

Article

Wide Area Coordinated Control of Multi-FACTS Devices to Damp Power System Oscillations

Shiyun Xu ^{1,2,*}, Ying Yang ^{3,*}, Kaixiang Peng ⁴, Linlin Li ⁴, Tasawar Hayat ^{2,5} and Ahmed Alsaedi ²¹ China Electric Power Research Institute, Beijing 100192, China² Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia; tahaksag@yahoo.com (T.H.); aalsaedi@kau.edu.sa (A.A.)³ College of Engineering, Peking University of China, Beijing 100871, China⁴ Key Laboratory of Knowledge Automation for Industrial Processes of Ministry of Education, School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China; kaixiang@ustb.edu.cn (K.P.); linlin.li@uni-due.de (L.L.)⁵ Department of Mathematics, Quaid-i-Azam University, 45320 Islamabad, Pakistan

* Correspondence: xushiyun@foxmail.com (S.X.); yy@mech.pku.edu.cn (Y.Y.)

Received: 3 November 2017; Accepted: 5 December 2017; Published: 14 December 2017

Abstract: Aiming at damping the inter-area oscillations of power systems, the present study proposes a wide-area decentralized coordinated control framework, where the upper-level controller is designed to coordinate the lower-level multiple FACTS devices. Based on the polytopic differential inclusion method, the derived controller adopts a decentralized structure and it is guaranteed to be robust to meet the demand of operation under multiple operating conditions. Since time delay of wide area signal transmission is inevitable, in what follows, the quantum evolution algorithm (QEA) method is introduced to find an optimal solution of the time-delay coordinated controller. In this regard, the stability of the system with a prescribed time delay is guaranteed and the system damping ratio is increased. Effectiveness and applicability of the proposed controller design methods have been demonstrated through numerical simulations.

Keywords: wide-area measurement system; FACTS devices; coordinated control; time delay; damping controllers; robustness

1. Introduction

With the rapid development of power systems, in recent years, the complexity of system structure and operating modes has been greatly increased [1] and the continuous and increasing demand of electrical energy consumption has greatly influenced the power system performance. In the operation of power systems, the insufficient damping of electromechanical oscillations is known to be a major constraint [2]. Such oscillations can be distinguished into two types, local oscillations that occur when generators in the same area oscillate with respect to each other and inter-area oscillations occurring among machines in different areas. If no adequate damping is available, the oscillations may cause operational limitations of power transmission capacity, or bring about the system separation, which in some cases may lead to blackouts [3–5].

Flexible AC Transmission System (FACTS) devices, including static var compensator (SVC), thyristor controlled series compensator (TCSC), static synchronous compensator (STATCOM) and so on, possess the rapid and reliable regulation property [6]. Since the construction of modern power grid demands the improvement of power flow distribution, system stability and transmission capacity in a flexible and reliable way, FACTS devices have been put into practical application and achieved satisfying control effects either act by modulating the reactive power or the active power or both [7].

FACTS controllers are located in the network where the controllability and observability of the inter-area oscillations are better. Generally speaking, FACTS devices exist in the power system are individually designed and installed for different targets due to local control and lack of coordination. Accordingly, a coordinated action among various FACTS devices is needed for the damping of inter-area oscillations and how to coordinately control the multi-FACTS devices to achieve greater effectiveness and at the same time, avoid adverse interaction that may occur between FACTS controllers have become an important research topic. Combined with modern control theory, the multi-FACTS coordinated control (MFCC) aiming at different control objectives has gained rich achievements over the past years [8,9]. In order to further improve system stability with the help of global information from wide-area measurement systems (WAMS) [10], more recently, the study of wide-area coordinated control is gradually increasing. However, it is noted that the study of MFCC based on WAMS is comparatively less, among which, reference [11] designs a controller that coordinates multiple robust FACTS damping controllers based on a BMI sequential approach. It indicates that MFCC can remarkably enhance system stability and at the same time, eliminate the negative interaction among devices and they also demonstrate the necessity of coordination.

Among the existing literature of MFCC, the control strategies are normally designed over a single dynamic model obtained from the linearization of system equations around one of the specific equilibria. Since system parameter matrices will change along with the variation of operating conditions, the derived control strategy should be guaranteed for each operating condition simultaneously, which leads to the complicatedness of calculation process. In this regard, how to meet the demand of operation under multiple cases is an important issue to be solved. On the other hand, time delays caused by the usage of communication networks to transfer the remote signal in the data transferring process is inevitable in WAMS, which will degrade the system performance or may even cause instability of the closed-loop system [12,13]. As a consequence, in a coordinated control strategy, it is of great significance to minimize the effect of time-delay [14]. Common methods of designing controllers to deal with the delay impact include equivalent treatment of time delay, robust control based on Linear Matrix Inequality (LMI) [15,16] and so on. However, for such robust coordinated controllers, the effect of time delay has seldom been taken into account in the previous literatures.

Based on the above considerations, the present study proposes a wide-area decentralized coordinated control framework for multiple FACTS devices. Aiming at realizing different control objectives, the upper-level coordinated controller is designed as both a robust dynamic output feedback controller and a time-delay output feedback controller. The polytopic differential inclusion method is introduced such that the derived dynamic output feedback controller is robust to various operating conditions. Moreover, the system damping ratio has been taken into account in the controller design strategy such that the system is capable to be operated under strong damping modes. In order to design the time-delay MFCC such that the stability of the system with a prescribed time delay is guaranteed, the sufficient condition of time-delay stability criterion proposed in [17] can be utilized. However, since the unknown objective parameters are coupled in the matrix inequalities, they cannot be solved by the LMI control toolbox in Matlab (R2016a, MathWorks, Natick, MA, USA). Aimed at deriving the controller parameter matrix, the quantum evolution algorithm (QEA) method is introduced to find an optimal solution. In this regard, the stability of the system with a prescribed time delay is guaranteed and the system damping ratio is increased. Validity and applicability of the proposed coordinated control algorithms are demonstrated in a two-area four-generator system. Simulation results demonstrate that the under robust coordinated control, the controlled power system successfully runs in strong damping modes in four different operating conditions and the algorithm exhibits good control effect in a wide range of time-delay.

2. Problem Formulation

In order to make preparation for the MFCC design, this section presents the MFCC framework based on WAMS. In the proposed framework, as is shown in Figure 1, a coordinated controller receives

the WAMS information and carries out calculation based on the state and output variable measured by WAMS data platform. The control instruction is then derived to be assigned to each FACTS device.

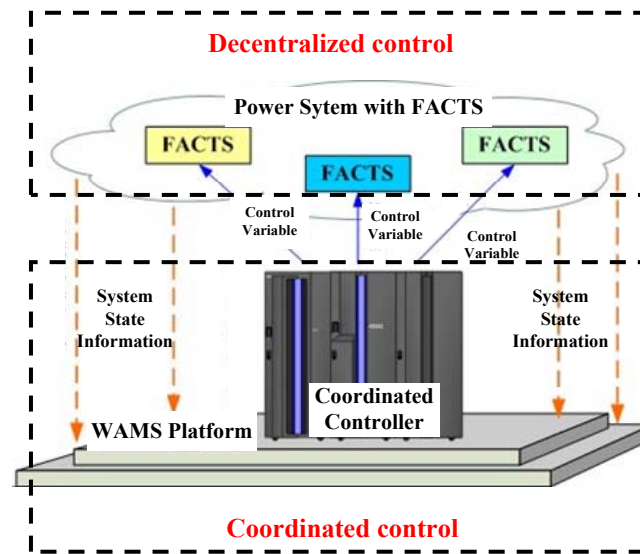


Figure 1. Coordinated control among FACTS devices based on WAMS.

By allocating the derived control signal as auxiliary input variables, the coordinated control among FACTS devices is then realized. For each FACTS device, the coordinated control variable u is received as a part of the controller input signal. It will be transmitted to the controller together with the local variable. Compared with local control, the wide-area coordination scheme is able to achieve global control through coordination control in a better way but it requires the entire system information, which may to some extent, influence the speed and accuracy of control.

In what follows, we give a brief description of some aspects involved in the power system model used in this work. For the sake of simplicity, the dynamic devices considered in this study mainly include generators and FACTS devices.

2.1. Generator Model

The generator is represented as a dynamic model equipped with a rapid excitation, whose model is described as:

$$\begin{aligned}
 \dot{\delta}_i &= \omega_0(\omega_i - 1) \\
 \dot{\omega}_i &= \frac{1}{2H_i}[P_{m_i} - E'_{d_i}I_{d_i} - E'_{q_i}I_{q_i} - D_i(\omega_i - 1)] \\
 \dot{E'_{q_i}} &= \frac{1}{T'_{d0_i}}[E'_{fd_i} - (x_{d_i} - x'_{d_i})I_{d_i} - E'_{q_i}] \\
 \dot{E'_{fd_i}} &= \frac{1}{T_A}[-E_{fd_i} + K_A(V_{ref_i} - V_t)]
 \end{aligned} \tag{1}$$

for $i = 1, 2, \dots, n$, where n is the number of synchronous generators. Referred to the generator i , δ_i is the rotor angle, ω_i is the rotor speed with respect to a synchronous reference, E'_{q_i} is the quadrature-axis transient voltage, E_{fd_i} is the field voltage, V_t is the terminal bus voltage magnitude. The definitions of the electrical quantities in Equation (1) can be found in [18].

2.2. TCSC Model

TCSC is a device constituted by a series capacitor bank with fixed value and a Thyristor-Controlled Reactor (TCR) [19]. It is installed directly in the transmission system and its equivalent reactance can be varied by adjusting the fire angle of the thyristors. In order to guarantee the additional damping supply to the inter-area oscillations of interest, a supplementary controller is required. TCSC is usually

represented by a first order linear model in small signal stability studies [20], which is also adopted in the present study. The block diagram of the adopted TCSC with a supplementary controller is given in Figure 2.

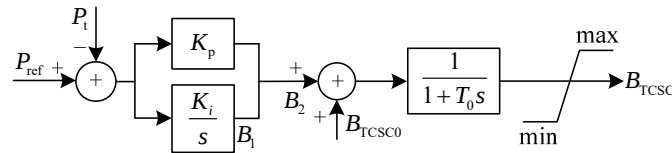


Figure 2. Dynamic model of TCSC.

The dynamic model based on Figure 2 can be written as:

$$\begin{cases} \dot{B}_1 = K_i(P_{\text{ref}} - P_t) \\ B_2 = B_1 + K_p(P_{\text{ref}} - P_t) \\ \dot{B}_{\text{TCSC}} = (B_2 + B_{\text{TCSC}0})/T_0 \end{cases} \quad (2)$$

where B_{TCSC} is the deviation of the equivalent TCSC reactance with respect to the nominal value, $B_{\text{TCSC}0}$ is the reference for the desired reactance deviation (from its nominal value) in steady state, B_2 is the stabilizing signal from the proposed supplementary controller, B_1 is an intermediate variable and T_0 is the device time constant. K_p and K_i are the gains of the PI control loop; P_{ref} and P_t are the active power reference and the active power of the TCSC control line, respectively.

2.3. SVC Model

SVC is one of the most widely applied FACTS devices which can maintain voltage stability and at the same time, improve the system damping. In this study, the control block of SVC mathematical formulation is shown in Figure 3.

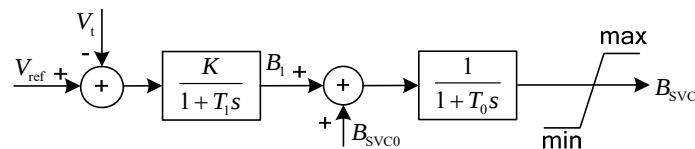


Figure 3. Dynamic model of SVC.

whose dynamical model is described as:

$$\begin{cases} \dot{B}_1 = \frac{1}{T_1}[-B_1 + K(V_{\text{ref}} - V_t)] \\ \dot{B}_{\text{SVC}} = \frac{1}{T_0}[B_1 - B_{\text{SVC}}] \end{cases} \quad (3)$$

where B_{SVC} is the equivalent susceptance output of SVC, $B_{\text{SVC}0}$ is the steady-state susceptance of SVC, B_1 is an intermediate variable and K is the gain of controller measurement. T_1 and T_0 are time constants, V_{ref} is the reference voltage and V_t is the measurement voltage of the SVC control point.

By integrating Equations (1)–(3), we derive the power system model composed by multiple generators and FACTS devices. After linearization around an equilibrium point, the state-space power system model can be represented by a linear time invariant (LTI) model given by a set of linear equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (4)$$

where x is the n -dimensional state vector, u is the p -dimensional system control input vector and y is the q -dimensional system output vector. A , B , C are given system parameter matrices with appropriate dimensions.

Aiming at realizing different control objectives, in this study, the upper-level wide-area coordinated controller is firstly designed as a robust dynamic output feedback controller. By further taking the time-delay into account, a time-delay output feedback controller is then proposed.

3. Main Results

During the practical operation of power systems, the state matrix A in Equation (4) may vary along with the changing of operating modes. In order to ensure the controller robustness to the variation of operating conditions, in this section, the wide-area MFCC adopts the dynamic output feedback control strategy for better dynamic characteristics, which is given as:

$$\begin{cases} \dot{x}_C = A_C x_C + B_C y \\ u = C_C x_C \end{cases} \quad (5)$$

where x_C is the n -dimensional state vector of controller and A_C , B_C , C_C are parameter matrices to be determined. Combining with the system dynamical Equation (4), the closed-loop controlled system is derived as:

$$\dot{\tilde{x}} = \tilde{A} \tilde{x} \quad (6)$$

where:

$$\tilde{x} = \begin{bmatrix} x \\ x_C \end{bmatrix}, \tilde{A} = \begin{bmatrix} A & B C_C \\ B_C C & A_C \end{bmatrix} \quad (7)$$

3.1. Robust MFCC Design

Based on the Lyapunov method [21], the problem of stabilizing system Equation (4) by the output feedback controller Equation (5) can be solved if and only if there exist matrices A_C , B_C , C_C and $\tilde{P} > 0$ for system Equation (6) such that the following matrix inequality holds:

$$\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} < 0 \quad (8)$$

In system Equations (6) and (7), the parameter matrix $\tilde{A}$ varies along with the variation of operating conditions. If system stability under different operating conditions is satisfied simultaneously, matrix inequality Equation (8) should be guaranteed for each operating condition, which leads to the complicatedness of calculation process. In order to solve the above-mentioned problem, the robust damping controller design method proposed in [22] treats the operating condition variation as uncertainties of nominal systems and the polytopic modeling method is introduced aiming at satisfying the robustness requirements.

Here, a polytopic model is composed by a series of p typical operating points. More specifically, under the i th operating condition, parameter matrices of system Equation (6) are presented as A_i , $i = 1, \dots, p$, which forms vertices of the polytope. The parameter matrices of state equations under the above p operating conditions compose a set:

$$\Phi = \{A_1, A_2, \dots, A_m\} \quad (9)$$

Construct a polytope Ω whose vertices are composed by elements of set Φ :

$$\Omega = \left\{ \sum_{i=1}^m s_i A_i, A_i \in \Phi, \sum_{i=1}^m s_i = 1, s_i \in R, s_i \geq 0 \right\} \quad (10)$$

Then for each vertex system, the closed-loop control system with a dynamic output feedback controller can be written as:

$$\dot{\tilde{x}} = \tilde{A}_i \tilde{x}, \tilde{A}_i = \begin{bmatrix} A_i & B_C C \\ B_C C & A_C \end{bmatrix} \quad (11)$$

where variables are defined the same as in Equations (6) and (7). In this regard, Equation (8) can be interpreted as finding a positive definite matrix $\tilde{P}$ and appropriate control parameter matrices A_C, B_C, C_C such that the following inequalities hold for $i = 1, \dots, p$:

$$\tilde{A}_i^T \tilde{P} + \tilde{P} \tilde{A}_i < 0 \quad (12)$$

Based on the polytopic property, unknown matrices A_C, B_C, C_C that satisfy Equation (12) can simultaneously stabilize, not limited to the chosen p vertex systems but all of the linear models included in the polytope. In other words, calculation procedure can be greatly simplified by utilizing the polytopic model [22].

Remark 1. Power systems are huge dimensional nonlinear dynamical systems and the system state-space matrices dimensions derived from linearization may leads to great difficulties in calculation. In practice, only several particular modes are useful for analysis, thus reducing the original systems into lower dimensional systems is commonly adopted as the first step to controller design and in this regard, critical system operating modes can be remained. In this study, the Hankel reduction method [23] is chosen for system reduction, which ensures that the errors of system Hankel singular value are in a relatively small range between the non-reduced and reduced systems.

In practical situations, power systems may possibly be operated under weak damping modes. Generally speaking, if and only if the damping ratios of all operation modes are larger than the damping ratio threshold ζ_0 (which is practically chosen as 0.03 or 0.05), then the system is said to be operated under a strong damping mode. However, condition Equation (12) does not guarantee a global minimum damping ratio of the system. Accordingly, stability criterion based on damping ratio can be realized by the pole placing method given in Theorem 1 [24].

Theorem 1. For a given minimum damping ratio $\zeta_{\min}$ of closed-loop system Equations (6) and (7), if and only if there exists a positive definite matrix $\tilde{P}$ such that the following matrix inequality holds

$$\begin{bmatrix} \sin \sigma(\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A}) & \cos \sigma(\tilde{A}^T \tilde{P} - \tilde{P} \tilde{A}) \\ * & \sin \sigma(\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A}) \end{bmatrix} < 0 \quad (13)$$

where $\sigma = \arccos \zeta_0$, then system Equations (6) and (7) is said to be asymptotically stable and meanwhile, $\zeta_{\min} \geq \zeta_0$ is guaranteed.

Remark 2. Due to the long distances among the generators and FACTS devices, it is desirable to implement a decentralized structure for damping controllers [25]. On the other hand, in matrix inequality Equation (13), the parameter matrix $\tilde{A}$ that includes unknown controller parameter matrix variables A_C, B_C, C_C is coupled with the unknown matrix variable $\tilde{P}$. Accordingly, Equation (13) turns out to be nonlinear and can only be solved by iteration, which leads to calculation time consumption and low efficiency. In this regard, reference [26] proposes a decoupling method of decentralized coordinated controller design, which transforms Equation (13) into an LMI that is conveniently solvable through Matlab LMI control toolbox.

In this study, the above-mentioned method is extended to the design of a robust MFCC algorithm for multiple operating modes, which is carried out in the next theorem. Choose p typical operating points and carry out Hankel order reduction, then we derive the state matrix parameters A_i, B and C of the closed-loop control system Equation (11) for $i = 1, \dots, p$.

Theorem 2. If there exist a positive symmetric matrix $Y > 0$, symmetric matrices P, X and matrices L, F, S such that LMIs Equations (14) and (15) holds for $i = 1, \dots, p$

$$\begin{bmatrix} \sin \sigma(A_i Y + Y A_i^T + BL + L^T B^T) & \cos \sigma(Y A_i^T - A_i Y + L^T B^T - BL) \\ * & \sin \sigma(A_i Y + Y A_i^T + BL + L^T B^T) \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} P & P \\ P & X \end{bmatrix} > 0, \quad \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & \Theta_{14}^T & \Theta_{24} \\ * & * & \Theta_{11} & \Theta_{12} \\ * & * & * & \Theta_{22} \end{bmatrix} < 0 \quad (15)$$

where $\Theta_{11} = \sin \sigma(PW_i + W_i^T P)$, $\Theta_{12} = \sin \sigma(PA_i + W_i^T X + C^T F^T + S)$, $\Theta_{13} = \cos \sigma(W_i^T P - PW_i)$, $\Theta_{14} = \cos \sigma(-PA_i + W_i^T X + C^T F^T + S)$, $\Theta_{22} = \sin \sigma(XA_i + W_i^T X + FC + C^T F^T)$, $\Theta_{24} = \cos \sigma(-XA_i + A_i^T X - FC + C^T F^T)$, with $W_i = A_i + BC_C$, then the close-loop system with coordinated controller is said to be asymptotically stable under operating modes $i = 1, \dots, p$ and the MFCC parameter variables can be obtained by $A_C = U^{-1}S^T P^{-T}P$, $B_C = (P - X)^{-1}F$, $C_C = LY^{-1}$.

Proof. Set A_C, B_C, C_C as diagonal matrices, then the controller added to each generator is related to its own input and output. Define diagonal matrices

$$\tilde{P} = \begin{bmatrix} X & U \\ U^T & X_c \end{bmatrix}, \quad \tilde{P}^{-1} = \begin{bmatrix} Y & V \\ V^T & Y_c \end{bmatrix} \quad (16)$$

and matrix variables $M = VA_C^T U^T$, $P = Y^{-1}$, $F = UB_C$, $S = Y^{-1}M$, $L = C_C V^T$. In combination with Equation (13), Equation (15) is equivalent to the following LMIs

$$AY + YA^T + BL + L^T B^T < 0 \quad (17)$$

$$\begin{bmatrix} PA + PBC_C + A^T P + C_C^T B_C P & PA + A^T X + C_C^T B^T X + C^T F^T + S \\ * & A^T X + XA + FC + C^T F^T \end{bmatrix} < 0 \quad (18)$$

where $*$ denotes the symmetric part of the matrix, thus completes the proof. $\square$

3.2. Time-Delay MFCC Design

As is previously mentioned, time delay degrades the dynamic performance and even violates the stability of a control system. However, the wide-area robust coordinated control algorithm proposed in the previous subsection has not taken the influence of time-delay into consideration. If the time-delay is relatively large, the controller may no longer stabilize the system. Moreover, during the design procedure of a wide-area measurement based controller, it is of great importance to estimate the maximum allowed time delay τ_0 that will not cause the loss of system stability. Aiming at eliminating the time-delay effect, in what follows, the time-delay MFCC will be designed as a dynamic and static output feedback controller, respectively.

A. Dynamic output feedback controller design

Suppose there is a constant time delay τ existing in the system output feedback of the output dynamic feedback controller Equation (5), namely, the controller is given by

$$\begin{cases} \dot{x}_C(t) = A_C x_C(t) + B_C y(t - \tau) \\ u(t) = C_C x_C(t) \end{cases} \quad (19)$$

The closed-loop system that includes the power system with FACTS devices, the time-delay MFCC given in Equation (19), and the wide-area signal transmission time delay τ is shown as in Figure 4.

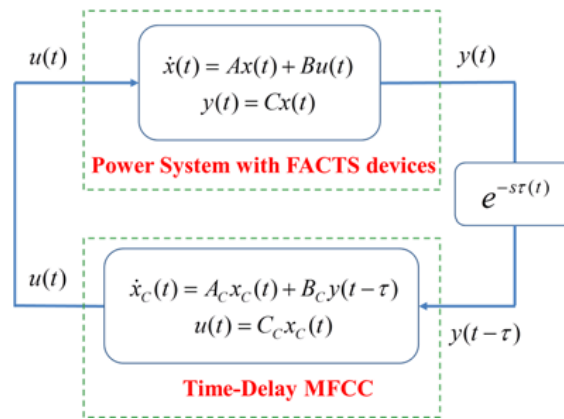


Figure 4. Closed-loop time-delay system under dynamic output feedback control.

By combining with the system equation, the closed-loop power system with time-delay is in the following form

$$\dot{\tilde{x}}(t) = A_0 \tilde{x}(t) + A_\tau \tilde{x}(t - \tau) \quad (20)$$

where $\tilde{x}$ is the same as in Equation (7) and the parameter matrices

$$A_0 = \begin{bmatrix} A & BC_C \\ A_C & 0 \end{bmatrix}, A_\tau = \begin{bmatrix} 0 & 0 \\ B_C C & 0 \end{bmatrix} \quad (21)$$

The following theorem can be used to determine the time-delay margin τ_0 .

Theorem 3. For the time-delay system Equation (20), if there exist positive definite matrices P , Q , V and a matrix W such that the following LMI holds

$$\begin{bmatrix} (A_0 + A_\tau)^T P + P(A_0 + A_\tau) + W^T A_\tau + A_\tau^T W + Q & -W^T A_\tau & A_0^T A_\tau^T V & 0 \\ * & -Q & A_\tau^T A_\tau^T V & 0 \\ * & * & -V & 0 \\ * & * & * & -V \end{bmatrix} < -\tau_0 \begin{bmatrix} 0 & 0 & 0 & (W + P) \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \quad (22)$$

then for all $\tau < \tau_0$, system (17) is asymptotically stable.

The proof of Theorem 3 can be found in [15]. By solving Equation (22) through the LMI control toolbox, the time-delay margin τ_0 can be conveniently derived.

B. Static output feedback controller considering time-varying delay

In order to eliminate the time delay effect, the MFCC designed in this part adopts a static output feedback control structure, where the system output variable y is the controller input. Then the output feedback coordinated controller considering time delay is given as

$$u(t) = Ky(t - \tau(t)) \quad (23)$$

where K is the coefficient matrix of controller, $\tau(t)$ is a time-varying delay, which is a continuous function of time and satisfies

$$0 \leq \tau(t) \leq \alpha, |\dot{\tau}(t)| \leq \beta \leq 1 \quad (24)$$

where α and β are upper bounds of time delay and its rate, respectively. For a constant time delay, $\beta = 0$, $\tau(t) = \tau = \alpha$.

The closed-loop system that includes the power system with FACTS devices, the time-delay MFCC given in Equation (24), and the time-varying wide-area signal transmission delay $\tau(t)$ is shown as in Figure 5. State equations of the closed-loop power system model with time-delay can be described by

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)) \quad (25)$$

where $A_d = BKC$ and the main task of designing a time-delay coordinated controller is to find a suitable K that stabilizes system Equation (25).

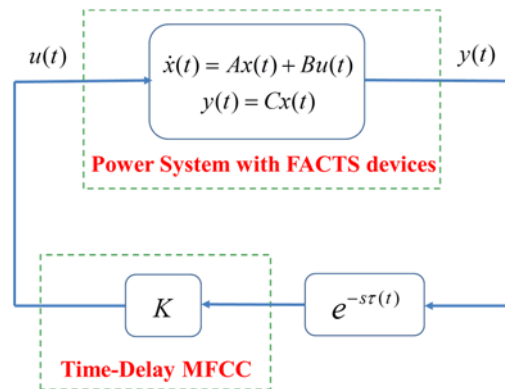


Figure 5. Closed-loop time-delay system under static output feedback control.

In order to guarantee the stability of time-delay system Equation (25), many methods have been proposed based on the Lyapunov theory. Among these methods, the free-weighting matrices method proposed in [27] has less conservativeness and the main idea is recalled briefly in the following theorems.

Theorem 4. If there exist matrices $P = P^T > 0$, $Q = Q^T > 0$, $Z = Z^T > 0$, $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0$, and any matrices N_1 and N_2 with appropriate dimensions such that the following matrix inequalities hold

$$\begin{bmatrix} PA + A^T P + N_1 + N_1^T + Q + \alpha X_{11} & PA_d - N_1 + N_2^T + \alpha X_{12} & \alpha A^T Z \\ * & -N_2 - N_2^T - (1 - \beta)Q + \alpha X_{22} & \alpha A_d^T Z \\ * & * & -\alpha Z \end{bmatrix} < 0 \quad (26)$$

$$\begin{bmatrix} X_{11} & X_{12} & N_2 \\ * & X_{22} & N_1 \\ * & * & Z \end{bmatrix} \geq 0 \quad (27)$$

then the system Equation (25) with time-varying delay is said to be asymptotically stable.

For a time-invariant delay system, set $X_{12} = 0$, $X_{22} = 0$ and $T = 0$ in Theorem 4, then we yield the following Corollary 1.

Corollary 1. If there exist matrices $P = P^T > 0$, $Q = Q^T > 0$, $Z = Z^T > 0$, $X \geq 0$, and any matrix N with appropriate dimension such that the following matrix inequalities hold

$$\begin{bmatrix} PA + A^T P + N + N^T + Q + \tau X & PA_d - N & \tau A^T Z \\ * & -Q & \tau A_d^T Z \\ * & * & -\tau Z \end{bmatrix} < 0 \quad (28)$$

$$\begin{bmatrix} X & N \\ * & Z \end{bmatrix} \geq 0 \quad (29)$$

then the system Equation (25) with a constant time delay is said to be asymptotically stable.

Remark 3. Theorem 4 and Corollary 1 present sufficient conditions to determine the stability of closed-loop systems with time-varying and constant time delay, respectively. However, since the unknown parameters are coupled in nonlinear matrix Equations (26) and (28), they cannot be solved by the LMI control toolbox in Matlab (R2016a, MathWorks, Natick, MA, USA). In fact, Corollary 1 only provides a sufficient condition of stability of Equation (6) for a given τ . In order to derive the controller parameter K , the QEA optimum algorithm is introduced in the following part.

Based on the concept and principles of quantum computing such as a quantum bit and superposition of states, QEA has been widely applied to seek the appropriate controller coefficients in control systems (one can refer to [28] for more detailed information). QEA combines the features of the quantum computation and the evolutionary algorithms, which has unique advantages in deriving the optimal solution, that is, the small population scale, fast convergence and capability of global optimization and so on. The basic QEA optimization procedure is shown as in Figure 6.

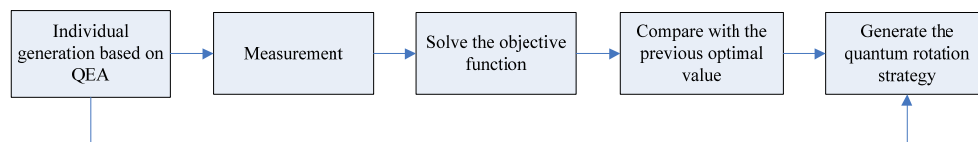


Figure 6. QEA optimization procedure.

After the iteration, the optimal value of the objective function can be derived within certain evolution algebra. Details of the algorithm and iterative parameter setting methods can be found in [20], where one only needs to set the range of K during application.

During the application of the QEA optimization, the constraint condition can be set as the existence of solution of Equations (26) and (27), such that the asymptotically stability of the obtained system is guaranteed. In what follows, for the sake of simplicity, the time-delay is chosen to be a constant number, and accordingly, the constraint condition can be set as the existence of solution of Equations (28) and (29) in Corollary 1. However, since multiple sets of feasible solutions may exist, therefore, by taking the practical demand of power systems into consideration, the objective function here is set to maximize the minimum system damping ratio. In this regard, the larger the minimum system damping ratio is, the more stable the system will be. At the same time, in order to make the control strategy reasonable, we set a minimum threshold of damping ratio (e.g., 7%). Once the threshold is exceeded, the optimization procedure will come to an end.

Before solving system damping ratio, the eigenvalue λ should be derived. The characteristic equation of time-delay system Equation (25) is

$$\det(\lambda I - A - A_d e^{-\tau\lambda}) = 0 \quad (30)$$

Equation (24) cannot be solved directly since it is a transcendental equation. To this end, in this study, the PDE discretization method is adopted for the approximation analysis. The delay differential equations of system Equation (25) can be converted to a set of Hyperbolic Partial Differential Equations (H-PDE) under the interconnected boundary conditions. As a consequence, the eigenvalues of the augmented matrix that obtained through fine discretization on PDE approximate to those of Equation (28). One can find more specific algorithm introduced in [29]. It should be noted that the number of Chebyshev discrete nodes needs to be set in this algorithm. Since we only focus on the

electromechanical modes, it would be precise enough by setting this number within the range of 20~30. In summary, the flow chart of time-delay MFCC design algorithm is shown in Figure 7.

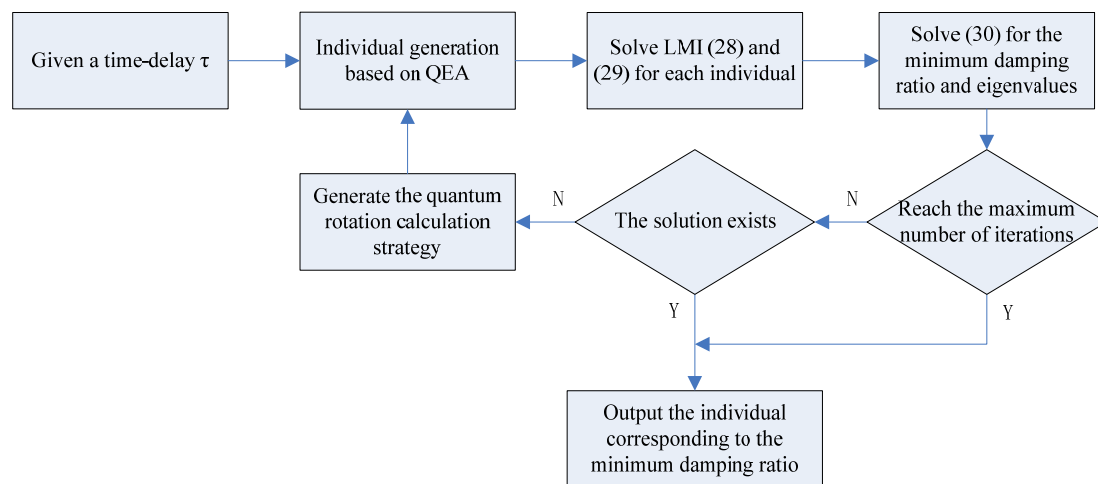


Figure 7. Flow chart of the controller design algorithm.

4. Numerical Simulations

In order to demonstrate the effectiveness of the two MFCC design algorithms proposed in the previous sections, consider the following two-area four-generator system shown as in Figure 8, where the system parameters are the same as in [18]. Since the voltage of Bus 7 is the lowest based on current system operation, an SVC is equipped to increase voltage and at the same time, a TCSC is equipped in the bus of inter-area system tie-line to remain the stability of power and damp inter-area oscillations.

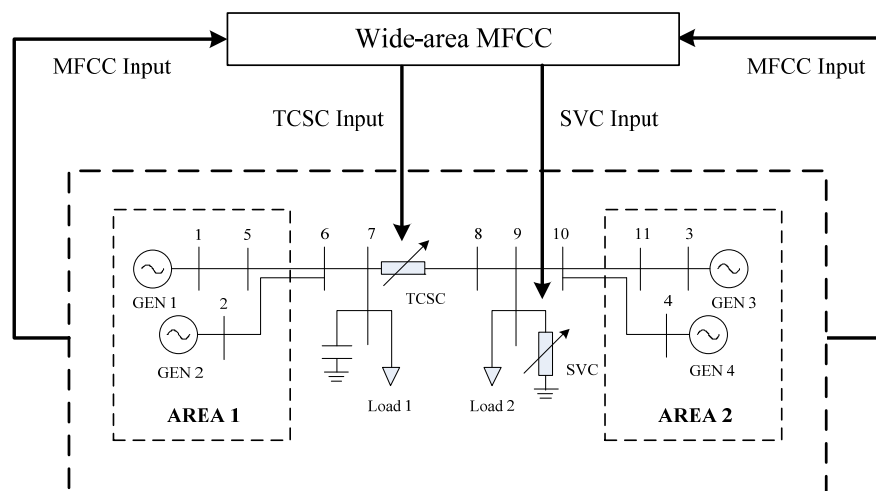


Figure 8. A four-machine, two area test system equipped with TCSC and SVC.

In virtue of the standard parameter tuning method for SISO controller design [30], the parameters of the SVC can be derived as $K = 100$, $T_1 = 0.05$, $T_0 = 0.01$; while the parameters of the TCSC can be derived as $K_P = 0.8$, $K_i = 10$, $T_0 = 0.01$. Characteristics under the above basic operation mode is derived as follows.

It is known from Table 1 that there is an inter-area oscillation mode of low damping in the system, thus in what follows, the proposed MFCC will be applied to improve the system damping and the effectiveness will be verified. Choose the velocity difference $\Delta\omega_1$ and $\Delta\omega_3$ of the most

relevant generators GEN1 and GEN3 for the system output, which is derived through the network communication system and carry out calculation.

Table 1. Modes Characteristics under the basic operation mode.

Modes	$-0.018 \pm j3.528$	$-0.728 \pm j6.324$	$-0.789 \pm j6.356$
Frequency/Hz	0.562	1.006	1.012
Damping Ratio	0.524%	11.443%	12.319%

Since system Equation (6) is a 24th-order dynamic model, it brings about great difficulties to the calculation of coordinated controllers. In order to speed up calculation, the Hankel reduction method introduced in Section 3.1 is applied to reduce the system into a 7th-order model. Comparison of the singular value before and after system reduction is shown in Figure 9. It can be observed from Figure 8 that the two curves almost perfectly in a wide range of frequencies.

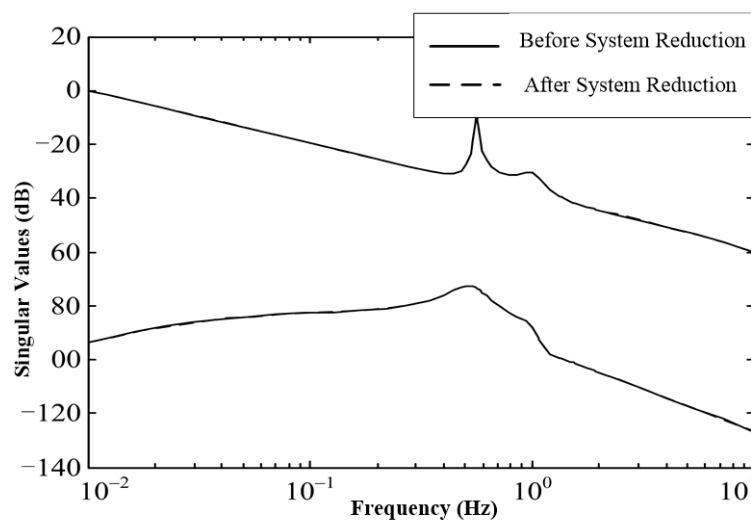


Figure 9. Comparison of Singular Values Before and After System Reduction.

In what follows, the effectiveness of the robust coordinated controller will be verified under three different operating conditions, built from variations of the load levels (shown as Load 1 and Load 2 in Figure 4) in both areas. The base case was also taken as a vertex system, the other two vertex systems are selected as increasing Load 1 by 100 MW while decreasing Load 2 by 100 MW, as well as decreasing Load 1 by 100 MW while increasing Load 2 by 100 MW, respectively. In this regard, the electromechanical modes under these three operating modes are given in Table 2.

Table 2. Modes Characteristics under three different cases.

	Description	Modes	Frequency/Hz	Damping Ratio
Case 1	Base Case	$-0.018 \pm j3.528$	0.562	0.524%
		$-0.728 \pm j6.324$	1.006	11.443%
		$-0.789 \pm j6.356$	1.012	12.319%
Case 2	Load 1: +100 MW Load 2: -100 MW	$-0.008 \pm j3.362$	0.535	0.231%
		$-0.728 \pm j6.291$	1.002	11.495%
		$-0.779 \pm j6.340$	1.009	12.194%
Case 3	Load 1: -100 MW Load 2: +100 MW	$-0.028 \pm j3.628$	0.578	0.7614%
		$-0.725 \pm j6.350$	1.011	11.339%
		$-0.795 \pm j6.364$	1.012	12.402%

It can be observed from Table 2 that, there exists weak damping of inter-area oscillation modes under the three operating modes. Set the damping ratio threshold as $\zeta_0 = 0.1$. The FACTS coordinated controller design algorithm in Theorem 2 is utilized to increase the system damping ratio, which guarantees that the minimum damping ratio is larger than 0.1. By solving Equations (14) and (15), the controller transfer function is derived as follows:

$$\begin{aligned} H_{11}(s) &= \frac{-0.181 \times 10^3 (s + 2.543 \pm 6.240j)(s + 1.327 \pm 1.769j)(s + 0.042)(s + 0.856)}{(s + 3.165 \pm 6.160j)(s + 2.807 \pm 3.079j)(s + 1.55642 \pm 1.219j)(s - 0.017)} \\ H_{12}(s) &= \frac{-1.055 \times 10^3 (s + 2.240 \pm 6.772j)(s + 2.432 \pm 4.172j)(s + 0.320)(s + 0.025)}{(s + 3.165 \pm 6.160j)(s + 2.807 \pm 3.079j)(s + 1.55642 \pm 1.219j)(s - 0.017)} \\ H_{21}(s) &= \frac{0.135 \times 10^3 (s + 0.630 \pm 6.553j)(s + 1.626 \pm 2.041j)(s + 0.030)(s + 1.260)}{(s + 3.165 \pm 6.160j)(s + 2.807 \pm 3.079j)(s + 1.55642 \pm 1.219j)(s - 0.017)} \\ H_{22}(s) &= \frac{1.475 \times 10^3 (s + 1.700 \pm 7.307j)(s + 2.421 \pm 3.526j)(s + 0.015)(s + 0.409)}{(s + 3.165 \pm 6.160j)(s + 2.807 \pm 3.079j)(s + 1.55642 \pm 1.219j)(s - 0.017)} \end{aligned} \quad (31)$$

where $H_{11}(s)$ and $H_{12}(s)$ are the transfer functions from the input value of the coordinated controller $\Delta\omega_1$ to the SVC and TCSC controller input, respectively; while $H_{21}(s)$ and $H_{22}(s)$ are the transfer function from the input value of the coordinated controller $\Delta\omega_3$ to the SVC and TCSC controller input, respectively.

Effectiveness of the control strategy is verified in the following operating conditions. The new electromechanical modes of the system under FACTS coordinated control are given in the following table and it can be observed from Table 3 that, the damping ratios under each electromechanical mode are larger than 10%, which satisfies the requirement of controller design.

Table 3. Characteristics of modes in three load cases with FACTS coordination.

	Description	Modes	Frequency/Hz	Damping Ratio
Case 1	Base Case	$-0.675 \pm j3.482$	0.554	19.0319%
		$-0.715 \pm j6.401$	1.018 7	11.0982%
		$-0.816 \pm j6.241$	0.993 3	12.958%
Case 2	Load 1: -100 MW	$-0.699 \pm j3.329$	0.529 9	20.5334%
	Load 2: $+100$ MW	$-0.711 \pm j6.371$	1.013 9	11.0872%
		$-0.809 \pm j6.232$	0.991 8	12.8706%
Case 3	Load 1: $+100$ MW	$-0.633 \pm j3.552$	0.565 3	17.5471%
	Load 2: -100 MW	$-0.716 \pm j6.420$	1.021 8	11.0898%
		$-0.817 \pm j6.247$	0.994 2	12.9673%

Moreover, the time delay margin can be calculated by the LMI Equation (22) given in Theorem 3. In virtue of the LMI robust control toolbox of Matlab (R2016a, MathWorks, Natick, MA, USA), the minimum time delay margin under each operation conditions of the controller are derived as $\tau_0 \approx 350$ ms. The theoretical results can be verified through exerting a large perturbation on the system. Choose a three-phase short-cut fault at bus 8 at 1 s, which is cleared at 1.1 s. Figure 10 depicts the time variation of inter-area tie-line power oscillations for different time delays of 100 ms, 200 ms, 300 ms and 330 ms, respectively.

It can be observed from Figure 9 that, the designed MFCC is tolerant to time delay less than 350 ms under different operating modes, which satisfies the demand of WAMS signal transmission.

In what follows, the effectiveness of the time-delay coordinated controller will be demonstrated. Assume that signal transmission time delay through WAMS is about 200 ms and the range of parameters in the QEA is set as $[10, 10]$. The minimum damping ratio threshold value is chosen as 7%. By carrying out the time-delay MFCC algorithm in Section 3.2, we arrive at the final optimization result of controller parameter K .

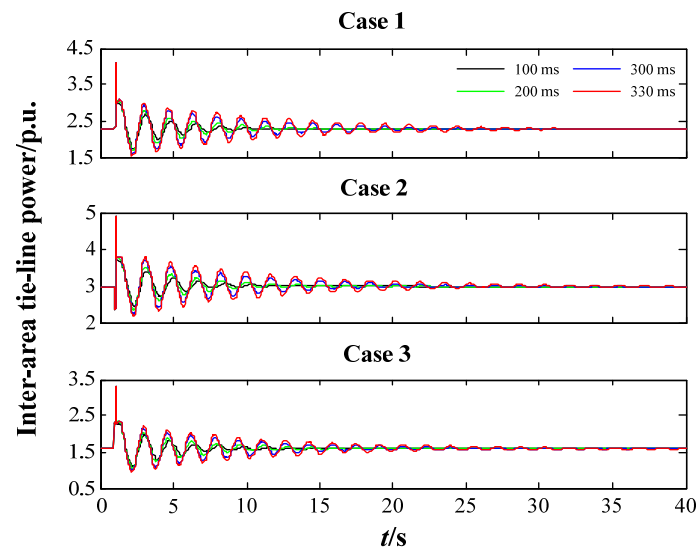


Figure 10. Power oscillation of the tie-line under different delay in three load cases.

In order to demonstrate the effectiveness of the derived controller, set the system disturbance as follows: increase the reference voltage of SVC by 0.01 p.u. at 1 s; and reduce it by 0.01 p.u. at 2 s. Figure 10 shows the curve of the inter-area tie-line power response, which compares the simulation results with and without coordinated control. It can be observed from Figure 11 that, since the minimum threshold of damping ratio is set, the system under coordinated control is able to be operated under strong damping mode with the controller K obtained by the algorithm proposed in this study. In consequence, considering the existence of signal transmission delay, the control performance of the MFCC is better than the individual control of FACTS without coordination.

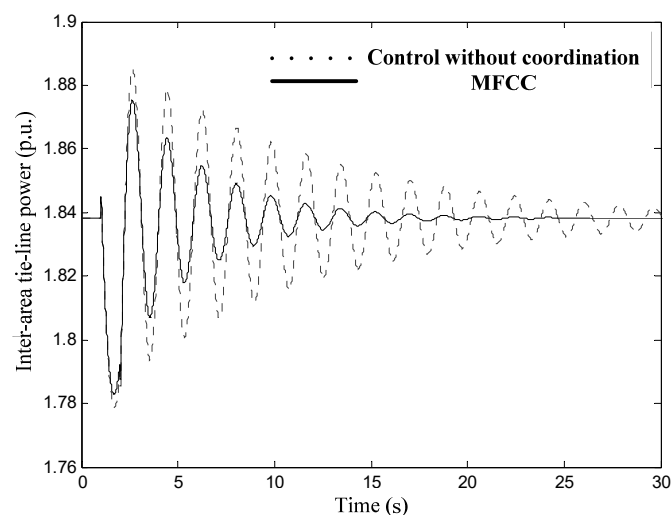


Figure 11. Power oscillation of the tie-line under 200 ms delay.

In order to determine the time-delay margin within which the coordinated controller can stabilize the system, Figure 11 depicts the curves of the minimum damping ratio of coordinated control system under different time delays, where the dashed horizontal line 0.0276 is the minimum damping ratio of system without coordinated control. It can be observed from Figure 12 that, since the MFCC algorithm is designed based on the time-delay of 200 ms, the minimum damping ratio reaches its maximum value under this time delay, which is about 0.07. Define the damping ratio above 0.03 as strong damping ratio, then for time delay less than 570 ms, the system will be operated under strong damping modes.

If the time delay is less than 600 ms, coordinated control can make the system damping better than that of control without coordination. It indicates that the algorithm proposed in this part has a relatively large delay margin, which guarantees that the system can be stabilized by the designed controller within the margin, that is, it demands less for the WAMS signal delay estimate.

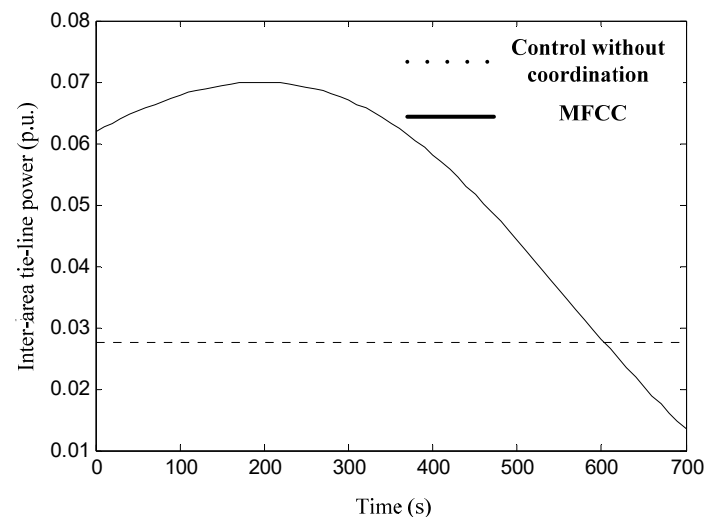


Figure 12. Ratio of power system under different delays.

5. Conclusions

The present study proposes a wide-area decentralized coordinated control framework for power systems with multiple FACTS devices, where the upper-level coordinated controller is designed as both a robust dynamic output feedback controller and a time-delay output feedback controller. The polytopic differential inclusion method is introduced during the dynamic output feedback controller design procedure and the derived controller is capable to be operated under strong damping modes and also remain robust to various operating conditions. The time-delay MFCC is designed in virtue of the output feedback signals from WAMS. In order to find an optimal solution, the quantum evolution algorithm (QEA) method is introduced. In this regard, the stability of the system with a prescribed time delay is guaranteed and the system damping ratio is increased. Validity and applicability of the proposed coordinated control algorithms are demonstrated in a two-area four-generator system. Simulation results demonstrate that the under robust coordinated control, the controlled power system successfully runs in strong damping modes in four different operating conditions and the algorithm exhibits good control effect in a wide range of time-delay.

Acknowledgments: We thank Liuqiang Huang for discussions and general support. This research has been supported by the key project of smart grid technology and equipment of national key research and development plan of china (2016YFB0900600) as well as the National Natural Science Foundation of China (71403285 and 71403017).

Author Contributions: S.X. and Y.Y. proposed the coordinated control strategy and the main theories; K.P. and L.L. helped with the optimization algorithm and performed numerical simulations; T.H. and A.A. contributed materials and analysis tools; S.X. wrote the paper.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Klein, M.; Rogers, G.; Kundur, P. A fundamental study of inter-area oscillations in power systems. *IEEE Trans. Power Syst.* **1991**, *6*, 914–921. [[CrossRef](#)]
2. Kamwa, I.; Grondin, G.; Hebert, Y. Wide-area measurement based stabilizing control of large power systems—A decentralized/hierarchical approach. *IEEE Trans. Power Syst.* **2001**, *16*, 136–153. [[CrossRef](#)]

3. Paserba, J. *Analysis and Control of Power System Oscillations*; CIGRE Special Publication 38.01.07, Technical Brochure III; International Council on Large Electric Systems (CIGRE): Paris, France, 1996.
4. Tedesco, F.; Casavola, A. Fault-tolerant distributed load/frequency supervisory strategies for networked multi-area microgrids. *Int. J. Robust Nonlinear Control* **2014**, *24*, 1380–1402. [[CrossRef](#)]
5. Magdy, E.; Sallam, A.; McCalley, J.; Fouad, A.A. Damping controller design for power system oscillation. *IEEE Trans. Power Syst.* **1996**, *11*, 767–773.
6. De Oliveira, R.V.; Kuiava, R.; Ramos, R.A.; Bretas, N.G. Automatic tuning method for the design of supplementary damping controllers for flexible alternating current transmission system devices. *IET Gener. Transm. Distrib.* **2009**, *3*, 919–929. [[CrossRef](#)]
7. Simfukwe, D.D.; Pal, B.C.; Jabr, R.A.; Martins, N. Robust and low-order design of flexible AC transmission systems and power system stabilisers for oscillation damping. *IET Gener. Transm. Distrib.* **2012**, *6*, 445–452. [[CrossRef](#)]
8. Sanchez-Gasca, J.J. Coordinated control of two FACTS devices for damping interarea oscillations. *IEEE Trans. Power Syst.* **1998**, *13*, 428–434. [[CrossRef](#)]
9. Noroozian, L.; Angquist, L.; Ghandhari, M.; Andersson, G. Improving power system dynamics by series-connected FACTS devices. *IEEE Trans. Power Deliv.* **1997**, *12*, 1635–1641. [[CrossRef](#)]
10. Phadke, A.G. Synchronized phasor measurements in power system. *IEEE Comput. Appl. Power* **1993**, *6*, 10–15. [[CrossRef](#)]
11. Deng, J.; Li, C.; Zhang, X.-P. Coordinated Design of Multiple Robust FACTS Damping Controllers: A BMI-Based Sequential Approach with Multi-Model Systems. *IEEE Trans. Power Syst.* **2015**, *30*, 3150–3159. [[CrossRef](#)]
12. Leon, A.E.; Solsona, J.A. Power Oscillation Damping Improvement by Adding Multiple Wind Farms to Wide-Area Coordinating Controls. *IEEE Trans. Power Syst.* **2014**, *29*, 1356–1364. [[CrossRef](#)]
13. Jiang, L.; Yao, W.; Wu, Q.H.; Wen, J.Y.; Cheng, S.J. Delay-dependent stability for load frequency control with constant and time-varying delays. *IEEE Trans. Power Syst.* **2012**, *27*, 932–941. [[CrossRef](#)]
14. Wei, Y.; Jiang, L.; Wen, J.; Wu, Q.H.; Cheng, S. Wide-area damping controller of FACTS devices for inter-area oscillations considering communication time delays. *IEEE Trans. Power Syst.* **2014**, *29*, 318–329.
15. Boyd, S.; El Ghaoui, L.; Feron, E.; Balakrishnan, V. *Linear Matrix Inequalities in System and Control Theory*; Society for Industrial and Applied Mathematics (SIAM): Philadelphia, PA, USA, 1994.
16. Gahinet, P.; Nemirovski, A. *LMI Control Toolbox for Use with MATLAB*; Mathworks Inc.: Natick, MD, USA, 1995.
17. Wu, M.; He, Y. *Robust Control of Time-Delay System*; Science Press: Beijing, China, 2007.
18. Anderson, P.M.; Fouad, A.A. *Power System Control and Stability*; IEEE Press: Piscataway, NJ, USA, 1994.
19. Hingorani, N.G.; Gyugyi, L. *Understanding FACTS: Concepts and Technology of Flexible AC Transmission Systems*; IEEE Press: New York, NY, USA, 2000.
20. Del Rosso, D.; Canizares, C.A.; Dona, V.M. A study of TCSC controller design for power system stability improvement. *IEEE Trans. Power Syst.* **2003**, *18*, 1487–1496. [[CrossRef](#)]
21. Zhou, K.; Doyle, J.C.; Glover, K. *Robust and Optimal Control*; Prentice Hall: Upper Saddle River, NJ, USA, 1995.
22. Pal, B.C.; Coonick, A.H.; Jaimoukha, I.M.; El-Zobaidi, H. A linear matrix inequality approach to robust damping control design in power systems with superconducting magnetic energy storage device. *IEEE Trans. Power Syst.* **2000**, *15*, 356–362. [[CrossRef](#)]
23. Kung, S.; Lin, D.W. Optimal Hankel-norm model reductions: Multivariable systems. *IEEE Trans. Autom. Control* **1981**, *26*, 832–852. [[CrossRef](#)]
24. Chiali, M.; Bretas, P.G. H_∞ design with pole placement constraints: An LMI approach. *IEEE Trans. Autom. Control* **1996**, *41*, 358–367. [[CrossRef](#)]
25. Xu, S.Y.; Sun, H.D.; Li, B.Q.; Bu, G.Q. Wide-Area Robust Decentralized Coordinated Control of HVDC Power System Based on Polytopic System Theory. *Math. Probl. Eng.* **2015**, *2015*, 510216. [[CrossRef](#)]
26. Oliveira, M.C.; Geromel, J.C.; Bernussou, J. Design of dynamic output feedback decentralized controllers via a separation procedure. *Int. J. Control* **2000**, *73*, 371–381. [[CrossRef](#)]
27. Wu, M.; He, Y.; She, J.H.; Liu, G.P. Delay-dependent criteria for robust stability of time-varying delay systems. *Automatica* **2004**, *40*, 1435–1439. [[CrossRef](#)]

28. Han, K.; Kim, H. Quantum-inspired evolutionary algorithm for a class of combinatorial optimization. *IEEE Trans. Evol. Comput.* **2002**, *6*, 580–593. [[CrossRef](#)]
29. Bellen, A.; Maset, S. Numerical solution of constant coefficient linear delay differential equations as abstract Cauchy problems. *Numer. Math.* **2000**, *84*, 351–374. [[CrossRef](#)]
30. Astrom, K.J.; Wittenmark, B. *Computer-Controlled Systems-Theory and Design*, 3rd ed.; Tsinghua University Press: Beijing, China, 2002; pp. 100–110.



© 2017 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).