Article

Analysis of Analytical Models Developed under the Uniaxial Strain Condition for Predicting Coal Permeability during Primary Depletion

Chuanming Li ¹, Zhiqiang Wang ², Lei Shi ² and Ruimin Feng ^{3,*}

¹ Anhui Province Key Laboratory of Mining Response and Disaster Prevention and Control in Deep Coal Mine, Anhui University of Science and Technology, Huainan 232001, China; chuanmingli @126.com

² College of Resources & Safety Engineering, China University of Mining and Technology, Beijing 100083, China; wzhiqianglhm@126.com (Z.W.); cumtshilei@163.com (L.S.)

³ Department of Mining and Mineral Resources Engineering, Southern Illinois University, Carbondale, IL 62901, USA

* Correspondence: ruimin.feng@siu.edu; Tel.: +1-618-305-0677

Received: 2 October 2017; Accepted: 10 November 2017; Published: 12 November 2017

Abstract: The stress-dependent permeability of coal during coalbed methane production has been extensively studied both experimentally and theoretically. However, how permeability changes as a function of stress variation is somewhat unclear to date, and currently used analytical models fail to accurately predict permeability evolution with gas depletion. Considering that the role played by changes in in situ stress in permeability evolution is critical, a comprehensive theoretical study was first conducted, through which it was found that coal permeability is determined by mean effective stress. Moreover, the influence of matrix shrinkage on cleat deformation and then coal permeability was overestimated by currently used models, leading to inaccuracy of the predicted permeability. By taking both mean effective stress and the influence of matrix shrinkage on cleat deformation into account, a new permeability model was developed under the uniaxial strain condition in order to precisely estimate permeability evolution during gas depletion. An in-depth investigation and comparison among four commonly-used permeability models, the Palmer and Mansoori (P&M) model, Improved P&M model, Shi and Durucan (S&D) model, and Cui and Bustin (C&B) model, was then conducted. It was experimentally verified that a good match can be achieved between the lab data and the results predicted by the proposed model. Permeability variation of coalbed reservoirs associated with gas depletion is a consequence of two opposing effects: mechanical compaction and matrix shrinkage. In comparison, it was found that the coefficients of these two effects incorporated in those four models have a significant impact on permeability variation; and the accuracy of the values of initial cleat porosity and cleat compressibility, the bridges connecting permeability, and those two effects in analytical models, is extremely critical to permeability estimations. This study can shed light on improving the accuracy of analytical coal permeability models and the prediction of gas production.

Keywords: mean effective stress; coalbed methane; permeability; modeling

1. Introduction

Coal is a highly heterogeneous media, characterized by a dual porosity structure that consists of a coal matrix and cleat/fracture system [1]. The cleat system is formed by closely spaced natural fractures with a coal matrix in between. These cleats, namely, the face cleat and butt cleat, determine the mechanical properties of coal and provide flow paths for methane

production [2]. Therefore, coal permeability is dependent upon the physical characteristics of cleats and the stresses applied on these cleats. Cleat spacing is typically affected by coal rank and bed thickness [3]. Besides the cleat system, another set of fractures, bedding planes, can be observed in coal. However, bedding planes do not normally have a role in conducting fluids due to the large stress caused by overburden weight [4].

A common naturally fractured reservoir geometry, a collection of matchsticks, was first extended to stressed coalbeds by Seidle et al. [5]. As shown in Figure 1, each stick represents one coal matrix block and the space is representative of the cleats. This geometry model has been considered the best physical representation of coalbed methane (CBM) reservoirs given that a good agreement has been obtained between theoretical and laboratorial permeability data when using this model [6]. Based on coal geometry being represented as a bundle of matchsticks, many analytical models have been proposed to predict dynamic changes in coal permeability during primary gas production under the uniaxial strain condition, which is regarded as the best replicated in-situ boundary conditions of CBM reservoirs [7–11]. Of the existing models, the most commonly used are the ones presented by Palmer and Mansoori [12] (P&M model), Shi and Durucan [13] (S&D model), and Cui and Bustin [14] (C&B model). Gu and Chalaturnyk [15] and Palmer [16] have reviewed commonly used analytical permeability models, and made a classification based on whether a model is strain-based or stress-based. For the strain-based models, the coal/rock strain is the cause for any change in flow behavior; In CBM reservoirs, gas desorption-induced matrix shrinkage and mechanical compaction induced by an increase in effective stress with gas depletion are the two factors influencing the volumetric strain of the coal matrix, and in turn, changes in cleat porosity and cleat permeability. For the strain-based models, coal strain is the cause for any change in flow behavior; but stress-based models take the stress applied on coal as the fundamental cause, and then changes in strain, effective stress, and coal permeability are computed. Therefore, the P&M model belongs to the strain-based category, while the S&D and C&B models are in the stress-based category. However, it should be noted that it is still controversial whether coal permeability is determined by effective horizontal stress (S&D model) or mean effective stress (C&B model).

Figure 1. Geometry model of Coalbed Methane Reservoirs. (a) Stereogram; (b) Top view; (c) Coal structure model.

To estimate the coal permeability behavior of CBM reservoirs as a function of stress variation, lots of laboratory tests have been carried out. Patching [17] reported a decrease in permeability of three orders of magnitude under different confining stress conditions. Permeability data obtained under the hydrostatic stress condition by Somerton et al. [18] showed that permeability was highly stress-dependent. Robertson and Christiansen [19] carried out a series of permeability experiments under constant confining stress conditions. Feng et al. [20] presented coal permeability evolution with gas production under the uniaxial strain condition. From previous studies, we know that it is still uncertain how changes in in situ stress/strain with gas depletion impact gas flow behavior in CBM reservoirs. Establishment of the relationship between in situ stress/strain and coal permeability is of great importance for accurately predicting long-term gas production. Moreover, matrix blocks are not fully separated by fractures, since there are minerals filled in the fracture system. Based

on this understanding, a more accurate geometry model representing coal structure was proposed, in which there are coal matrix “bridges” connecting matrix blocks [21,22]. When the coal matrix swells, these bridges limit the displacement of coal matrix toward into fractures. The role of these bridges in permeability model development should be considered.

To gain an in-depth insight into the influence of in situ stress change with pressure drawdown on coal permeability variation under the uniaxial strain condition, this study is aimed at the following aspects:

- To derive a new permeability model by taking mean effective stress as the fundamental cause of permeability variation during gas depletion;
- To investigate the influence of matrix shrinkage on cleat deformation in the proposed permeability model by incorporating an effective deformation coefficient of coal matrix;
- To compare four commonly-used analytical permeability models, the P&M, Improve P&M, S&D, and C&B models.

2. Review of Sorption-Induced Strain and Permeability Models

2.1. Modeling of Sorption-Induced Strain

Coal matrix swelling/shrinkage induced by gas sorption/desorption is a well-known phenomenon and is the cause of profound changes in coal porosity and permeability during coalbed methane recovery and CO₂ sequestration. Coal permeability is primarily determined by cleat aperture, the size of which is a function of effective stress, i.e., with pressure drawdown, increased effective stress would lead to the closure of cleat aperture. On the contrary, coal matrix shrinkage with gas depletion would increase the size of the aperture. Hence, the change in coal cleat aperture and permeability is a consequence of competition between these two processes [2].

The research on coal matrix swelling/shrinkage has attracted lots of attention since the 1980's due to its impact on the change in cleat porosity and permeability. Many experiments have investigated coal deformation characteristics under isothermal conditions [23–28]. Even though matrix volumetric strain has been quantified with gas ad-/desorption, a clarified theoretical explanation between shrinkage strain and the amount of gas adsorbed is still not presented in each one of these laboratory studies.

A linear dependence of swelling on adsorbed gas content has been reported by [1,29]. By assuming that the swelling is proportional to the amount of gas adsorbed and that the adsorbed gas is related to pressure by Langmuir's equation, Seidle and Huitt [26] described the relation between swelling and pressure as:

$$\varepsilon_v = \varepsilon_g \frac{V_L p}{p + p_L} \quad (1)$$

where ε_v is the sorption-induced coal matrix strain at pressure p , ε_g is the matrix swelling coefficient, and V_L and p_L are Langmuir constants. Following this, Chikatamarla et al. [30] conducted a series of shrinkage and swelling experiments on four coal samples with different gases, and their measurements showed that the volumetric strain and pressure can be described using a Langmuir-type equation and the volumetric strain is approximately linearly proportional to the amount of gas adsorbed. Cui and Bustin [14] also approximated the sorption-induced volumetric strain as a linear function of the adsorbed gas volume, considering that non-linear fitting between these two does not significantly improve our fundamental understanding on the hydromechanical behavior, particularly when many other parameters are poorly constrained.

Levine [24] used the Langmuir-isotherm model to fit the sorption strain data for Illinois basin coal because of the finding that the swelling behavior followed the same form as the adsorption isotherm. This model was then quantitatively incorporated into a number of analytical permeability models to describe the impact of sorption-induced swelling/shrinkage of the coal matrix [12,13,19,25,31–33]. The model is given as:

$$\varepsilon_s = \varepsilon_l \frac{p}{p + p_e} \quad (2)$$

where ε_s is the sorption-induced coal matrix strain at pore pressure p , ε_l is the maximum strain value when the gas pressure tends to infinity, and p_e is the pore pressure at which the coal matrix strain is half of the maximum strain value.

By applying an energy balance approach, the first theoretical model developed to estimate the sorption-induced strain at adsorption and strain equilibrium was presented by Pan and Connell [34], with the assumption that the surface energy change caused by adsorption is equal to the elastic energy change of the coal solid. This model combines adsorption and pressure compression strains, expressed as follows:

$$\varepsilon = -\frac{\Phi \rho_s}{E_s} f(x, \nu_s) - \frac{p}{E_s} (1 - 2\nu_s) \quad (3)$$

where Φ is the surface potential of sorption, ρ_s is the density of the solid adsorbent, E_s is Young's modulus of the solid phase, and ν_s is Poisson's ratio. The function $f(x, \nu_s)$ represents the pore structural model parameter, given as:

$$f(x, \nu_s) = \frac{[2(1 - \nu_s) - (1 + \nu_s)cx][3 - 5\nu_s - 4(1 - 2\nu_s)cx]}{(3 - 5\nu_s)(2 - 3cx)} \quad (4)$$

where $c = 1.2$ and $x = a/l$. The parameter ' a ' represents the cylindrical radius of the selected pore structure model and ' l ' is its length. The modeled results showed good agreement with experimental observations of swelling. However, this model associates some uncertainties, such as Young's modulus and Poisson's ratio of solid grain, pore structure geometry term, which are difficult to estimate in a laboratory.

A new theoretical model for the change in matrix volumetric strain associated with gas de-/adsorption was proposed by Liu and Harpalani [35], which incorporates the knowledge of geo-mechanics and chemistry of a surface and the interface theory. The volumetric strain of the coal matrix for a sorbing gas is composed of two parts: mechanical strain and sorption-induced strain. The strain induced by a mechanical effect is derived based on Hooke's law, and the sorption-induced strain is directly proportional to the change in surface energy. The volumetric strain of the coal matrix was assumed to be the consequence of purely adding these two parts, expressed as:

$$\varepsilon = -\frac{3(1 - 2\nu)}{E} \int_0^p dp + \frac{3V_L \rho_s RT}{E_A V_0} \int_0^p \frac{1}{p_L + p} dp \quad (5)$$

where E is Young's modulus, ν is Poisson's ratio, V_L and P_L are Langmuir constants, ρ_s is the density of the solid adsorbent (coal), E_A is the modulus of the solid expansion, V_0 is the gas molar volume ($22.4 \text{ m}^3/\text{kmol}$), R is the universal gas constant, and T is the absolute temperature. This proposed model was validated by laboratory volumetric strain data available in open literature. The results of their study showed a great agreement between modeled and experimental data. Moreover, the advantage of this proposed model is that it is capable of describing mixed gas sorption behavior, so the model can be applied to CO_2 sequestration operations and enhanced coalbed methane (ECBM), as well as CBM reservoirs where some amount of other gases, like CO_2 and nitrogen, are contained.

2.2. Summary of Sorption-Induced Strain

Since the late 80s of the last century, the dynamic changes in coal permeability during CBM production have been studied and described using many analytical models. A comprehensive review and a comparison of the models have been presented by Palmer [16] and Pan and Connell [36]. Four analytical permeability models are involved and presented in Appendix A: Original P&M model, Improved P&M model, S&D model, and C&B model, since they are derived under the uniaxial strain condition regarded as the best to replicate the in-situ condition. To avoid duplication of review work, a summary and comparison is presented here:

- Application of collection of matchsticks geometry. The four models listed above are based on matchstick representation. Besides, all four use a geo-mechanics approach, making them more transparent and easy to understand.
- Boundary conditions: uniaxial strain and constant vertical stress. The original P&M, improved P&M, S&D, and C&B models were developed under the uniaxial strain condition. Another assumption employed by all models is that the vertical stress is constant over the life of producing wells because the burial depth of CBM reservoirs does not change with depletion.
- Strain-/stress-based models. Gu and Chalaturnyk [15] offered a distinction based on whether a model is strain-based or stress-based. The original P&M model and improved P&M model are based on the change in volumetric strain due to the desorption of methane, thus resulting in changes in cleat porosity and permeability. The other two are based on the variation of effective stress resulting from desorption-induced volumetric strain, and then permeability.
- Cleat volume compressibility, C_f . This input parameter in stress-based models is the bridge connecting effective stress and permeability. However, the definition of cleat volume compressibility differs from each other. In the S&D model, C_f is defined with respect to the change in effective horizontal stress [37]; however, in the C&B model, it is defined with respect to the mean effective stress. Moreover, a poor history match was obtained using the S&D model due to the constant C_f value applied, so the molders modified the model for a variable value of C_f [38]. However, this change is still purely based on history matching, without scientific backing.
- Matrix volumetric strain. All four models estimate the matrix shrinkage as a result of desorption in a manner similar to thermal contraction of a material, which is based on an empirical approach without a theoretical explanation to clarify the relationship between volumetric strain and the effect of adsorption; hence, our fundamental understanding on desorption-induced matrix strain should be improved.
- Cleat aperture. In previous studies, the decrease in the dimension of the coal matrix due to shrinkage was regarded as equal to the increase in the dimension of the cleat aperture. However, matrix strain measured in the laboratory under the unconfined condition neglects the restriction of filling material between two adjacent matrix blocks on the matrix deformation. All four models do not take the interaction between adjacent matrixes into account, meaning that a larger matrix strain is applied in analytical models.

3. Formulation of Stress-Dependent Permeability

3.1. In-Situ Stress Decomposition

The in-situ stress state of coalbed reservoirs can be expressed as the following stress tensor, which can be further split into three components: mean stress, pore pressure, and deviatoric stress. Based on the geo-mechanics principles, this stress state can be thought to have two roles, one purely extensional determined by the coupled effect of the component I and II, and one purely distortional determined by the component III.

$$\begin{bmatrix} \sigma_{xx} - \alpha p & 0 & 0 \\ 0 & \sigma_{yy} - \alpha p & 0 \\ 0 & 0 & \sigma_{zz} - \alpha p \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}}_I - \alpha \underbrace{\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}}_{II} + \underbrace{\begin{bmatrix} \frac{2\sigma_{xx} - \sigma_{yy} - \sigma_{zz}}{3} & 0 & 0 \\ 0 & \frac{2\sigma_{yy} - \sigma_{xx} - \sigma_{zz}}{3} & 0 \\ 0 & 0 & \frac{2\sigma_{zz} - \sigma_{xx} - \sigma_{yy}}{3} \end{bmatrix}}_{III} \quad (6)$$

where σ_{xx} , σ_{yy} , and σ_{zz} are the applied stresses in the horizontal and vertical direction; $\sigma_m = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$, which is simply the average of the three principal stresses; and α is the Biot's coefficient.

It is not obvious that the deviatoric component given in the last matrix represents pure shear, since there are nonzero components on its diagonal. However, a stress transformation using Euler angles can give the stress state in an intuitive form, as shown in Equation (7). The role of the deviatoric

component is to transform the coal matrix into a different shape (Figure 2); however, based on the matchstick assumption, it is clear that the porosity of the coal sample has no change. Hence, the coal permeability variation during gas depletion is mainly determined by component I and component II, i.e., the mean effective stress.

$$\begin{bmatrix} \frac{\sigma_{xx}-\sigma_{zz}}{3} & 0 & 0 \\ 0 & \frac{\sigma_{xx}-\sigma_{zz}}{3} & 0 \\ 0 & 0 & \frac{2(\sigma_{zz}-\sigma_{xx})}{3} \end{bmatrix} = \begin{bmatrix} 0 & \sigma_{zz}-\sigma_{xx} & \sigma_{xx}-\sigma_{zz} \\ \sigma_{zz}-\sigma_{xx} & 0 & \sigma_{xx}-\sigma_{zz} \\ \sigma_{xx}-\sigma_{zz} & \sigma_{xx}-\sigma_{zz} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & 0 & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & 0 \end{bmatrix} \quad (7)$$

Figure 2. Coal matrix distortion after gas depletion. (a) Stereogram; (b) Top view.

3.2. Development of a New Permeability Model

3.2.1. Strain Induced by Mean Effective Stress

Considering that CBM reservoirs are composed of a coal matrix and cleat system, and loaded by the resultant stress from mean stress (σ_m) and pore pressure (p), the volumetric strain of the coal bulk and cleat system can be expressed as the following two equations without taking the effect of coal matrix shrinkage/swelling induced by gas sorption into account:

$$\tilde{\varepsilon}_v = \frac{\tilde{V}}{V} = -\frac{1}{K}(\tilde{\sigma}_m - \alpha \tilde{p}) \quad (8)$$

$$\tilde{\varepsilon}_p = \frac{\tilde{V}_p}{V_p} = -\frac{1}{K}(\tilde{\sigma}_m - \beta \tilde{p}) \quad (9)$$

where the sign “~” indicates incremental values; ε_v and ε_p represent the coal bulk strain and cleat strain, respectively; V and V_p are the bulk volume and cleat volume, respectively; and K and K_p are the bulk modulus of coal and bulk modulus of the cleat system, respectively.

The Biot’s coefficient α , and the effective coefficient of pore system β are, respectively, defined as the following two equations [22].

$$\alpha = 1 - \frac{K}{K_m} \quad (10)$$

$$\beta = 1 - \frac{K_p}{K_m} \quad (11)$$

where K_m is the modulus of the coal matrix.

Based on Betti-Maxwell’s reciprocal theorem, $\frac{\partial V}{\partial p}|_{\sigma_m} = \frac{\partial V_p}{\partial \sigma_m}|_p$, we obtain:

$$K_p = \frac{\phi}{\alpha} K \quad (12)$$

where ϕ is the porosity.

3.2.2. Coal Matrix Strain Induced by Gas Sorption

There have been a few sorption-induced strain models developed as reviewed above. In this study, we follow a commonly-used Langmuir-type model, as shown in Equation (2). Coal matrix deformation has a significant impact on the changes in bulk volume and cleat volume, assumed to entirely contribute to the fracture deformation in previous studies [12,13,26]. However, its contribution to the fractures has been significantly overestimated [21,25]. A factor ξ , known as the “effective deformation coefficient”, is introduced to measure the degree of influence of the coal matrix deformation on cleat deformation. The factor ξ is a parameter mainly related to the distribution of cleats and the filling minerals of the cleats, ranging between 0 and 1. If there is no cleat system in coal, this factor is equal to zero. When two surfaces of the cleat are smooth, an ideal state described as the matchsticks geometry model is seen, and the factor is equal to one. Mathematically, cleat deformation resulting from coal matrix shrinkage/swelling can be expressed as:

$$\tilde{V}_p = \xi \tilde{V}_m = \xi V_m \tilde{\varepsilon}_s \quad (13)$$

where V_m is the volume of coal matrix under an unconstrained condition.

3.2.3. Establishment of Permeability Model

Based on the definition of porosity, the porosity change of a deforming coal reservoir can be described as:

$$\tilde{\phi} = \phi \left(\frac{\tilde{V}_p}{V_p} - \frac{\tilde{V}}{V} \right) \quad (14)$$

The bulk volume deformation is the sum of deformation caused by effective stress and coal matrix shrinkage:

$$\tilde{\varepsilon}_v = \frac{\tilde{V}}{V} = -\frac{1}{K}(\tilde{\sigma}_m - \alpha \tilde{p}) + \xi \tilde{\varepsilon}_s \quad (15)$$

Similarly, the pore volume deformation is the sum of deformation caused by effective stress acting on the cleat system and the effective cleat deformation caused by matrix shrinkage.

Combining Equations (12)–(15), we have:

$$\frac{\tilde{\phi}}{\phi} = -\frac{1}{K_p}(\tilde{\sigma}_m - \beta \tilde{p}) + \frac{1}{K}(\tilde{\sigma}_m - \alpha \tilde{p}) \quad (16)$$

Substituting Equations (10)–(12) into Equation (16), considering $K_m \gg K \gg K_p$, and simplifying Equation (16), we obtain:

$$\frac{\tilde{\phi}}{\phi} = \left(1 - \frac{\alpha}{\phi} \right) \frac{\tilde{\sigma}_m - \tilde{p}}{K} \quad (17)$$

For coal reservoirs, $\phi \ll 1$, $\phi = 0.1$ – 0.5% [12],

$$\frac{\phi}{\phi_0} = 1 - \frac{\phi - \alpha}{\phi_0} \tilde{\varepsilon}_{eff,v} = 1 + \frac{\alpha}{\phi_0} \tilde{\varepsilon}_{eff,v} \quad (18)$$

The change of volumetric strain during gas depletion is a consequence of the two effects: matrix shrinkage and mechanical compression, as shown in Figure 3.

Figure 3. Coal deformation under the uniaxial strain condition during depletion.

Considering the effect of mechanical compression under the uniaxial stress condition and applying Hooke's law, we have:

$$\tilde{\varepsilon}_v = \tilde{\varepsilon}_z = \frac{\tilde{\sigma}_v}{M} \quad (19)$$

where M is the constrained modulus.

The relationship between the constrained modulus (M), Young's modulus (E), and Poisson's ratio (ν) can be expressed as:

$$M = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \quad (20)$$

Applying Hooke's law to the strain induced by matrix shrinkage, the stress-strain relationship induced by the matrix shrinkage/swelling can be given as:

$$\begin{cases} \tilde{\varepsilon}_x = \tilde{\varepsilon}_y = \frac{1}{E} [\tilde{\sigma}_x - \nu(\tilde{\sigma}_y + \tilde{\sigma}_z)] = -\frac{\zeta}{3} \tilde{\varepsilon}_s \\ \tilde{\varepsilon}_z = \frac{1}{E} [\tilde{\sigma}_z - \nu(\tilde{\sigma}_x + \tilde{\sigma}_y)] \end{cases} \quad (21)$$

Considering $d\sigma_x = d\sigma_y$ and $d\sigma_z = 0$ under the unconstrained boundary condition, we have:

$$\tilde{\varepsilon}_{eff,v} = \tilde{\varepsilon}_z - \frac{2}{3} \zeta \tilde{\varepsilon}_s = -\frac{2\zeta(1-2\nu)}{3(1-\nu)} \tilde{\varepsilon}_s \quad (22)$$

Combining Equations (20)–(22), the total volumetric strain caused by the change in effective stress can be expressed as:

$$\tilde{\varepsilon}_{eff,v} = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \tilde{\sigma}_v - \frac{2\zeta(1-2\nu)}{3(1-\nu)} \tilde{\varepsilon}_s \quad (23)$$

Substituting Equation (23) into Equation (18), and then integrating, we have:

$$\frac{\phi}{\phi_0} = 1 + \frac{\alpha}{\phi_0} \left[\frac{(1+\nu)(1-2\nu)}{E(1-\nu)} (p - p_0) - \frac{2\zeta(1-2\nu)}{3(1-\nu)} (\varepsilon_s - \varepsilon_{s0}) \right] \quad (24)$$

The cubic relationship between permeability and porosity was used to predict coal permeability evolution [13,39], as follows:

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^3 \quad (25)$$

Combining Equations (25) and (26), the coal permeability model that considers the effect of the effective stress and coal matrix shrinkage is given as:

$$\frac{k}{k_0} = \left\{ 1 + \frac{\alpha}{\phi_0} \left[\frac{(1+\nu)(1-2\nu)}{E(1-\nu)} (p - p_0) - \frac{2\zeta(1-2\nu)}{3(1-\nu)} (\varepsilon_s - \varepsilon_{s0}) \right] \right\}^3 \quad (26)$$

From the derivation process of the analytical model, it is evident that the model is derived based on the changes in mean effective stress, and developed by applying the boundary condition of uniaxial strain. Compared with currently-used models, two more parameters are involved in the proposed model; one is Biot's coefficient (α), and the other one is the effective deformation coefficient (ζ). If both coefficients are taken as unity, the model can be reduced to the P&M model. Therefore, the P&M model can be treated as a special case of the proposed model. By incorporating these two parameters into the analytical model, the application of the P&M model can be extended to coal permeability predictions under different stressed conditions. However, it should be noted that additional efforts are required to experimentally determine both two parameters before an accurate prediction of coal permeability can be achieved by this model.

4. Model Validation and Comparison

4.1. Experimental Results

To best replicate the in-situ conditions and be consistent with the founding principles used for the development of the widely used analytical permeability models [12,14,32], uniaxial strain was maintained by passively adjusting horizontal stress during gas depletion. Horizontal stress with an initial value of 9.6 MPa was reduced gradually until radial strain equilibrium was attained at each pressure level, ensuring that the lateral dimension remains constant. A constant vertical stress of 14.5 MPa was applied on the coal specimen since the overburden depth in the field remains constant. The initial reservoir pressure was estimated to be 8.3 MPa, and the reservoir temperature was kept constant at 95 °F.

The specimen used for tests was from San Juan Basin of the US; After the specimen was stressed to the reservoir conditions and reached equilibrium, helium was injected into the system at 0.5 MPa for one day to bleed out the air trapped in the specimen. After that, methane was injected into the coal core to flush helium for another day and then pressurized to the desired pressure level of 8.3 MPa. Once both stress and strain equilibrium were attained, permeability of the specimen at the initial pressure level was measured. The pore pressure was reduced to the next pressure level, and permeability was measured once the new equilibrium state was attained. During the experiment, the horizontal stress was decreased in a stepwise manner until the gas was depleted. The detailed experimental procedure can be referred to in Feng et al. [2], and the permeability results are shown in Figure 4. It is evident that a continuous increasing trend can be seen during gas production, which means that coal permeability increases with gas depletion.

Figure 4. Methane permeability measured under the uniaxial strain condition.

4.2. Model Validation

The list of input parameters required by the analytical models are presented in Table 1. Petrophysical parameters (Young's modulus and Poisson's Ratio) can be measured or estimated in the

laboratory or derived from measurements of pore volume compressibility in the field. The Langmuir Model is a widely accepted model describing gas sorption and coal matrix swelling characteristics for coalbed reservoirs, and many ad-/desorption experiments [7,35,40–42] have been conducted for estimations of coal swelling/shrinkage under different reservoir pressures. Initial cleat porosity of coal is a controversial parameter in analytical modeling. Palmer and Mansoori [12] pointed out that initial cleat porosity values range from 0.1 to 0.5%, being from historical matches of primary production in the San Juan Basin. In order to match the field data, an extremely small initial cleat porosity (0.05%) was applied by Clarkson et al. [41]. Although small values of initial porosity have been suggested and applied by modelers, there is a certain level of discomfort with these values. Possible values of each variable obtained from open literature are presented in Table 1.

Table 1. Input parameters used to match the experimental data.

Input Parameters	Selected Value	Sources
Young's Modulus E , MPa	3793	Liu [28]; Palmer and Mansoori [12]
Poisson's Ratio ν , Dimensionless	0.29	Liu [28]
Initial Porosity ϕ_0 , Dimensionless	0.1–0.5%	Palmer and Mansoori [12]
ε_l , Dimensionless	0.023	Zahner [43]
p_{ε} , MPa	4.5	Zahner [43]

Using the values of input parameters given in Table 1, the variation in permeability during the primary production of gas was estimated using the proposed model (Equation (26)). An effort has been carried out to match the experimental data with different initial cleat porosities by fixing all the other parameters existing in the analytical model given that the model prediction of variation is very sensitive to the value of initial cleat porosity (Figure 5). By comparing the influence of different initial cleat porosities on permeability variation, it is concluded that a relatively good match can be achieved by only varying the value of initial cleat porosity using the proposed model. From Figure 4, we can know that the initial porosity of 0.18% is a reasonable value for modeling in the study, which is also in the range of 0.1–0.5%, as indicated by Palmer and Mansoori [12]. To analyze the sensitivity of different effective deformation coefficients (ξ value) on permeability variation, the modeled and experimental results are displayed in Figure 6. It is evident that the change in ξ value has a great impact on the permeability variation, and this impact becomes more significant when reservoir pressure decreases.

The primary purpose for developing analytical models is to calibrate uncertainties that are required as input parameters for commercial software in order to accurately predict long-term gas production. For example, in the improved P&M model, there are two uncertainties, “ g ” and “ f ”, both of which cannot be directly measured through experiments. One commonly-used approach to determine the values of the uncertainties is conducting experiments and fitting the lab data by the analytical models. Through this fitting exercise, the uncertainties can be determined. However, as discussed above, the new parameters (Biot's coefficient and ξ value) incorporated in the proposed model may have a significant influence on permeability. To accurately determine the uncertainties in the analytical models, more factors influencing permeability prediction should be considered and incorporated into the analytical models.

Figure 5. Measured and modeled variation in permeability using the proposed model for different initial cleat porosities.

Figure 6. Measured and modeled variation in permeability using the proposed model for different ξ values.

4.3. Model Comparison

Four widely-used analytical models developed under the uniaxial strain condition are presented in Appendix A. The primary variables determining the variation of coal permeability with pressure drawdown are shown in Table 2. It is clear that two terms govern the permeability variation during reservoir pressure drawdown: compaction term resulting from the increase of effective stress with the role of decreasing coal permeability, and matrix shrinkage term resulting from gas desorption with the role of increasing coal permeability. Correspondingly, the coefficients for these two terms are listed in Table 2, and these two terms play an opposite role in permeability variation. Poisson's ratio of coal ranges from 0.23 to 0.4 [24]; hence, for the compaction term, it is easy to know $\frac{1+\nu}{3(1-\nu)} > \frac{\nu}{(1-\nu')}$, which means that the permeability decrease predicted by C&B model is always larger than that by S&D model. The original P&M model has the same coefficient as the proposed model; however, the relationship between the improved P&M model and other models is uncertain due to the uncertainty 'g' factor. According to the variation of different Poisson's ratios, the relationship of the magnitude of the compaction term in each model can be expressed as:

$$\left\{ \begin{array}{l} \frac{(1+\nu)(1-2\nu)}{1-\nu} |P\&M| > \frac{1+\nu}{3(1-\nu)} |C\&B| > \frac{\nu}{1-\nu} |S\&D|, \nu < \frac{1}{3} \\ \frac{1+\nu}{3(1-\nu)} |C\&B| > \frac{(1+\nu)(1-2\nu)}{1-\nu} |P\&M| > \frac{\nu}{1-\nu} |S\&D|, \frac{1}{3} < \nu < \frac{\sqrt{3}-1}{2} \\ \frac{1+\nu}{3(1-\nu)} |C\&B| > \frac{\nu}{1-\nu} |S\&D| > \frac{(1+\nu)(1-2\nu)}{1-\nu} |P\&M|, \nu > \frac{\sqrt{3}-1}{2} \end{array} \right. \quad (27)$$

Table 2. Coefficients of different variables affecting permeability variation.

Model	Analytical Equation of Primary Variable	Coef. Δp	Coef. $\Delta \varepsilon$
Shi and Durucan (2004)	$\Delta \sigma_h = -\frac{\nu}{1-\nu}(p-p_0) + \frac{E\varepsilon_l}{3(1-\nu)}\left(\frac{p}{p+p_e} - \frac{p_0}{p_0+p_e}\right)$	$-\frac{\nu}{1-\nu}$	$\frac{1}{3(1-\nu)}$
Cui and Bustin (2005)	$\Delta \sigma_m = -\frac{1+\nu}{3(1-\nu)}(p-p_0) + \frac{2E\varepsilon_l}{9(1-\nu)}\left(\frac{p}{p+p_e} - \frac{p_0}{p_0+p_e}\right)$	$-\frac{1+\nu}{3(1-\nu)}$	$\frac{2}{9(1-\nu)}$
Palmer and Mansoori (1996)	$\phi - \phi_0 = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)}(p-p_0) - \frac{2(1-2\nu)\varepsilon_l}{3(1-\nu)}\left(\frac{p}{p+p_e} - \frac{p_0}{p_0+p_e}\right)$	$\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}$	$\frac{2(1-2\nu)}{3(1-\nu)}$
Improved Palmer et al. (2007)	$\phi - \phi_0 = \frac{g(1+\nu)(1-2\nu)}{E(1-\nu)}(p-p_0) - \frac{2(1-2\nu)\varepsilon_l}{3(1-\nu)}\left(\frac{p}{p+p_e} - \frac{p_0}{p_0+p_e}\right)$	$\frac{g(1+\nu)(1-2\nu)}{E(1-\nu)}$	$\frac{2(1-2\nu)}{3(1-\nu)}$
Proposed model	$\phi - \phi_0 = \alpha \left[\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}(p-p_0) - \frac{2\xi(1-2\nu)}{3(1-\nu)}(\varepsilon_s - \varepsilon_{s0}) \right]$	$\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}$	$\frac{2\xi(1-2\nu)}{3(1-\nu)}$

For the coefficient of the shrinkage term, the only in-equation that can be determined is $\frac{1}{3(1-\nu)} > \frac{2}{9(1-\nu)}$, indicating that the matrix shrinkage predicted by the S&D model is greater than the C&B model. The relationship of the magnitude of the strain induced by matrix shrinkage in different analytical models due to gas desorption can be expressed as:

$$\left\{ \begin{array}{l} \frac{2(1-2\nu)}{3(1-\nu)} |P\&M| > \frac{1}{3(1-\nu)} |S\&D| > \frac{2}{9(1-\nu)} |C\&B|, \nu < \frac{1}{4} \\ \frac{1}{3(1-\nu)} |S\&D| > \frac{2(1-2\nu)}{3(1-\nu)} |P\&M| > \frac{2}{9(1-\nu)} |C\&B|, \frac{1}{4} < \nu < \frac{1}{3} \\ \frac{1}{3(1-\nu)} |S\&D| > \frac{2}{9(1-\nu)} |C\&B| > \frac{2(1-2\nu)}{3(1-\nu)} |P\&M|, \nu > \frac{1}{3} \end{array} \right. \quad (28)$$

The combined effect of these two competing terms is a function of the Poisson's ratio only. The P&M model and improved P&M model can be expressed as Equation (A4), which yields a model that is the same as the proposed model (when $\xi = 1$, $g = 1$). This illustrates that the P&M model and improved P&M model are special cases of the proposed model. Using the values of the input parameters given in Table 1, variation of the compaction and shrinkage terms was estimated using different models and the influence of the combined effect on the permeability variation is presented in Figure 7. The points of intersection indicate the magnitude of the coupled effect, representing the magnitude of primary variables between different analytical models. The point located at a higher position has a smaller role in permeability variation compared to other points at a lower position. For example, in Figure 7a, the order of the magnitude of the coupled effect on coal permeability from different models is $4 > 3 > 2 > 1$. From Figure 7b, it is clear to see that the role of shrinkage term in permeability variation is suppressed with the decrease of the effective deformation coefficient (ξ value).

Figure 7. (a) The variation of two competing terms with gas depletion: compaction term (straight lines) and shrinkage term (nonlinear curves); note: the lines with same color represent the results from the same analytical model; (b) The results from the proposed model.

4.4. Model Performance and Evaluation

The coupled effect of the compaction term and shrinkage term in different analytical models can be easily determined by the value of Poisson's ratio, and this coupled effect has a significant influence on permeability variation. However, there are some other input parameters impacting coal permeability during pressure drawdown, such as cleat compressibility and the initial porosity of the coalbed reservoir.

In stress-based models, pore/cleat compressibility is the factor bridging permeability and stress change, so the accuracy of the cleat compressibility becomes one of the most important input parameters during the modeling exercise. The advantage of strain-based models is that the controversial parameter, pore/cleat compressibility, is avoided in these models. However, the value of initial cleat porosity is critical for an accurate prediction of permeability. Moreover, in the improved P&M model, there are two uncertainties, g and f , which are difficult to determine experimentally. The g and f values are in the range of 0–1 [12]. The modelers suggest that a small g value should be taken ($g \leq 0.3$), whose role is to suppress the compaction term in the analytical model.

Using the values of the input parameters given in Table 1, the permeability variation was predicted by the original P&M model [12] and the improved version [31]. The estimated permeability variation was compared with the experimental results, as shown in Figure 8a,b. Taking the same initial porosity value ($\phi_0 = 0.18\%$) and f value ($f = 0.8$), the improved model estimates a higher permeability increase than the original model. As expected, the compaction term in the improved model is suppressed by introducing the factor " g ", making a predicted permeability value larger than that of the original model. However, both the original and modified P&M model may overestimate the permeability if using the input parameters given in Table 1, because the effective deformation coefficient of the coal matrix is not taken into consideration. The modeler [12] indicated that permeability variation is critically dependent upon the initial cleat porosity. A good match can be achieved by increasing the porosity value to be 0.26%, as shown in Figure 8a. Moreover, due to the suppression effect of the g value on the compaction term, the estimated permeability predicted by the improved version is larger than our laboratory values, so by tuning " f " to be 0.1, the match can be improved. Even if a perfect match can be reached by changing " g " and " f " values, the modeling exercise lacks the desired scientific backing, going against the basic principle of rigorous analytical modeling. Clarifying the physical meaning of " g " and " f " parameters has a significant meaning on the accurate prediction of permeability variation, especially for the unexploited coal reservoirs.

Coal permeability associated with gas depletion was estimated by substituting the same values of the input parameters given in Table 1 into the S&D model and C&B model. The modeled results, along with the experimental results, are as shown in Figure 8c. Good agreement between them can be achieved if appropriate cleat compressibility is determined. Furthermore, the match could be further

improved by using varying C_f values during gas depletion as indicated by Shi and Durucan [13]. A modeling effort was made by Liu et al. [44], and their result showed that the S&D model matches the lab data really well by using a bi-modal distribution of C_f . However, scientific backing is needed for improving the quality of the match by increasing or decreasing the cleat compressibility during gas production.

Figure 8. Comparison of experimental and modeled results using different analytical permeability models. (a) Original P&M model; (b) Improved P&M model; and (c) S&D and C&B model; (d) the lab data and the best fit curves from different analytical models.

To further evaluate the analytical models, the lab data and the best fit curves from the analytical models are presented in Figure 8d. From the figure, we can see that the predicted permeability trends from all four models are consistent with the lab data. However, in this study, the matching degree between the lab data and the predicted values from both the P&M and improved P&M model is better than that of the S&D and C&B model. It can also be observed that the predicted permeability curve from the improved P&M model almost overlaps with that of the P&M model, which is attributed to the “ g ” factor. From Palmer et al. [31], “ g ” is a factor representing the influence of cleat orientation on predicted permeability. Therefore, the overlap of these two curves illustrates that the cleats of the tested coal specimen are distributed horizontally in this study. As indicated by Palmer et al. [31], when all cleats are horizontal, the improved P&M model can be simplified to the P&M model. To quantitatively compare those four models (Figure 8d) and the proposed model (Figures 5 and 6), the deviation of the predicted results for the best fits from the lab data at each measured pore pressure level are listed in Table 3. To some extent, average deviation at these pore pressure levels can reflect the accuracy of the analytical models when taking the lab data as the baseline. It is clear that the average deviation of the

predicted permeability obtained from the proposed model is smaller than that predicted by all the other models.

Table 3. Deviation of the predicted permeability results from the lab data with gas depletion.

Pore Pressure, MPa		8.3	6.9	5.2	4.1	3.1	2.1	1.0	0.5	Average Deviation (%)
Lab data, k/k_0		1.0	1.6	2.3	3.8	4.3	8.3	11.2	14.7	
P&M model	k/k_0	1.0	1.3	1.9	2.7	4.1	6.6	11.6	16.5	12.6
	Deviation, %	0	21.2	14.1	26.7	4.5	19.6	3.1	11.7	
C&B model	k/k_0	1.0	1.2	1.6	2.1	3.1	5.2	10.6	13.9	21.6
	Deviation, %	0	26.7	28.4	42.7	26.4	37.4	5.5	5.7	
S&D model	k/k_0	1.0	1.6	3.1	4.4	6.2	8.7	11.9	17.1	15.9
	Deviation, %	0	1.4	36.5	17.4	44.9	5.6	5.6	15.9	
Proposed model	k/k_0	1.0	1.7	2.1	2.7	3.8	6.5	11.5	16.1	9.8
	Deviation, %	0	2.8	5.8	27.7	9.9	21.0	2.0	9.1	

5. Summary and Conclusions

An analytical permeability model was first proposed for an investigation of the influence of the coal matrix shrinkage on fracture deformation. The dynamic permeability was then modeled using different analytical models developed under the uniaxial strain condition. The modeling results from different models for permeability variation were finally compared with the laboratory data. Based on the work completed, a few important conclusions can be made, as follows:

1. The proposed model decreases the effect of coal matrix shrinkage on cleat/fracture deformation and coal permeability by incorporating an effective deformation coefficient in the shrinkage term of the model. This effect, as expected, becomes significant in the low-pressure range.
2. The proposed model is capable of accurately capturing the overall rising trend of coal permeability with gas depletion. Initial cleat porosity and cleat compressibility, being the critical input parameters, are the bridges connecting those two competing terms and permeability variation.
3. For stress-based analytical models, the permeability predicted by the S&D model is always larger than the C&B model if the same input values are substituted when the same cleat compressibility factor is used.

Acknowledgments: The study was supported by the National Natural Science Foundation of China (51504005), University Science Research Project of Anhui Province (KJ2015A091), and Open foundation from the Provincial Key Laboratory of Mining Effects and Disasters Preventing Under Deep Mining (KLMME12103).

Author Contributions: Ruimin Feng designed and performed the experiments; Zhiqiang Wang and Lei Shi conducted the theoretical analysis; and Chuanming Li wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Analytical Permeability Models

Appendix A.1. Original and Improved Palmer and Mansoori model

Palmer and Mansoori [12] developed the original model incorporating the fundamentals of geo-mechanics. The change in cleat porosity as a function of pore pressure was derived as:

$$\frac{\phi}{\phi_0} = 1 + \frac{C_m}{\phi_0}(p - p_0) + \frac{\varepsilon_l}{\phi_0} \left(\frac{K}{M} - 1 \right) \left(\frac{p}{p + p_\varepsilon} - \frac{p_0}{p_0 + p_\varepsilon} \right) \quad (\text{A1})$$

with

$$C_m = \frac{1}{M} - \left(\frac{K}{M} + f - 1 \right) \gamma, \quad M = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \quad \text{and} \quad K = \frac{E}{3(1 - 2\nu)}$$

where E is Young's Modulus, ν is Poisson's Ratio, K is the bulk modulus, M is the constrained axial modulus, γ is the grain compressibility, p is the reservoir pressure, and f is a fraction between 0 and 1.

The cubic equation was used to establish the relationship between permeability (k) and porosity (ϕ).

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^3 \quad (\text{A2})$$

A variable g ($g < 0.3$) was then introduced into the original model by Palmer et al. [31] in the compaction term in order to best match some of the field permeability data from the San Juan basin, US.

$$C_m = \frac{g}{M} - \left(\frac{K}{M} + f - 1 \right) \gamma \quad (\text{A3})$$

By assuming that the value of grain compressibility (γ) is equal to zero, Equations (A1)–(A3) are combined, and we have:

$$\phi - \phi_0 = \frac{g(1+\nu)(1-2\nu)}{E(1-\nu)}(p - p_0) - \frac{2(1-2\nu)\varepsilon_l}{3(1-\nu)} \left(\frac{p}{p + p_\varepsilon} - \frac{p_0}{p_0 + p_\varepsilon} \right) \quad (\text{A4})$$

Appendix A.2. Shi and Durucan Model

Shi and Durucan [32] proposed a stress-based analytical model, in which the change in effective stress is regarded as the prevailing factor determining cleat permeability variation during primary gas production.

$$\frac{k}{k_0} = \exp(-3C_f \Delta\sigma_h) \quad (\text{A5})$$

where C_f the value of cleat compressibility, and the change in effective horizontal stress is expressed as:

$$\Delta\sigma_h = \sigma_h - \sigma_{h0} = -\frac{\nu}{1-\nu}(p - p_0) + \frac{E\varepsilon_l}{3(1-\nu)} \left(\frac{p}{p + p_\varepsilon} - \frac{p_0}{p_0 + p_\varepsilon} \right) \quad (\text{A6})$$

Appendix A.3. Cui and Bustin Model

In the Cui and Bustin [14] model, changes in the cleat permeability of coal varies exponentially with changes in the mean effective stress.

$$\frac{k}{k_0} = \exp(-3C_f \Delta\sigma_m) \quad (\text{A7})$$

where $\Delta\sigma_m$ is given by:

$$\Delta\sigma_m = \sigma_m - \sigma_{m0} = -\frac{1+\nu}{3(1-\nu)}(p - p_0) + \frac{2E\varepsilon_l}{9(1-\nu)} \left(\frac{p}{p + p_\varepsilon} - \frac{p_0}{p_0 + p_\varepsilon} \right) \quad (\text{A8})$$

References

1. Harpalani, S.; Schraufnagel, R.A. Shrinkage of coal matrix with release of gas and its impact on permeability of coal. *Fuel* **1990**, *69*, 551–556. [CrossRef]
2. Feng, R.; Harpalani, S.; Pandey, R. Laboratory measurement of stress-dependent coal permeability using pulse-decay technique and flow modeling with gas depletion. *Fuel* **2016**, *177*, 76–86. [CrossRef]
3. Laubach, S.; Marrett, R.; Olson, J.; Scott, A. Characteristics and origins of coal cleat: A review. *Int. J. Coal Geol.* **1998**, *35*, 175–207. [CrossRef]
4. Liu, S.; Wang, Y.; Harpalani, S. Anisotropy characteristics of coal shrinkage/swelling and its impact on coal permeability evolution with CO₂ injection. *Greenh. Gases Sci. Technol.* **2016**, *6*, 615–632. [CrossRef]
5. Seidle, J.; Jeanstone, M.; Erickson, D. Application of matchstick geometry to stress dependent permeability in coals. In Proceedings of the SPE Rocky Mountain Regional Meeting, Casper, WY, USA, 18–21 May 1992.

6. Ma, Q.; Harpalani, S.; Liu, S. A simplified permeability model for coalbed methane reservoirs based on matchstick strain and constant volume theory. *Int. J. Coal Geol.* **2011**, *85*, 43–48. [[CrossRef](#)]
7. Mitra, A.; Harpalani, S.; Liu, S. Laboratory measurement and modeling of coal permeability with continued methane production: Part 1—Laboratory results. *Fuel* **2012**, *94*, 110–116. [[CrossRef](#)]
8. Zimmerman, R.W. Pore volume and porosity changes under uniaxial strain conditions. *Transp. Porous Media* **2017**, *119*, 481–498. [[CrossRef](#)]
9. Feng, R. An optimized transient technique and flow modeling for laboratory permeability measurements of unconventional gas reservoirs with tight structure. *J. Nat. Gas Sci. Eng.* **2017**, *46*, 603–614. [[CrossRef](#)]
10. Feng, R.; Harpalani, S.; Liu, J. Optimized pressure pulse-decay method for laboratory estimation of gas permeability of sorptive reservoirs: Part 2—Experimental study. *Fuel* **2017**, *191*, 565–573. [[CrossRef](#)]
11. Feng, R.; Liu, J.; Harpalani, S. Optimized pressure pulse-decay method for laboratory estimation of gas permeability of sorptive reservoirs: Part 1—Background and numerical analysis. *Fuel* **2017**, *191*, 555–564. [[CrossRef](#)]
12. Palmer, I.; Mansoori, J. How permeability depends on stress and pore pressure in coalbeds: A new model. In Proceedings of the SPE Annual Technical Conference and Exhibition, Denver, CO, USA, 6–9 October 1996.
13. Shi, J.Q.; Durucan, S. A model for changes in coalbed permeability during primary and enhanced methane recovery. *SPE Reserv. Eval. Eng.* **2005**, *8*, 291–299. [[CrossRef](#)]
14. Cui, X.; Bustin, R. Volumetric strain associated with methane desorption and its impact on coalbed gas production from deep coal seams. *AAPE Bull.* **2005**, *89*, 1181–1202. [[CrossRef](#)]
15. Gu, F.; Chalaturnyk, R. Numerical simulation of stress and strain due to gas sorption/desorption and their effects on in situ permeability of coalbeds. *J. Can. Pet. Technol.* **2006**, *45*. [[CrossRef](#)]
16. Palmer, I. Permeability changes in coal: Analytical modeling. *Int. J. Coal Geol.* **2009**, *77*, 119–126. [[CrossRef](#)]
17. Patching, T. Retention and release of gas in coal—A review. *Can. Min. Met. Bull.* **1970**, *63*, 1302–1308.
18. Somerton, W.; Söylemezoglu, I.; Dudley, R. Effect of stress on permeability of coal. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* **1975**, *12*, 129–145. [[CrossRef](#)]
19. Robertson, E.P.; Christiansen, R.L. A permeability model for coal and other fractured, sorptive-elastic media. *SPE J.* **2008**, *13*, 314–324. [[CrossRef](#)]
20. Feng, R.; Harpalani, S.; Pandey, R. Evaluation of various pulse-decay laboratory permeability measurement techniques for highly stressed coals. *Rock Mech. Rock Eng.* **2017**, *50*, 297–308. [[CrossRef](#)]
21. Liu, H.-H.; Rutqvist, J. A new coal-permeability model: Internal swelling stress and fracture—Matrix interaction. *Transp. Porous Media* **2010**, *82*, 157–171. [[CrossRef](#)]
22. Liu, J.; Chen, Z.; Elsworth, D.; Miao, X.; Mao, X. Evolution of coal permeability from stress-controlled to displacement-controlled swelling conditions. *Fuel* **2011**, *90*, 2987–2997. [[CrossRef](#)]
23. Harpalani, S.; Mitra, A. Impact of CO₂ injection on flow behavior of coalbed methane reservoirs. *Transp. Porous Media* **2010**, *82*, 141–156. [[CrossRef](#)]
24. Levine, J. Model study of the influence of matrix shrinkage on absolute permeability of coal bed reservoirs. *Geol. Soc. Lond. Spec. Publ.* **1996**, *109*, 197–212. [[CrossRef](#)]
25. Robertson, E.P. Modeling permeability in coal using sorption-induced strain data. In Proceedings of the SPE Annual Technical Conference and Exhibition, Dallas, TX, USA, 9–12 October 2005.
26. Seidle, J.R.; Huitt, L. Experimental measurement of coal matrix shrinkage due to gas desorption and implications for cleat permeability increases. In Proceedings of the International Meeting on Petroleum Engineering, Beijing, China, 14–17 November 1995.
27. Wang, G.; Wei, X.; Wang, K.; Massarotto, P.; Rudolph, V. Sorption-induced swelling/shrinkage and permeability of coal under stressed adsorption/desorption conditions. *Int. J. Coal Geol.* **2010**, *83*, 46–54. [[CrossRef](#)]
28. Liu, S. *Estimation of Different Coal Compressibilities of Coalbed Methane Reservoirs under Replicated In Situ Condition*; Southern Illinois University Carbondale: Carbondale, IL, USA, 2012.
29. Fan, J.; Feng, R.; Wang, J.; Wang, Y. Laboratory investigation of coal deformation behavior and its influence on permeability evolution during methane displacement by CO₂. *Rock Mech. Rock Eng.* **2017**, *50*, 1725–1737. [[CrossRef](#)]
30. Chikatamarla, L.; Cui, X.; Bustin, R.M. Implications of volumetric swelling/shrinkage of coal in sequestration of acid gases. In Proceedings of the International Coalbed Methane Symposium, Tuscaloosa, AL, USA, 3–7 May 2004.

31. Palmer, I.; Mavor, M.; Gunter, B. Permeability changes in coal seams during production and injection. In Proceedings of the International Coalbed Methane Symposium, University of Alabama, Tuscaloosa, AL, USA, 23–24 May 2007.
32. Shi, J.; Durucan, S. Drawdown induced changes in permeability of coalbeds: A new interpretation of the reservoir response to primary recovery. *Transp. Porous Media* **2004**, *56*, 1–16. [[CrossRef](#)]
33. Wang, S.; Elsworth, D.; Liu, J. A mechanistic model for permeability evolution in fractured sorbing media. *J. Geophys. Res. Solid Earth* **2012**, *117*. [[CrossRef](#)]
34. Pan, Z.; Connell, L.D. A theoretical model for gas adsorption-induced coal swelling. *Int. J. Coal Geol.* **2007**, *69*, 243–252. [[CrossRef](#)]
35. Liu, S.; Harpalani, S. A new theoretical approach to model sorption-induced coal shrinkage or swelling. *AAPG Bull.* **2013**, *97*, 1033–1049. [[CrossRef](#)]
36. Pan, Z.; Connell, L. Modelling permeability for coal reservoirs: A review of analytical models and testing data. *Int. J. Coal Geol.* **2012**, *92*, 1–44. [[CrossRef](#)]
37. Shi, J.-Q.; Pan, Z.; Durucan, S. Analytical models for coal permeability changes during coalbed methane recovery: Model comparison and performance evaluation. *Int. J. Coal Geol.* **2014**, *136*, 17–24. [[CrossRef](#)]
38. Shi, J.Q.; Durucan, S. Exponential growth in san juan basin fruitland coalbed permeability with reservoir drawdown: Model match and new insights. *SPE Reserv. Eval. Eng.* **2010**, *13*, 914–925. [[CrossRef](#)]
39. Walder, J.; Nur, A. Porosity reduction and crustal pore pressure development. *J. Geophys. Res. Solid Earth* **1984**, *89*, 11539–11548. [[CrossRef](#)]
40. Cui, X.; Bustin, R.M.; Chikatamarla, L. Adsorption-induced coal swelling and stress: Implications for methane production and acid gas sequestration into coal seams. *J. Geophys. Res. Solid Earth* **2007**, *112*. [[CrossRef](#)]
41. Clarkson, C.R.; Pan, Z.; Palmer, I.; Harpalani, S. Predicting sorption-induced strain and permeability increase with depletion for coalbed-methane reservoirs. *SPE J.* **2010**, *15*, 152–159. [[CrossRef](#)]
42. Zhang, H.; Liu, J.; Elsworth, D. How sorption-induced matrix deformation affects gas flow in coal seams: A new fe model. *Int. J. Rock Mech. Min. Sci.* **2008**, *45*, 1226–1236. [[CrossRef](#)]
43. Zahner, B. Application of material balance to determine ultimate recovery of a san juan fruitland coal well. In Proceedings of the SPE Annual Technical Conference and Exhibition, San Antonio, TX, USA, 5–8 October 1997.
44. Liu, S.; Harpalani, S.; Pillalamarri, M. Laboratory measurement and modeling of coal permeability with continued methane production: Part 2—Modeling results. *Fuel* **2012**, *94*, 117–124. [[CrossRef](#)]

© 2017 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).