# Supplementary Materials: The Techno-Economics of Small-Scale Residential Heating in Low Carbon Futures

**Avinash Vijay and Adam Hawkes**

## 1. Heating System Model

This section deals with the equations that govern operation of the residential heating system. The model makes use of a mixed integer linear program. Reasons behind this choice are explained in the main article. The structure of the optimization problem can be summarised as:

- Objective:
    - Minimise operating cost (For CHP this includes the revenue gained from electricity export)
- Subject to:
    - Electrical load balance
    - Thermal load balance
    - Heat to power ratio (For CHP)
    - Thermal energy storage operation
    - Component operational limits

Details regarding mathematical modelling of these constraints are presented below. Descriptions of the symbols used are presented in Appendix A.

### *1.1. Objective*

For the electric heater and heat pump, this only consists of operating costs as shown in Equation (S1).

$$\lambda = \sum_{t \in T} (\beta_t) \tag{S1}$$

For micro-CHP units the objective includes revenue from the export of electricity as shown in Equation (S2).

$$\lambda = \sum_{t \in T} (\beta_t - \gamma_t) \tag{S2}$$

### *1.2. Operational Cost*

Operational costs are the cost of natural gas and electricity imported. The cost of natural gas includes a fuel cost and a carbon cost. For the electric heater this describes consumption of the boiler and the overall electrical demand including heater usage as shown in Equation (S3):

$$\beta_t = \left(\frac{B_t}{\eta_B}\right)\left(\alpha_{gas} + \alpha_{CO2} e_{gas}\right) + P_t^{imp} \alpha_t^{imp} \tag{S3}$$

For the heat pump, only electricity is consumed as this does not include a boiler as shown in Equation (S4):

$$\beta_t = P_t^{imp} \alpha_t^{imp} \tag{S4}$$

For the micro-CHP unit, gas is consumed by both the boiler and the micro-CHP unit. Electricity imported is added to cost as shown in Equation (S5):

$$\beta_t = \left(\frac{P_t^{th}}{\eta_{th}} + \frac{B_t}{\eta_B}\right)\left(\alpha_{gas} + \alpha_{CO2} e_{gas}\right) + P_t^{imp} \alpha_t^{imp} \tag{S5}$$

### *1.3. Export Revenue*

Export revenue is the product of electricity exported and the export price of electricity summed over all periods as shown in Equation (S6):

$$\gamma_t = P_t^{exp} \alpha_t^{wholesale} \quad \text{(S6)}$$

### *1.4. Electrical Load Balance*

Generation and import are on the left side of the equation. Demand and export are on the right side of the equation. The components for the electric heater, heat pump and micro-CHP are shown in Equation (S7), Equation (S8) and Equation (S9), respectively:

$$P_t^{imp} = P_t^{EH} + D_t^{el} \quad \text{(S7)}$$

$$P_t^{imp} = P_t^{HP} + D_t^{el} \quad \text{(S8)}$$

$$P_t^{imp} + P_t^{el} = P_t^{exp} + D_t^{el} \quad \text{(S9)}$$

### *1.5. Thermal Load Balance*

Generation and heat supplied by installed technology are on the left side of the equation. Demand and storage are on the right side of the equation. The components for the electric heater, heat pump and micro-CHP are shown in Equation (S10), Equation (S11) and Equation (S12), respectively:

$$B_t + \eta_{EH} P_t^{EH} = D_t^{th} + R_t^+ - R_t^- \quad \text{(S10)}$$

$$COP_{HP} P_t^{HP} = D_t^{th} + R_t^+ - R_t^- \quad \text{(S11)}$$

$$B_t + P_t^{th} = D_t^{th} + R_t^+ - R_t^- \quad \text{(S12)}$$

### *1.6. Heat to Power Ratio*

This governs the relation between thermal and electrical power supplied by a micro-CHP unit as shown in Equation (S13):

$$P_t^{el} = \eta_{el} \left( \frac{P_t^{th}}{\eta_{th}} \right) \quad \text{(S13)}$$

### *1.7. Thermal Energy Storage*

These equations govern thermal storage operation. Current state of charge is a function of previous state of charge and the charging/discharging level in the current period as shown in Equation (S14). The storage unit is not allowed to charge and discharge simultaneously according to Equation (S15). Charging and discharging limits are governed by Equation (S16) and Equation (S17). The state of charge at the beginning of the planning horizon is the same as that seen at the end according to Equation (S18). Storage capacity is defined by Equation (S19):

$$S_t = S_{t-1} + \eta_{ch} R_t^+ - \frac{R_t^-}{\eta_{disch}} \quad \text{(S14)}$$

$$\phi_t^+ + \phi_t^- \le 1 \quad \text{(S15)}$$

$$0 \le R_t^+ \le \overline{S} \phi_t^+ \quad \text{(S16)}$$

$$0 \le R_t^- \le \overline{S} \phi_t^- \quad \text{(S17)}$$

$$S_1 = S_{T_{end}} \quad \text{(S18)}$$

$$0 \le S_t \le \overline{S} \quad \text{(S19)}$$

*1.8. Component Operational Limits*

Each of the components: boiler, electric heater, heat pump and micro-CHP have associated capacity limits. These are described by Equations (S20) to (S23):

$$0 \leq B_t \leq \overline{B} \tag{S20}$$

$$0 \leq P_t^{EH} \leq \overline{P^{EH}} \tag{S21}$$

$$0 \leq P_t^{HP} \leq \overline{P^{HP}} \tag{S22}$$

$$0 \leq P_t^{el} \leq \overline{P^{el}} \tag{S23}$$

*1.9. Performance Indicators*

This study focuses on the comparison of economic and environmental performance. Economic performance is measured in terms of the Equivalent Annual Cost (EAC) which is defined by Equation (S24) for heating systems other than micro CHP and Equation (S25) for micro CHP:

$$\Phi = \Psi \frac{r}{1 - \frac{1}{(1+r)^L}} + \sum_{k \in K} w_k \left( \sum_{t \in T} (\beta_t) \right) \tag{S24}$$

$$\Phi = \Psi \frac{r}{1 - \frac{1}{(1+r)^L}} + \sum_{k \in K} w_k \left( \sum_{t \in T} (\beta_t - \gamma_t) \right) \tag{S25}$$

Environmental performance is measured in terms of annual carbon emissions which is defined by Equation (S26) for heating systems other than micro CHP and Equation (S27) for micro CHP:

$$E = \sum_{k \in K} w_k \left( \sum_{t \in T} \left( e_{gas} \left( \frac{B_t}{\eta_B} \right) + e_{el} P_t^{imp} \right) \right) \tag{S26}$$

$$E = \sum_{k \in K} w_k \left( \sum_{t \in T} \left( e_{gas} \left( \frac{B_t}{\eta_B} + \frac{P_t^{th}}{\eta_{th}} \right) + e_{el} P_t^{imp} - e_{el} P_t^{el} \right) \right) \tag{S27}$$

## 2. Data Used in the Simulations

Capital costs of different generator types are from a tool developed by the UK government [1]. Lifespan values for generator types are from [2]. Future electricity system scenario assumptions such as energy mix, fuel prices and renewable output are the same as those used in [3], using the National Grid Gone Green scenario as a basis. The intensity of carbon emissions for electricity generation for the year 2035 is from the same National Grid reference [4] used in [3]. Natural gas prices are from the UK government's projections for the year 2035 [5]. Carbon prices are based on the non-traded price of carbon data in the UK government's valuation of energy use and greenhouse gas emissions [6]. The carbon intensity of natural gas is from a report on greenhouse gas reporting released by the UK government [7]. The uplift of wholesale price to give retail price is based on projections generated by the UK government [5]. Capital costs for heating technologies in the year 2035 are within the ranges specified in [8] and cross-referenced with the UK TIMES model [9]. Technology learning for the fuel cell, Stirling engine and heat pump has been discussed in existing literature [8,10,11]. As these learning curve forecasts tend to be especially optimistic about reduction in capital cost, this article makes use of realistic estimates used in the UK TIMES model [9]. Further details and individual values are presented in Table S1.

**Table S1.** Heating system parameters in the year 2035. Note that a complete fuel cell micro-CHP system requires a fuel cell, fuel cell reformer, TES and boiler. Likewise a complete Stirling engine system requires a Stirling engine, TES and a boiler.

| Parameter | Description | Units | Range | Chosen Value | Ref. |
|---|---|---|---|---|---|
| $\alpha_{CO2}$ | Non-traded price of carbon | £/tonne | 113 | 113 | [6] |
| $\alpha_{gas}$ | Natural gas price | p/kWh | 4.5 | 4.5 | [5] |
| $\eta_B$ | Efficiency of natural gas boiler | - | 0.8-0.9 | 0.86 | [12]–[16] |
| $\eta_{ch}, \eta_{disch}$ | Charging/discharging efficiency of TES | - | 0.4-0.97 | 0.9 | [17]–[19] |
| $\eta_{el}$ (Fuel cell) | Electrical efficiency | - | 0.3-0.45 | 0.45 | [20]–[22] |
| $\eta_{th}$ (Fuel cell) | Thermal efficiency | - | 0.39-0.48 | 0.45 | [20]–[22] |
| $\eta_{el}$ (Stirling engine) | Electrical efficiency | - | 0.125-0.22 | 0.10 | [23]–[25] |
| $\eta_{th}$ (Stirling engine) | Thermal efficiency | - | 0.6-0.8 | 0.80 | [23]–[25] |
| $\Psi$ (Boiler) | Upfront cost | £ | 2200-2500 | 2328.1 | [26], [11], [9] |
| $\Psi$ (TES) | Upfront cost | £ | 50-1300 | 415.4 | [9], [27], [28] |
| $\Psi$ (Heat pump) | Upfront cost | £ | 6653-35363 | 7587.3 | [9], [8] |
| $\Psi$ (Fuel cell) | Upfront cost | £ | 1775-15152 | 5037 | [21], [9], [8], [10], [29] |
| $\Psi$ (Fuel cell reformer) | Upfront cost | £ | 240-3800 | 717.4 | [9], [30], [31] |
| $\Psi$ (Stirling engine) | Upfront cost | £ | 750-26500 | 4193.4 | [9], [11], [21], [32] |
| $\overline{B}$ | Capacity of natural gas boiler | kW | - | 30 | - |
| $COP_{HP}$ | Coefficient of Performance of the heat pump | - | 2-5.8 | 3 | [33]–[35] |
| $e_{el}$ | Carbon intensity of electrical grid | kgCO2/ kWh | - | 0.0753 | [4] |
| $e_{gas}$ | Carbon intensity of natural gas | kgCO2/ kWh | - | 0.1840 | [7] |
| $L$ (All heating options) | Lifespan | years | 10-15 | 15 | [20], [21], [23] |
| $\overline{P^{EH}}$ | Capacity of resistive heater | kW | - | 3.5 | - |
| $\overline{P^{HP}}$ | Capacity of heat pump | kW | - | 27 | - |
| $\overline{P^{el}}$ | Electrical capacity of CHP unit | kW | - | 1 | - |
| $r$ | Discount rate | % | - | 3 | - |
| $\overline{S}$ | Capacity of thermal energy storage | kWh | - | 5 | - |

## Appendix A

List of symbols

**SETS**

| | |
|---|---|
| $J$ | Set of all generators $j \in J$ |
| $T$ | Set of all time periods $t \in T$ |
| $K$ | Set of all sample days $k \in K$ |

**PARAMETERS**

| | |
|---|---|
| $\alpha_{gas}$ | Natural gas price (p/kWh) |
| $\alpha_{CO2}$ | Non-traded price of carbon (£/tonne) |
| $\alpha_t^{imp}$ | Retail price of electricity (p/kWh) |
| $\alpha_t^{wholesale}$ | Wholesale price of electricity (p/kWh) |
| $\eta_B$ | Efficiency of natural gas boiler |
| $\eta_{ch}$ | Charging efficiency of thermal energy storage |
| $\eta_{disch}$ | Discharging efficiency of thermal energy storage |
| $\eta_{el}$ | Electrical efficiency of CHP unit |
| $\eta_{th}$ | Thermal efficiency of CHP unit |
| $\eta_{EH}$ | Efficiency of resistive heater |
| $\Gamma_t$ | Feasible operating region of pumped storage in period t |
| $E$ | Annual carbon emissions (kg) |
| $\Pi_{j,t}$ | Feasible operating region of generator j in period t |
| $\Phi$ | Equivalent Annual Cost (£) |
| $\Psi$ | Upfront cost of heating technology (£) |
| $\mathrm{A}^{wholesale}$ | Mean value of wholesale price of electricity (p/kWh) |
| $\overline{B}$ | Capacity of natural gas boiler (kW) |
| $COP_{HP}$ | Coefficient of Performance of the heat pump |
| $D_t$ | Electricity demand in the dispatch model in period t (MW) |
| $D_t^{el}$ | Electricity demand in the heating system model (kW) |
| $D_t^{th}$ | Thermal demand in the heating system model (kW) |
| $e_{gas}$ | Carbon intensity of natural gas (kg/kWh) |
| $e_{el}$ | Carbon intensity of electrical grid (kg/kWh) |
| $k_{wholesale}$ | Ratio between wholesale price and retail price of electricity |
| $L$ | Lifetime of heating technology (years) |
| $\overline{P^{el}}$ | Electrical capacity of CHP unit (kW) |
| $\overline{P^{EH}}$ | Capacity of resistive heater (kW) |
| $\overline{P^{HP}}$ | Capacity of heat pump (kW) |
| $r$ | Discount rate (%) |
| $\overline{S}$ | Capacity of thermal energy storage (kWh) |
| $T_{end}$ | End of the planning horizon for the heating system model |
| $w_k$ | Weight of sample day k |

**BINARY VARIABLES**

| | |
|---|---|
| $\phi_t^+$ | Charging mode indicator for thermal energy storage |
| $\phi_t^-$ | Discharging mode indicator for thermal energy storage |

**CONTINUOUS VARIABLES**

| | |
|---|---|
| $\beta_t$ | Operating cost of the heating system (£) |
| $\gamma_t$ | Export revenue generated by the heating system (£) |
| $\lambda$ | Objective function in the heating system (£) |
| $\Phi$ | Equivalent Annual Cost of heating technology (£) |
| $B_t$ | Thermal output of the boiler in period t (kWh) |
| $c_{j,t}^{carbon}$ | Carbon cost for unit j in period t (£) |
| $c_{j,t}^{CU}$ | Capacity-utilization cost for unit j in period t (£) |
| $c_{wind,t}^{CU}$ | Capacity-utilization cost for wind power in period t (£) |
| $c_{solar,t}^{CU}$ | Capacity-utilization cost for solar power in period t (£) |
| $c_{j,t}^{fuel}$ | Fuel cost for unit j in period t (£) |
| $c_{j,t}^{O\&M}$ | Operation and maintenance cost for unit j in period t (£) |
| $c_{j,t}^{startup}$ | Start-up cost for unit j in period t (£) |
| $E$ | Annual carbon emissions (kg) |
| $p_{j,k}$ | Electricity generated by unit j at period t (MW) |
| $p_t^{solar}$ | Electricity injected from solar sources in period t (MW) |
| $p_t^{wind}$ | Electricity injected from wind sources in period t (MW) |
| $P_t^{exp}$ | Electricity exported by CHP unit in period t (kW) |
| $P_t^{el}$ | Electricity generated by the CHP unit (kW) |
| $P_t^{EH}$ | Electricity consumed by resistive heater in period t (kW) |
| $P_t^{HP}$ | Electricity consumed by heat pump in period t (kW) |
| $P_t^{imp}$ | Electricity imported in period t (kW) |
| $P_t^{th}$ | Thermal output of CHP unit in period t (kW) |
| $q_t$ | Electricity released or consumed by pumped storage in period t (MW) |
| $R_t^+$ | Charging of thermal energy storage (kW) |
| $R_t^-$ | Discharge from thermal energy storage (kW) |
| $S_t$ | Storage level in the thermal energy storage (kWh) |

## References


1. BEIS. 2050 Pathways—The 2050 Calculator. 2013. Available online: https://www.gov.uk/guidance/2050-pathways-analysis (accessed on 28 July 2017).
2. Tidball, R.; Bluestein, J.; Rodriguez, N.; Knoke, S. *Cost and Performance Assumptions for Modeling Electricity Generation Technologies*; ICF International: Fairfax, VA, USA, 2010.
3. Vijay, A.; Fouquet, N.; Staffell, I.; Hawkes, A. The value of electricity and reserve services in low carbon electricity systems. *Appl. Energy* **2017**, *201*, 111–123.
4. National Grid. *UK Future Energy Scenarios*; National Grid: Warwick, UK, 2015.
5. Department for Business, Energy and Industrial Strategy (BEIS). *Updated Energy & Emissions Projections*; BEIS: London, UK, 2016.
6. Department for Business, Energy and Industrial Strategy (BEIS). *Green Book Supplementary Guidance: Valuation of Energy Use and Greenhouse Gas Emissions for Appraisal*; BEIS: London, UK, 2017.

7. Department for Business, Energy and Industrial Strategy (BEIS). *Greenhouse Gas Reporting—Conversion Factors 2016*; BEIS: London, UK, 2016.
8. Sweett Group for DECC. *Research on the Costs and Performance of Heating and Cooling Technologies*; Sweett Group: London, UK, 2013.
9. Daly, H.; Fais, B. *UK TIMES Model Overview*; UCL Energy Institute: London, UK, 2014.
10. Staffell, I.; Green, R. The cost of domestic fuel cell micro-CHP systems. *Int. J. Hydrogen Energy* **2013**, *38*, 1088–1102.
11. Weiss, M.; Junginger, M.; Patel, M.K.; Blok, K. A review of experience curve analyses for energy demand technologies. *Technol. Forecast. Soc. Chang.* **2010**, *77*, 411–428.
12. Sun, Z.G.; Wang, R.Z.; Sun, W.Z. Energetic efficiency of a gas-engine-driven cooling and heating system. *Appl. Therm. Eng.* **2004**, *24*, 941–947.
13. Kong, X.Q.; Wang, R.Z.; Huang, X.H. Energy optimization model for a CCHP system with available gas turbines. *Appl. Therm. Eng.* **2005**, *25*, 377–391.
14. Lazzarin, R.; Noro, M. Local or district heating by natural gas: Which is better from energetic, environmental and economic point of views? *Appl. Therm. Eng.* **2006**, *26*, 244–250.
15. Qu, M.; Abdelaziz, O.; Yin, H. New configurations of a heat recovery absorption heat pump integrated with a natural gas boiler for boiler efficiency improvement. *Energy Convers. Manag.* **2014**, *87*, 175–184.
16. De Paepe, M.; D'Herdt, P.; Mertens, D. Micro-CHP systems for residential applications. *Energy Convers. Manag.* **2006**, *47*, 3435–3446.
17. Hahne, E.; Chen, Y. Numerical study of flow and heat transfer characteristics in hot water stores. *Sol. Energy* **1998**, *64*, 9–18.
18. Lavan, Z.; Thompson, J. Experimental study of thermally stratified hot water storage tanks. *Sol. Energy* **1977**, *19*, 519–524.
19. Verda, V.; Colella, F. Primary energy savings through thermal storage in district heating networks. *Energy* **2011**, *36*, 4278–4286.
20. International Energy Agency. *Energy Technology Perspectives*; International Energy Agency: Paris, France, 2010.
21. Parsons Brinckerhoff, Update of Non-PV Data for Feed in Tariff. Available online: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/42912/5900-update-of-nonpv-data=for-feed-in-tariff-.pdf (accessed on 20 November 2017).
22. Korsgaard, A.R.; Nielsen, M.P.; Kær, S.K. Part one: A novel model of HTPEM-based micro-combined heat and power fuel cell system. *Int. J. Hydrogen Energy* **2008**, *33*, 1909–1920.
23. Orr, H. *IMPROSUME Publication Series#4—Distributed Generation and Storage Technical Portfolio—An Overview of the Most Prominent Generation and Storage Technologies and Their Governing Principles*; Norwegian Centres of Expertise: Oslo, Norway, 2012.
24. Bianchi, M.; de Pascale, A.; Melino, F. Performance analysis of an integrated CHP system with thermal and Electric Energy Storage for residential application. *Appl. Energy* **2013**, *112*, 928–938.
25. Barbieri, E.S.; Melino, F.; Morini, M. Influence of the thermal energy storage on the profitability of micro-CHP systems for residential building applications. *Appl. Energy* **2012**, *97*, 714–722.
26. Carbon Trust. *Micro CHP (Combined Heat and Power) Accelerator—Final Report*; Carbon Trust: London, UK, 2011.
27. International Renewable Energy Agency (IRENA). *Thermal Energy Storage Technology Brief*; International Renewable Energy Agency: Abu Dhabi, UAE, 2013.
28. Mondol, J.D.; Smyth, M.; Zacharopoulos, A.; Hyde, T. Experimental performance evaluation of a novel heat exchanger for a solar hot water storage system. *Appl. Energy* **2009**, *86*, 1492–1505.
29. Nelson, D.B.; Nehrir, M.H.; Wang, C. Unit sizing and cost analysis of stand-alone hybrid wind/PV/fuel cell power generation systems. *Renew. Energy* **2006**, *31*, 1641–1656.
30. Battelle. *Manufacturing Cost Analysis of 1 kW and 5 kW Solid Oxide Fuel Cell (SOFC) for Auxilliary Power Applications*; Battelle: Columbus, OH, USA, 2014.
31. Körner, A. *Technology Roadmap: Hydrogen and Fuel Cells*; International Energy Agency: Paris, France, 2015.
32. Electric Power Research Institute (EPRI). *Stirling Engine Assessment 1007317*; EPRI: Palo Alto, CA, USA, 2002.
33. Kelly, N.J.; Cockroft, J. Analysis of retrofit air source heat pump performance: Results from detailed simulations and comparison to field trial data. *Energy Build.* **2011**, *43*, 239–245.

34. Bertsch, S.S.; Groll, E.A. Two-stage air-source heat pump for residential heating and cooling applications in northern U.S. climates. *Int. J. Refrig.* **2008**, *31*, 1282–1292.
35. Tong, Y.; Kozai, T.; Nishioka, N.; Ohyama, K. Greenhouse heating using heat pumps with a high coefficient of performance (COP). *Biosyst. Eng.* **2010**, *106*, 405–411.