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Equity Returns and the Output Shocks in a Dynamic Stochastic General Equilibrium Framework

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Abstract: We conducted a study analyzing the impact of productivity shocks on equity returns in the U.S. economy from Q1 1960 to Q1 2022 using an RBC DSGE model. Our results suggest that while initial productivity shocks lead to higher equity returns, this effect fades within eight quarters. Nonetheless, such shocks can still provide valuable signals for investors to strategically allocate their investments in sectors that may benefit the most. Our study also found that the responses of key macroeconomic variables, including real GDP, are consistent with those observed in other calibration based DSGE models of the U.S. in previous research.

Keywords: DSGE; impulse responses; total factor productivity shocks; real business cycle

JEL Classification: E00; E13; E19

1. Introduction

The association of productivity and equity returns has been debated by scholars and professionals. Madsen and Davis (2006), Davis and Madsen (2001, 2008), Brown and Rowe (2007), and Ibbotson and Chen (2003) have addressed the subject by deploying various methodologies. Their findings indicated that productivity growth is one of many factors that are associated with equity returns. Other researchers have established that equity returns are tied to the real GDP growth. For instance, Arnott and Bernstein (2002), Arnott and Asness (2003), Bernstein and Arnott (2003), and Cornell (2010) offer insights on the subject.

Solow (1956) argues that the growth of the per capita GDP in the long run is associated with exogenous technological innovations and not just due to capital stock expansion. The size of the capital stock does lead to a higher standard of living. However, the law of diminishing returns takes its toll on the marginal productivity of the capital stock and stunts the GDP growth. When the marginal product of capital stock falls to equality with its marginal cost, an equilibrium is reached, and capital stock will not grow. At this point, the economy reaches a steady state growth path where the per capital GDP ceases to grow. At this stage, the only way GDP per capita can grow is through technological innovations or shocks which lead to a rising marginal product of capital. The advent of radio, the internet, and innovations in the auto industry through autonomous and electric vehicles are examples of such positive technological shocks.

So long as the technological innovation continues, so does the growth in per capita GDP. Historically, equity returns are tied to the earnings of a corporation. Corporate earnings before tax have maintained a stationary relationship with the U.S. GDP over time. Excluding the times of great recessions or other cataclysmic economic shocks, these earnings are in the range of 11–13% of the GDP. The stationarity of this ratio implies that the growth rates of corporate earnings and the GDP are roughly equal. Therefore, in

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theory, the reaction of the GDP and corporate earnings, and thus equity returns to positive technological shocks, are expected to be similar.

Our paper contributes to this body of literature by formally modeling the narrative above. We investigate the role of technological shocks that lead to productivity growth in a DSGE framework. This approach seeks to examine the association of the shocks to productivity growth with the GDP, labor employment, and other macroeconomic variables of interest as well as equity returns.

We found that equity returns, similar to the GDP and returns on corporate capital investments, reacted favorably to positive shocks to productivity. We attribute this reaction to the optimizing behavior of households, and firms. Consistent with equity return's positive response to positive productivity shocks in the economy, initially, households save and invest more in the market, which in turn is the source of corporate capital investment.

Both investments and capital input impulse responses show a positive trend up to around eight quarters, where investment responses drop to zero and capital accumulation peaks. Our methodology is a deviation from prior work in that we rely on the DSGE model and impulse responses.

Dynamic stochastic general equilibrium (DSGE) models are macroeconomic models that attempt to capture the entire economy's behavior, rather than analyzing the behavior of a single market or sector, as in partial equilibrium models. Here are some of the advantages of DSGE models compared to other partial equilibrium models:

DSGE models are more comprehensive: DSGE models can account for the interactions between different sectors and markets in the economy, which partial equilibrium models do not consider. In other words, DSGE models incorporate a broader range of economic variables and their interdependencies.

DSGE models are more dynamic: DSGE models take into account how the economy evolves over time, while partial equilibrium models are static and assume that the economy is always in equilibrium. This means that DSGE models can capture the impact of economic shocks and policy changes over time.

DSGE models can handle uncertainty: DSGE models incorporate stochastic elements, which means they can account for random fluctuations in the economy, such as changes in technology, consumer preferences, or government policies. This enables DSGE models to analyze the effects of policy changes on the economy under different scenarios, taking into account the inherent uncertainty in the system.

DSGE models are more realistic: DSGE models incorporate a range of economic behaviors and constraints, such as household preferences, firms' production technologies, and government policies, making them more realistic and able to capture complex interactions that may not be captured by simpler partial equilibrium models.

In summary, DSGE models are more comprehensive, dynamic, realistic, and can handle uncertainty better than partial equilibrium models, making them a useful tool for analyzing the behavior of the entire economy.

The benchmark DSGE model is based on the work of Smets and Wouters (2007), who continued the work of other researchers, particularly Christiano et al. (2005). The basic model which we estimate in this paper is founded on the real business cycle model (RBC). In this model, households' intertemporal consumption decisions, firms' investment decisions, capital accumulation, labor supply decisions, and shocks to total factor productivity are formulated.

To grasp the complexities, advantages, and disadvantages of DSGE models, we encourage readers to consider the work of Korinek (2018) and Gorodnichenko and Ng (2010).

In the next section, we offer a brief review of the relevant literature. Section 3 briefly explains the RBC DSGE model. Data are described in Section 4. Empirical results are the topic of Section 5. A summary and conclusions are the subject of Section 6.

2. Review of the Relevant Literature

Researchers have examined the association of the productivity growth and equity returns. Davis and Madsen (2001) analyzed the data for nine OECD countries, including the U.S., for 80 years through 1990, and explored the link of productivity to share returns. They estimated vector autoregressive models (VARs) for total factor productivity (TFP), labor productivity, and capital productivity. TFP was found to be a more reliable indicator of equity returns than capital or labor productivity. VARs suggest that for Germany, Japan, and Italy, productivity shocks help explain the forecast variance of equity returns, and thus are reliable predictors of equity returns. Davis and Madsen (2008), in their follow-up study, analyzed data spanning 80 years for 11 OECD economies. Their investigation shows that the association of capital productivity with share returns is more pronounced than that of labor and TFP.

Madsen and Davis (2006) investigated the temporary association between equity price fundamentals and productivity growth and technological innovations in their data for 11 OECD economies. Meanwhile, Brown and Rowe (2007) demonstrated that productive value firms earn risk-adjusted positive and statistically significant returns, and that value-oriented managers who wish to achieve abnormal returns must seek productive equities. Lastly, Ibbotson and Chen (2003) found that the growth in corporate earnings followed the growth of overall economic productivity when studying the 1926–2001 US equity returns.

In recent years, dynamic stochastic general equilibrium (DSGE) models have become the main tool of macroeconomic modeling. Economists find these models useful because they incorporate the time dimension, stochastic uncertainty, and rely on general equilibrium.

The Dynamic Stochastic General Equilibrium (DSGE) models are designed for infinite time horizon and are stochastic, accommodating technological shocks as a source of uncertainty (Kydland and Prescott 1982). All DSGE models including the real business cycle (RBC) consider productivity shocks as the primary source of uncertainty (Costa 2016). The micro-foundations of the DSGE models incorporate the rational expectations in optimizing models, enhancing model validity (Clarida et al. 2002). New Keynesian DSGE models demonstrate that expansionary monetary policies are ultimately not effective tools of stimulating the real economy and could lead to inflationary pressures in the long run (Smets and Wouters 2003). DSGE models allow policy makers to assess the welfare effects of economic policies as they are built on utility maximizing decisions of the economic agents (Woodford 2003).

However, the DSGE approach also suffers from shortcomings, including the complexity of infinite time horizon models, which are virtually impossible to solve (Korinek 2018; Gorodnichenko and Ng 2010). Researchers have resorted to using two-period horizon models, which are dynamic but not over an infinite time horizon. Despite these limitations, the DSGE models have proven to be useful in macroeconomic modeling and policy analysis.

DSGE models fall into two categories. Models in the first group are designed by central banks and others to simulate the macroeconomy for policy purposes. These models provide information regarding the reaction of the economy to economic and policy shocks. Papers by Poutineau and Vermandel (2015), Edge et al. (2009b, 2009c), Burriel et al. (2010), Bhattarai and Trzeciakiewicz (2017), Lindé (2018), and Sharma and Behera (2022) are notable in this group.

Poutineau and Vermandel (2015), investigated the association of business cycles in the eurozone and cross-border interbank and corporate transactions with peripheral economies in a DSGE framework. Their estimated DSGE model for quarterly data spanning 1991–2003 showed that economic shocks are magnified by these transactions. For instance, cross-border lending is a catalyst that transmits the financial crisis between two groups of countries. The degree of sensitivity of economic variables to cross-border shocks is not identical. For instance, current account balances and investments are more sensitive to financial shocks than interest rates.

Lindé (2018) offered a detailed analysis of the effectiveness of DSGE models in monetary and fiscal policy analysis. Blanchard (2018) was skeptical about the usefulness of DSGEs to serve as macro-econometric policy models and recommended their use in con-

junction with existing time series-based models. Despite their flaws, Lindé (2018) argued that DSGE models are superior to other existing models for policy purposes.

Edge et al. (2007a, 2007b, 2009a) have been among the pioneering researchers and proponents of DSGE models who have estimated a DSGE model of the U.S. economy with the objective of addressing policy issues and forecasting at the Federal Reserve Board (FRB). Edge et al. (2009b, 2009c) investigated the capability of their DSGE-based model (FRB/EDO) in analyzing the association of business investment shocks and business cycle fluctuations. They showed that DSGE models are capable of analyzing the critical role of investment shocks, investment-specific technological change and shocks to intertemporal IS-curves, and long-run growth. The New-Keynesian DSGE models are particularly powerful tools for examining the effects of various policies, including monetary policies, on the U.S. economy.

Edge et al. (2009b) showed that DSGE models are flexible, internally consistent formulations of the macroeconomy founded on the microeconomic principles of optimization. Models by Christiano et al. (2005) and Smets and Wouters (2003, 2004) are good examples.

The second group of papers examines the predictive abilities of DSGE models in advanced and emerging economies. This category includes works by Smets and Wouters (2004), Schorfheide et al. (2010), Burriel et al. (2010), Edge and Gürkaynak (2011), Alpanda et al. (2011), Fernández-de-Córdoba and Torres (2011), Del Negro and Schorfheide (2013), Wickens (2014), Wouters (2015), Kolasa and Rubaszek (2015), Balcilar et al. (2015), Martínez-Martín et al. (2019), Cai et al. (2019), and Ahmad and Haider (2019). However, since our paper does not focus on the forecasting aspect of DSGE models, we do not provide an extensive discussion on this topic. The conclusion was that DSGE model forecasts generally are not superior to other time series methodologies.

3. The Real Business Cycle DSGE Model

The details of derivations and algebraic manipulations of DSGE model equations are quite standard and readily available and accessible (Costa 2016). Therefore, for the purpose of brevity, we will skip the details of the derivations for each block of the model here.

For simplicity, we assumed a simple closed economy without a government or a monetary authority. Furthermore, there are no frictions to create adjustment costs. That leaves two sectors, households and firms. In the following, we offer a brief explanation of each sector's decision process.

The general equilibrium in this basic DSGE model hinges on the optimization decisions of consumers and firms. The intertemporal decision of labor and leisure is predicated on consumer utility maximization subject to budget constraints. We assume an additively separable utility function of consumption (C) and labor (H), and without habit formation. There are 40 h per week that can be allocated between labor (H) and leisure (L), where the former produces disutility while leisure adds to utility. Furthermore, in this γsimple model, the population growth rate is zero and wages are flexible in a perfectly competitive labor market. The utility function is assumed to be quasiconcave with positive diminishing marginal utilities of C and H. Based on the above assumptions, we define the objective function, i.e., the present value of the utility function as follows:

$$\underset{C_t, L_t}{Max} E_t \sum_{t=0}^{\infty} \beta^t (C_t^{\sigma}, H_t^{\gamma}), \quad \beta > 0, \tag{1}$$

where E is the expected value operator, β is the intertemporal discount factor, σ is the risk aversion coefficient, and γ is the marginal disutility of labor supply.

The household budget constraint consists of compensation for the labor hours at the competitive wage rate and return to capital. Assuming that household income at time t is consumed or saved and invested, the household intertemporal budget constraint is written as

$$C_t + S_t = W_t H_t + R_t K_t \tag{2}$$

where W stands for the wage rate and R is the return on capital, a portion of which is the dividend share of the household as payment for the invested capital. We assume that the household savings are invested in firms and that capital formation over time rises by household investments (equal to savings), and there is depreciation. The equation of capital formation is therefore written as

$$K_{t+1} = I_t + (1 - \delta) \times K_t \tag{3}$$

where δ is the capital depreciation rate.

Maximizing the utility function subject to the budget constraint results in Euler Equations (4) and (5),

$$\frac{C_{t+1}}{C_t} = \beta(R_{t+1} + 1 - \delta). \tag{4}$$

The supply of labor hours is given by

$$H_t = \frac{W_t - \gamma C_t}{W_t}. \tag{5}$$

Firms in the economy are assumed to operate in a competitive market and possess a homogeneous of degree one Cobb–Douglas production function given by Equation (6).

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}. \tag{6}$$

In Equation (6), A represents the total factor productivity, Y_t is the output at time t, K_t is the capital input, H_t represents hours of work per unit of time, and α is the elasticity of output with respect to capital. The production function is strictly quasiconcave, and marginal products of capital and labor are positive.

Typical firms in the competitive market maximize profit, and in doing so, they determine the levels of labor and capital input. Assuming that price level is constant at 1, the profit function is written as Equation (8).

$$\underset{K,H}{Max} \Pi_t = A_t K_t^{\alpha} H_t^{1-\alpha} - W_t H_t - R_t K_t. \tag{7}$$

The input demand functions for K_t and H_t are obtained as Equations (8) and (9).

$$R_t = \alpha \frac{Y_t}{K_t}, \tag{8}$$

$$W_t = (1 - \alpha) \frac{Y_t}{H_t}. \tag{9}$$

Equilibrium Condition

The equilibrium condition consists of the following equations that determine the interaction of households and firms. In equilibrium, the aggregate demand is equal to the aggregate supply. Thus, $Y_t = C_t + I_t$, where I_t is equal to household saving in time t. The model that describes the economy is given in the block of equations as follows.

Household block

$$\frac{C_{t+1}}{C_t} = \beta(R_{t+1} + 1 - \delta) \tag{10}$$

$$H_t = \frac{W_t - \gamma C_t}{W_t} \tag{11}$$

Firm block

$$K_{t+1} = I_t + (1 - \delta) \times K_t \tag{12}$$

$$Y_t = A_t K_t^{\alpha} H_t^{(1-\alpha)} \tag{13}$$

$$R_t = \alpha \frac{Y_t}{K_t} \tag{14}$$

$$EQR_t = 0.3R_t \tag{15}$$

$$W_t = (1 - \alpha) \frac{Y_t}{H_t} \tag{16}$$

In equilibrium, our model requires that the well-known condition that the GDP is either consumed or saved and invested hold as follows.

$$Y_t = C_t + I_t \tag{17}$$

To the model of equations, we add the total factor productivity shocks that in the RBC model are assumed to stem from technological innovations. Following the literature, we enter the shocks equation in an autoregressive of order one format.

$$ln(A_{t+1}) = \rho ln(A_t) + e_{t+1}.$$

The system of model equations is nonlinear. Linear approximation of the nonlinear equations can be derived by log-linearization using Uhlig's method. However, in this paper, we estimate the model equations in logarithms of variables.

The steady state of a DSGE model relies on the foundations of Walras' Law. Debreu (1952) proved that there will be an equilibrium in an economic system that agents interact. Arrow and Debreu (1954) further proved the theoretical existence of equilibrium in a system that includes production and consumption decisions of consumers and firms. Walras ([1874] 1996) provided the principles that suggest the existence of a unique equilibrium. Wald (1951) provided a more formal approach demonstrating the path toward a stable steady-state equilibrium in the competitive market. Arrow and Hurwicz (1958) and Arrow et al. (1959) further developed the notion of a steady-state general equilibrium based on the previous work of Arrow and Debreu.

Once the competitive general equilibrium is established, the model variables at the steady state are also defined. Steady state is achieved as Walras's law implies that the total excess demand in the economy is zero. Setting the general price level equal to one and assuming a homogeneous of degree zero aggregate excess demand function, some algebraic manipulation leads to the steady-state levels of model variables indicated by subscript (s). The model is in steady state as variables become stationary, i.e., the expected value of each variable is constant over time. More formally, $E_t x_{t+1} = x_t = x_{t-1} = x_s$.

In the standard RBC model, the TFP (A) is assumed to be exogenously determined; normally, A = 1 at the steady state. The model in steady state is described by Equations (20)–(28). The system of equations in the model determines the steady-state values of seven endogenous variables Y_t, C_t, I_t, K_t, W_t, H_t, and R_t. We provide the steady-state levels of each variable in the empirical section.

$$C_s = \beta(1 - \delta + R_s) \tag{18}$$

$$K_s = I_s + (1 - \delta) \times K_s \tag{19}$$

$$Y_s = K_s^{\alpha} H_s^{1-\alpha} \tag{20}$$

$$I_s = \delta \times K_s \tag{21}$$

$$R_s = \alpha \frac{Y_s}{K_s} \tag{22}$$

$$EQR_s = 0.30 \times \alpha \frac{Y_s}{K_s} \tag{23}$$

$$W_s = (1 - \alpha) \frac{Y_s}{H_s} \tag{24}$$

$$H_s = W_s \gamma C_s \tag{25}$$

$$Y_s = C_s + I_s \tag{26}$$

4. Data

The data set for the study consists of quarterly observations of consumption expenditures, private investments, employment, real GDP, potential real GDP, average weekly hours of work, and S&P 500 returns spanning the first quarter of 1960 to the first quarter of 2022.

The GDP gap is measured as the logarithm of the ratio of the real GDP to the potential real GDP. Consumption, investment, and other gaps are measured as the gap between the actual variable and the trend using the Hodrick–Prescott methodology.

To estimate the model, some parameter calibration is necessary. We partially calibrated the model based on parameter values recommended in the literature (Costa 2016).

$\beta = 0.98$	Intertemporal Consumption
$\alpha = 0.33$	Capital Share
$\delta = 0.028$	Capital Depreciation Rate
$\gamma = 2.5$	Elasticity of Leisure

5. Empirical Results

Figure 1 presents a graph of key variables. It is shown that variables are stationary with a mean of zero, though there are occasional structural breaks in hours worked. Graphs support the ADF statistics presented in Table 1.

Table 2 presents the variances of maximum-likelihood estimators of the model variables. The delta method is based on a one-step Taylor series expansion of a model variable around its mean. For instance, a function of variable a may be expanded as follows:

$$\text{f(a) = f(E(a)) + (a } - \text{ E(a))f}'\text{(E(a)); therefore,}$$

$$\text{Var(f(a)) = Var(a)}*[\text{f}'\text{(E(a))}]^2.$$

Table 2 shows that variances are statistically significant. Therefore, in the final estimation, variable distributions are well-defined.

The estimation results also indicated that the U.S. model reaches stable steady-state values for the model variables. Table 3 shows the model variable steady-state values.

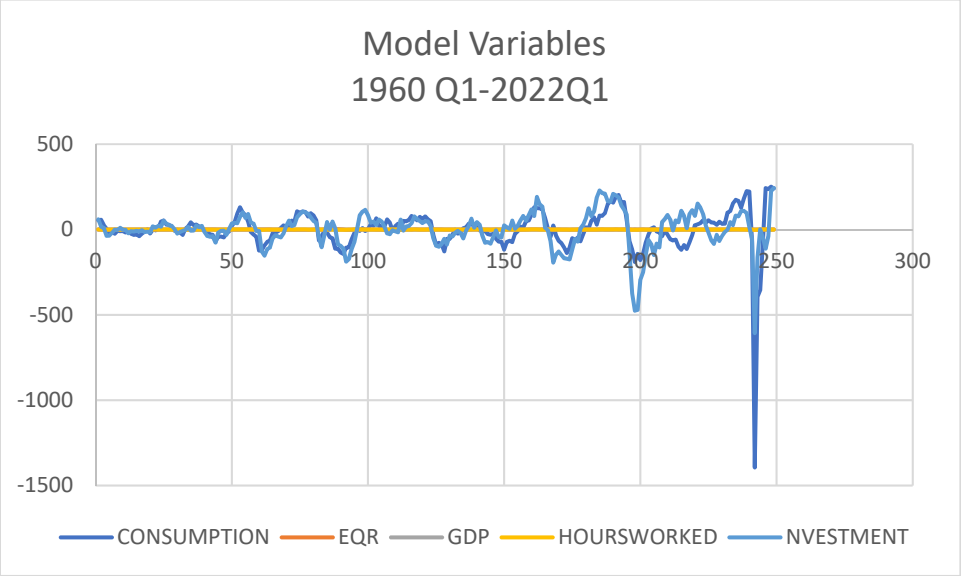


Figure 1. Key Model variables. Notes: All model variables except the equity returns and GDP are detrended by the Hodrick-Prescott methodology. Equity returns are percentage changes in the S&P 500. The GDP gap is measured as the logarithm of the ratio of the real GDP to the potential real GDP.

Table 1. Presents the summary statistics for the key observed model variables.

Variable	n	Mean	Std. Dev.	Min	Max	ADF (with Break Point)
c	249	1.43×10^{-9}	1.407	−11.161	3.700	−8.584 [a]
i	249	-1.86×10^{-9}	6.514	−22.733	15.268	−5.822 [a]
h	249	-2.60×10^{-10}	0.934	−5.179	1.577	−8.508 [a]
y	249	−1.045	2.419	−11.696	5.540	−6.725 [a]
eqr	249	1.68×10^{-13}	0.590	−1.776	2.441	−7.538 [a]

Notes: All model variables except the equity returns and GDP are detrended by the Hodrick-Prescott methodology. Equity returns are percentage changes in the S&P 500. The GDP gap is measured as the logarithm of the ratio of the real GDP to the potential real GDP. The ADF stands for the Augmented Dickey-Fuller unit root test accounting for break points. [a] statistically significant at 1% level based on Vogelsang (1993) one-sided critical values.

Table 2. Estimated variances of model variables estimated by the Delta-method of Taylor series approximation.

		Coefficient
C_t	var(C_t)	2.433 [b]
		(1.015)
R_t	var(R_t)	3.897 [a]
		(0.644)
W_t	var(W_t)	2.912 [a]
		(1.124)
H_t	var(H_t)	1.152 [a]
		(0.125)
I_t	var(I_t)	65.913 [a]
		(8.773)
EQR_t	var(EQR_t)	3.897 [a]
		(0.644)
Y_t	var(Y_t)	5.893 [a]
		(1.589)

[a,b] indicate that Z statistic is significant at 1% and 5%, respectively.

Table 3. Location of model steady-state variables.

Variable	Calibrated	Estimated
Y_s	0.74	0.64
C_s	0.57	0.52
I_s	0.17	0.12
K_s	2.86	4.37
H_s	0.36	0.25
W_s	1.34	1.72
R_s	0.09	0.05
A	1	1
EQR_s	0.02	0.01

Figure 2 summarizes the responses of key variables in the model to productivity shocks and the 95% confidence interval of the shocks represented by the shaded area. Starting with the top left graph, a positive productivity shock raises productivity immediately, and the responses become statistically insignificant at around 12 quarters or three years. Therefore, the U.S. economy benefits up to three years from positive productivity shocks that normally stem from technological innovations.

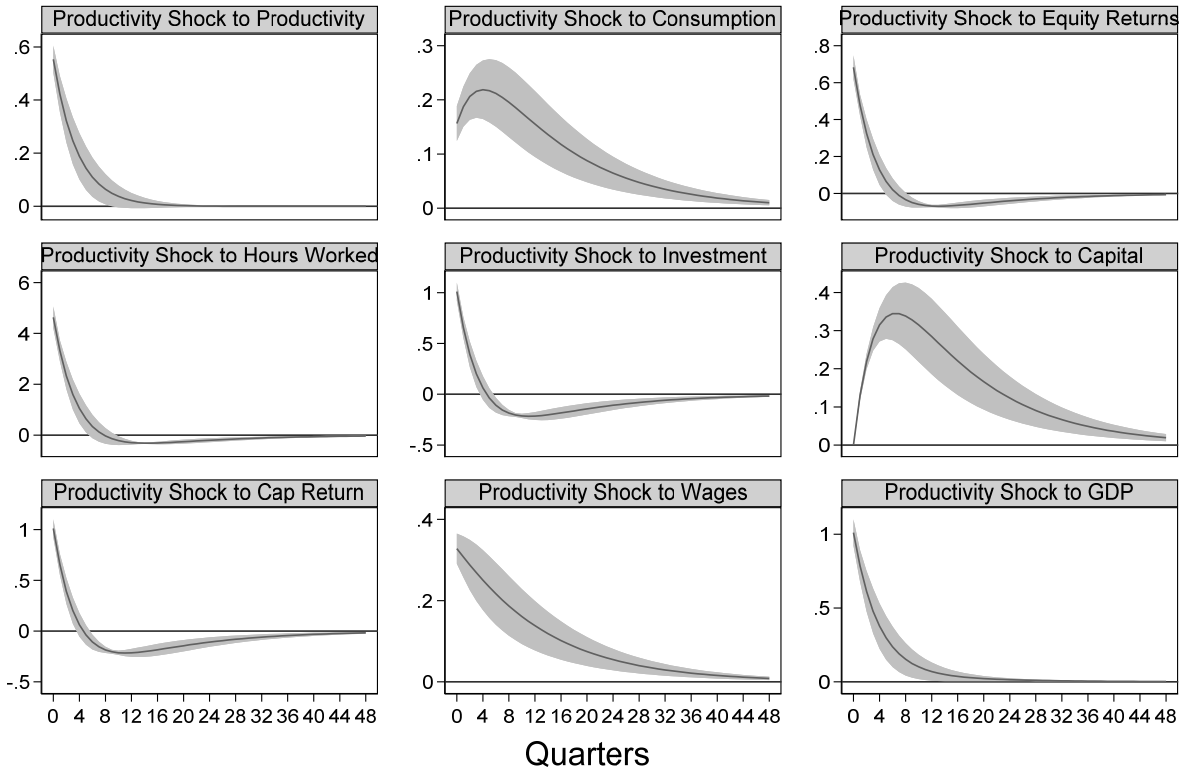


Figure 2. Impulse Responses and 95 Percent Confidence Intervals of Model Variables to Productivity Shocks.

The positive productivity shock triggers a rising trend in consumption. This trend reverses in around eight quarters or two years. However, the consumption response to the productivity shock does not die down for many quarters. The response of consumption to productivity shocks is a result of wages and returns to households from equity investments.

Impulse responses of real wages and invested capital to productivity shocks are associated with the rise in the marginal productivity of capital and labor. Thus, as the marginal productivity of both input factors rises, demand for labor and capital follows. Assuming no growth in the labor force and its participation rate, wages rise. Rising wages

do not level off for around 11 quarters, indicating the prolonged effects of increases in labor productivity.

It is clear from impulse responses of equity returns that a rising marginal product of capital triggers increases in firm profitability and thus increased shares of profits that compensate investors in equity markets. Cochrane (1991) shows that the first order conditions in profit maximization by producers relate the investment return to asset returns. Firms will adjust their investment and production plans until the investment return equals a target portfolio return. The producer's first order conditions require that the investment return and the typical portfolio return should be equal under various conditions. This can be confirmed for investment returns and a production function that includes the adjustment costs of investment. It can be shown that given a production function and producer's first order conditions, return on a firm's investment is the return on the firm's stock. Generalizing this finding to all firms, it can be shown that returns from firms' production in the entire economy are a predictor of their own stock returns. Assuming constant prices in an RBC model, equity returns could be predicted by the returns on capital investment from a production function. This can be proven as the arbitrage process for all firms would ensure that investments in productive or financial assets in general equilibrium would move toward equality. In the absence of monetary effects, we make a further simplifying assumption without any loss of generality that the marginal productivity of capital in the economy and stock returns follow a linear relationship.

Impulse responses also indicate that initially, the supply of labor hours by households rises. However, impulse responses become statistically insignificant in around 8 quarters or two years. This observation is plausible because rising wages and dividend receipts raise the opportunity cost of leisure hours. Therefore, households' reduced labor hours due to the income effect of higher wages on the opportunity cost of leisure and that leisure is a normal good in the consumer utility function.

Consistent with equity return's positive response to positive productivity shocks in the economy, initially, households save and invest more, which in turn adds to the stock of capital. Both investments and capital input (K) show a positive trend up to around eight quarters, where investment responses drop to zero and the capital accumulation peaks. Beyond eight quarters, and with no new investments resulting from new productivity shocks, depreciation of capital stock reduces the available capital.

The real GDP gap relative to real potential GDP also positively responds to positive productivity shocks. The impulse responses of the real GDP do not die off for about 24 quarters or six years.

Our findings are consistent with the findings of others (Costa 2016; Vermandel 2014). These scholars simulated RBC DSGE models with similar calibration. Our estimations are based on U.S. data. Figure 3 presents the impulse responses (IRF) from our model which we calibrated. Steady state variable values from the calibrated model reported in Table 3 are qualitatively similar to the estimated model. The IRFs corroborate the findings of the estimated models. However, similar to other calibrated RBC models, no confidence intervals are simulated. The findings of this paper and other simple calibrated RBC models further emphasize the role of technological innovations and productivity improvements for continued economic growth.

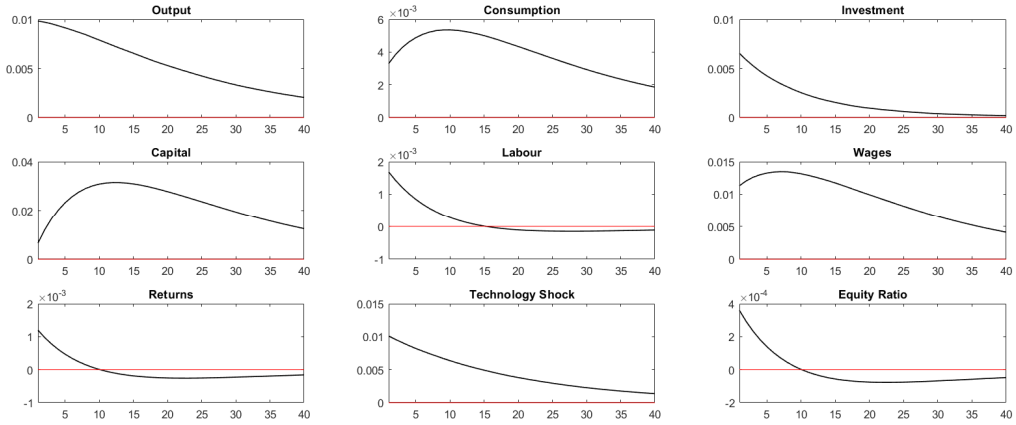


Figure 3. Impulse Responses of Model variables to Productivity Shocks, Calibrated Model.

6. Summary and Conclusions

This paper aims to analyze how equity returns respond to productivity shocks. To achieve this goal, we estimated an RBC DSGE model of the U.S. economy using data from the first quarter of 1960 to the first quarter of 2022.

Positive productivity shocks, which refer to unexpected enhancements in production efficiency, have been linked to economic growth, capital formation, and equity markets. Such shocks can raise output without a corresponding increase in inputs, thus improving the economy's overall efficiency.

According to a study by Stiroh (2002), positive productivity shocks can have a substantial impact on economic growth, with a one percent increase in productivity leading to a 0.5 to 1 percent increase in GDP. Additionally, these shocks can lead to an increase in capital formation as firms invest in new technologies and equipment to take advantage of the productivity gains. Finally, positive productivity shocks have been found to have a positive effect on equity markets as investors anticipate increased profitability and earnings growth. This has been supported by empirical studies, such as the work of Greenwald et al. (2014), who found that positive productivity shocks had a significant positive effect on stock prices. Overall, positive productivity shocks are an important driver of economic growth and can have positive effects on both capital formation and equity markets.

There are several advantages to investigating equity returns in a DSGE framework. First, this model is dynamic and therefore easily incorporates the time dimension in the decision process of economic agents. Secondly, it embodies the risks and uncertainties that surround economic decisions, especially over time. Finally, the DSGE model is founded on the micro foundations of macroeconomics to study the general equilibrium ramifications of decisions by consumers, firms, and investors. For instance, the RBC DSGE model in this paper allows us to examine the equity market responses to firms' and consumers' decisions, their effects on the real economy, rather than financial engineering or market manipulations.

We estimated an RBC DSGE model of the U.S. economy using data from Q1 1960 to Q1 2022, which showed that positive productivity shocks have a lasting effect on equity returns for approximately eight quarters. These findings are consistent with our calibrated model and previous studies, such as those of Costa (2016) and Vermandel (2014), which used calibrated RBC DSGE models.

Our results highlight the critical role of technological innovation and productivity improvements in sustaining economic growth. By estimating an RBC DSGE model, we demonstrate that equity returns are linked to productivity shocks resulting from technological breakthroughs. These findings align with Solow's (1956) argument that long-run per capita GDP growth is tied to exogenous technological advancements, rather than solely capital stock expansion.

As the only way to sustain growth in per capita GDP is through technological innovation or shocks leading to higher marginal product of capital, an increase in per capita income is likely to raise the aggregate demand. Technological advancements also enhance the efficiency of goods and services production, which could contribute to an increase in equity returns. The combined impact of these forces may be driving the observed equity return patterns.

Positive productivity shocks resulting from technological innovations offer investors and wealth managers a valuable opportunity to allocate their investments strategically toward sectors of the economy that stand to benefit the most. These shocks can be viewed as a signal for identifying specific areas of the economy where technological improvements are driving productivity gains, which may therefore yield higher returns.

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