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A Duty Cycle Space Vector Modulation Strategy for a Three-to-Five Phase Direct Matrix Converter

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Abstract: The duty cycle space vector (DCSV) modulation strategy is of universal significance, and the method can be utilized for different modulation approaches. In this paper, the vectors of input voltages and currents are equivalently represented by a complex two-dimensional space vector, and the vectors of output voltages and currents are equivalently represented by two two-dimensional space vectors. Then, input–output relationships in both the d1-q1 space and the d3-q3 space are obtained. Because the desired output voltages are only mapped onto a reference voltage space vector in the d1-q1 space, the reference in the d3-q3 space is regarded as zero, in order to reduce harmonics of output voltages to the greatest extent. Then, the duty cycle space vector modulation strategy of the three-to-five phase direct matrix converter (DMC) is deduced. Considering the influence of the zero vector on system performance, the duty cycles are decomposed and recomposed to obtain the space vector pulse width modulation (SVPWM) strategy based on the duty cycle space vector. Finally, the accuracy and feasibility of the theory are verified through experiments.

Keywords: five-phase matrix converter; modulation strategy; duty cycle space vector; space vector pulse width modulation

1. Introduction

Multiphase drive systems involving more than three phases have been receiving increasing attention because of their inherent advantages over the traditional three-phase motor drives. They reduce the amplitude and increase the frequency of torque pulsations, reduce the rotor harmonic current losses, reduce the current per phase without increasing the voltage per phase, and lower the DC link current harmonics [1–4]. Therefore, it is necessary to develop multiphase power electronic converters to supply such multiphase systems. Consequently, the multiphase matrix converter (MC) has been widely studied and used in many different applications such as wind energy conversion systems, diesel generators, microturbines, electric aircraft, and ship propulsion [5–10].

The three-to-five phase direct matrix converter is a typical multiphase power converter, and its topology is shown in Figure 1. It has become a focus of research in the field of power transmission and conversion due to its excellent performance. The advantages of DMCs include their sinusoidal input and output currents, simple and compact power circuits due to the lack of electrolytic capacitors, bidirectional power flow, and controllable input power factors [11,12]. However, modulation strategies become more complex due to the increasing number of output phases.

Recently, many modulation strategies for the multiphase MC have been developed, including direct duty ratio pulse width modulation (DPWM), space vector pulse width modulation, and carrier-based pulse width modulation (CBPWM) [13–17]. The pulse width modulation (PWM) technique is presented in [13] for a generalized three-to- k phase MC topology based on direct duty ratio control. SVPWM is discussed in [14] for a three-to-five phase MC considering only outer large length

space vectors. However, for SVPWM implementation, only 93 vectors (90 active and 3 zero vectors) can be used, and the complete space vector model of three-to-five phase MC topology is presented in [15]. To simplify the SVPWM, the CBPWM method has been adopted by researchers. The CBPWM technique is proposed in [16] to achieve the maximum voltage transfer ratio (VTR) in the linear and over-modulation modes. A novel method of CBPWM based on space vector modulation analysis is presented in [17], using only one symmetrical triangular carrier signal to generate the PWM signals for all of the switches. Obviously, the performance of DMC is highly dependent on the control algorithms.

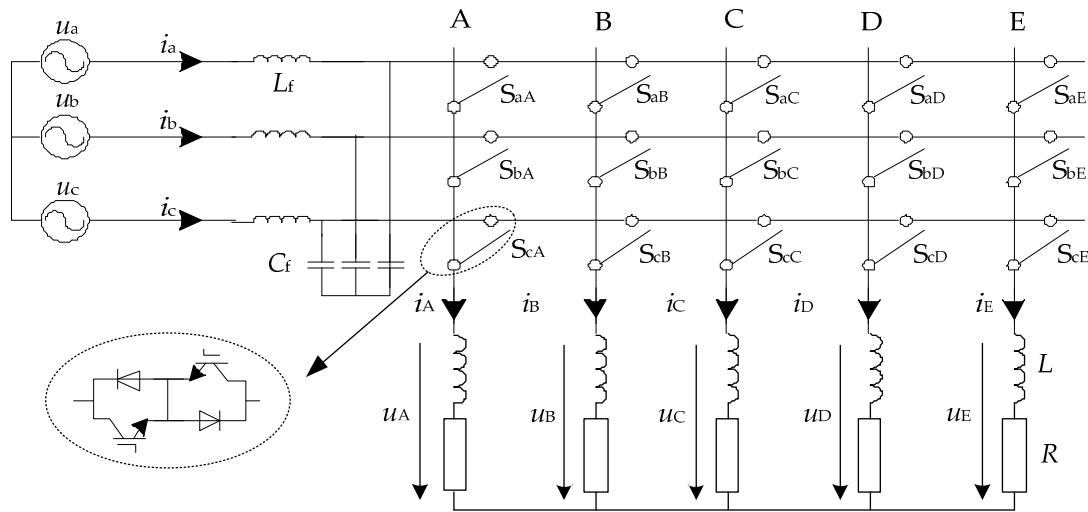


Figure 1. Topology of a three-to-five phase MC.

In this paper, a new generalized strategy—duty cycle space vector modulation—is proposed to control the three-to-five phase DMC. According to this strategy, the duty cycles have been deduced and the general expression of them has been obtained. In addition, the proposed strategy is a general solution to the modulation problem of the three-to-five phase DMC. In addition, this modulation strategy can be easily applied to multiphase DMCs in the future. Then, the previously proposed modulation (DPWM, SVPWM, CBPWM) strategies are considered as a particular case of the DCSV. In this paper, the SVPWM based on DCSV is obtained by decomposing and recombining the duty cycles of 15 switches. Finally, in order to show the validity of the proposed DCSV strategy, an experimental platform is established.

2. Modeling of the Three-to-Five Phase DMC

The topology of the three-to-five phase DMC is shown in Figure 1. Fifteen bidirectional switches (BDSs) are used to connect five output phases to three input phases, and they are defined as S_{xX} (the subscripts of x and X are employed to describe the inputs and outputs, respectively), where $x \in \{a, b, c\}$, $X \in \{A, B, C, D, E\}$; u_a , u_b , u_c and i_a , i_b , i_c are three-phase input voltages and currents, respectively, and u_A , u_B , u_C , u_D , u_E and i_A , i_B , i_C , i_D , i_E are five-phase output voltages and currents, respectively. L_f and C_f are the inductor and capacitor of the input filter, respectively, which smooth the current waveform.

The output voltages and input currents of the three-to-five phase DMC are expressed respectively as the following forms:

$$\begin{pmatrix} u_A \\ u_B \\ u_C \\ u_D \\ u_E \end{pmatrix} = \begin{pmatrix} d_{aA} & d_{bA} & d_{cA} \\ d_{aB} & d_{bB} & d_{cB} \\ d_{aC} & d_{bC} & d_{cC} \\ d_{aD} & d_{bD} & d_{cD} \\ d_{aE} & d_{bE} & d_{cE} \end{pmatrix} \begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} = D \begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} d_{aA} & d_{bA} & d_{cA} \\ d_{aB} & d_{bB} & d_{cB} \\ d_{aC} & d_{bC} & d_{cC} \\ d_{aD} & d_{bD} & d_{cD} \\ d_{aE} & d_{bE} & d_{cE} \end{pmatrix}^T \begin{pmatrix} i_A \\ i_B \\ i_C \\ i_D \\ i_E \end{pmatrix} = \mathbf{D}^T \begin{pmatrix} i_A \\ i_B \\ i_C \\ i_D \\ i_E \end{pmatrix} \quad (2)$$

where T represents transposition; the variables d_{xX} ($x \in \{a, b, c\}$, $X \in \{A, B, C, D, E\}$) are duty cycles of S_{xX} ; and $\mathbf{D}$ is the modulation matrix.

In order to avoid short circuits on the input side and open circuits on the output side, only one switch in every output phase is kept in the ON state at any time. Thus, the constraints of the duty cycles are expressed as $d_{aX} + d_{bX} + d_{cX} = 1$.

3. Space Vector Expression of the Three-to-Five Phase DMC

3.1. Space Vector of the Input Side

In the three-phase symmetric system, except for the $(6n \pm 3)$ th harmonics corresponding to zero sequence components, all the $(6n \pm 1)$ th harmonics of three-phase variables are equivalently represented by a two-dimensional complex space vector [18]. Thus, the space vector of the input side of the three-to-five phase DMC can be represented as follows:

$$\mathbf{U}_i = \frac{2}{3} \left(u_a + u_b e^{j\frac{2}{3}\pi} + u_c e^{j\frac{4}{3}\pi} \right) \quad (3)$$

$$\mathbf{I}_i = \frac{2}{3} \left(i_a + i_b e^{j\frac{2}{3}\pi} + i_c e^{j\frac{4}{3}\pi} \right) \quad (4)$$

where $\mathbf{U}_i$ is the space vector of input voltages, and $\mathbf{I}_i$ is the space vector of input currents.

3.2. Space Vector of the Output Side

Similarly, the concept of the space vector can be extended to the five-phase symmetric system. Each five-dimensional variable has four freedom degrees (except zero sequence components), and can be mapped to two spaces (d1-q1 space and d3-q3 space) according to Park's transformation of the five-dimensional space vector. Therefore, two orthogonal space vectors synchronously rotating in the d1-q1 and d3-q3 spaces can equivalently express five-phase variables except for the zero sequence components. All the $(10n \pm 1)$ th harmonics of five-phase variables can be equivalently expressed as AC components with the frequency of $(10n \pm 1)\omega$ in the stationary d1-q1 space, whereas the $(10n \pm 3)$ th harmonics of five-phase variables can be equivalently expressed as AC components with the frequency of $(10n \pm 3)\omega$ in the stationary d3-q3 space [19]. Thus, the space vectors of the output side of the three-to-five phase DMC can be represented as follows:

$$\mathbf{U}_{o1} = \frac{2}{5} \left(u_A + u_B e^{j\frac{2}{5}\pi} + u_C e^{j\frac{4}{5}\pi} + u_D e^{j\frac{6}{5}\pi} + u_E e^{j\frac{8}{5}\pi} \right) \quad (5)$$

$$\mathbf{I}_{o1} = \frac{2}{5} \left(i_A + i_B e^{j\frac{2}{5}\pi} + i_C e^{j\frac{4}{5}\pi} + i_D e^{j\frac{6}{5}\pi} + i_E e^{j\frac{8}{5}\pi} \right) \quad (6)$$

$$\mathbf{U}_{o3} = \frac{2}{5} \left(u_A + u_B e^{j\frac{6}{5}\pi} + u_C e^{j\frac{2}{5}\pi} + u_D e^{j\frac{8}{5}\pi} + u_E e^{j\frac{4}{5}\pi} \right) \quad (7)$$

$$\mathbf{I}_{o3} = \frac{2}{5} \left(i_A + i_B e^{j\frac{6}{5}\pi} + i_C e^{j\frac{2}{5}\pi} + i_D e^{j\frac{8}{5}\pi} + i_E e^{j\frac{4}{5}\pi} \right) \quad (8)$$

where $\mathbf{U}_{o1}$ and $\mathbf{I}_{o1}$ are the space vectors of the output voltages and currents in the d1-q1 space, respectively; and $\mathbf{U}_{o3}$ and $\mathbf{I}_{o3}$ are in the d3-q3 space.

4. Duty Cycle Space Vector Approach

4.1. Single-Phase Duty Cycle Space Vector

The duty cycle space vector of an arbitrary output phase is defined in [20], and is expressed as follows:

$$D_X = \frac{2}{3} \left(d_{aX} + d_{bX} e^{j\frac{2}{3}\pi} + d_{cX} e^{j\frac{4}{3}\pi} \right) \quad (9)$$

Due to the constraint $d_{aX} + d_{bX} + d_{cX} = 1$, the expression of each duty cycle can be derived as follows:

$$d_{xX} = \frac{1}{3} + D_X \cdot e^{j(k-1)\frac{2}{3}\pi} \quad (10)$$

where $x = a, b, c; k = 1, 2, 3$.

4.2. Five-Phase Duty Cycle Space Vector

The duty cycle space vector D_X , instead of the modulation matrix D , can be used to express the relationships of output voltages and input currents. Then, Equations (1) and (2) can be rewritten in the d1-q1 space as follows:

$$u_{o1} = \frac{3}{10} U_i \left(D_A^* + D_B^* e^{j\frac{2}{5}\pi} + D_C^* e^{j\frac{4}{5}\pi} + D_D^* e^{j\frac{6}{5}\pi} + D_E^* e^{j\frac{8}{5}\pi} \right) + \frac{3}{10} U_i^* \left(D_A + D_B e^{j\frac{2}{5}\pi} + D_C e^{j\frac{4}{5}\pi} + D_D e^{j\frac{6}{5}\pi} + D_E e^{j\frac{8}{5}\pi} \right) \quad (11)$$

$$I_{i1} = \frac{1}{2} I_o \left(D_A + D_B e^{j\frac{8}{5}\pi} + D_C e^{j\frac{6}{5}\pi} + D_D e^{j\frac{4}{5}\pi} + D_E e^{j\frac{2}{5}\pi} \right) + \frac{1}{2} I_o^* \left(D_A + D_B e^{j\frac{2}{5}\pi} + D_C e^{j\frac{4}{5}\pi} + D_D e^{j\frac{6}{5}\pi} + D_E e^{j\frac{8}{5}\pi} \right) \quad (12)$$

Likewise, the input–output relationships in the d3-q3 space can be rewritten as follows:

$$u_{o3} = \frac{3}{10} U_i \left(D_A^* + D_B^* e^{j\frac{6}{5}\pi} + D_C^* e^{j\frac{2}{5}\pi} + D_D^* e^{j\frac{8}{5}\pi} + D_E^* e^{j\frac{4}{5}\pi} \right) + \frac{3}{10} U_i^* \left(D_A + D_B e^{j\frac{6}{5}\pi} + D_C e^{j\frac{2}{5}\pi} + D_D e^{j\frac{8}{5}\pi} + D_E e^{j\frac{4}{5}\pi} \right) \quad (13)$$

$$I_{i3} = \frac{1}{2} I_o \left(D_A + D_B e^{j\frac{4}{5}\pi} + D_C e^{j\frac{8}{5}\pi} + D_D e^{j\frac{2}{5}\pi} + D_E e^{j\frac{6}{5}\pi} \right) + \frac{1}{2} I_o^* \left(D_A + D_B e^{j\frac{8}{5}\pi} + D_C e^{j\frac{2}{5}\pi} + D_D e^{j\frac{6}{5}\pi} + D_E e^{j\frac{4}{5}\pi} \right) \quad (14)$$

Five new variables— D_1, D_2, D_3, D_4 , and D_0 —as defined by the Equations (11)–(14), can be defined as follows:

$$\begin{aligned} D_1 &= \frac{1}{5} \left(D_A + D_B e^{j\frac{2}{5}\pi} + D_C e^{j\frac{4}{5}\pi} + D_D e^{j\frac{6}{5}\pi} + D_E e^{j\frac{8}{5}\pi} \right) \\ D_2 &= \frac{1}{5} \left(D_A + D_B e^{j\frac{4}{5}\pi} + D_C e^{j\frac{8}{5}\pi} + D_D e^{j\frac{2}{5}\pi} + D_E e^{j\frac{6}{5}\pi} \right) \\ D_3 &= \frac{1}{5} \left(D_A + D_B e^{j\frac{6}{5}\pi} + D_C e^{j\frac{2}{5}\pi} + D_D e^{j\frac{8}{5}\pi} + D_E e^{j\frac{4}{5}\pi} \right) \\ D_4 &= \frac{1}{5} \left(D_A + D_B e^{j\frac{8}{5}\pi} + D_C e^{j\frac{6}{5}\pi} + D_D e^{j\frac{4}{5}\pi} + D_E e^{j\frac{2}{5}\pi} \right) \\ D_0 &= \frac{1}{5} (D_A + D_B + D_C + D_D + D_E). \end{aligned} \quad (15)$$

The inverse transformation expressions of Equation (15) are

$$\begin{aligned} D_A &= D_1 + D_2 + D_3 + D_4 + D_0 \\ D_B &= D_1 e^{j\frac{8}{5}\pi} + D_2 e^{j\frac{6}{5}\pi} + D_3 e^{j\frac{4}{5}\pi} + D_4 e^{j\frac{2}{5}\pi} + D_0 \\ D_C &= D_1 e^{j\frac{6}{5}\pi} + D_2 e^{j\frac{2}{5}\pi} + D_3 e^{j\frac{8}{5}\pi} + D_4 e^{j\frac{4}{5}\pi} + D_0 \\ D_D &= D_1 e^{j\frac{4}{5}\pi} + D_2 e^{j\frac{8}{5}\pi} + D_3 e^{j\frac{2}{5}\pi} + D_4 e^{j\frac{6}{5}\pi} + D_0 \\ D_E &= D_1 e^{j\frac{2}{5}\pi} + D_2 e^{j\frac{4}{5}\pi} + D_3 e^{j\frac{6}{5}\pi} + D_4 e^{j\frac{8}{5}\pi} + D_0. \end{aligned} \quad (16)$$

Then, the following equations can be obtained by substituting Equation (16) into Equations (11) and (12):

$$\begin{aligned} u_{o1} &= \frac{3}{2} U_i D_4^* + \frac{3}{2} U_i^* D_1 \\ I_{i1} &= \frac{5}{2} I_o D_4 + \frac{5}{2} I_o^* D_1. \end{aligned} \quad (17)$$

Equation (17) shows the input–output relationship of the three-to-five phase DMC effectively and compactly. In order to obtain the desired input power factor, suppose that $\bar{\psi}$ is the desired phase angle for the input current space vector. Then, Equation (17) can be rewritten in the following form:

$$\begin{aligned} \mathbf{U}_{o1} &= \frac{3}{2} \mathbf{U}_i \mathbf{D}_4^* + \frac{3}{2} \mathbf{U}_i^* \mathbf{D}_1 \\ (\mathbf{I}_o \mathbf{D}_4 + \mathbf{I}_o^* \mathbf{D}_1) \cdot \mathbf{j} \bar{\psi} &= 0. \end{aligned} \quad (18)$$

The general solution of Equation (18), which is valid for any value of the parameter λ_1 , is

$$\begin{aligned} \mathbf{D}_1 &= \frac{\mathbf{U}_{o1} \bar{\psi}}{3(\mathbf{U}_i \cdot \bar{\psi})} + \frac{\lambda_1}{\mathbf{U}_i^* \mathbf{I}_o} \\ \mathbf{D}_4 &= \frac{\mathbf{U}_{o1}^* \bar{\psi}}{3(\mathbf{U}_i \cdot \bar{\psi})} - \frac{\lambda_1}{\mathbf{U}_i^* \mathbf{I}_o}. \end{aligned} \quad (19)$$

Similarly, from substituting Equation (16) into Equations (13) and (14), $\mathbf{D}_2$ and $\mathbf{D}_3$ also can be deduced as follows:

$$\begin{aligned} \mathbf{D}_3 &= \frac{\mathbf{U}_{o3} \bar{\psi}}{3(\mathbf{U}_i \cdot \bar{\psi})} + \frac{\lambda_2}{\mathbf{U}_i^* \mathbf{I}_o} \\ \mathbf{D}_2 &= \frac{\mathbf{U}_{o3}^* \bar{\psi}}{3(\mathbf{U}_i \cdot \bar{\psi})} - \frac{\lambda_2}{\mathbf{U}_i^* \mathbf{I}_o}. \end{aligned} \quad (20)$$

In Equations (19) and (20), λ_1 and λ_2 are the degrees of freedom, which can be utilized only if the phase angle of the output current is known. In practical applications, the method of selecting $\lambda_1 = \lambda_2 = 0$, which does not take the output load into account, is still universal [20]. As seen from Equations (19) and (20), when the reference voltage is synthesized in the d1-q1 space, the space vectors in the d3-q3 space—which can cause serious distortion of the currents—are synthesized at the same time. Since the desired output voltage is a sinusoidal waveform, the space vector of $\mathbf{U}_{o3}$ in the d3-q3 space should be regarded as zero [17]. Then, Equations (19) and (20) can be rewritten as

$$\begin{aligned} \mathbf{D}_1 &= \frac{q}{3 \cos \varphi_i} e^{j\alpha_o} e^{j\beta_i}; \quad \mathbf{D}_2 = 0 \\ \mathbf{D}_3 &= 0; \quad \mathbf{D}_4 = \frac{q}{3 \cos \varphi_i} e^{-j\alpha_o} e^{j\beta_i} \end{aligned} \quad (21)$$

where $\cos \varphi_i$ is the input power factor.

Then, from substituting Equation (21) into Equation (16), the general expression for duty cycle space vectors of the three-to-five phase DMC can be obtained:

$$\mathbf{D}_X = \frac{2}{3} q \frac{\cos[\alpha_o - (k-1)\frac{2\pi}{5}]}{\cos \varphi_i} e^{j\beta_i} + \mathbf{D}_0 \quad (22)$$

According to Equations (22) and (10), when the input power factor is fixed at 1, the expressions of 15 duty cycles of the three-to-five phase DMC can be obtained as follows:

$$d_{xX} = \frac{1}{3} + \frac{2}{3} q \cos\left[\alpha_o - (k-1)\frac{2\pi}{5}\right] \cos\left[\beta_i - (l-1)\frac{2\pi}{3}\right] + \mathbf{D}_0 \cdot e^{j(l-1)\frac{2}{3}\pi} \quad (23)$$

where $k = 1, 2, 3, 4, 5$ and $l = 1, 2, 3$; q is the voltage transfer ratio; α_o is the angle of the space vector of the output voltage; β_i is the angle of the space vector of the input current; and $\mathbf{D}_0$ is an arbitrary zero sequence vector. This expression is a new and compact solution for determining the modulation method for the three-to-five phase DMC.

4.3. Discussion on the Permissible Maximum Value of the VTR

The expression of five duty cycle vectors at any moment is described by Equation (22).

As can be seen from Figure 2, the end of the five duty cycle vectors is on a line segment. The effect of $\mathbf{D}_0$ is to move the duty cycle vectors, which demonstrates a strong positive relationship between the lengths of these line segments and the voltage transfer ratio (VTR) q . The maximum difference among

the five cosine functions is $2\sin(2\pi/5)$, and the maximum length of any line segment in the triangle is 1. Therefore, the following expression can be obtained:

$$\frac{2}{3}q \frac{2\sin(\frac{2\pi}{5})}{\cos \varphi_i} \leq 1. \quad (24)$$

When the input side is the unity power factor, the maximum voltage transfer ratio is obtained, and $q = 0.7886$.

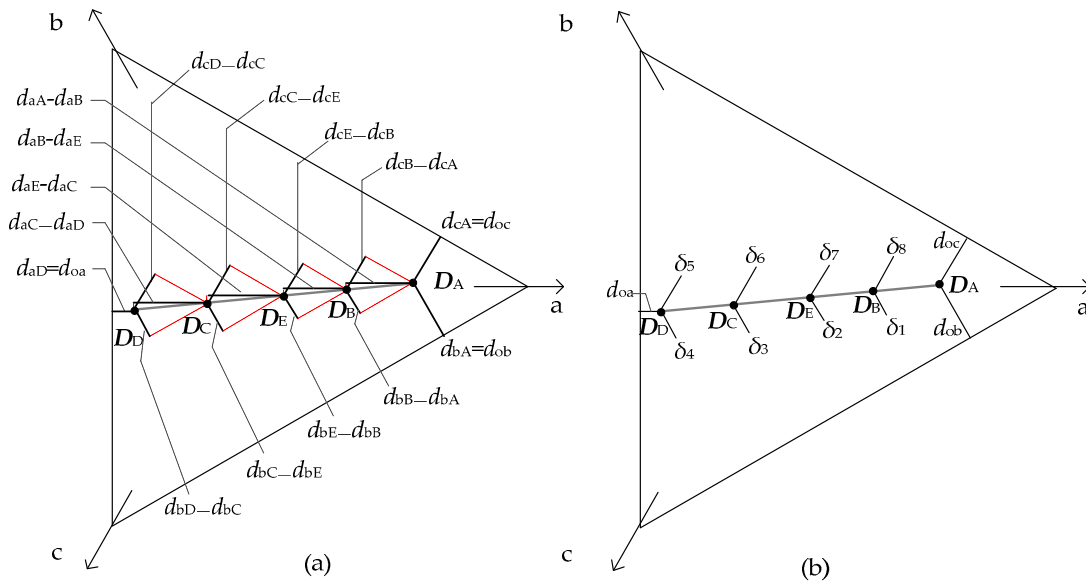


Figure 2. Decomposition and simplification of duty cycles of the three-to-five phase DMC: (a) decomposition of five duty-cycle space vectors; (b) simplification of a decomposed duty cycle.

5. SVPWM Based on DCSV

5.1. Decomposition and Simplification of Duty Cycles of the Three-to-Five Phase DMC

From Equation (1), the relationships between line-to-line output voltages and phase input voltages can be obtained as follows:

$$\begin{pmatrix} u_{AB} \\ u_{BC} \\ u_{CD} \\ u_{DE} \\ u_{EA} \end{pmatrix} = \begin{pmatrix} d_{aA} - d_{aB} & d_{bA} - d_{bB} & d_{cA} - d_{cB} \\ d_{aB} - d_{aC} & d_{bB} - d_{bC} & d_{cB} - d_{cC} \\ d_{aC} - d_{aD} & d_{bC} - d_{bD} & d_{cC} - d_{cD} \\ d_{aD} - d_{aE} & d_{bD} - d_{bE} & d_{cD} - d_{cE} \\ d_{aE} - d_{aA} & d_{bE} - d_{bA} & d_{cE} - d_{cA} \end{pmatrix} \begin{pmatrix} u_a \\ u_b \\ u_c \end{pmatrix} \quad (25)$$

Equation (25) shows that the line-to-line output voltages only depend on the difference between the two duty cycles. From Equation (23), which gives 15 duty cycles of switches, the difference between the two duty cycles in Equation (25) is irrelevant to D_0 . For instance, u_{AB} , u_{BC} , u_{CD} , u_{DE} , and u_{EA} only depend on $D_{AB}(D_A - D_B)$, $D_{BC}(D_B - D_C)$, $D_{CD}(D_C - D_D)$, $D_{DE}(D_D - D_E)$, and $D_{EA}(D_E - D_A)$. According to the geometrical representation proposed in [17], five duty cycle vectors (D_A , D_B , D_C , D_D , D_E) of the three-to-five phase DMC can be decomposed, as shown in Figure 2a.

As shown in Figure 2a, there is an equilateral triangle and its height is 1. The points represent the ends of five duty cycle vectors (D_A , D_B , D_C , D_D , and D_E). The size of the duty cycles is the distance between each point and the equilateral triangle's edge. Thus, duty cycles of 15 switches are obtained. By the example given in Figure 2a, the constraint on the angle of the input current is expressed as $-\pi/6 \leq \beta_i < \pi/6$, while the relationships between the five output voltages are expressed

as $u_D < u_C < u_E < u_B < u_A$. D_D, D_C, D_E, D_B , and D_A are arranged sequentially in the positive direction of the β_i . The 15 duty cycles in the modulation matrix can be expressed by 12 differences in duty cycles, and the relationships are as follows:”

$$\begin{cases} d_{aA} = d_{aD} + (d_{aC} - d_{aD}) + (d_{aE} - d_{aC}) + (d_{aB} - d_{aE}) + (d_{aA} - d_{aB}); & d_{aE} = d_{aD} + (d_{aC} - d_{aD}) + (d_{aE} - d_{aC}); \\ d_{aB} = d_{aD} + (d_{aC} - d_{aD}) + (d_{aE} - d_{aC}) + (d_{aB} - d_{aE}); & d_{aC} = d_{aD} + (d_{aC} - d_{aD}); & d_{aD} = d_{aD}; \\ d_{bA} = d_{bA}; & d_{bB} = d_{bA} + (d_{bB} - d_{bA}); & d_{bC} = d_{bA} + (d_{bB} - d_{bA}) + (d_{bE} - d_{bB}) + (d_{bC} - d_{bE}); \\ d_{bD} = d_{bA} + (d_{bB} - d_{bA}) + (d_{bE} - d_{bB}) + (d_{bC} - d_{bE}) + (d_{bD} - d_{bC}); & d_{bE} = d_{bA} + (d_{bB} - d_{bA}) + (d_{bE} - d_{bB}); \\ d_{cA} = d_{cA}; & d_{cB} = d_{cA} + (d_{cB} - d_{cA}); & d_{cC} = d_{cA} + (d_{cB} - d_{cA}) + (d_{cE} - d_{cB}) + (d_{cC} - d_{cE}); \\ d_{cD} = d_{cA} + (d_{cB} - d_{cA}) + (d_{cE} - d_{cB}) + (d_{cC} - d_{cE}) + (d_{cD} - d_{cC}); & d_{cE} = d_{cA} + (d_{cB} - d_{cA}) + (d_{cE} - d_{cB}). \end{cases} \quad (26)$$

Given the known geometric properties of equilateral triangles, it is easy to obtain the following relation:

$$\begin{aligned} d_{aC} - d_{aD} &= (d_{bD} - d_{bC}) + (d_{cD} - d_{cC}) \\ d_{aE} - d_{aC} &= (d_{bC} - d_{bE}) + (d_{cC} - d_{cE}) \\ d_{aB} - d_{aE} &= (d_{bE} - d_{bB}) + (d_{cE} - d_{cB}) \\ d_{aA} - d_{aB} &= (d_{bB} - d_{bA}) + (d_{cB} - d_{cA}). \end{aligned} \quad (27)$$

Taking Equation (27) into account, 15 duty cycles of the three-to-five phase DMC can be gained using eight differences in duty cycles, as shown in Figure 2b. Then, the eight differences in duty cycles can be redefined as follows:

$$\begin{aligned} \delta_1 &= d_{bB} - d_{bA}; & \delta_2 &= d_{bE} - d_{bB}; \\ \delta_3 &= d_{bC} - d_{bE}; & \delta_4 &= d_{bD} - d_{bC}; \\ \delta_5 &= d_{cD} - d_{cC}; & \delta_6 &= d_{cC} - d_{cE}; \\ \delta_7 &= d_{cE} - d_{cB}; & \delta_8 &= d_{cB} - d_{cA}. \end{aligned} \quad (28)$$

Then, defining the equalities $d_{aD} = d_{oa}$, $d_{bA} = d_{ob}$, and $d_{cA} = d_{oc}$, and substituting Equations (27) and (28) into Equation (26) leads to

$$\begin{cases} d_{aA} = d_{oa} + \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8; & d_{aB} = d_{oa} + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7; \\ d_{aC} = d_{oa} + \delta_4 + \delta_5; & d_{aD} = d_{oa}; & d_{aE} = d_{oa} + \delta_3 + \delta_4 + \delta_5 + \delta_6; \\ d_{bA} = d_{ob}; & d_{bB} = d_{ob} + \delta_1; & d_{bC} = d_{ob} + \delta_1 + \delta_2 + \delta_3; \\ d_{bD} = d_{ob} + \delta_1 + \delta_2 + \delta_3 + \delta_4; & d_{bE} = d_{ob} + \delta_1 + \delta_2; \\ d_{cA} = d_{oc}; & d_{cB} = d_{oc} + \delta_8; & d_{cC} = d_{oc} + \delta_6 + \delta_7 + \delta_8; \\ d_{cD} = d_{oc} + \delta_5 + \delta_6 + \delta_7 + \delta_8; & d_{cE} = d_{oc} + \delta_7 + \delta_8. \end{cases} \quad (29)$$

Since the average value of the output voltages in one switching period is decided by the turn-on time of switches rather than the order of the switching sequence, the method in this paper rearranges the switching sequence, as given by Equation (29). Therefore, the arrangement of the switching sequence can be expressed as shown in Figure 3.

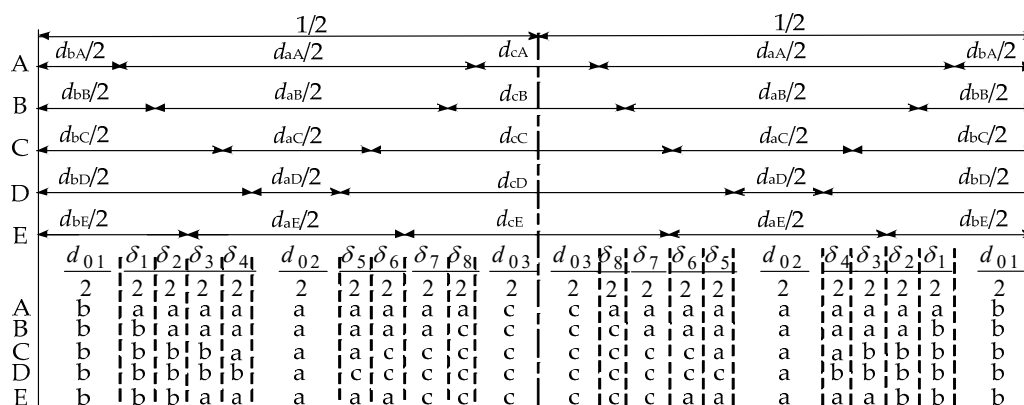


Figure 3. Arrangement of a switching sequence.

5.2. Segment Partition for Output Voltage and Input Current

As shown in Figure 2, the decomposition and simplification of the duty cycles will not be changed when $-\pi/6 \leq \beta_i < \pi/6$ and the relationship between the five output voltages does not change. Therefore, one period of input currents can be divided into six segments, while one period of expected five output voltages can be divided into 10 segments, as shown in Figure 4, where θ_j ($0 \leq \theta_j < \pi/5$) and θ_k ($0 \leq \theta_k < \pi/3$) are the relative position angles of the output voltage and input current in each segment, respectively.

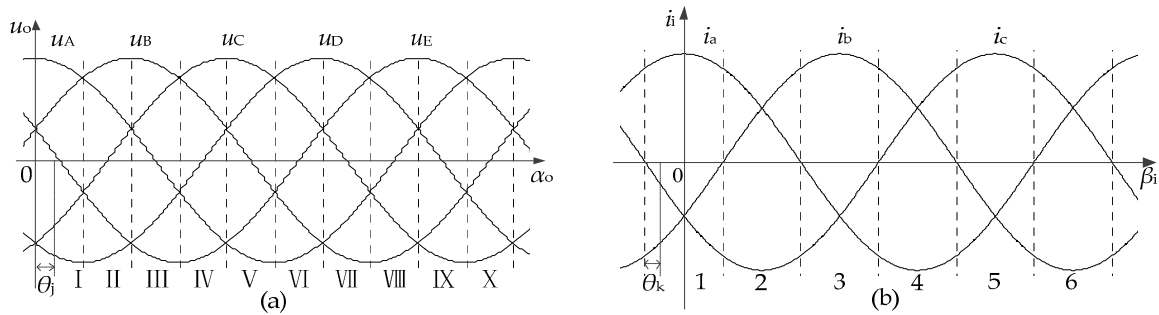


Figure 4. Segment partition (a) output voltages and (b) input currents.

5.3. General Expression of the Recombined Duty Cycles

The case illustrated in Figure 2 refers to the vectors of output voltage and input current lying in the corresponding sector 1, while $\theta_j = \alpha_o$ and $\theta_k = \beta_i + \pi/6$. By substituting Equation (23) into Equation (28), the expressions of eight recombined duty cycles can be obtained as follows:

$$\begin{cases} \delta_1 = \frac{4}{3}q \sin \frac{\pi}{5} \sin(\frac{\pi}{5} - \theta_j) \cos(\theta_k + \frac{\pi}{6}); & \delta_2 = \frac{4}{3}q \sin \frac{2\pi}{5} \sin \theta_j \cos(\theta_k + \frac{\pi}{6}); \\ \delta_3 = \frac{4}{3}q \sin \frac{2\pi}{5} \sin(\frac{\pi}{5} - \theta_j) \cos(\theta_k + \frac{\pi}{6}); & \delta_4 = \frac{4}{3}q \sin \frac{\pi}{5} \sin \theta_j \cos(\theta_k + \frac{\pi}{6}); \\ \delta_5 = \frac{4}{3}q \sin \frac{\pi}{5} \sin \theta_j \cos(\theta_k - \frac{\pi}{2}); & \delta_6 = \frac{4}{3}q \sin \frac{2\pi}{5} \sin(\frac{\pi}{5} - \theta_j) \cos(\theta_k - \frac{\pi}{2}); \\ \delta_7 = \frac{4}{3}q \sin \frac{2\pi}{5} \sin \theta_j \cos(\theta_k - \frac{\pi}{2}); & \delta_8 = \frac{4}{3}q \sin \frac{\pi}{5} \sin(\frac{\pi}{5} - \theta_j) \cos(\theta_k - \frac{\pi}{2}); \end{cases} \quad (30)$$

where $\theta_j = \alpha_o - (s-1)\pi/5$, $\theta_k = \beta_i - (t-1)\pi/3 + \pi/6$, and s and t represent the segment numbers of the output voltage and input current, respectively, as shown in Figure 4.

The expressions of eight recombined duty cycles in other segment combinations also can be required, and are related to the parity of the sum of the input current and the output voltage segment. The expressions of the duty cycles are shown in Equation (30) when the sum of segments is even, while they are shown in Equation (31) when the sum is odd. The relationship between odd (denoted by δ') and even (denoted by δ) can be expressed as follows:

$$\begin{aligned} \delta'_1 &= \delta_4; & \delta'_2 &= \delta_3; & \delta'_3 &= \delta_2; & \delta'_4 &= \delta_1; \\ \delta'_5 &= \delta_8; & \delta'_6 &= \delta_7; & \delta'_7 &= \delta_6; & \delta'_8 &= \delta_5. \end{aligned} \quad (31)$$

Due to the constraint $d_{aX} + d_{bX} + d_{cX} = 1$ on all duty cycles, the duty cycle of the zero space vector can be obtained:

$$d_0 = d_{01} + d_{02} + d_{03} = 1 - (\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8) \quad (32)$$

As seen from Equations (30)–(32), all active duty cycles and the zero duty cycle would be obtained easily if output voltages and input currents could be received. Thus, the method of SVPWM based on DCSV presented in this paper is simpler than the traditional SVPWM.

6. Experimental Results

To validate the accuracy and feasibility of the proposed modulation strategy, the DMC topology was built in the laboratory, as shown in Figure 5. The control system was implemented using the 32-bit digital signal processor (DSP) TMS320F28335 for calculating modulation waves and the field programmable gate array (FPGA) XC6SLX9 for implementing the switching sequences and the commutation process. Metal-oxide-semiconductor field-effect transistor (MOSFET) power switches were used to implement the power circuit. The MOSFET model was 2SK1940, with maximal turn-on and turn-off times of 135 ns and 330 ns, respectively. Therefore, the dead time was set to 500 ns in the program. In the experiment, the reference output voltage as set in the program beforehand. The DSP was applied to collect the input voltage, and generate PWM signals according to the modulation strategy. PWM signals were transmitted from the DSP to the FPGA, and four-step commutation [21] was achieved through logic programming in the FPGA. The clamp circuit was designed to protect the DMC against overvoltage of the input and/or output of the matrix converter. Experimental tests were carried out using a three-phase power supply ($f_{in} = 50$ Hz). The input filter's parameters were $R = 2 \Omega$, $L_f = 0.2$ mH, and $C_f = 20 \mu\text{F}$, and the five-phase loads were five-phase resistance–inductance (RL) balanced loads ($R = 16 \Omega$, $L = 12$ mH). The switching frequency of the bidirectional power switch was fixed at 10 kHz.

The experiments were divided into four cases. In one case, the input phase voltage value U_{in} was 80 Vrms, the output voltage frequency f_{out} was 20 Hz, and the voltage transfer ratio (VTR) was 0.5. The experimental waveforms are shown in Figure 6. The input phase voltage u_a and input phase current i_a in Figure 6a clearly show the unity power factor at the input side. However, there is a displacement angle between the input voltage and input current because of the input LC filter. Figure 6b shows the waveforms of the line-to-line output voltage u_{AB} and zoom of u_{AB} . Figure 6c shows the fast Fourier transform (FFT) analysis of u_{AB} , and the output voltage total harmonic distortion (THD) is 8%. It is evident that the output line-to-line voltage contains very few low-order harmonics. Figure 6d shows the output currents, which are balanced sinusoidal and symmetrical waveforms. From Figure 6b–d, it is confirmed that the fundamental output voltages are sinusoidal and symmetrical. Figure 6e shows the output line-to-line voltage u_{AB} , output phase voltage u_A and output phase current i_A , synchronously. There is a displacement angle between u_{AB} and u_A , which is about 54° . There is also a small displacement angle between u_A and i_A because of the resistance–inductance load.

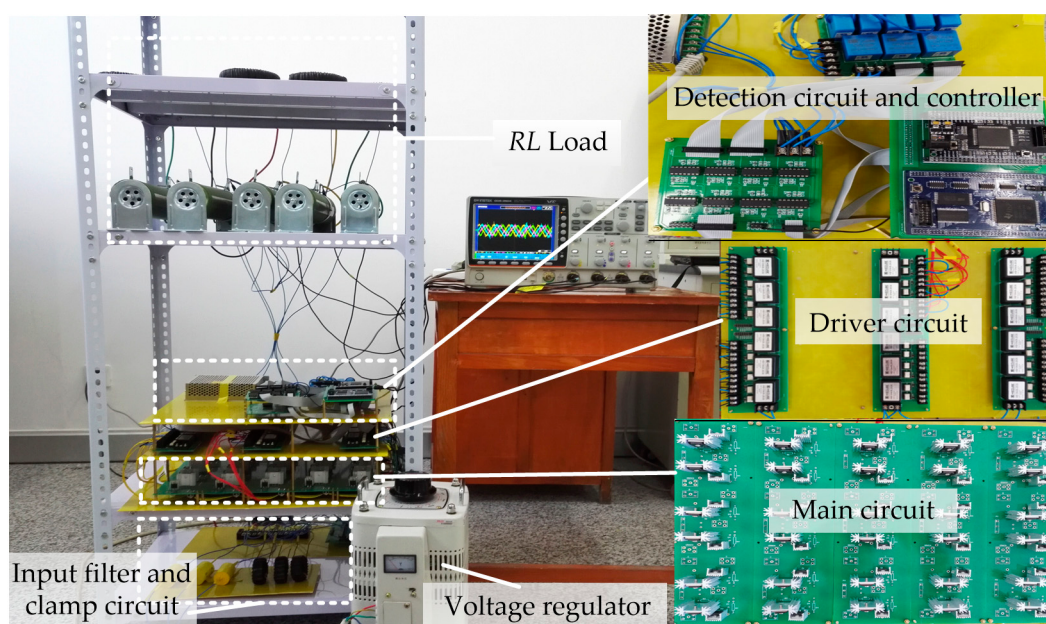


Figure 5. Experimental prototype.

Figure 7 shows the experimental results with $U_{in} = 80$ Vrms, $f_{out} = 20$ Hz, and $q = 0.6$. The waveforms in Figure 7 are similar to those in Figure 6, but the amplitude of input and output currents is just slightly larger because the VTR was changed from 0.5 to 0.6. It is clear that the proposed method has the ability to control the output voltage magnitude.

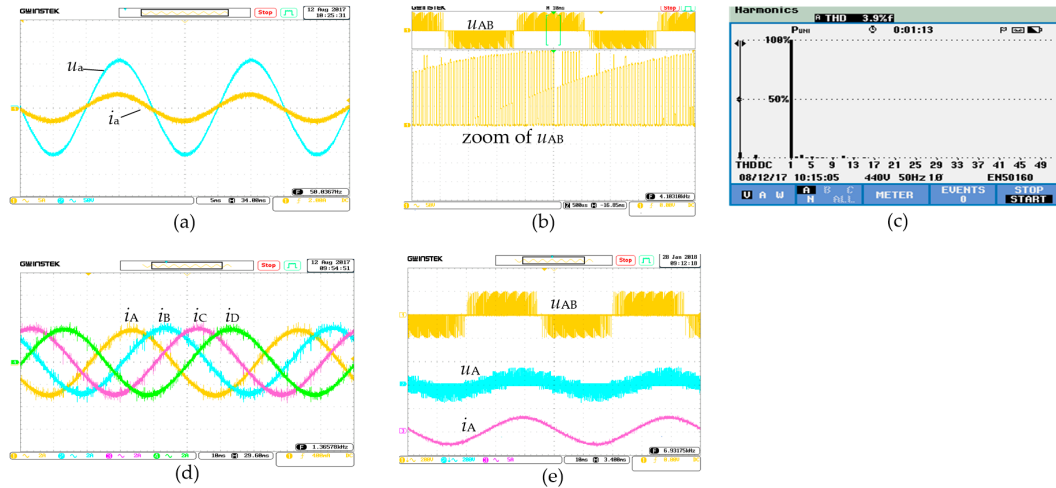


Figure 6. Experimental waveforms with $U_{in} = 80$ Vrms, $f_{out} = 20$ Hz, and $q = 0.5$: (a) input voltage u_a and input current i_a ; (b) output line-to-line voltage u_{AB} and zoom of u_{AB} ; (c) the FFT analysis of the u_{AB} ; (d) output currents i_A , i_B , i_C , i_D ; (e) output line-to-line voltage u_{AB} , output phase voltage u_A , and output phase current i_A .

For $U_{in} = 100$ Vrms and $f_{out} = 100$ Hz, experimental results are shown in Figures 8 and 9 with different VTRs of 0.5 and 0.6, respectively. Compared with Figures 6 and 7, the biggest difference is in the waveforms shown in Figure 8b,d,e and Figure 9b,d,e, because the output frequency is changed from 20 Hz to 100 Hz. It is clear that the proposed method has the ability to control the output voltage frequency to coincide with the reference values.

From the above experimental results, it is evident that performance of the three-to-five phase DMC with the proposed method in this paper can be achieved.

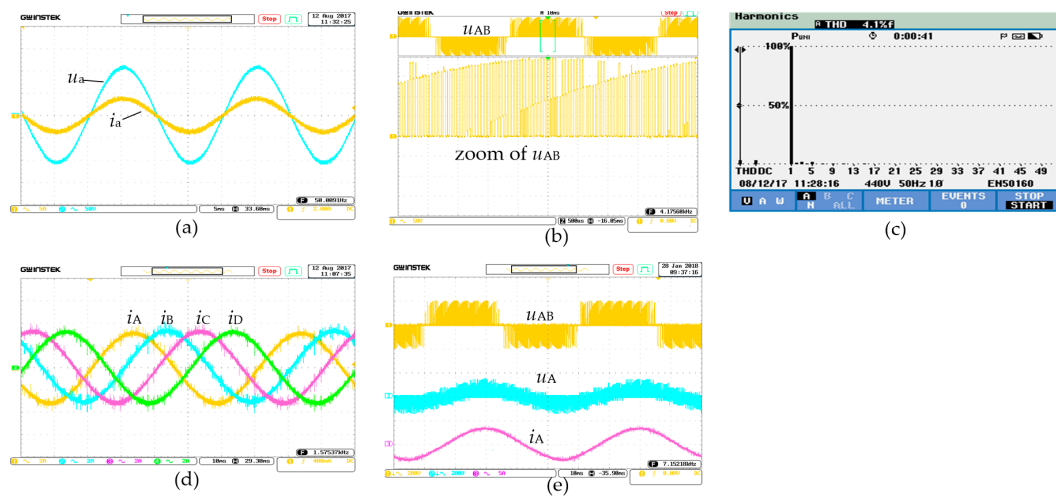


Figure 7. Experimental waveforms with $U_{in} = 80$ Vrms, $f_{out} = 20$ Hz, and $q = 0.6$: (a) input voltage u_a and input current i_a ; (b) output line-to-line voltage u_{AB} and zoom of u_{AB} ; (c) the FFT analysis of the u_{AB} ; (d) output currents i_A , i_B , i_C , i_D ; (e) output line-to-line voltage u_{AB} , output phase voltage u_A , and output phase current i_A .

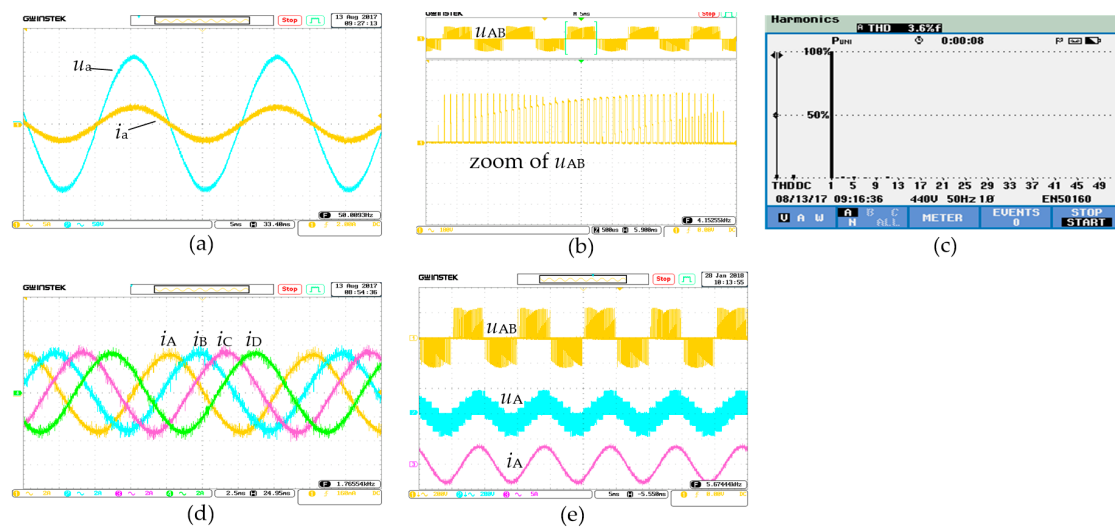


Figure 8. Experimental waveforms with $U_{in} = 100$ Vrms, $f_{out} = 100$ Hz, and $q = 0.5$: (a) input voltage u_a and input current i_a ; (b) output line-to-line voltage u_{AB} and zoom of u_{AB} ; (c) the FFT analysis of the u_{AB} ; (d) output currents i_A , i_B , i_C , i_D ; (e) output line-to-line voltage u_{AB} , output phase voltage u_A , and output phase current i_A .

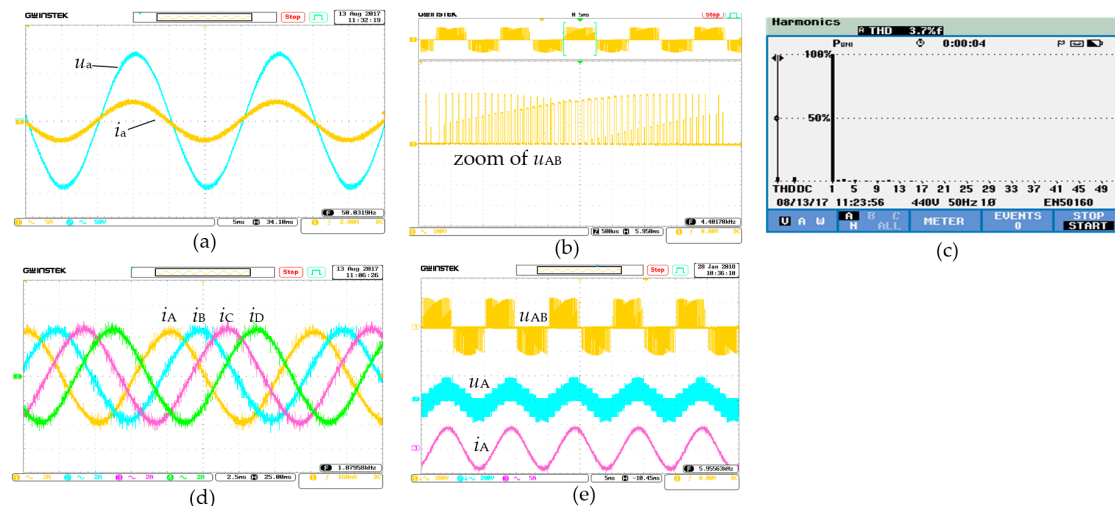


Figure 9. Experimental waveforms with $U_{in} = 100$ Vrms, $f_{out} = 100$ Hz, and $q = 0.6$: (a) input voltage u_a and input current i_a ; (b) output line-to-line voltage u_{AB} and zoom of u_{AB} ; (c) the FFT analysis of the u_{AB} ; (d) output currents i_A , i_B , i_C , i_D ; (e) output line-to-line voltage u_{AB} , output phase voltage u_A , and output phase current i_A .

7. Conclusions

In this paper, a novel method for the three-to-five phase direct matrix converter, named the DCSV strategy, was proposed. For this method, the general expression of the turn-on time of each switch was obtained. This method is of universal significance as it can realize different modulation strategies through suitable use of the zero configurations. In addition, an original SVPWM strategy based on the proposed DCSV was derived. The DCSV also has the ability to realize a reference load voltage of up to 0.7886 for the input voltage in the linear modulation region. A symmetrical switching sequence was obtained by using zero vectors and active vectors, and the fewest commutations were realized in one switching period. Finally, the feasibility of this novel method was validated through experimental results.

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Author Contributions: Rutian Wang and Xue Wang conceived the theory; Chuang Liu performed the experiments and analyzed the data; Xiwen Gao contributed materials and analysis tools; Rutian Wang and Xue Wang wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

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