.

4. The Solution to Source-Load Coordinated Restoration Model Considering Double Uncertainty

IGDT is adopted to convert uncertainty multi-objective optimization model to a certain one, which tolerates the most uncertainty and still meets the desired requirement. When compared with the fuzzy random method, IGDT neither needs distribution characteristics nor membership function of uncertainties. Finally, the specific restoration scheme can be solved by NSGA-II.

4.1. Information Gap Decision Theory

IGDT can build a robust model based on different requirements owing to the different preferences of decision-maker. The robustness is defined as the immunity of the minimum requirement satisfaction at presence of uncertain parameters [22].

For a given uncertainty optimization model:

$$\begin{cases} \max B(\bar{X}, d) \\ s.t. \quad H(\bar{X}, d) = 0 \\ \quad \quad G(\bar{X}, d) \leq 0 \end{cases} \quad (18)$$

where \bar{X} is the uncertainty set describing the uncertain input parameters. d is the decision variables set. $B(\bar{X}, d)$ represents the optimization objective. $H(\bar{X}, d) = 0$ and $G(\bar{X}, d) \leq 0$ are quality and inequality constrains, respectively.

The uncertainty is defined as the distance between what is known and what may happen in reality [22]. Therefore, the envelope bound model is used to represent the prior information about the uncertain parameters \bar{X} , as follows [22]:

$$\begin{cases} \bar{X} \in U(a, X_E) \\ U(a, X_E) = \{ \bar{X} : |(\bar{X} - X_E) / X_E| \leq a \} \end{cases} \quad (19)$$

where X_E is the predicted value of the uncertain parameters \bar{X} ; a is the uncertainty radius of \bar{X} ; $U(a, X_E)$ is the set of all values of \bar{X} whose deviation from X_E is nowhere greater than $a\bar{X}$.

The uncertainty optimization model of Equation (18) can be converted to the certainty model based on the predicted value of the uncertain parameters \bar{X} as follows:

$$\begin{cases} \max B(X_E, d) \\ \text{s.t.} & H(X_E, d) = 0 \\ & G(X_E, d) \leq 0 \end{cases} \quad (20)$$

By solving Equation (20), the basic value of objective function is B_0 . However, it is obvious that the optimal solution of Equation (20) is not reliable while the value of \bar{X} may be different from the predicted value X_E . In order to guarantee the optimization effect, decision maker sets a minimum requirement B' as not surpassing a predefined limit.

$$B' = (1 - \delta)B_0 \quad \delta \in [0, 1) \quad (21)$$

where δ is a deviation factor, which is the degree of deviation between minimum requirement and the optimal solution of the certainty model. The more δ is, the bigger value of uncertainty radius of \bar{X} can be tolerated.

Based on these, the uncertainty optimization model of Equation (18) can be finally converted to solve the maximum value of uncertainty radius of \bar{X} when the minimum requirement is always satisfied. The robust model can be expressed as:

$$\begin{cases} \max a \\ \text{s.t.} & \min B(X_E, d) \geq B' \\ & B' = (1 - \delta)B_0 \\ & H(X_E, d) = 0 \\ & G(X_E, d) \leq 0 \end{cases} \quad (22)$$

The decision scheme of Equation (22) can tolerate the maximum value of uncertainty radius of \bar{X} and meet the minimum requirement. Owing to the uncertainty set \bar{X} in the model has been replaced by predicted value X_E , the original objective function and constraints need to be modified to correspond with the actual requirements.

4.2. Source-Load Coordinated Restoration Model Based on IGDT

The uncertainty optimization model and envelope bound model of double uncertainty have been established above. Therefore, the maximum generating power output B_1 and the weighted amount of load pick-up B_2 are calculated by Equations (8) and (9), based on the predicted value t_{Esuj} and P_{Elk} . Dispatcher set B_c and B_d as the minimum requirements of generating output and the amount of load pick-up, respectively.

$$\begin{cases} B_c = (1 - \delta_1)B_1 \\ B_d = (1 - \delta_2)B_2 \end{cases} \quad (23)$$

According to the idea of IGDT, two objective functions are converted to constraints in a new robust model, as follows:

$$\min \left[\sum_{j=1}^m P_{NBSUj}(t) \cdot (t - \overline{t_{suj}} - 2t_{Nsj} - \frac{P_{NBSUj}(t)}{2K_{Nj}}) - \sum_{j=1}^m (t - t_{Nsj})P_{Nstj} \right] \geq B_c \quad (24)$$

$$\min \sum_{l=1}^{N_k} \sum_{k=1}^{N_p} \omega_{lk} P_{Loadlk}(t) \geq B_d \quad (25)$$

The generating power output is minimal when the output of each NBSU is the minimum. Therefore, Equation (24) should be modified as:

$$\min\left(\sum_{j=1}^m P_{NBSUj}(t) \cdot (t - (1 + \alpha_1)t_{Esuj} - 2t_{Nsj} - \frac{P_{NBSUj}(t)}{2K_{Nj}}) - \sum_{j=1}^m (t - t_{Nsj})P_{Nstj}\right) \geq B_c \quad (26)$$

The weighted amount of load pick-up is minimal when the amount of load restored in each feeder is the minimum. Therefore, Equation (25) should be modified as:

$$\min\sum_{l=1}^{N_k} \sum_{k=1}^{N_p} \omega_{lk} P_{Elk}(t - t_{clk})(1 - \alpha_2) \geq B_d \quad (27)$$

Besides, the constraints also need to be modified.

Similarly, Equation (11) should be modified as the cranking power constraint to NBSU j when the output of each NBSU is the minimum and the amount of load that is restored in each feeder is the maximum.

$$\sum_{i=1}^n P_{BSUi}(t) + \sum_{j=1}^m (1 - \alpha_1)P_{NBSUj}(t) - \sum_{l=1}^{N_k} \sum_{k=1}^{N_p} (1 + \alpha_2)P_{Loadlk}(t) - \sum_{j=1}^m (1 - u_j(t)) \cdot u_j(t + \Delta t)P_{Nstj} \geq 0 \quad (28)$$

Equation (13) should be modified as maximum power constraint to load pick-up at one time when the amount of load restored in each feeder is the maximum.

$$(1 + \alpha_2)P_{Elk} \leq \Delta f_{\max} \left(\sum_{i=1}^n \frac{P_{BSUni}}{f_{di}} + \sum_{j=1}^m \frac{u_j(t)P_{NBSUnj}}{f_{dj}} \right) \quad (29)$$

Equations (12) and (14)–(17) do not need further modification, owing to no relationship with double uncertainty. In summary, source-load coordinated restoration model based on IGDT can be expressed as:

$$\begin{cases} \max & \alpha_1, \alpha_2 \\ \text{s.t.} & 0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1 \\ & \text{Equations (12) and (14) to Equations (17) and (26) to Equation (29)} \end{cases} \quad (30)$$

Source-load coordinated restoration scheme calculated by this model is robust to double uncertainty. More specifically, the restoration scheme can guarantee that the generating power output and the amount of load pick-up are no less than $(1 - \delta_1)B_1$ and $(1 - \delta_2)B_2$ when the length of cranking time fluctuates within $(1 - a_1, 1 + a_1)$ and the amount of load pick-up fluctuates within $(1 - a_2, 1 + a_2)$.

4.3. The Solution to Source-Load Coordinated Restoration Model Based on IGDT

NSGA-II which is one of the most effective and efficient algorithms for solving multi-objective problems is applied to solve the certain model with robustness [25]. Its main characteristics are the following:

- (1) Two fitness functions, given by a_1 and a_2
- (2) Integer codification: each individual is described by the number of s ($s = m + l \times k$) variables that are describing the time when NBSUs start to be cranked and loads are restored. Although time is not continuous and integer, Haffner S built conversion that can transform the codified variables back to the original variables [26], as follows:

$$x_i = x_{\min} + y_i \Delta x_i \quad 0 \leq y_i \leq \frac{x_{\max} - x_{\min}}{\Delta x_i}, y_i \in Z \quad (31)$$

where x_i is original variable, x_{imax} and x_{imin} are upper and lower values of x_i . y_i is integer variable. This paper set 1 min as Δx_i .

- (3) Use of non-dominated sorting and crowding distance [27].

Based on NSGA-II algorithm, the specific solving progress shown in Figure 4 is as follows:

- (1) Input basic system data.
- (2) Set up the deviation factors δ_1 and δ_2 , basic parameters of NSGA-II including number of individuals in the population n_{pop} , maximum number of generation g_{max} , crossover probability p_c , mutation probability p_m .
- (3) Generate $3n_{pop}$ individuals randomly and among them select n_{pop} with distinct characteristics and constraints satisfaction as parent population P_t .
- (4) Create offspring population Q_t from P_t by using the tournament selection, crossover and mutation operators.
- (5) Combine Q_t and P_t to create intermediate population $R_t = P_t \cup Q_t$. Check constraint conditions of individuals and punish them by subtracting a large number on the fitness values a_1 and a_2 both.
- (6) Perform a non-dominated sorting and crowding distance calculation to R_t and select the first n_{pop} individuals as new parent population P_{t+1} .
- (7) Check whether the result has reached the maximum number of iterations. If not, turn to (4), else turn to (8).
- (8) Output the Pareto-optimal front.

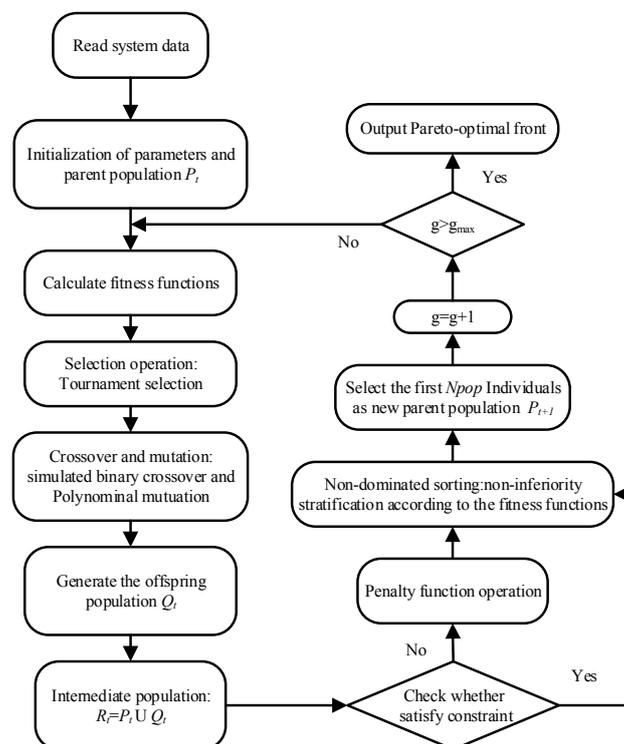


Figure 4. The flowchart of solution to source-load coordinated restoration model based on information gap decision theory (IGDT).

5. Case Study

The IEEE 39-bus shown in Figure 5 is used as a case study to verify the effectiveness of the proposed method. The optimization program was developed with the programming tool of

MATLAB. Suppose that the unit at bus 33 is used as the BSU, with rated power 600 MW, $\cos \varphi = 0.9$, $Q_{Bmax} = 0.3S_N$, $K_Q = 0.45$, and it is successfully restarted at the instant 0 h as the beginning time of the system restoration. The rest buses are NBSUs and loads to be restored. All of the units are subjected to a maximum critical hot start time of 1.5 h. The numbers and parameters of the units are assumed as that in Table 1. The parameters of the loads are assumed as that in Table 2. The feeder of load is represented as $X(y)$, in which X is the node number and y is the feeder number. The power factors for all the loads are set to 0.8.

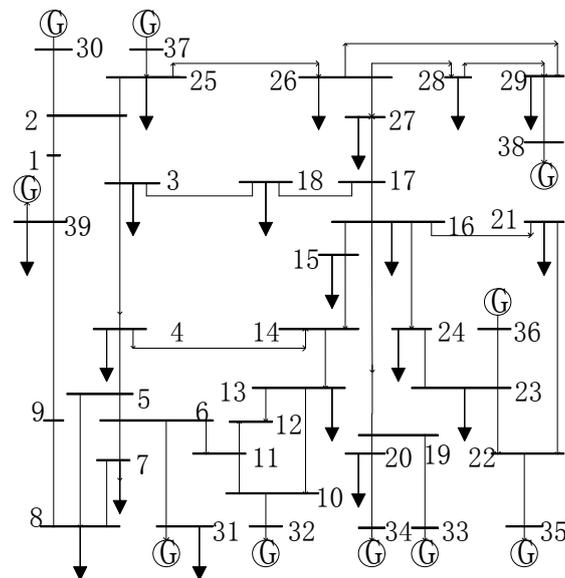


Figure 5. New England 10-unit 39-bus power system.

Table 1. The parameters of the units.

Unit	Rated Power/MW	Cranking Power/MW	Ramping Rate/MW/h	Predicted Cranking Time/h
30	350	17.5	127.27	0.67
31	600	30.0	160.00	0.50
32	250	12.5	108.23	0.67
33	600	0.0	260.00	0.00
34	300	15.0	179.64	1.10
35	250	12.5	149.70	0.83
36	330	16.5	120.00	1.10
37	320	16.0	160.00	1.00
38	300	15.0	200.00	0.83
39	300	15.0	179.64	0.83

The parameters of NSGA-II are set as follows: the number of individuals in the population is 200, maximum number of generation is 20, crossover probability is 0.8 and the mutation probability is 0.1. Besides, $\Delta x_i = 1$ min. The entire restoration time for system is 2 h. The charging time for each transmission line is 2 min.

Table 2. The parameters of the loads.

Load	Predicted Amount/MW	Weight	Load	Predicted Amount/MW	Weight
3(1)	16.1	0.5400	20(4)	17	0.5012
3(2)	16.1	0.5260	21(1)	13.7	0.5496
4(1)	12.5	0.5556	21(2)	13.7	0.5276
4(2)	12.5	0.5572	23(1)	24.7	0.5696
4(3)	12.5	0.5204	24(1)	15.4	0.5468
4(4)	12.5	0.5568	24(2)	15.4	0.5792
7(1)	23.3	0.4820	25(1)	22.4	0.5000
8(1)	17.4	0.4832	26(1)	13.9	0.5356
8(2)	17.4	0.5836	27(1)	14.0	0.5884
8(3)	17.4	0.5016	27(2)	14.0	0.5272
12(1)	8.5	0.5728	28(1)	20.6	0.4728
15(1)	16.0	0.5908	29(1)	14.1	0.5224
15(2)	16.0	0.5480	29(2)	14.1	0.4940
16(1)	10.9	0.5560	31(1)	9.2	0.5800
16(2)	10.9	0.5472	31(2)	9.2	0.5236
16(3)	10.9	0.5612	39(1)	22.0	0.5360
18(1)	15.8	0.5836	39(2)	22.0	0.5756
20(1)	17	0.5184	39(3)	22.0	0.4800
20(2)	17	0.5208	39(4)	22.0	0.5656
20(3)	17	0.5496	39(5)	22.0	0.5480

5.1. Result of Source-Load Coordinated Restoration

Firstly, the maximum generating power output B_1 and the weighted amount of load pick-up B_2 are calculated by Equations (8) and (9) based on the predicted value t_{Esuj} and P_{Elk} . The unique solution can be obtained, in which the generating power output is 797.25 MW, the weighted amount of load pick-up is 343.23. The units and loads coordinately restored from 0 min to 85 min. After 85 min, only loads are restored further. The source-load coordinated restoration scheme without considering double uncertainty is shown in Table 3. The interval between two units begin to restart is set as a time-step.

Table 3. The source-load coordinated restoration scheme without considering double uncertainty.

Time-Step	Time/min	Unit Restoration	Load Restoration	Amount of Load Pick-Up/MW
1	0–23	31	8(2), 15(1), 18(1), 27(1)	62.8
2	23–36	38	24(2), 31(1), 39(2)	46.6
3	36–47	39	23(1), 16(3)	35.6
4	47–54	34	20(3)	17.0
4	54–61	37	4(4), 39(4)	34.5
5	61–65	30	12(1)	8.5
7	65–73	35	4(2), 16(1), 39(5)	45.4
8	73–77	36	4(1)	12.5
9	77–85	32	15(2), 16(2), 21(1)	40.6

Then, double uncertainty is considered by setting the deviation factors δ_1 and δ_2 to determine minimum requirements B_c and B_d . For example, δ_1 is assumed as 0.03 and δ_2 is assumed as 0.2. Through solving the multi-objective optimization model that is built in this paper, the Pareto-optimal front can be obtained, as shown in Figure 6.

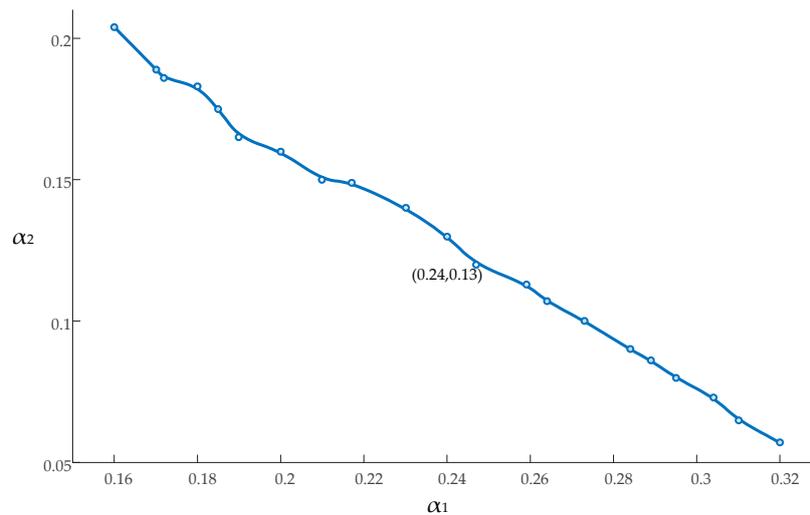


Figure 6. Pareto-optimal front.

The bigger values of uncertainty radius a_1 and a_2 are, the more robust restoration process is. In order to achieve the balance between a_1 and a_2 , the midpoint of Pareto-optimal (0.24, 0.13) is selected, the corresponding source-load coordinated restoration scheme is shown in Table 4.

Table 4. The source-load coordinated restoration scheme considering double uncertainty.

Time-Step	Time/min	Unit Restoration	Load Restoration	Amount of Load Pick-Up/MW
1	0–23	31	8(2), 31(1), 18(1), 27(1)	56.4
2	23–34	39	15(1), 16(3)	26.9
3	34–43	38	39(2)	22.0
4	43–49	30	12(1)	8.5
4	49–56	34	39(4)	22.0
5	56–64	35	23(1), 24(2)	40.1
7	64–70	36	16(1), 16(2)	21.8
8	70–78	32	4(1), 4(2), 4(4)	12.5
9	78–84	37	39(5)	40.6

The amount of load pick-up is 257.27 MW to balance the active power in system, accounting for eight percent of the generating output when the system is in normal operation.

5.2. Analysis of Optimization Characteristics

In order to further investigate the optimization characteristics of this method, the load restoration and unit restoration are optimized with the consideration of uncertainty separately. The generating power output and the weighted amount of load pick-up are compared under different deviation factors.

Firstly, the optimization characteristic of unit restoration is investigated. δ_2 is assumed as 0, δ_1 is changed to determine the minimal requirement of generating power output. The midpoint of Pareto-optimal is selected to analyze. The values of uncertainty radius a_1 and corresponding unit restoration scheme are shown in Table 5. It can be seen that the generating power output is decreasing with the increase of the deviation factor δ_1 .

Table 5. The unit restoration scheme considering double uncertainty.

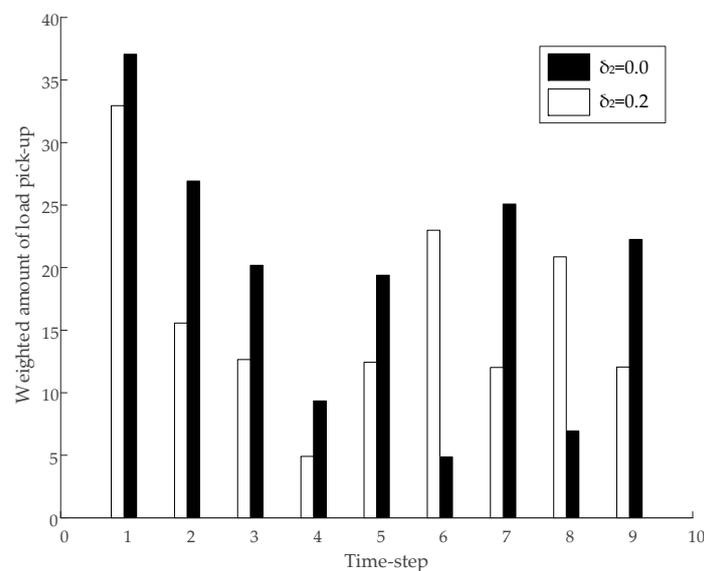
δ_1	a_1	Unit Restoration		Generating Power Output/MW
0.03	0.24	Sequence Time/min	31, 39, 38, 30, 34, 35, 36, 32, 37 23, 34, 43, 49, 56, 64, 70, 78, 84	774.59
0.06	0.53	Sequence Time/min	31, 32, 39, 36, 30, 34, 35, 37, 38 23, 32, 43, 49, 55, 61, 67, 73, 85	750.39
0.09	0.77	Sequence Time/min	38, 31, 39, 30, 35, 34, 32, 37, 36 17, 36, 47, 53, 62, 68, 75, 80, 85	731.49

Then, the optimization characteristic of load restoration is investigated similarly. δ_1 is assumed as 0, δ_2 is changed to determine the minimal requirement of weighted amount of load pick-up. The first time-step is taken as an example. The values of uncertainty radius a_2 and corresponding load restoration scheme are shown in Table 6.

Table 6. The load restoration scheme in first time-step considering double uncertainty.

δ_2	a_2	Load Restoration	Amount of Load Pick-Up/MW
0.2	0.13	8(2), 31(1), 18(1), 27(1)	56.4
0.4	0.25	4(2), 15(1), 27(1)	42.5
0.6	0.47	27(1), 27(2)	28.0
0.75	0.83	12(1), 31(1)	17.7

It can be seen that the load restoration schemes can be obtained through the optimization model that is proposed under different uncertainty radius a_2 . The dispatcher can look up the table to select the appropriate load restoration scheme according to the approximate fluctuation range of load pick-up based on historical data. A comparison between load restoration scheme considering double uncertainty and that considering ideal situation is shown in Figure 7.

**Figure 7.** A comparison of load restoration scheme considering double uncertainty.

It can be seen from Figure 7 that except time-step 6 and 8, the weighted amount of load pick-up considering double uncertainty is lower than the ideal value not only in a single time-step, but also in the entire restoration process. The essence of IGDT is to sacrifice part of the load pick-up and enhance the robustness of the system restoration.

Finally, the relationship between uncertainty radius and deviation factors is analyzed. $\delta_1 - \alpha_1$ and $\delta_2 - \alpha_2$ are shown in Figure 8.

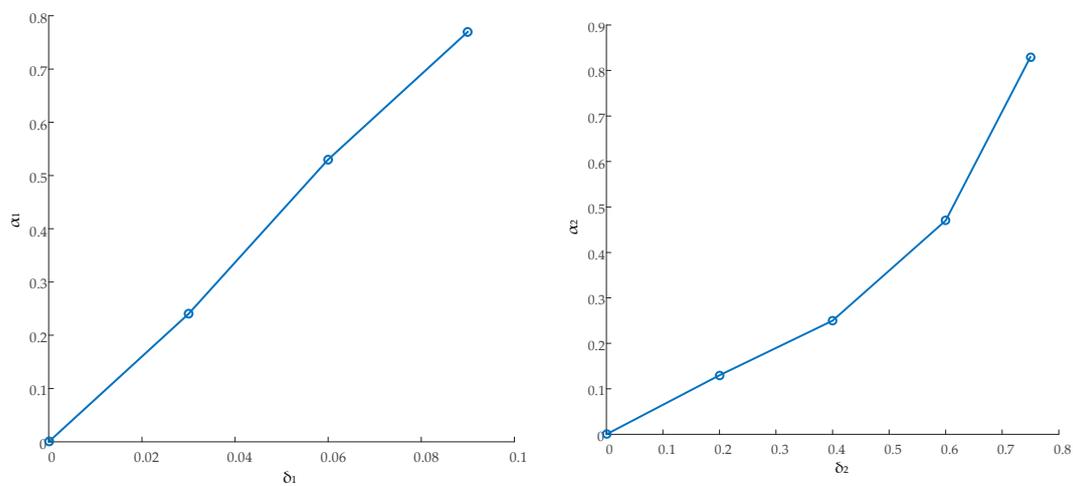


Figure 8. The relationship between uncertainty radius and deviation factors.

The deviation factor has a positive correlation with the maximum uncertainty radius. Specifically, the smaller requirement of generating output is, the longer the start-up time of a unit can be tolerated. The smaller requirement of weighted amount of load pick-up is, the greater fluctuation of load pick-up in each feeder can be tolerated.

5.3. Comparison of Optimization Characteristics

To further verify the advantages of the proposed source-load coordinated restoration method based on IGDT, the proposed method in this paper is compared to fuzzy chance constrained programming. Assuming that the start-up time of generating unit and the amount of load pick-up are fluctuated around the predicted value in (0.8, 1.2). Double uncertainty is expressed by triangular fuzzy parameters and confidence intervals are all assumed as 0.8. The weighted amount of load pick-up in each time-step based on different methods is recorded, as shown in Figure 9. If the constraints are not satisfied, then the weighted amount of load pick-up is 0.

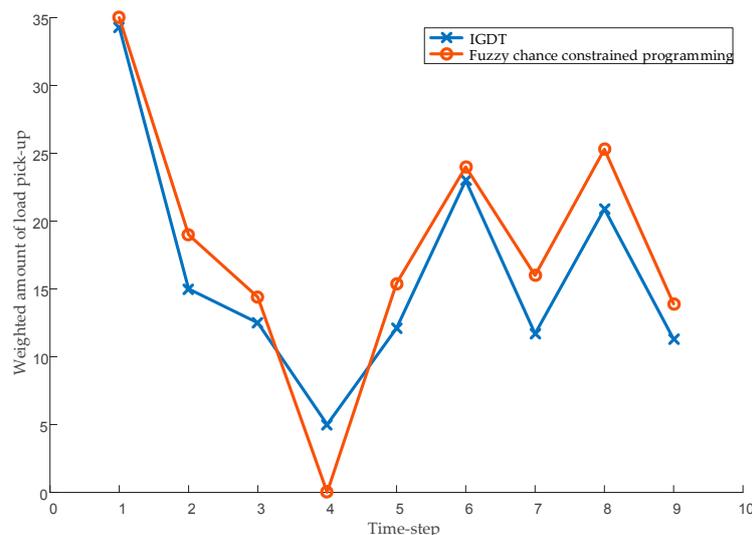


Figure 9. A comparison of load restoration scheme based on different methods.

The source-load coordinated restoration scheme based on fuzzy chance constrained programming is shown in Table 7.

Table 7. The source-load coordinated restoration scheme based on fuzzy chance constrained programming.

Time-Step	Time/min	Unit Restoration	Load Restoration	Amount of Load Pick-Up/MW	Frequency Deviation/Hz
1	0–25	31	8(2), 15(1), 18(1), 27(1)	62.8	0.486
2	25–38	32	16(1), 23(1)	35.6	0.320
3	38–44	30	39(2)	22.0	0.231
4	44–56	39	4(2), 24(2), 4(1), 39(4), 12(1), 31(1)	90.1	0.513
5	56–60	35	16(2), 4(4)	23.4	0.193
6	60–66	34	39(5), 24(1)	37.4	0.297
7	66–70	37	3(1), 15(2)	32.1	0.195
8	70–75	36	20(3), 39(1)	39.0	0.223
9	75–81	38	21(1), 16(3), 4(3)	38.1	0.204

It can be seen that the source-load coordinated restoration scheme based on fuzzy chance constrained programming can not meet the security constraints in time-step 4. The descending value of frequency is more than 0.5 Hz, which exceeds the limit.

In summary, the source-load coordinated restoration method based on IGDT can tolerate the fluctuation of the start-up time of the generating unit and the amount of load pick-up. Although the weighted amount of load pick-up is smaller than fuzzy chance constrained programming, it is more robust in the overall restoration process.

6. Conclusions

An optimal source-load coordinated restoration model considering double uncertainty is proposed, in which unit and load are restored coordinately in the manner of different priorities. Firstly, double uncertainty including the start-up time of generating units and the amount of load pick-up is analyzed to build time-domain characteristics of unit and load. On this basis, a coupled multi-objective optimization model is established with uncertainties; the units to be restarted can be determined by maximizing the generating power output, while the loads to be restored can be determined by maximizing the weighted amount of load pick-up. Then, IGDT is adopted to convert uncertainty multi-objective optimization model to a certain one with robustness. NSGA-II is applied to solve the proposed model. Finally, The test results on the IEEE 39-bus system show that unit and load can be effectively and coordinately restored, which tolerates the most uncertainty and meet the minimal requirements. The proposed method improves the robustness of restoration and provides a useful way for decision making of the restoration scheme.

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