## Article

# A Mixed WLS Power System State Estimation Method Integrating a Wide-Area Measurement System and SCADA Technology 

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#### Abstract

To address the issue that the phasor measurement units (PMUs) of wide area measurement system (WAMS) are not sufficient for static state estimation in most existing power systems, this paper proposes a mixed power system weighted least squares (WLS) state estimation method integrating a wide-area measurement system and supervisory control and data acquisition (SCADA) technology. The hybrid calculation model is established by incorporating phasor measurements (including the node voltage phasors and branch current phasors) and the results of the traditional state estimator in a post-processing estimator. The performance assessment is discussed through setting up mathematical models of the distribution network. Based on PMU placement optimization and bias analysis, the effectiveness of the proposed method was proved to be accurate and reliable by simulations of different cases. Furthermore, emulating calculation shows this method greatly improves the accuracy and stability of the state estimation solution, compared with the traditional WLS state estimation.


Keywords: power system; state estimation; wide-area measurement system; weighted least squares; mixed algorithm

## 1. Introduction

Electric power is essential to modern society. Economic prosperity, national security, and standard of living depend on reliable electric power systems, and it is very important for the power systems to obtain operating condition information about the state of the electric grid [1]. In the 1970s, Schweppe firstly proposed the idea of state estimation in power systems [2]. Power system state estimation constitutes the core of the on-line system monitoring, analysis and control functions [3,4]. State estimation acts like a filter between the raw measurements received from the system and all the application functions that require the most reliable data for the current system operation state, and it typically includes bad data processing, state estimation solutions, parameter and topology error processing, and other analyses [5].

In modern power systems, the control center receives the system-wide device information and measurement mainly through a supervisory control and data acquisition (SCADA) system [6,7]. However, the information and measurement data provided by SCADA may not always be accurate and reliable. On the other hand, the collected measurements may not allow direct extraction of the corresponding real-time AC operation state of the system. These concerns drive the development of power system technology. Among all the newly-developed applications that aim at satisfying those new technological demands, the so-called wide area measurement system (WAMS) opens a new avenue for power system stability analysis and control, and it has been attracting increasing attention in recent years, since it is a powerful tool for power system monitoring, protection and control, and has been widely used in the energy management systems of power systems [8-11]. WAMS measurements
have high precision and short updating cycles and can measure the node states directly, but due to the high costs of PMU devices and their placement problems, how to utilize WAMS measurements and SCADA information in state estimation effectively must be faced and solved now [12,13].

At the same time, power system state estimation uses a SCADA system to collect the real-time measurement data, which involves injection measurements, line flow measurements and voltage magnitude measurements [14,15]. However, the accuracy of SCADA system is low, and it cannot directly measure the voltage, current and phase angle. Therefore, the accuracy and state estimation effectiveness will be effected greatly [16-18]. The use of phasor measurements in power system state estimation is investigated in [19,20]. In [21], a post processing linear estimator incorporating the phasor measurements and the results of the traditional state estimator is studied. In [22,23], the inclusion of real and imaginary part of the complex current measurement in an existing WLS estimator is described, and their convergence and accuracy performance was presented. In [24], the effect of node voltage phasor measurement of synchronous phasor measurement unit (PMU) on state estimation algorithm is studied. Its disadvantage is that it cannot take advantage of the majority of measurements in wide area measurement systems. A linear state estimation algorithm using the node voltage phasors and branch current phasors is proposed in [25,26]. The algorithm has a fast computation speed due to the linear measurement equations and the constant Jacobian matrix [27,28]. But it requires the installation of enough PMU measuring to ensure the system is predictable. In [29], the placement of the PMUs on the buses has been presented using intelligent algorithms, and a system observability analysis method in [30] is proposed to ensure fast and reliable results. It is known that, the PMUs of WAMS are not sufficient for static state estimation in most of the existing power systems because of the high price [31-33]. So, the state estimation with enough PMU measurements cannot be carried out normally at present. Therefore, a mixed power system weighted least squares (WLS) state estimation method, which uses the PMU measurements to supplement SCADA measurements, has a practical significance.

This paper is organized as follows: the wide-area state estimation model and the proposed mixed WLS theory is analyzed in Section 2. The performance assessment through simulations using IEEE 14, 30, and 118 bus systems is validated in Section 3. Conclusions are drawn in Section 4.

## 2. A State Estimation Method Integrating WAMS and SCADA

In this section, a state estimation model with PMU and SCADA data is elaborated. The mixed measurements of the Jacobian matrix and residual vector are formed by introducing the branch current phasors. Motivated by the desire to improve precision of state estimation, an alternative solution and detailed procedures are proposed.

### 2.1. Model Analysis of Power System State Estimation

In power state estimation methods, the most commonly used measurements are line power flow, bus power injection and voltage magnitude measurements. In some cases, especially for state estimation of distribution buses, the line current magnitude measurements may be taken into consideration as well. Where PMUs exist, there will be two new types of measurements utilized by power system state estimation methods. One is voltage phasor measurements, which include voltage magnitudes and angles of the system bus, and the other is current phasor measurements, which include current magnitudes and angles of the transmission lines or transformers [34]. All types of measurements can be expressed in terms of the system state using Equation (1):

$$
Z=\left[\begin{array}{c}
z_{1}  \tag{1}\\
z_{2} \\
\vdots \\
z_{m}
\end{array}\right]=\left[\begin{array}{c}
h_{1}\left(x_{1}, x_{2}, \cdots, x_{n}\right) \\
h_{2}\left(x_{1}, x_{2}, \cdots, x_{n}\right) \\
\vdots \\
h_{m}\left(x_{1}, x_{2}, \cdots, x_{n}\right)
\end{array}\right]+\left[\begin{array}{c}
e_{1} \\
e_{2} \\
\vdots \\
e_{m}
\end{array}\right]=h(X)+e
$$

where $Z$ is the measured value, $h(x)$ is a nonlinear function, $X$ is the system state vector, including the voltage magnitudes and phase of all the buses excluding the reference bus phase angle, and $e$ is the measurement error of measurement $i$.

The sum of the squares of the measurement errors is used as objective function:

$$
\begin{equation*}
J(X)=[Z-h(X)]^{T} R^{-1}[Z-h(X)] \tag{2}
\end{equation*}
$$

where $R$ is the covariance matrix of the measurement error $e$. The optimization problem is to minimize $J(X)$, i.e.,

$$
\begin{equation*}
\frac{\partial J(X)}{\partial X}=0 \tag{3}
\end{equation*}
$$

Using Taylor series method, and ignoring the higher order derivative, one has:

$$
\begin{equation*}
h\left[X^{(k+1)}\right]=h\left(X^{(k)}\right)+\left.\frac{\partial h(X)}{\partial X}\right|_{X=X^{(k)}} \Delta X \tag{4}
\end{equation*}
$$

where $k$ is the iteration index, $X^{(k)}$ is the solution vector at the $k$-th iteration. Here $H(X)=\frac{\partial h(X)}{\partial X}$ can be named as Jacobian matrix, and it can be expressed as:

$$
H(X)=\frac{\partial h(X)}{\partial X}=\left[\begin{array}{cccc}
\frac{\partial h_{1}(x)}{x_{1}} & \frac{\partial h_{1}(x)}{x_{2}} & \ldots & \frac{\partial h_{1}(x)}{x_{n}}  \tag{5}\\
\frac{\partial h_{2}(x)}{x_{1}} & \frac{\partial h_{2}(x)}{x_{2}} & \ldots & \frac{\partial h_{2}(x)}{x_{n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial h_{m}(x)}{x_{1}} & \frac{\partial h_{m}(x)}{x_{2}} & \ldots & \frac{\partial h_{m}(x)}{x_{n}}
\end{array}\right]
$$

Using the Newton iterative method, and substituting Equation (4) into Equation (3), the linear equation can be described as:

$$
\begin{equation*}
H^{T} R^{-1}\left[Z-h\left(X^{(k)}\right)-H \Delta X^{(k)}\right]=0 \tag{6}
\end{equation*}
$$

Defining $\Delta Z=Z-h\left(X^{(k)}\right)$, so:

$$
\begin{equation*}
H^{T} R^{-1} H \Delta X^{(k)}=H^{T} R^{-1} \Delta \mathrm{Z} \tag{7}
\end{equation*}
$$

$G=H^{T} R^{-1} H$ is called the gain matrix. It is sparse positive definite and symmetric if the system is fully observable. State estimation involves the iterative solution of Equation (11), and the initial state estimation data, so the related equivalent circuit of different power system components, admittance matrix, power injection, power flow, should be researched firstly.

In a power system, transmission lines can be seen as middle-length transmission and the conductance do not need to take into account, then the equivalent circuit of transmission lines is as shown in Figure 1. In this figure, $Z$ indicates impedance and $Y / 2$ indicates admittance. Based on Kirchhoff's law, we can get:

$$
\left[\begin{array}{c}
\dot{U}_{k}  \tag{8}\\
\dot{I}_{k}
\end{array}\right]=\left[\begin{array}{cc}
1+\frac{Z Y}{2} & Z \\
Y\left(1+\frac{Z Y}{4}\right) & 1+\frac{Z Y}{2}
\end{array}\right]\left[\begin{array}{c}
\dot{U}_{m} \\
\dot{I}_{m}
\end{array}\right]
$$

According to tap changing and phase shifting transformers, Figure 2 is the typical equivalent circuit. The two transformer terminal buses are commonly designated as the impedance side and the tap side bus, respectively.


Figure 1. Equivalent circuit for transmission line.


Figure 2. Equivalent circuit for a transformer.
From a transformer's features, it is easy to know $\stackrel{\bullet}{U}_{k}=\left(\dot{U}_{m}-\stackrel{\bullet}{I}_{m} / Y\right) \times a, \stackrel{\bullet}{I}_{k}=-\stackrel{\bullet}{I}_{m} / \stackrel{\bullet}{a}$, and the node injection current can be determined by:

$$
\left[\begin{array}{c}
\bullet_{I}  \tag{9}\\
\dot{\bullet}_{m} \\
I_{m}
\end{array}\right]=\left[\begin{array}{cc}
Y /|\stackrel{\bullet}{a}|^{2} & -Y /{ }^{*} \\
-Y / \stackrel{\bullet}{a} & Y
\end{array}\right]\left[\begin{array}{c}
\dot{U}_{k} \\
\dot{U}_{m}
\end{array}\right]
$$

where the tap ratio $\dot{a}$. is complex, and $\stackrel{*}{a}$ represents the conjugate complex of $\dot{a}$.
Generators are normally presented in the power system. The equivalent circuit of generators is divided into voltage and current equivalent circuits, as shown in Figure 3a,b.

The reactance of generators is expressed by Equation (10), where $X_{G} \%$ is the reactance percent value of generators, $U_{G N}$ is the rated voltage of generators, $S_{N}$ is the nominal power of generators:

$$
\begin{equation*}
X_{G}=\frac{X_{G} \% U_{G N}^{2}}{100 S_{N}} \tag{10}
\end{equation*}
$$

Using Kirchhoff's current law at each bus to build the network model for the entire power system, the following equation can be obtained:

$$
\dot{I}=\left[\begin{array}{c}
\bullet  \tag{11}\\
\dot{I}_{1} \\
\dot{I}_{2} \\
\vdots \\
\dot{I_{N}}
\end{array}\right]=\left[\begin{array}{cccc}
Y_{11} & \Upsilon_{12} & \cdots & \Upsilon_{1 N} \\
Y_{11} & \Upsilon_{11} & \cdots & \Upsilon_{11} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{11} & \Upsilon_{11} & \cdots & \Upsilon_{11}
\end{array}\right]\left[\begin{array}{c}
\bullet \\
\dot{U}_{1} \\
\dot{U}_{2} \\
\vdots \\
\dot{U}_{N}
\end{array}\right]=Y \cdot \dot{U}
$$

where $\dot{I}, \dot{U}$ are respectively the net current injection phasor and the voltage phasor at bus $k$. Matrix $Y$ is the bus admittance matrix, and when considering the transformer, the following equation can be given:

$$
\begin{gather*}
Y_{k k}^{\text {new }}=Y_{k k}+y /|\stackrel{\bullet}{a}|^{2} \\
Y_{k m}^{n e w}=Y_{k m}-y / \stackrel{*}{a}  \tag{12}\\
Y_{m k}^{n e w}=Y_{m k}-y / \dot{a} \\
Y_{m m}^{n e w}=Y_{m m}+y
\end{gather*}
$$

In Figure 4, the general two-port model for the power network branch. In the figure, $g_{i j}+j b_{i j}$ is the admittance of the series branch connecting buses $i$ and $j, g_{s i}+j b_{s i}$ is the admittance of the shunt branch connected at bus $i$, then we can get all kinds of measurements $h(x)$ which are expressed by voltage, angle, and other network parameters.


Figure 3. Equivalent circuit for a generator. (a) Voltage equivalent circuit for a generator. (b) Current equivalent circuit for a generator.


Figure 4. Two-port model of power network branch.
The node voltage $\dot{U}_{i}, \dot{U}_{j}$ can be shown as Equation (13) with voltage amplitude $U_{i}, U_{j}$ and the phase angle is $\delta_{i}, \delta_{j}$ :

$$
\begin{gather*}
\dot{U}_{i}=U_{i} e^{j \delta i}=U_{i}\left(\cos \delta_{i}+j \sin \delta_{i}\right) \\
\dot{U}_{j}=U_{j} e^{j \delta j}=U_{j}\left(\cos \delta_{j}+j \sin \delta_{j}\right) \tag{13}
\end{gather*}
$$

Based on Kirchhoff's current law, $\dot{I}_{i}$ is the node injection current can be determined by $\dot{I}_{i}=\sum_{j=1}^{n} Y_{i j} \dot{u}_{j, i}, i, 2, \ldots n$, where $n$ is the number of nodes in the network. The power can be calculated with the following equation:

$$
\begin{equation*}
P_{i}+j Q_{i}=\dot{U}_{i} I_{i}^{*}=\dot{U}_{i} \sum_{j=1}^{n} \stackrel{*}{Y}_{i j} \stackrel{*}{U}_{j}=U_{i} \sum_{j=1}^{n} U_{j}\left(G_{i j}-B_{i j}\right)\left(\cos \delta_{i j}-j \sin \delta_{i j}\right) \tag{14}
\end{equation*}
$$

where $\delta_{i j}=\delta_{i}-\delta_{j}, Y_{i j}=G_{i j}+B_{i j}, G_{i j}, B_{i j}$ is in matrix form.

With Equation (14), the $P_{i}$ and $Q_{i}$ can be rearranged as:

$$
\begin{align*}
& P_{i}=U_{i} \sum_{j=1}^{n} U_{j}\left(G_{i j} \cos \delta_{i j}+B_{i j} \sin \delta_{i j}\right) \\
& Q_{i}=U_{i} \sum_{j=1}^{n} U_{j}\left(G_{i j} \sin \delta_{i j}-B_{i j} \cos \delta_{i j}\right) \tag{15}
\end{align*}
$$

From Figure 4, we can get:

$$
\begin{equation*}
\stackrel{\bullet}{I j}_{i j}=\left(\dot{U}_{i}-\stackrel{\bullet}{U}_{j}\right)\left(g_{i j}+j b_{i j}\right)+\stackrel{\bullet}{U}_{i}\left(g_{s i}+j b_{s i}\right) \tag{16}
\end{equation*}
$$

so, the power flow from bus $i$ to $j$ can be described as follows:

$$
\begin{align*}
P_{i j}+j Q_{i j} & =\stackrel{\bullet}{U}_{i} I_{i j}^{*} \\
& =\stackrel{\bullet}{U}_{i}\left[\left(\dot{U}_{i}-\stackrel{\bullet}{U}_{j}\right)\left(g_{i j}+j b_{i j}\right)+\stackrel{\bullet}{U}_{i}\left(g_{s i}+j b_{s i}\right)\right]^{*} \\
& =U_{i} e^{j \delta_{i}}\left\{U_{i} e^{-j \delta_{i}}\left[g_{i j}+g_{s i}-j\left(b_{i j}+b_{s i}\right)\right]-U_{j} e^{-j \delta_{j}}\left(g_{i j}-j b_{i j}\right)\right\}  \tag{17}\\
& =U_{i}^{2}\left(g_{i j}+g_{s i}\right)-U_{i} U_{j}\left(g_{i j} \cos \delta_{i j}+b_{i j} \sin \delta_{i j}\right) \\
& +j\left[-U_{i}^{2}\left(b_{i j}+b_{s i}\right)+U_{i} U_{j}\left(b_{i j} \cos \delta_{i j}-g_{i j} \sin \delta_{i j}\right)\right]
\end{align*}
$$

Similar to Equation (15), the real and reactive power flow can be denoted as:

$$
\begin{align*}
& P_{i j}=U_{i}^{2}\left(g_{i j}+g_{s i}\right)-V_{i} V_{j}\left(g_{i j} \cos \delta_{i j}+b_{i j} \sin \delta_{i j}\right) \\
& Q_{i j}=-U_{i}^{2}\left(b_{i j}+b_{s i}\right)+V_{i} V_{j}\left(g_{i j} \cos \delta_{i j}-b_{i j} \sin \delta_{i j}\right) \tag{18}
\end{align*}
$$

### 2.2. Proposed Power System State Estimation Method

PMUs are used to measure data such as voltages, current, angle and frequency, and can provide real-time information about the state of large portions of national and transnational grids and allow one to obtain a state estimate which is a true snapshot of the power system, making the technology for real-time supervision of power systems clearly at hand [35], but because of the high relative cost and inconvenience of installation, we cannot install PMUs in each part of power grid, and we still need state estimation technology using SCADA and other technologies to get the full state of power grid, and the algorithm and application of state estimation will still need to be explored, proposed and tested in the coming years. This section analyzes a power network state estimation method which includes PMUs.

As shown in Figure 5, there is a PMU in node $i$ and no PMU in node $j$, and the branch current is then determined through Equation (19):

$$
\begin{align*}
\stackrel{\bullet}{I}_{i j} & =\left(\dot{U}_{i}-\dot{U}_{j}\right) \cdot y_{i j}+\dot{U}_{i} \cdot y_{s i} \\
& =U_{i}\left(\cos \delta_{i}+j \sin \delta_{i}\right)\left[\left(g_{i j}+g_{s i}\right)+j\left(b_{i j}+b_{s i}\right)\right]-U_{j}\left(\cos \delta_{j}+j \sin \delta_{j}\right)\left(g_{i j}+j b_{i j}\right)  \tag{19}\\
& =U_{i}\left[\left(g_{i j}+g_{s i}\right) \cos \delta_{i}-\left(b_{i j}+b_{s i}\right) \sin \delta_{i}\right]-U_{j}\left(g_{i j} \cos \delta_{j}-b_{i j} \sin \delta_{j}\right) \\
& +j\left\{U_{i}\left[\left(g_{i j}+g_{s i}\right) \sin \delta_{i}+\left(b_{i j}+b_{s i}\right) \cos \delta_{i}\right]-U_{j}\left(g_{i j} \sin \delta_{j}+b_{i j} \cos \delta_{j}\right)\right\}
\end{align*}
$$

so, the real and imaginary parts of the branch current $\stackrel{\bullet}{I}_{i j}$ can be denoted as:

$$
\begin{align*}
& I_{i j}^{r}=U_{i}\left[\left(g_{i j}+g_{s i}\right) \cos \delta_{i}-\left(b_{i j}+b_{s i}\right) \sin \delta_{i}\right]-U_{j}\left(g_{i j} \cos \delta_{j}-b_{i j} \sin \delta_{j}\right) \\
& I_{i j}^{i m}=U_{i}\left[\left(g_{i j}+g_{s i}\right) \sin \delta_{i}+\left(b_{i j}+b_{s i}\right) \cos \delta_{i}\right]-U_{j}\left(g_{i j} \sin \delta_{j}+b_{i j} \cos \delta_{j}\right) \tag{20}
\end{align*}
$$

The mixed measurement state estimation model, which introduces node voltage phasors and branch current phasors through PMU [36], can be denoted as:

To have the state estimation of a power network, one of the key problems is how to solve the Jacobian matrix. From Figure 6, the Jacobian matrix $H_{(m \times n)}$ is composed of $H_{t(s \times n)}, H_{I(p \times n)}, H_{U(q \times n)}$, they are respectively the Jacobian sub-matrices of traditional measurements, PMU branch current phasor measurements and PMU node voltage phasor measurements. The Jacobian matrix $H_{(m \times n)}$ can be described by:

$$
H=\left[\begin{array}{c}
H_{t}  \tag{21}\\
H_{I} \\
H_{U}
\end{array}\right]=\left[\begin{array}{ccccccccc}
\frac{\partial U_{t i}}{\partial \delta} & \frac{\partial P_{i}}{\partial \delta} & \frac{\partial P_{i j}}{\partial \delta} & \frac{\partial Q_{i}}{\partial \delta} & \frac{\partial Q_{i j}}{\partial \delta} & \frac{\partial I_{i j}^{r}}{\partial \delta} & \frac{\partial I_{i j}^{i m}}{\partial \delta} & \frac{\partial U_{p i}}{\partial \delta} & \frac{\partial \delta_{P i}}{\partial \delta} \\
\frac{\partial U_{t i}}{\partial U} & \frac{\partial P_{i}}{\partial U} & \frac{\partial P_{i j}}{\partial U} & \frac{\partial Q_{i}}{\partial U} & \frac{\partial Q_{i j}}{\partial U} & \frac{\partial I_{i j}^{r}}{\partial U} & \frac{\partial I_{i j}^{i m}}{\partial U} & \frac{\partial U_{p i}}{\partial U} & \frac{\partial \delta_{p i}}{\partial U}
\end{array}\right]^{T}
$$

where $m$ is number of conventional and phasor measurements, $n$ is the number of state variables; $s, p$, $q$ are number of conventional, PMU current, and PMU voltage measurements, respectively.

From Equation (21), the Jacobian matrix is composed of partial derivatives, and all these partial derivatives would be calculated based on Kirchhoff's laws and circuit theory. As a result, the state output from the classical state estimator $H_{t(s \times n)}$ is given in Supplementary. The elements of $H_{I(p \times n)}$ and $H_{U(q \times n)}$ can be determined by:

$$
\begin{align*}
& \frac{\partial I_{i j}^{r}}{\partial \delta_{i}}=-U_{i}\left[\left(g_{i j}+g_{s i}\right) \sin \delta_{i}+\left(b_{i j}+b_{s i}\right) \cos \delta_{i}\right] \\
& \frac{\partial I_{i j}^{r}}{\partial \delta_{j}}=U_{i}\left(g_{i j} \sin \delta_{j}+b_{i j} \cos \delta_{j}\right) \\
& \frac{\partial I_{i j}^{r}}{\partial U_{i}}=\left(g_{i j}+g_{s i}\right) \cos \delta_{i}-\left(b_{i j}+b_{s i}\right) \sin \delta_{i}  \tag{22}\\
& \frac{\partial I_{i j}^{r}}{\partial U_{j}}=-\left(g_{i j} \cos \delta_{j}-b_{i j} \sin \delta_{j}\right) \\
& \frac{\partial I_{i j}^{i m}}{\partial \delta_{i}}=U_{i}\left[\left(g_{i j}+g_{s i}\right) \cos \delta_{i}-\left(b_{i j}+b_{s i}\right) \sin \delta_{i}\right] \\
& \frac{\partial \frac{i m}{i m}}{\partial \delta_{j}}=-U_{j}\left(g_{i j} \cos \delta_{j}-b_{i j} \sin \delta_{j}\right)  \tag{23}\\
& \frac{\partial I_{i j}^{i m}}{\partial U_{i}}=\left(g_{i j}+g_{s i}\right) \sin \delta_{i}+\left(b_{i j}+b_{s i}\right) \cos \delta_{i} \\
& \partial I_{i j}^{i m} \\
& \partial U_{j}
\end{align*}=-\left(g_{i j} \sin \delta_{j}+b_{i j} \cos \delta_{j}\right) \quad,
$$

It well known that with PMU measurement, the node voltage magnitudes, node voltage phase angles, and branch currents can be obtained directly.


Figure 5. Power network model including PMUs.


Figure 6. The mixed calculation model including PMUs.

### 2.3. The Mixed WLS Power System State Estimation Algorithm and Program Flow

Based on the above analysis, the proposed mixed state estimation involves the iterative solution of the normal equations given by Equation (7). As shown in Figure 7, the iterative program flow can be described as following. In the implementation, the initial value of state variables including voltage magnitude and voltage phase angle, are respectively set to 1 and 0 . Firstly, PMUs placements in the node system are optimized, and if there is not any PMUs in the network, the traditional WLS state estimation will be utilized. Otherwise, building the mixed Jacobian matrix $H$ according to Equation (21), and then the maximal $\left|\Delta X^{(k)}\right|_{\max }$ can be obtained by Equation (7). After a few calculations of iteration, the accuracy of current prediction becomes much better.


Figure 7. Flow chart of the proposed power system state estimation.

## 3. Case Studies and Discussion

In this section, the IEEE $14,30,118$ bus test systems are used to verify the presented approaches. To easily calculate and intuitively display the results, usually it is better to transform the active and reactive power injection measurements, $P_{i}, Q_{i}$, and active and reactive power flows $P_{i j}, Q_{i j}$ to per-unit values. In this paper, the reference value as 100 MW or 100 MVAR is selected to transform the above data to per-unit system, and the units of phase angle are degree. All other measurement units are p.u.

Measurements from PMUs have a high accuracy, and their amplitude errors are from $0.1 \%$ to $1 \%$, and the range of phase angle is from $0.01^{\circ}$ to $0.05^{\circ}$. Then the covariance matrix of traditional measurement errors $R_{t}$ and PMU voltage phasor measurement errors can be calculated by $\operatorname{diag}\left(\sigma_{1}^{2}, \sigma_{2}^{2}, \mathrm{~K}, \sigma_{m}^{2}\right)$.

There is the relation, i.e., $I_{i j} e^{j \theta_{i j}}=I_{i j}\left(\cos \theta_{i j}+j \sin \theta_{i j}\right)=I_{i j}^{r}+j I_{i j}^{i m}$. According to the error propagation theory [37], the standard deviations of $I_{i j}^{r}$ and $I_{i j}^{i m}$ can be calculated by Equation (24):

$$
\begin{gather*}
\sigma_{I_{i j}^{r}}=\sqrt{\left(\frac{\partial I_{i j}^{r}}{\partial I_{i j}}\right)^{2} \cdot \sigma_{I_{i j}}^{2}+\left(\frac{\partial I_{i j}^{r}}{\partial \theta_{i j}}\right)^{2} \cdot \sigma_{\theta_{i j}}^{2}}=\sqrt{\left(\cos \theta_{i j}\right)^{2} \cdot \sigma_{I_{i j}}^{2}+\left(I_{i j} \sin \theta_{i j}\right)^{2} \cdot \sigma_{\theta_{i j}}^{2}}  \tag{24}\\
\sigma_{I_{i j} m}=\sqrt{\left(\frac{\partial I_{i j}^{i m}}{\partial I_{i j}}\right)^{2} \cdot \sigma_{I_{i j}}^{2}+\left(\frac{\partial I_{i j}^{i m}}{\partial \theta_{i j}}\right)^{2} \cdot \sigma_{\theta_{i j}}^{2}}=\sqrt{\left(\sin \theta_{i j}\right)^{2} \cdot \sigma_{I_{i j}}^{2}+\left(I_{i j} \cos \theta_{i j}\right)^{2} \cdot \sigma_{\theta_{i j}}^{2}}
\end{gather*}
$$

so, the covariance matrix of PMU branch current phasor measurement errors $R_{I}$ can be calculated as well. The mixed error covariance matrix $R_{m}$ defined by Equation (24) can be obtained [36].

$$
R_{m}=\left[\begin{array}{lll}
R_{t} & &  \tag{25}\\
& R_{U} & \\
& & R_{I}
\end{array}\right]
$$

In order to check the estimation accuracy and computational efficiency of the proposed state estimation method, this paper compares it with results from three common methods. Method 1: the traditional weighted least squares (WLS) state estimation method with only SCADA measurements. Method 2: PMU installed at partial nodes of power systems, but the PMU measurements with voltage phasors without participating in state estimation. In this method, the PMU measurement results are used as the state solution of test system. Method 3: PMU equipped at partial nodes of power systems, and the PMU measurements with voltage phasors participates in state estimation.

In this paper, the tolerance to converge is set to $10^{-4}$, meaning that if there is:

$$
\begin{equation*}
\max \left|\Delta x_{k}\right| \leq \varepsilon=0.0001 \tag{26}
\end{equation*}
$$

the simulation program calculation will finish its iteration and we get the state estimation results.

### 3.1. IEEE 14 Bus System

In the IEEE14 bus test system, the parameter configuration includes traditional measurements and PMU measurements. Figure 8 is the measurement configuration diagram of the IEEE 14 bus system [38]. The PMU is installed on buses 2, 7 and 9. In addition, traditional measurements include, the power flows in $1-2,2-3,4-7,4-9,5-6,6-13$ and $13-14$, power injections in buses $2-11$ and the voltage amplitude in bus 1. Four methods based on Figure 8 are designed to complete the state estimation, and simulation is carried out by MATLAB. The results are shown in Tables 1 and 2. It should be noted that the proposed method is a fixed reference angle method, but in the calculation results, in order to intuitively reflect all the state of the entire system, the phase angles of all the buses are listed in this paper.


Figure 8. Simulation measurement configuration of the IEEE 14 bus system.
Table 1. Bus voltage amplitude estimation of the four methods used in the 14 bus system.

| Bus Voltage | True Value | Bus Voltage Amplitude Estimation of Simulation Methods |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Method 1 | Method 2 | Method 3 | Proposed Method |
| $U_{1}$ (p.u) | 1.0600 | 1.0639 | 1.0639 | 1.0717 | 1.0715 |
| $U_{2}$ (p.u) | 1.0450 | 1.0418 | 1.0450 | 1.0526 | 1.0542 |
| $U_{3}$ (p.u) | 1.0100 | 1.0028 | 1.0027 | 1.0162 | 1.0208 |
| $U_{4}$ (p.u) | 1.0177 | 1.0025 | 1.0022 | 1.0185 | 1.0219 |
| $U_{5}$ (p.u) | 1.0195 | 1.0046 | 1.0041 | 1.0194 | 1.0238 |
| $U_{6}$ (p.u) | 1.0700 | 1.0131 | 1.0104 | 1.0379 | 1.0541 |
| $U_{7}$ (p.u) | 1.0615 | 1.0283 | 1.0615 | 1.0554 | 1.0526 |
| $U_{8}$ (p.u) | 1.0900 | 1.0597 | 1.0595 | 1.0873 | 1.0737 |
| $U_{9}$ (p.u) | 1.0559 | 1.0102 | 1.0559 | 1.0418 | 1.0433 |
| $U_{10}$ (p.u) | 1.0510 | 1.0038 | 1.0034 | 1.0352 | 1.0402 |
| $U_{11}$ (p.u) | 1.0569 | 1.0054 | 1.0037 | 1.0340 | 1.0450 |
| $U_{12}$ (p.u) | 1.0552 | 0.9446 | 0.9396 | 0.9664 | 1.0037 |
| $U_{13}$ (p.u) | 1.0504 | 0.9954 | 0.9921 | 1.0231 | 1.0381 |
| $U_{14}$ (p.u) | 1.0355 | 1.0022 | 0.9945 | 1.0480 | 1.0520 |

Table 1 presents the bus voltage amplitude estimation results of the four methods in the 14 bus system. The second column indicates the true value of the bus voltage amplitude, and the third, fourth, fifth and the sixth columns show voltage state estimated values using different methods. Table 2 is the bus voltage angle estimation results of the four methods in the 14 bus system.

Table 2. Bus voltage angle estimation of four methods used in 14 bus system.

| Angle of Bus Voltage | True Value | Bus Voltage Angle Estimation of Simulation Methods |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Method 1 | Method 2 | Method 3 | Proposed Method |
| $\delta_{1}\left({ }^{\circ}\right)$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $\delta_{2}\left({ }^{\circ}\right)$ | -4.9826 | -4.7910 | -4.9826 | -4.8901 | -4.9031 |
| $\delta_{3}\left({ }^{\circ}\right)$ | -12.7251 | -12.5248 | -12.5305 | -12.5569 | -12.5344 |
| $\delta_{4}\left({ }^{\circ}\right)$ | -10.3129 | -9.9161 | -9.9247 | -10.1391 | -10.1433 |
| $\delta_{5}\left({ }^{\circ}\right)$ | -8.7739 | -8.3690 | -8.3679 | -8.5936 | -8.6314 |
| $\delta_{6}\left({ }^{\circ}\right)$ | -14.2209 | -14.0831 | -14.0803 | -14.4525 | -14.4183 |
| $\delta_{7}\left({ }^{\circ}\right)$ | -13.3596 | -12.9912 | -13.3596 | -13.3109 | -13.3390 |
| $\delta_{8}\left({ }^{\circ}\right)$ | -13.3596 | -12.9479 | -13.0011 | -13.3244 | -13.4862 |
| $\delta_{9}\left({ }^{\circ}\right)$ | -14.9385 | -14.6890 | -14.9385 | -14.9608 | -14.9120 |
| $\delta_{10}\left({ }^{\circ}\right)$ | -15.0973 | -14.8962 | -14.9297 | -15.1827 | -15.1385 |
| $\delta_{11}\left({ }^{\circ}\right)$ | -14.7906 | -14.6393 | -14.6578 | -14.9714 | -14.9295 |
| $\delta_{12}\left({ }^{\circ}\right)$ | -15.0756 | -13.9067 | -13.7922 | -14.8306 | -15.1238 |
| $\delta_{13}\left({ }^{\circ}\right)$ | -15.1563 | -15.1183 | -15.1214 | -15.4914 | -15.4195 |
| $\delta_{14}\left({ }^{\circ}\right)$ | -16.0336 | -16.0200 | -16.0287 | -16.7292 | -16.5855 |

The state estimation average relative error absolute value can be defined as [39]:

$$
\begin{equation*}
\text { mape }=\sum_{i=1}^{n}\left|\frac{y_{i}-y_{e i}}{y_{i}}\right| / n \tag{27}
\end{equation*}
$$

where $y_{i}$ is the real value, $y_{e i}$ is the estimation value.
Then in this report, the state estimation average accuracy can be obtained as follows:

$$
\begin{equation*}
\text { accuracy }=(1-\text { mape }) \times 100 \% \tag{28}
\end{equation*}
$$

Figures 9 and 10 are the result of percentage bus voltage amplitude and angle estimation errors using the data from Tables 1 and 2. As can be seen from Table 3, the maximum percentage voltage error of the proposed method is $4.8849 \%$, which represents a great improvement when compared with the three other methods. On the whole, we can see that Method 1 has the worst state estimation effect and the proposed method has the best estimation effect. It also should be pointed out the elapsed time of the proposed method shows no lessening, but we can accept it. This shows that when adding PMU data to conventional SCADA measurement inputs, the state estimator will raise its redundancy and improve its accuracy. Especially, the proposed mixed state estimation has more marked improvement.

Table 3. The results of state estimation of four methods in 14 bus system.

| Method No. | Voltage <br> Average <br> Accuracy (\%) | Angle <br> Average <br> Accuracy (\%) | Voltage <br> Maximum <br> Error (\%) | Angle <br> Maximum <br> Error (\%) | Iteration <br> Number | Convergence <br> Time (s) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method 1 | 96.56 | 97.66 | 10.4809 | 7.7535 | 4 | 0.6201 |
| Method 2 | 96.96 | 98.26 | 10.9565 | 8.5134 | 5 | 0.6961 |
| Method 3 | 98.31 | 98.62 | 8.4163 | 4.3385 | 4 | 0.6527 |
| Proposed method | 98.67 | 98.88 | 4.8849 | 3.4419 | 4 | 0.7235 |



Figure 9. The percentage of bus voltage amplitude estimate error in 14 bus system.


Figure 10. The percentage of bus voltage angle estimate error in 14 bus system.
In the field of numerical analysis, on the one hand, the condition number of a function with respect to an argument measures the asymptotically worst case of how much the function can change in proportion to small changes in the argument. The "function" is the solution of a problem and the "arguments" are the data in the problem. Usually, the condition number is described as:

$$
\begin{equation*}
k(A)=\|A\| \cdot\left\|A^{-1}\right\| \tag{29}
\end{equation*}
$$

If complex square matrix $A$ is normal, i.e., $A^{*} A=A A^{*}, A^{*}$ is the conjugate transpose of $A$, then the condition number can be described as:

$$
\begin{equation*}
k(A)=\left|\frac{\lambda_{\max }(A)}{\lambda_{\min }(A)}\right| \tag{30}
\end{equation*}
$$

where $\lambda_{\max }(A), \lambda_{\min }(A)$ are maximal and minimal eigenvalues of $A$, respectively.
A problem with a low condition number is said to be well-conditioned, while a problem with a high condition number is said to be ill-conditioned.

On the other hand, quality of an estimate is inversely related to it is variance. Estimation quality is very important to state estimator. There are many parameters which can be looked as a reference. In this report, we select the following equation to describe the quality of the estimator:

$$
\begin{equation*}
Q_{\mathrm{d}}=\log \left(\frac{1}{\sqrt{\operatorname{det}(P)}}\right) \tag{31}
\end{equation*}
$$

Here numerically computed error covariance matrix $P$ can be defined as:

$$
\begin{align*}
P & =E\left[(X-\hat{X})(X-\hat{X})^{T}\right] \\
& =E\left[\left\{G^{-1} H^{T}(X) R^{-1}[Z-h(X)]\right\} \times\left\{G^{-1} H^{T}(X) R^{-1}[Z-h(X)]\right\}^{T}\right]  \tag{32}\\
& =E\left[G^{-1} H^{T}(X) R^{-1} e e^{T} R^{-1} H(X)\left(G^{-1}\right)^{T}\right] \\
& =G^{-1}
\end{align*}
$$

where $X$ is the true state vector, $\hat{X}$ is estimation state vector and $G=H^{T} R^{-1} H$ is the gain matrix.
Sometimes, it becomes difficult to compute the determinant of error covariance matrix numerically. Hence, trace of error covariance matrix can be utilized to quantify the quality of estimation as follows:

$$
\begin{equation*}
Q_{\mathrm{t}}=\log \left(\frac{1}{\operatorname{trace}(P)}\right) \tag{33}
\end{equation*}
$$

The gain matrix condition number $k_{\mathrm{a}}$, quality $Q_{\mathrm{t}}$ and $Q_{\mathrm{d}}$ of the four methods are shown in Table 4 and these results are plotted in Figure 11.

Table 4. The condition number and quality of the four methods in 14 bus system.

| Method No. | $\boldsymbol{k}_{\mathbf{a}}$ | Quality |  |
| :---: | :---: | :---: | :---: |
|  |  | $Q_{\mathbf{t}}$ | $Q_{\mathbf{d}}$ |
| Method 1 | $2.3527 \times 10^{4}$ | 5.2977 | 161.1225 |
| Method2 | $2.2532 \times 10^{4}$ | 5.3476 | 161.5726 |
| Method 3 | $7.5222 \times 10^{3}$ | 6.1806 | 165.3389 |
| Proposed method | $6.7418 \times 10^{3}$ | 6.3365 | 166.6500 |



Figure 11. The condition number and quality of the four methods in 14 bus system.

From the above analysis, we can see the proposed method has better condition number and estimation quality than the other three methods. In order to further validate the effectiveness, the following five cases with different PMU measurement configurations will be discussed to illustrate the performance of the proposed method. The simulation results of different cases are shown in Table 5. It can be observed from Table 5 that as the number of PMU configurations increases, we can get better results, which include voltage accuracy $\left(V_{\mathrm{a}}\right)$, angle accuracy $\left(A_{\mathrm{a}}\right)$, the condition number $\left(K_{\mathrm{a}}\right)$ and quality $\left(Q_{t}\right)$.

Table 5. The results of state estimation of different cases in 14 bus system.

| PMU <br> Numbers | Method 2 |  |  |  | Method 3 |  |  |  | Proposed Method |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{4}$ | $Q_{\text {t }}$ | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{3}$ | $Q_{\text {t }}$ | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{3}$ | $Q_{\text {t }}$ |
| 0 | 96.56 | 97.66 | 2.3527 | 5.2977 | 96.56 | 97.66 | $\Delta 2.3527$ | 5.2977 | 96.56 | 97.66 | $\Delta 2.3527$ | 5.2977 |
| 2 | 96.66 | 98.20 | 2.2631 | 5.3454 | 97.87 | 98.45 | 8.0496 | 6.0317 | 98.34 | 98.74 | 7.5969 | 6.1123 |
| 3 | 96.96 | 98.26 | 2.2532 | 5.3476 | 98.31 | 98.62 | 7.5222 | 6.1806 | 98.67 | 98.88 | 6.7418 | 6.3365 |
| 4 | 97.26 | 98.31 | 2.2361 | 5.3658 | 98.84 | 98.77 | 4.1822 | 6.7509 | 99.24 | 99.25 | 2.1143 | 7.3519 |
| 5 | 97.67 | 98.32 | 2.2191 | 5.3807 | 98.91 | 98.96 | 4.1183 | 6.8803 | 99.26 | 99.30 | 1.6745 | 7.6163 |

Comment: $\Delta 2.3527$ represents $2.3527 \times 10^{4}$.
In statistics, bias of an estimator is the difference between estimator expected value and the true value of the parameter being estimated [40]. Based on mathematical theory, one has:

$$
\begin{equation*}
\operatorname{Bias}[\hat{X}]=\sum_{k}^{N}\left(\hat{X}_{k}-X\right) P_{k} \tag{34}
\end{equation*}
$$

where $P_{k}$ is the distribution probability. In this paper, Monte Carlo simulations were utilized to research the statistical characteristic, and the estimation step number $N$ is 500 steps. The measurement inputs are randomly produced by the following equation:

$$
\begin{equation*}
z=z_{0}(\mu+\sigma \times \operatorname{randn}(1,1)) \tag{35}
\end{equation*}
$$

where $\mu_{0}+\sigma_{0} \times \operatorname{randn}(1,1)$ produces a normal distribution random number. The symbol $\mu_{0}$ is set to " 1 ", and the symbol $\sigma_{0}$ is set to " $10^{-3 "}$.

Figure 12 is the bias analysis of the proposed method using the Monte Carlo method. It's clear from the figure that the bias of proposed mixed state estimator is low.


Figure 12. Bias analysis of proposed method in 14 bus system.

### 3.2. IEEE 30 Bus System

In the IEEE 30 bus test system, the PMU is installed at buses 2, 4, 11, 14, 24, as shown in Figure 13. All the simulation conditions are same with that of IEEE 14 bus system. The percentage of voltage amplitude and angle estimate error is plotted in Figures 14 and 15. It can be observed from Figures 14 and 15 that the overall performance of estimator with proposed method is better when compared with traditional estimator with method 1,2, or 3. Other results are shown in Table 6.


Figure 13. Simulation measurement configuration of the IEEE 30 bus system.


Figure 14. The percentage of voltage amplitude estimate error in 30 bus system.


Figure 15. The percentage of bus voltage angle estimate error in 30 bus system.

Table 6. The results of state estimation of four methods in 30 bus system.

| Method No. | Voltage <br> Average <br> Accuracy (\%) | Angle <br> Average <br> Accuracy (\%) | Voltage <br> Maximum <br> Error (\%) | Angle <br> Maximum <br> Error (\%) | Iteration <br> Number | Convergence <br> Time (s) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method 1 | 97.81 | 99.10 | 3.9811 | 1.7199 | 4 | 0.6862 |
| Method 2 | 98.10 | 99.21 | 3.9864 | 1.7024 | 6 | 0.8209 |
| Method 3 | 99.38 | 99.68 | 2.6512 | 1.7514 | 4 | 0.7198 |
| Proposed method | 99.55 | 99.69 | 2.2068 | 1.7839 | 4 | 0.8611 |

The viability and effectiveness of the proposed method is established by means of installing different numbers of PMUs. As can be seen from Table 7, the more number of PMU the power system has, the better estimation accuracy and stability can obtain. For bias, it can be observed in Figure 16 that the estimator with proposed mixed method has the good unbiased performance.


Figure 16. Bias analysis of proposed method in 30 bus system.

Table 7. The results of state estimation of different cases in 30 bus system.

| PMU <br> Numbers | Method 2 |  |  |  | Method 3 |  |  |  | Proposed Method |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{5}$ | $Q_{\text {t }}$ | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{4}$ | $Q_{\text {t }}$ | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{4}$ | $Q_{\text {t }}$ |
| 0 | 97.81 | 99.10 | 1.7646 | 4.2216 | 97.81 | 99.10 | $\Delta 1.7646$ | 4.2216 | 97.81 | 99.10 | $\Delta 1.7646$ | 4.2216 |
| 3 | 97.89 | 99.14 | 1.7666 | 4.2231 | 98.73 | 99.39 | 5.6293 | 4.7760 | 99.23 | 99.52 | 5.1536 | 4.9021 |
| 4 | 97.98 | 99.19 | 1.7595 | 4.2274 | 98.97 | 99.61 | 4.9078 | 4.9859 | 99.30 | 99.67 | 4.9372 | 5.0490 |
| 5 | 98.10 | 99.21 | 1.7556 | 4.2287 | 99.38 | 99.68 | 4.3702 | 5.1709 | 99.55 | 99.69 | 3.5528 | 5.3508 |
| 6 | 98.17 | 99.23 | 1.7550 | 4.2299 | 99.53 | 99.70 | 1.8397 | 5.6703 | 99.71 | 99.75 | 1.4980 | 5.9185 |

Comment: $\Delta 1.7646$ represents $1.7646 \times 10^{5}$.

### 3.3. IEEE 118 Bus System

In the IEEE 118 bus test system, the PMU is installed at buses $14,20,21,22,42,43,44,45$, as shown in Figure 17. All the simulation conditions are the same as those of the IEEE 14 bus system. The results of state estimation of the different methods are observed from Figures 18 and 19. It can be seen from Figures 18 and 19 that the overall accuracy of the proposed estimator is better when compared with the others. As can be seen from the Table 8, although the percentage of angle maximum error of proposed method shows no lessening relative to Method 3, it drops a lot when compared with Methods 1 and 2. Furthermore, the accuracy and the number of iterations using the proposed method are greatly improved in a large scale system. Similarly, it can be observed from the Table 9 and Figure 20 that the proposed method the effectiveness and accuracy of the calculation model is also verified by the 118 bus system.


Figure 17. Simulation measurement configuration of the IEEE 118 bus system.


Figure 18. The percentage of voltage amplitude estimate error in 118 bus system.


Figure 19. The percentage of bus voltage angle estimate error in 118 bus system.
Table 8. The results of state estimation of four methods in 118 bus system.

| Method No. | Voltage <br> Average <br> Accuracy (\%) | Angle <br> Average <br> Accuracy (\%) | Voltage <br> Maximum <br> Error (\%) | Angle <br> Maximum <br> Error (\%) | Iteration <br> Number | Convergence <br> Time (s) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method 1 | 96.71 | 92.50 | 11.0607 | 25.5349 | 9 | 2.7302 |
| Method 2 | 96.75 | 93.16 | 11.3506 | 26.4268 | 13 | 2.8076 |
| Method 3 | 98.57 | 98.84 | 7.9859 | 4.0330 | 6 | 2.7386 |
| Proposed method | 98.68 | 98.99 | 6.9800 | 6.1623 | 6 | 2.9110 |

Table 9. The results of state estimation of different cases in 118 bus system.

| PMU <br> Numbers | Method 2 |  |  |  | Method 3 |  |  |  | Proposed Method |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{a} / 10^{6}$ | $Q_{\text {t }}$ | $V_{\mathrm{a}} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{6}$ | $Q_{\text {t }}$ | $V_{\text {a }} / \%$ | $A_{\mathrm{a}} / \%$ | $K_{\mathrm{a}} / 10^{6}$ | $Q_{\text {t }}$ |
| 0 | 96.71 | 92.50 | 2.9259 | 4.4244 | 96.71 | 92.50 | 2.9259 | 4.4244 | 96.71 | 92.50 | 2.9259 | 4.4244 |
| 5 | 96.70 | 93.09 | 2.9170 | 4.4240 | 98.05 | 98.52 | 2.2396 | 4.9731 | 98.20 | 98.71 | 2.2051 | 5.0014 |
| 7 | 96.75 | 93.16 | 2.9103 | 4.4251 | 98.57 | 98.84 | 2.1208 | 5.0519 | 98.68 | 98.99 | 2.0694 | 5.0957 |
| 9 | 96.78 | 93.16 | 2.9059 | 4.4256 | 98.60 | 98.67 | 1.1202 | 5.4080 | 98.75 | 99.15 | $\Delta 9.8727$ | 5.5620 |
| 11 | 96.80 | 93.31 | 2.9041 | 4.4264 | 98.64 | 99.07 | $\Delta 5.8952$ | 5.7853 | 98.85 | 99.30 | $\Delta 5.5307$ | 5.8530 |

Comment: $\Delta 5.8952$ represents $5.8952 \times 10^{5}$. Similarly $\Delta 9.8727$ and $\Delta 5.5307$.


Figure 20. Bias analysis of proposed method in 118 bus system.

Note that the convergence time of the proposed method is slightly bigger than that of the other three methods in Table 10. This is because of the proposed method utilizes more PMU measurements in the calculation, but the convergence time can meet the basic assessment requirements of the State Grid, i.e., the calculation time of a single state estimate is less than 5 seconds. The computer configuration of the simulations is as follows: processor i5-4460, CPU 3.20 GHz , internal memory 4 GB . The computer system has Windows 1064 -bit operating system. With improved online computer memory capacity and processing speed, the convergence time of the proposed method could be reduced greatly.

Table 10. The convergence time of proposed method in 30 and 118 bus system.

| PMU Numbers |  | Convergence Time (s) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | IEEE 118 Bus | IEEE 30 Bus System |  |  | IEEE 118 Bus System |  |  |
| IEEE 30 Bus |  | Method 2 | Method 3 | Proposed <br> Method | Method 2 | Method 3 | Proposed <br> Method |
| 3 | 5 | 0.6731 | 0.6054 | 0.7267 | 2.4155 | 2.3908 | 2.5581 |
| 4 | 7 | 0.8016 | 0.6941 | 0.8493 | 2.8076 | 2.7386 | 2.9110 |
| 5 | 9 | 0.8209 | 0.7198 | 0.8611 | 3.4137 | 3.1181 | 3.6407 |
| 6 | 11 | 0.8198 | 0.7065 | 0.8627 | 3.1096 | 3.0013 | 3.3277 |

## 4. Conclusions

With the growing number of PMUs planned for installation in the near future, it is anticipated that the use of PMU measurement data for the state estimation of power systems will become a standard procedure. The method proposed in this paper analyzes in details the state estimation model by mixed measurements with WAMS and SCADA and presents a "mixing" state estimation method algorithm through introducing the PMU measurement into the traditional state estimation. In addition, how to calculate the error covariance of measurements is investigated. Performance of the proposed method was tested on IEEE 14, 30 and 118 bus systems and related experiments. At the same time, the condition number, estimation quality and bias of developed estimator are also described. In addition, the proposed mixed state estimation is an unbiased estimation can be obtained. The results of simulations proved that the proposed method can improve the accuracy of power system state estimation, and it has better condition number, estimation quality and high reliability. Further work will be focused on analyzing the influences of factors such as bad data and different PMU placement to the results of the proposed method and on the staged PMU placement strategy using minimum number of PMUs to achieve full observability.

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Author Contributions: Tao Jin coordinated the main theme of this paper and wrote the manuscript. Xueyu Shen developed the simulation models and completed the simulations. All the authors read and approved the final manuscript.
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