Journal of
Risk and Financial Management

MDPI

Review

On Asymmetric Correlations and Their Applications in Financial Markets

Linyu Cao [1], Ruili Sun [1,*], Tiefeng Ma [2] and Conan Liu [3]

[1] College of Mathematics and Information Science, Zhengzhou University of Light Industry, Zhengzhou 450001, China
[2] School of Statistics, Southwestern University of Finance and Economics, Chengdu 611130, China
[3] UNSW Business School, University of New South Wales, Sydney, NSW 2052, Australia
* Correspondence: sunruili2009@163.com

Abstract: Progress on asymmetric correlations of asset returns has recently advanced considerably. Asymmetric correlations can cause problems in hedging effectiveness and overstate the value of diversification. Furthermore, considering the asymmetric correlations in portfolio construction significantly enhances performance. The purpose of this paper is to trace developments and identify areas that require further research. We examine three aspects of asymmetric correlations: first, the existence of asymmetric correlations between asset returns and their significance tests; second, the test on the existence of asymmetric correlations between different markets and financial assets; and third, the root cause analysis of asymmetric correlations. In the first part, the contents of extreme value theory, the H statistic and a model-free test are covered. In the second part, commonly used models such as copula and GARCH are included. In addition to the GARCH and copula formulations, many other methods are included, such as regime switching, the Markov switching model, and the multifractal asymmetric detrend cross-correlation analysis method. In addition, we compare the advantages and differences between the models. In the third part, the causes of asymmetry are discussed, for example, higher common fundamental risk, correlation of individual fundamental risk, and so on.

Keywords: asymmetric correlation; statistical test; copula; GARCH

Citation: Cao, Linyu, Ruili Sun, Tiefeng Ma, and Conan Liu. 2023. On Asymmetric Correlations and Their Applications in Financial Markets. *Journal of Risk and Financial Management* 16: 187. https://doi.org/10.3390/jrfm16030187

Academic Editors: Ştefan Cristian Gherghina and Robert Brooks

Received: 9 January 2023
Revised: 18 February 2023
Accepted: 7 March 2023
Published: 9 March 2023

1. Introduction

Several recent studies corroborating asset returns have three asymmetric characteristics: the asymmetries in volatility, correlations, and betas. Notably, Black (1976) was the first researcher to consider asymmetry in volatility. Since then, asymmetric GARCH-type models have become popular when investigating the characteristics of financial time series, and a significant number of asymmetric GARCH models have been proposed (Choy et al. 2012). In addition, there is notable relevance between beta coefficients and asset pricing theories, and beta coefficients help to understand the riskiness of the associated asset stocks (Hong et al. 2007); see Ball and Kothari (1989), Conrad et al. (1991), and Bekaert and Wu (2000) for literature covering asymmetries in the betas.

This paper focuses on asymmetric correlations, the study of which is important for three reasons. Firstly, hedging mainly depends on the correlations between assets and financial instruments, and the existence of asymmetric correlations may lead to problems in hedging effectiveness (Hong et al. 2007). Second, in an optimal portfolio selection problem, if all stocks tend to fall with the decline of the market, the value of diversification may be exaggerated without considering the increase of downside correlations (Ang and Chen 2002). Third, taking the asymmetric correlations into account enhances the portfolio performance significantly (Taamouti and Tsafack 2009).

Let $\{r_{1t}, r_{2t}\}$ denote two assets returns during time period t. For convenience of computation and statistical analysis, the returns are normalized to zero mean and unit variance. Using the same notation as in Longin and Solnik (2001), Ang and Chen (2002), and Hong et al. (2007), we define the exceeding correlation at a given level c as follows:

$$\rho^{+}(c) = corr(r_{1t}, r_{2t} | r_{1t} > c, r_{2t} > c),$$
$$\rho^{-}(c) = corr(r_{1t}, r_{2t} | r_{1t} < -c, r_{2t} < -c),$$

where $\rho^{+}(c)$ measures the correlation between two returns above a certain exceedance level c, and $\rho^{-}(c)$ measures the correlation below a certain exceedance level c. Additionally, $\rho^{+}(c)$ represents the correlation during market upturns, and $\rho^{-}(c)$ denotes the correlation during downturns.

If $\rho^{+}(c) = \rho^{-}(c)$ for all $c \geq 0$, the correlation between the positive returns are the same as those with negative returns. This is called symmetric correlation. However, if $\rho^{+}(c) \neq \rho^{-}(c)$ for all $c \geq 0$, then there are asymmetric correlations. Specifically, certain literature on co-movement also indicates asymmetric correlations.

The asymmetric correlations of Austrian, Belgian, and Italian government bonds with US government bonds from January 1976 to March 2010 are shown in Figure 1, taken from Ozsoy (2013). On the one hand, the three curves share some similar patterns, indicating these countries' exhibit larger conditional correlations on the negative standardized exceedances than those on the positive standardized exceedances. On the other hand, they differ from each other with Belgium's conditional correlations intersecting Austria's at standardized exceedances of 0 and about 0.038, and with Italy's conditional correlations at the bottom.

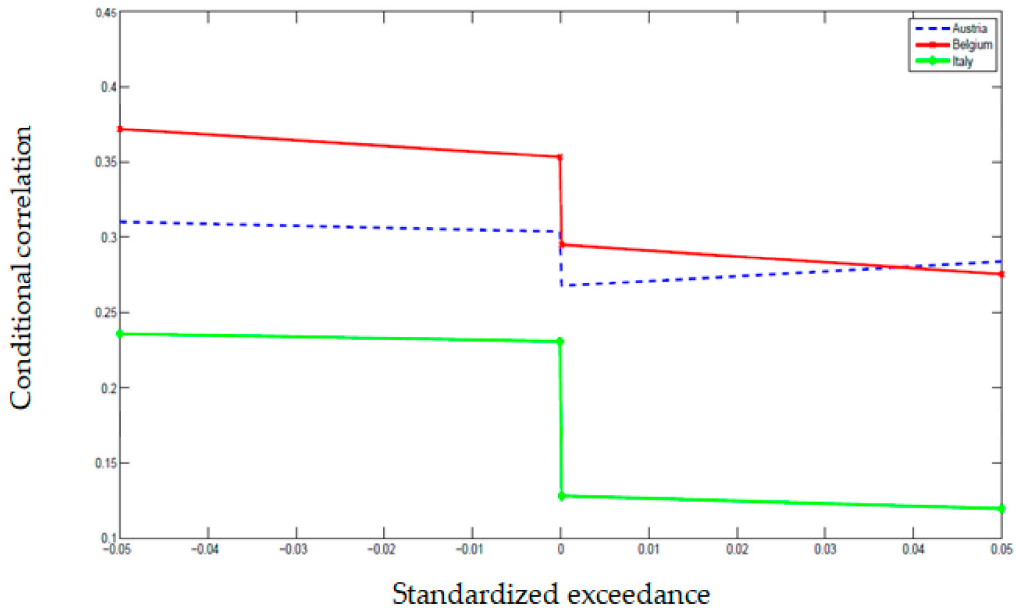

Figure 1. Asymmetric correlations.

This paper is organized as follows. In Section 2, we outline the existence of asymmetric correlations of asset returns and its significance test. In Section 3, we review the test on the existence of asymmetric correlations between different markets and financial assets. In Section 4, we present the root cause analysis of asymmetric correlations. In Section 5, we provide our conclusions of this study and directions for future research.

2. Existence of Asymmetric Correlations

From the introduction, it is clear that asymmetric correlations are a crucial topic in the research of portfolio selection-related issues. Therefore, in this section, we review the discovery of asymmetric correlations and their existence tests. We then summarize and provide some problems worthy of comprehensive study.

2.1. Literatures Review

In this subsection, we consider the existing research on asymmetric correlations. Some important literature is shown in Table 1.

Table 1. Selected work on existence of asymmetric correlations.

Author (Year)	Paper Title
Longin and Solnik (2001)	Extreme Correlation of International Equity Markets
Ang and Chen (2002)	Asymmetric Correlations of Equity Portfolios
Campbell et al. (2002)	Increased Correlation in Bear Markets
Hong et al. (2007)	Asymmetries in Stock Returns: Statistical Tests and Economic Evaluation
Pan et al. (2014)	Testing Asymmetric Correlations in Stock Returns via Empirical Likelihood Method
Jondeau (2016)	Asymmetry in Tail Dependence in Equity Portfolios

Erb et al. (1994) considered the behavior of correlation over time and predicting correlation to be of importance. Therefore, the changing international correlations in the G7 countries were investigated, and the results showed that correlations during recessions were higher than those during periods of growth, and that correlations were not symmetrical in up and down markets.

In order to verify the hypothesis that the correlation between international equity markets increases during fluctuation periods, Longin and Solnik (2001) used extreme value theory to model the tail of multivariate distribution, derived the extreme correlation distribution of the broad category distributions, and found that correlation was related to the market trend and that correlation increased in bear markets. Since Longin and Solnik (2001), asymmetric correlations have garnered more and more research attention.

However, Forbes and Rigobon (2002) found that a correlation calculated conditional on some variables was a biased estimator for the corresponding unconditional correlation.

Ang and Chen (2002) found that correlations between U.S. stocks and the aggregate U.S. market were much greater during declines than during market rallies. A new H statistic was developed to test conditional correlation asymmetries, which could correct for conditioning biases. Moreover, they established several empirical models about asymmetric correlation in the U.S. equity market. The results showed that mall stocks, value stocks, and past loser stocks had more asymmetric movements, and that stocks with lower betas exhibited greater asymmetric correlations by controlling for size.

To overcome estimator bias for implied correlation, Campbell et al. (2002) derived the quantile correlation estimator, which, based on the quantiles of the multivariate distribution, used the unbiased quantile correlation estimates to explore the correlations in international equity markets, and found that correlation in international equity returns increased significantly in bear markets.

Hong et al. (2007) emphasized that the H statistic proposed by Ang and Chen (2002) only answered the question of whether the asymmetry could be explicated by a given mode. Therefore, Hong et al. (2007) provided a model-free test for asymmetric correlations of stock returns in which stocks fluctuated with the market more often when the market fell than when it rose; the test also had a simple asymptotic chi-square distribution and could easily be applied to test the symmetries of beta and covariance. There existed significant asymmetries in size and momentum portfolios. To account for parameter and model uncertainties, a Bayesian framework was proposed to model them and evaluate their economic value. The results showed that taking the asymmetric characteristics of assets into consideration could significantly improve the performance of portfolio selection.

To investigate the robustness of recent empirical results that indicated a structural breakdown of correlation, Campbell et al. (2008) derived theoretical truncated and exceedance correlations, evaluated the performance of the truncated and exceedance correlation estimators, and found important asymmetry evidence of the conditional correlation functions.

Based on detrended fluctuation analysis (DFA), Alvarez-Ramirez et al. (2009) developed a DFA extension to study asymmetric correlations in nonstationary time-series, and the DFA version separated positive trends and negative trends to analyze the individual contributions to the overall scaling behavior. The results showed that the asymmetries of three different time-series were scale-dependent, and that there were different correlation properties depending on whether the signal trending was positive or negative.

Based on a conditional version of Kendall's tau and copula method, Manner (2010) proposed two tests for symmetric dependence; these tests outperformed the one proposed by Hong et al. (2007) in a Monte Carlo study. When the tests were applied to stock market returns and quarterly US GNP and unemployment data, the results showed that there was evidence of asymmetries and nonlinearities.

Livan and Rebecchi (2012) investigated the spectral properties of correlation matrices between distinct statistical systems, in which the correlation matrices were intrinsically nonsymmetrical, and extended the spectral analyses to the realm of complex eigenvalues. Random matrix theory was used to differentiate the noise and nontrivial correlation structures. The above results were applied to study the asymmetric correlation matrix of daily prices of the US and UK stock exchanges.

In order to analyze the asymmetric correlation of sovereign bond yield dynamics between eight Eurozone countries pair-wise, Dajčman (2013a) provided a dynamic version of the test proposed by Hong et al. (2007) and identified time periods when the correlation of Eurozone sovereign bond yield dynamics became asymmetric. They found that correlation between the positive and the negative yield dynamics between sovereign bonds became asymmetric after the start of the Eurozone debt crisis.

Pan et al. (2014) stressed that the model-free test proposed by Hong et al. (2007) seemed to be under-rejected in the size value and had low power in a finite sample. Therefore, they used an empirical likelihood method to conduct a model-free statistic that could test asymmetric correlations of stock returns, corrected the size performance using a bootstrap method, which improved the performance of Hong et al.'s (2007) test, and analyzed the asymmetric correlations of the China stock market and international stock markets, respectively. The results showed that asymmetric correlations occurred in the China stock market and international stock markets.

Jondeau (2016) considered that standard nonparametric measures of tail dependence had poor finite-sample properties in view of the limited number of observations in the tails of a joint distribution. Therefore, Jondeau (2016) developed a parametric model to measure and test asymmetry in tail dependence based on a multivariate noncentral t distribution. The proposed model accommodated situations in which the volatilities or the correlations between different asset returns changed over time. Applying the above model to real data, they found that the correlation between the international markets and Fama–French portfolios in bear markets was greater than that in bull markets.

Based on the statistic originally proposed by Hong et al. (2007), Alcock and Hatherley (2016) used a linear (β) dependence invariant metric to investigate the price of asymmetric dependence on the cross section of Wall Street stocks, and found that the existence of asymmetric dependence between the firm's returns and those of the market would lead to corresponding price discounts or premiums, and that failing to recognize the impact of asymmetric dependence of the cost of capital may cause low pricing or insufficient subscription of public capital offerings.

Miyazaki and Hamori (2016) implemented the model-free test proposed by Hong et al. (2007) to study the asymmetric cross-asset correlations of the gold market. The results showed that gold exhibited asymmetric correlation with stocks and the US dollar, and by dividing the sample into three characteristic periods, the exceedance correlation also exhibited significant time variation even under similar market stress of the same asset pairs.

Jiang et al. (2018a) emphasized that the test proposed by Hong et al. (2007) did not solve asymmetry problems beyond the second moment and had low power. Therefore, to measure the asymmetric co-movement between returns on a single asset and the market returns, they proposed a model-free entropy measure, which provided a direct test for asymmetry in the joint distribution, generalizing the correlation-based test proposed by Hong et al. (2007). The results showed that many common portfolios such as size, book value, and momentum portfolios had significant asymmetry in statistics.

Jiang et al. (2018b) considered that the test proposed by Hong et al. (2007) captured only linear dependence. To characterize the general asymmetric dependence between two random variables, they proposed a modified information measure, provided a test of asymmetric dependence and examined its finite sample performance. The results showed that common stock portfolios and market returns in the US and other similarly developed countries existed obvious asymmetric correlations, and when these markets were in a downturn, they exhibited higher correlation with each other.

2.2. Conclusions and Further Research

Erb et al. (1994) and Longin and Solnik (2001) played a pioneering role in the discovery of asymmetric correlations. However, Forbes and Rigobon (2002) found that there existed conditioning biases in the estimation of correlation. To correct the biases of correlation, Ang and Chen (2002), Campbell et al. (2002), and Hong et al. (2007) proposed a new H statistic, quantile correlation estimator, and a model-free test, respectively. Furthermore, Campbell et al. (2008) derived theoretical truncated and exceedance correlations to verify the robustness of recent empirical results. The other studies are mostly based on the research of Hong et al. (2007) and improve some of its shortcomings such as low power in a finite sample, or linear dependence.

However, there are still some problems worth considering and studying in the verification of the existence of asymmetric correlations. First, does the exceedance level c affect the results of all the test statistics mentioned above? If so, how does the exceedance level affect the results? How do we choose a reasonable and accurate exceedance level? Second, as pointed out by Dajčman (2013a), the model-free test proposed by Hong et al. (2007) depends on the time interval. The interesting question is whether the model-free test is consistent with the time interval and whether there are certain methods and criteria for the selection of time intervals.

3. Asymmetric Correlations between Different Markets and Financial Assets

With the discovery of asymmetric correlations, especially the corresponding asymmetric correlation test statistics, more and more scholars are beginning to pay attention to the asymmetric correlations of asset returns. In the research of asymmetric correlations, the two most used models are GARCH family models and copula. In the first two subsections, we focus on asymmetric GARCH family models and copula. In the third subsection, we introduce some other research methods related to asymmetric correlations. Finally, we make a summary and comparative analysis, and put forward some new and open issues worth studying.

3.1. Asymmetric GARCH Formulations

In this subsection, we first review the development of GARCH family models. Then, we represent the use of GARCH formulation in capturing the asymmetric correlations between different financial markets. Some pioneering research is summarized in Table 2, shown below.

Table 2. Selected works on correlation/covariance and GARCH.

Author (Year)	Paper Title
Engle (1982)	Autoregressive Conditional Heteroscedasticity and Estimates of UK Inflation
Bollerslev (1986)	Generalized Autoregressive Conditional Heteroscedasticity
Bollerslev (1990)	Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Model
Engle and Kroner (1995)	Multivariate Simultaneous Generalized ARCH
Tse and Tsui (2002)	A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-varying Correlations
Engle (2002)	Dynamic Conditional Correlation (DCC): A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models
Cappiello et al. (2006)	Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns
Wang and Nie (2016)	Research of Asymmetric Dynamics in the Correlations of the Chinese Stock Markets
Chen et al. (2021)	On a Bivariate Hysteretic AR-GARCH Model with Conditional Asymmetry in Correlations

Engle (1982) first introduced the autoregressive conditional heteroscedasticity (ARCH) model, and Bollerslev (1986) subsequently extended the ARCH model to the generalized autoregressive conditional heteroskedasticity (GARCH) model. Bollerslev et al. (1988) proposed a multivariate GARCH (MGARCH) model and used it to estimate the earnings of bills, bonds, and stocks. To ensure that the conditional covariance was positive definite, Bollerslev (1990) proposed the constant conditional correlations (CCC) MGARCH model. However, many researchers found that practical financial data violated certain assumptions of the CCC MGARCH model. Engle and Kroner (1995) proposed a BEKK method for multivariate ARCH processes and derived the sufficiency constraints to ensure the conditional covariance matrices were positive definite. Kroner and Ng (1998) compared the restrictions of VECH, BEKK, factor ARCH, and CCC GARCH models; introduced a group of robust conditional moment tests to check whether the model was specified properly; and proposed a generalized adoption model that allowed for asymmetric influences on the variances and covariances. Many researchers have found that the correlation is not invariant, which means the correlation is time-varying. Tse and Tsui (2002) proposed a MGARCH model whose correlation could be changed over time, in which they decomposed the conditional variance–covariance matrix into a product of two parts: one was a conditional variance matrix, and the other was a conditional correlation coefficient matrix. They also stuck each term of the conditional variance matrix to a single variable GARCH model and engineered each element of the conditional correlation coefficient matrix to follow an ARMA model. Meanwhile, Engle (2002) suggested a DCC MGARCH model to estimate time-varying correlations. Since then, GARCH family models and its generations, especially the asymmetric version of the DCC MGARCH model, have been widely used in asymmetric correlations measurement and testing. For additional GARCH family models, see, e.g., Liu and Heyde (2008), Liu and Neudecker (2009), and Dewick (2022).

Next, let us briefly introduce the asymmetry generalized dynamic conditional correlation multivariate GARCH (AG-DCC-MVGARCH) model. Assume r_t is the *p*-dimensional asset returns at time *t*. Then, r_t obeys the multivariate normal distribution

$$r_t \mid \Omega_{t-1} \sim N\left(0, H_t\right),$$

where Ω_{t-1} represents the information set at time $t-1$; H_t is the conditional variance–covariance matrix; and it can be decomposed as

$$H_t = D_t R_t D_t,$$

where D_t is a $p \times p$ diagonal variance matrix of asset returns; $D_t = diag\left\{\sqrt{h_{i,t}}\right\}$; $h_{i,t}$ is the time-varying variance obtained from the single-variable GARCH model; R_t is the time-varying conditional correlation coefficient matrix defined as

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1},$$
$$Q_t = \left(\overline{Q} - A'\overline{Q}A - B'\overline{Q}B - G'\overline{N}G\right) + A'\varepsilon_{t-1}\varepsilon_{t-1}{}'A + B'Q_{t-1}B + G'\eta_{t-1}\eta_{t-1}{}'G,$$

where Q_t^* is a diagonal matrix; $Q_t^* = \left[\sqrt{q_{i,i,t}}\right]$, $q_{i,i,t}$ is the corresponding diagonal element of Q_t; *A*, *B*, and *G* are $p \times p$ parameter matrices; $\varepsilon_{i,t} = \frac{r_{i,t}}{\sqrt{h_{i,t}}}$, $\overline{Q}$ is the unconditional covariance matrix of $\varepsilon_{i,t}$; $\eta_{i,t} = I\left[\varepsilon_{i,t}\right] \circ \varepsilon_{i,t}$, $I[\bullet]$ is the indicator function; $\circ$ is the Hadamard product; $\overline{N}$ is the unconditional variance–covariance matrix of $\eta_{i,t}$ and can capture the asymmetric characteristics of conditional correlation.

Butler and Joaquin (2002) used three popular bivariate distributions (the normal, RiskMetrics' restricted GARCH(1,1) distribution) to investigate the correlations with monthly returns observed in bear, calm, and bull markets. The results showed that the correlation during the market declines was obviously higher than that predicted by the normal distribution and RiskMetrics distributions, and the correlation during the bear market was significantly higher than that during bull market.

Kearney and Potì (2005) focused on country-level market index correlations, applied the symmetric and asymmetric version of the DCC MGARCH model to capture dynamic correlations, and found mixed evidence of asymmetric correlation reactions to news types simulated by the traditional asymmetric DCC MGARCH formulations.

Cappiello et al. (2006) implemented an asymmetric version of the DCC MGARCH model proposed by Engle (2002) to investigate asymmetric correlations in international capital stock and bond returns. The results illustrated that both bonds and international capital stock exhibited asymmetric correlation.

In the presence of asymmetry in the tail dependence, Tsafack (2009) considered that the DCC MGARCH model was a symmetric model, and that symmetrical portfolio models of this kind would cause investors to undervalue the value at risk (VaR) or expected shortfall (ES) of the portfolio, concluding that it was important to adopt an asymmetric portfolio model, e.g., the Gumbel copula, to deal with the asymmetric correlation problem.

To study the correlation between some notable indices and bonds in the United States, Yang et al. (2010) applied an asymmetric generalized DCC MGARCH model to a series of daily data, such as the S&P 500 and corporate bonds, and their real estate counterparts. They found that the correlation between REIT and stock returns exhibited asymmetries.

Horvath and Poldauf (2012) used multivariate GARCH models to investigate the comovements of certain stock markets among various countries. The results showed that during 2008–2010, the correlation between stock returns increased, and that the correlation between the stock markets in the US and China was basically zero before the crisis, but slightly increased during the crisis.

Choy et al. (2012) used a bivariate GARCH model with DCC and leverage effect to model financial data, and proposed a new modified multivariate t-distribution, which offered independent marginal Student-t distributions, to highlight the relationship between different stock returns. The empirical study showed that the correlations between the oil price shocks and stock returns from 2008 to 2009 increased significantly.

Chen (2013) employed the asymmetry generalized dynamic conditional correlation multivariate GARCH (AG-DCC-MVGARCH) model, quasi-maximum likelihood estimation, and LR test to investigate the asymmetric and dynamic correlation of stock returns in the US and China, and found that the correlation between different stock returns enhanced during bear markets.

Toyoshima and Hamori (2013) used the asymmetric DCC MGARCH model to describe the correlation of stock markets in Japan and Singapore, and found that financial integration had advanced due to the Japan–Singapore Economic Partnership Agreement, and that the investment portfolio in Asia had increased since the recent global financial crisis.

Gjika and Horváth (2013) used the asymmetric DCC MGARCH model to study stock market co-movements in central Europe. The results showed that the correlations increased over time, and that the stock markets exhibited asymmetric correlations to a certain degree.

Since the mean variance model was the most important model in the portfolio optimization, Kalotychou et al. (2014) explored its economic value in modeling conditional correlations and evaluated its dynamic strategies. They found that, by characterizing the change of correlation properly, fund managers could improve risk-adjusted returns by accurately capturing correlation time variation.

El Abed (2016) adopted a multivariate asymmetric DCC EGRACH framework to investigate the correlations of US dollar exchange rates and three European stock prices, and found that there were asymmetric responses in correlations, and that the correlation between exchange rates and stock prices increased during times of crisis.

Chen (2016) used the AG-DCC-MGARCH model to analyze the correlations among the four main stock markets in China and the impacts of the major economic events on the dynamics of the correlation coefficients of the four main stock markets. The results showed that the conditional correlations between Hong Kong and Shanghai, Hong Kong and Shenzhen, and Shanghai and Shenzhen were asymmetric.

Wang and Nie (2016) built EGARCH and an asymmetric version of the DCC MGARCH model to investigate dynamics and asymmetries in conditional variance and correlations in the Chinese stock markets. They found that A and B shares significantly existed dynamics and asymmetry in conditional correlation.

By generalizing the time-varying conditional correlation model proposed by Tse and Tsui (2002), Chen et al. (2021) suggested a new MHAR-A-GARCH-T model and used it to investigate the correlations with conditionally dynamic asymmetric structure. Moreover, by employing an adaptive Bayesian MCMC method, they found that adopting the asymmetric effects made a difference in estimation of dynamic correlations.

3.2. Copula Formulations

In this subsection, we first review the advancement of copula, and then introduce the application of copula in asymmetric correlations.

Sklar (1959) proposed copula to verify the structure of dependency, especially the latent nonlinear correlation. Many researchers find that copula works well in capturing the correlation of financial data, so it is widely used in correlation measurement of financial data (Embrechts 1999). Since then, different copulas have been developed and are used in financial data exploration (Mashal and Zeevi 2002; Van den Goorbergh et al. 2005;

Bartram et al. 2007; Chen and Tu 2013; Pastpipatkul et al. 2018). For more details about copula and its applications, see, e.g., Dewick and Liu (2022). Some important publications are listed in Table 3.

Table 3. Selected works on copula.

Author (Year)	Paper
Sklar (1959)	Fonctions Derépartitionàn Dimensions et Leurs Marges
Embrechts (1999)	An Introduction to Copulas
Mashal and Zeevi (2002)	Beyond Correlation: Extreme Co-movements between Financial Assets
Patton (2006)	Modelling Asymmetric Exchange Rate Dependence
Christoffersen et al. (2012)	Is the Potential for International Diversification Disappearing? A Dynamic Copula Approach

We assume $F\left(x_1,\cdots,x_p\right)$ is an arbitrary p dimension joint distribution function, $F_1\left(x_1\right),\cdots,F_p\left(x_p\right)$ are marginal distribution functions of $F\left(x_1,\cdots,x_p\right)$, and $C\left(u_1,\cdots,u_p\right)$ is a p dimension copula of $F\left(x_1,\cdots,x_p\right)$ if they satisfy the equation

$$F\left(x_1,\cdots,x_p\right)=C\left(F_1\left(x_1\right),\cdots,F_p\left(x_p\right)\right),$$

where the above function C is called a copula of F. Moreover, if the marginal distributions are continuous, then there is a unique copula corresponding to the joint distribution $F\left(x_1,\cdots,x_p\right)$, which can be obtained from

$$C\left(u_1,\cdots,u_p\right)=F\left(F_1^{-1}\left(u_1\right),\cdots,F_p^{-1}\left(u_p\right)\right).$$

On the contrary, the corresponding density function of joint distribution $F\left(x_1,\cdots,x_p\right)$ is calculated with

$$f\left(x_1,\cdots,x_p\right)=c\left(F_1\left(x_1\right),\cdots,F_p\left(x_p\right)\right)\prod_{i=1}^{p}f_i\left(x_i\right),$$

given the density functions exist, where $f_i\left(x_i\right)$ represents the marginal density functions and c is the density function of the copula and can be obtained by the equation

$$c\left(u_1,\cdots,u_p\right)=\frac{f\left(F_1^{-1}\left(u_1\right),\cdots,F_p^{-1}\left(u_p\right)\right)}{\prod_{i=1}^{p}f_i\left(F_i^{-1}\left(u_i\right)\right)}.$$

Patton (2004) considered the portfolio selection problem for investors with constant relative risk aversion, used models that could depict fourth order time-varying moments, and constructed time-varying dependence structure models allowing for different dependencies during bear markets and bull markets using copula theory. They found that the understanding of higher moments and asymmetric dependence would, in some cases, bring significant economic and statistical benefits to investors without short-selling restrictions.

Based on the GARCH model and regime-switching (RS) copula function, Wei and Zhang (2005) constructed the RS-copula–GARCH model to investigate the asymmetric tail dependence structure in Chinese stock markets and found that tail dependence structure of Shanghai and Shenzhen stock markets were asymmetric, and that RS-copula–GARCH model was superior to static copula model in describing dependence.

To test asymmetry of dependence between the German mark and the Japanese yen, Patton (2006) generalized the copulas theory to adopt conditioning variables and built conditional dependence models to fit the dependence of these exchange rates. The results showed that the exchange rates were more correlated when depreciating against the dollar than when appreciating.

To capture time-varying and nonlinear relationships among European stock markets, Bartram et al. (2007) used a time-varying copula model in which a GARCH formulation using a Gloston Jagannatha Runkle-generalized autoregressive conditional heteroskedasticity-moving average-t model was used to model the marginal distributions and the Gaussian copula was adopted to model the joint distribution. The results showed that market dependence increased after the introduction of the common currency only for large equity markets.

In order to investigate the dual dependence of exchange rates against the dollar, Boero et al. (2011) employed nonparametric plots and a robust semiparametric method to obtain the copula function. The results showed that the model captured asymmetric tail dependence well.

Using four parametric copulas to model the dependence structure at different investment horizons, Kang et al. (2010) reexamined the asymmetric correlations within hedge fund returns and market returns at a range of investment, and found that the dependence asymmetry was not limited to a specific time range but emerged clearly at all investment periods, and that the size of asymmetry was not invariable to the investment period, and its degree decreased significantly with the extension of investment period.

Garcia and Tsafack (2011) proposed an alternative RS copula of extreme dependence asymmetry. The copula-based model included one normal regime where dependence was symmetric and a second regime in which it was characterized by asymmetric dependence. Applying the above model to the capital stock and bond markets, they observed significant asymmetric behavior between different markets.

Christoffersen et al. (2012) considered that international equity markets were characterized by nonlinear dependence and asymmetries, proposed a new dynamic asymmetric copula model that allowed for asymmetric and dynamic tail dependence, and found that correlations had increased significantly in all markets.

To investigate the asymmetric dependence of financial data, Uhm et al. (2012) employed the copula approach for directional dependence. They found that the exchange rates correlation between the Republic of Korea and Japan was asymmetric due to the influence of the 2008 financial crisis and concluded that the direction-dependent copula method could be supplemented to interpret the asymmetric dependence.

Cho and Lee (2022) considered that default probabilities of credit portfolios were seriously affected by system risk, so they used the GJR-GARCH model and copula method to fit the volatility and dependence, respectively, proposing a new time-varying credit risk model. The results showed that the suggested model outperformed the existing model, and that there was strong evidence to show the existence of asymmetric correlation of asset returns.

3.3. Other Asymmetric Formulations

Except the GARCH and copula formulations, many other methods are used to explore the asymmetric correlations, such as regime switching, the Markov switching model, and the multifractal asymmetric detrend cross-correlation analysis method (MF-ADCCA).

In order to characterize the risk and return in risk arbitrage, Mitchell and Pulvino (2001) used piecewise linear regressions to analyze 4750 mergers from 1963 to 1998. The results showed that risk arbitrage returns in most environments were uncorrelated with market returns, and that the correlation between market returns and risk arbitrage returns increased dramatically during market downturn.

The existence of asymmetric correlation made investors question the correctness of international diversification. In order to investigate the above result, Ang and Bekaert (2002) introduced RS model to deal with the asset allocation problem within a dynamic international situation and found that international diversification was still useful under regime changes.

Yuan (2005) presented a rational expectations equilibrium model to cope with the determinants of asset market crises and contagion. They found that market return distributions were asymmetric and that correlations between different asset returns tended to increase during crashes.

Michayluk et al. (2006) examined the volatility spillover effects and the inherent correlation among the US- and UK-securitized real estate indices, and found the correlation of different markets exhibited asymmetry.

To verify whether asymmetric correlations existed and determine the explanation of asymmetry, Taamouti and Tsafack (2009) used the generalized impulse response function under an autoregressive model framework to quantify the relationship among return, volatility, and correlation, and tested the asymmetric correlations between return and volatility against correlation. The results showed that considering the asymmetric correlation between return and correlation could obtain improved financial gain.

Abid and Bahloul (2011) used the discrete time Markov switching model to analyze the behavior of equity returns correlations, investigated the effect of this behavior on international portfolio allocation, and found that the correlations in a bear market showed obvious difference with correlations in the bull market.

Lee et al. (2011) examined the performance of asset correlation with the market returns in the asymptotic single risk factor (ASRF) approach of the Basel II accord on regulatory capital requirement and found that asset correlations were asymmetric.

By comparing the equity market in Croatia in good (bull, clam) and bad (bear, turbulent) market conditions, Kunovac (2012) found that correlations between stock prices during bear markets more than doubled those exhibited during bull markets. In addition, they found that the losses might occur if the asymmetry was ignored in practice by the research of taking asymmetric correlation into consideration and assessing the performance of the portfolio selection model.

Cao et al. (2013) used asymmetric multifractal detrended fluctuation analysis to test the asymmetry of China's stock markets in the upward or downward trend. They found that asymmetric correlation was more obvious in large fluctuations.

Dajčman (2013b) examined the asymmetric correlation pair-wise between the Eurozone's stock market returns, and investigated if the results were sensitive to a time span of returns. The results showed the asymmetric correlation test relied on the time span of returns.

By using the Chinese market data, Cao et al. (2014) proposed the MF-ADCCA model to study the asymmetric correlations in stock and exchange market. The empirical results showed that there was asymmetric cross-correlation between Chinese stock market and the Chinese RMB exchange market.

Based on theoretical derivation, Chen et al. (2014) studied the time varying correlation between the Chinese stock market and the broader macroeconomy. The results showed that there was indeed asymmetric correlation between the Chinese stock market and global economies.

To verify whether the strength of the co-movements caused by market declines and market rallies were significantly different, Li (2014) developed a nonparametric test, and the proposed test could be applied to verify whether there were asymmetric co-movements resulting from a linear or nonlinear dependence. The results showed significant evidence of asymmetric co-movements in the stock markets of the U.S. and other developed countries.

To study the correlation of gold prices and oil prices with COVID-19, Mensi et al. (2020) used the asymmetric multifractal detrended fluctuation analysis (A-MF-DFA) method to investigate the impacts between them and found obvious evidence of asymmetric multifractality that increased as the fractality scale increased.

Kristjanpoller et al. (2020) used the MF-ADCCA approach to study asymmetric multifractality and found significant evidence of asymmetric multifractality in the cross-correlation between five main cryptocurrencies and six equity ETFs.

Based on the autoregressive distributed lag model, Thampanya et al. (2020) investigated the asymmetric influences of gold and cryptocurrency returns on the Thai stock market, and studied whether hedging functions of gold or cryptocurrency were still effective in the event of a stock market decline or rally. The results showed that gold and cryptocurrencies were not good tools for stock market hedging.

Given the fact that industry and market portfolios showed the asymmetry in correlations, Kim et al. (2021) developed a novel optimal consumption and portfolio selection framework and found that neglecting asymmetric correlations could cause loss to investors.

Xu et al. (2021) used the multifractal cross-correlation analysis method to investigate the asymmetric cross-correlations between international stock markets such as the China and US markets. The empirical results showed that the cross-correlations between markets were asymmetric, and that the cross-correlations were more stable and stronger in bear markets than those exhibited in bull markets.

Chuang et al. (2022) suggested nonparametric tests to verify asymmetric co-movements, applied them to daily return of SP 500 and 29 individual stocks, and found that most stock returns showed the showed asymmetric co-movements.

3.4. Conclusions and Further Research

Since Bollerslev et al. (1988) proposed the MGARCH model, MGARCH is widely used in the research of multiple asset returns. In particular, DCC model proposed by Engle (2002), a new family of multivariate GARCH models, constructs the model based on using the MGARCH model to study asymmetric correlation. Much of the research on asymmetric correlations based on GARCH model use the asymmetric version of DCC model.

Copula has unique advantages in the study of correlation, especially for nonlinear relationships. In the research of asymmetric correlations, copula is often combined with other models, such as GARCH model and regime switching.

In addition, regime switching, the Markov switching model, and the MF-ADCCA model are also used to investigate asymmetric correlations.

Through the review of research on asymmetric correlations, we compare the difference and advantages of the aforementioned models:

(a) GARCH family models are usually used to interpret covariance asymmetry. The most used GARCH family model with asymmetric correlation is an asymmetric version of DCC model proposed by Engle (2002). The asymmetric DCC MGARCH model could consider the asymmetric effects on conditional second moments, adopt asymmetric dynamics in the correlation as well as the asymmetric response in variances, and accommodate different news impact patterns for correlations between different assets. However, traditional GARCH family models are constructed using the conditionally normal distribution assumption of asset returns, have too many unknown parameters to estimate, and usually impose limited scope or significant parameter restrictions.

(b) Copula is a more effective measurement of dependence between multivariate variables; since the joint distribution is nonelliptical the conventional correlation cannot capture the dependence structure appropriately. In addition, when decomposing multivariate distribution into marginal distributions, copula can construct a better distribution of stock returns than existing multivariate distributions. However, copula needs the assumption of marginal distributions and needs to specify an affirmatory dependence structure about the asset returns.

(c) The multivariate regime switching model is a useful parametric alternative to copula models. In the regime switching model, the Markov switching model is a special case of regime switching model in which the discrete state variable follows a Markov chain process. The regime switching model is versatile and effective in capturing nonlinear relationships. However, the regime switching model assumes that the observations come from a mixture of parametric distributions and constant transition probabilities for the unobserved states. Furthermore, the identification of the number of regimes is also important but difficult. Both copula and regime switching models are usually combined with other models, such as the GARCH model.

(d) The multifractal asymmetric detrend cross-correlation analysis method is model-free and easy to implement. It can be used to analyze the nonlinear and highly volatile nature of and investigate asymmetric multifractality between financial time series data.

4. Root Cause Analysis of Asymmetric Correlations

4.1. Literature Review

As the asymmetric correlations garner the attention of many researchers, the root cause of asymmetric correlations also increases in popularity. To our knowledge, we classify the literature on the root cause of asymmetric correlations into four categories: the first is cashflow related causes, the second is firm-level return dispersions, the third is skewness-related causes, the fourth is other causes. Some important publications on root cause of asymmetric correlations are listed in Table 4.

Table 4. Selected works on root cause of asymmetric correlations.

Author (Year)	Paper
Yu and Wu (2001)	Economic Sources of Asymmetric Cross-correlation among Stock Returns
Demirer and Lien (2004)	Firm-level Return Dispersion and Correlation Asymmetry: Challenges for Portfolio Diversification
Ding et al. (2011)	Asymmetric Correlations in Equity Returns: a Fundamental-based Explanation
Albuquerque (2012)	Skewness in Stock Returns: Reconciling the Evidence on Firm Versus Aggregate Returns
Chung et al. (2019)	What Causes the Asymmetric Correlation in Stock Returns

Campbell (1991) and Vuolteenaho (2002) showed that stock returns could be decomposed into the following components: the expected return, shocks to expected cashflows, and shocks to discount rates. However, Vuolteenaho (2002) and Campbell and Vuolteenaho (2004) found that the first two components of stock returns were related, and pointed out that stock returns were caused by cashflow news. Therefore, cash flow related causes are first investigated. Yu and Wu (2001) suggested an alternative framework to explain and verify major causes of asymmetric cross-correlation and found the asymmetric cross-correlation was mostly attributed to differences in sensitivity of stock prices to market information and the differences in quality of cash flow information of differently sized firms. Chung et al. (2019) considered the latent causes of the asymmetric correlation in stock returns. They found that the correlation of firms' cash flow news variable and other accounting measures of firm performance was asymmetric, and that only the asymmetric correlation in firm performance could explain the asymmetric correlation in stock returns.

Unlike the cashflow-related causes, firm-level return dispersions were only studied by Demirer and Lien (2004). Demirer and Lien (2004) studied the question of whether firm-level return dispersions could explain asymmetric correlations in stock returns significantly. The results showed that asymmetric correlations were correlated with asymmetric firm-level return dispersions, and that portfolio managers need to take the asymmetry in return correlation and firm-level return dispersions into account.

Skewness of financial data is another cause of asymmetric correlations. Emphasizing that significant literature explained aggregate stock market returns, displayed negative skewness, and ignored the fact that firm stock returns displayed positive skewness, Albuquerque (2012) provided a unified theory, built a stationary asset pricing model of firm announcement events, and found that cross-sectional heterogeneity could result in asymmetric correlations in stock returns. Chung and Kim (2017) thought that asymmetric correlations led to negative skewness of portfolios, provided asymmetric correlation measurements by using portfolio skewness, and found that asymmetric correlation was generated at the asset level of individual firms.

The other causes of asymmetric correlations include increasing common fundamental risk, investor sentiment, variance and earning price ratio, and so on. However, they can only partially explain the asymmetric correlation. Ding et al. (2011) offered an explanation to the potential fundamental causes of the asymmetric correlations of stock portfolio returns. They found that several sources caused the asymmetry during market decline, such as increasing common fundamental risk, and they also concluded that these key factors were only part of the causes of asymmetric correlation. Wang et al. (2021) considered that the tests proposed to verify the existence of asymmetric correlations in previous literature could not be used in practical investment due to the natures of time-varying and unpredictable of asymmetric correlations. Therefore, they constructed a unified state–space model to measure in-sample and out-of-sample asymmetric correlations. They found that there were many factors that resulted in asymmetric correlations, such as investor sentiment, variance and price-to-earnings ratio, but they all could not fully explain the asymmetric correlation.

4.2. Conclusions and Further Research

Through the above review, we can see that researchers have conducted extensive research on the causes of asymmetric correlations and that various factors may cause or partially cause the asymmetric phenomenon of asset return. In our view, the financial market is rapidly changing. Therefore, during different periods of time, especially with the different financial policy guidance of each country, the causes for the asymmetry of asset returns may not be unique and certain; that is, different periods of time and different countries have different causes. There may not be a uniform determining cause for the asymmetric correlations of asset return, but there is a common applicable research framework, which can contain various causes and methods that need to be verified one by one according to the actual situation.

5. Conclusions

Since Markowitz (1952), the portfolio selection problem has been a hot topic. However, when the asset returns show asymmetry in the correlations, the portfolio selection problem should be reconsidered seriously. Therefore, asymmetric correlations of stock returns play an important role in portfolio selection and risk management. In this paper, we review the development and application of asymmetric correlations in financial markets and identify the directions for future research. This review focuses on three aspects: (a) the existence of asymmetric correlations between stock returns and its significance test; (b) the test on the existence of asymmetric correlations between different markets and financial assets; (c) the root cause analysis of asymmetric correlations.

Abundant empirical research verifies that the correlations of stock returns are higher in bear markets than in bull markets. Longin and Solnik (2001) are among the first to show the existence of asymmetric correlations after controlling for bias resulting from conditioning. The relevant methods and tools used on testing the existence of asymmetric correlations are extreme value theory, quantile, Kendall's tau, the copula method, detrended fluctuation analysis, etc. For the test on the existence of asymmetric correlations, GARCH family models and the copula method are two main methods. In addition, regime switching, the Markov switching model, and multifractal asymmetric detrend cross-correlation analysis method are also important tools. Asymmetric correlations also become a stylized fact of asset returns. In order to deepen the study of asymmetric correlations, the root causes of asymmetric correlations have also attracted the interest of researchers. According to the contents of root causes of asymmetric correlations, we divide them into four categories: the cash flow related causes, the firm-level return dispersions, the skewness related causes and other causes.

However, there are still many open issues worthy of consideration and research. For example, for the hypothesis testing of asymmetric correlations, how does the exceedance level affect the results of all the test statistics mentioned above, and how can we choose a reasonable and accurate exceedance level? In addition, Kang et al. (2010) found that the dependence of asymmetry was not limited to a specific time range but emerged clearly at all investment periods, that the size of asymmetry was not invariable to the investment period, and its degree decreased significantly with the extension of an investment period. Is there an appropriate way to measure the change degree of the size of asymmetry? Is the change degree of the size of asymmetry linear or nonlinear?

As mentioned at the beginning of this paper, the asset returns do exist asymmetrically in the volatility and correlations, but will the performance of portfolio selection be improved by taking the asymmetry in the volatility and correlations into account simultaneously? In addition, it is well known that the rolling window method proposed by DeMiguel and Nogales (2009) is widely used in testing the performance of portfolio selection. If we combine the above two methods and apply them to the portfolio selection problem, how can we detect the upturns and downturns of asset returns pairwise for a certain time window?

As to the cause of asymmetric correlations, how do we build a common applicable research framework, which can contain various causes and methods that need to be verified one by one according to the actual situation?

The asymmetric correlations only measure the asymmetry in terms of time; however, we consider that the asymmetry in correlation has two levels: the first is the time level, the second is the individual level, which means the asymmetry in different asset returns. For instance, on the stock market, the leader stock in one industry has a significant positive impact on other stocks, while other stocks in the same industry have a rather small positive impact on the leader stock. How do we measure the asymmetry at an individual level and combine the asymmetry in two levels of asset returns? Moreover, Chatterjee (2021) introduced a simple new rank correlation coefficient, which is not symmetric in two random variables. How can we use it in the portfolio selection problem?

Author Contributions: Original draft Preparation: L.C. and R.S.; Writing-review and editing: T.M. and C.L. All authors have read and agreed to the published version of the manuscript.

Funding: The first two authors received support from the National Natural Science Foundation of China (12201579).

Data Availability Statement: Not applicable.

Acknowledgments: The authors sincerely thank the editors and reviewers for their insightful comments which aided in improving the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

Abid, Fathi, and Slah Bahloul. 2011. Regime switching, asymmetric correlation and international portfolio choices. *International Journal of Monetary Economics and Finance* 4: 172–94.

Albuquerque, Rui. 2012. Skewness in stock returns: Reconciling the evidence on firm versus aggregate returns. *The Review of Financial Studies* 25: 1630–73.

Alcock, Jamie, and Anthony Hatherley. 2016. Characterizing the asymmetric dependence premium. *Review of Finance* 21: 1701–37.

Alvarez-Ramirez, Jose, Eduardo Rodriguez, and Juan Carlos Echeverria. 2009. A DFA approach for assessing asymmetric correlations. *Physica A: Statistical Mechanics and Its Applications* 388: 2263–70.

Ang, Andrew, and Geert Bekaert. 2002. International asset allocation with regime shifts. *Review of Financial Studies* 15: 1137–87.

Ang, Andrew, and Joseph Chen. 2002. Asymmetric correlations of equity portfolios. *Journal of Financial Economics* 63: 443–94.

Ball, Ray, and S. P. Kothari. 1989. Nonstationary expected returns: Implications for tests of market efficiency and serial correlation in returns. *Journal of Financial Economics* 25: 51–74.

Bartram, Söhnke, Stephen Taylor, and Yaw-Huei Wang. 2007. The euro and European financial market integration. *Journal of Banking and Finance* 31: 1461–81.

Bekaert, Geert, and Guojun Wu. 2000. Asymmetric volatility and risk in equity markets. *Review of Financial Studies* 13: 1–42.

Black, Fisher. 1976. Studies of stock market volatility changes. In *Proceedings of the 1976 Meeting of the Business and Economic Statistics Section*. Washington DC: American Statistical Association, pp. 177–81.

Boero, Gianna, Param Silvapulle, and Ainura Tursunalieva. 2011. Modelling the bivariate dependence structure of exchange rates before and after the introduction of the euro: A semi-parametric approach. *International Journal of Finance and Economics* 16: 357–74.

Bollerslev, Tim. 1986. Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31: 307–27.

Bollerslev, Tim. 1990. Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics* 72: 498–505.

Bollerslev, Tim, Robert F. Engle, and Jeffrey M. Wooldridge. 1988. A capital asset pricing model with time-varying covariances. *The Journal of Political Economy* 96: 116–31.

Butler, Kirt, and Domingo Joaquin. 2002. Are the gains from international portfolio diversification exaggerated? The influence of downside risk in bear markets. *Journal of International Money and Finance* 21: 981–81.

Campbell, John Y. 1991. A variance decomposition for stock returns. *The Economic Journal* 101: 157–79.

Campbell, John Y., and Tuomo Vuolteenaho. 2004. Bad beta, good beta. *American Economic Review* 94: 1249–75.

Campbell, Rachel A. J., Kees Koedijk, and Paul Kofman. 2002. Increased Correlation in Bear Markets. *Financial Analysts Journal* 58: 87–94.

Campbell, Rachel A. J., Catherine S. Forbes, Kees G. Koedijk, and Paul Kofman. 2008. Increasing correlations or just fat tails? *Journal of Empirical Finance* 15: 287–309.

Cao, Guangxi, Jie Cao, Longbing Xu. 2013. Asymmetric multifractal scaling behavior in the Chinese stock market: Based on asymmetric MF-DFA. *Physica A: Statistical Mechanics and Its Applications* 392: 797–807.

Cao, Guangxi, Jie Cao, Longbing Xu, and Lingyun He. 2014. Detrended cross-correlation analysis approach for assessing asymmetric multifractal detrended cross-correlations and their application to the Chinese financial market. *Physica A: Statistical Mechanics and Its Applications* 393: 460–69.

Cappiello, Lorenzo, Robert F. Engle, and Kevin Sheppard. 2006. Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics* 4: 537–72.

Chatterjee, Sourav. 2021. A new coefficient of correlation. *Journal of the American Statistical Association* 116: 2009–22.

Chen, Yun. 2013. Empirical study on asymmetric dynamic correlations among stock returns in the US, Hong Kong and Mainland China. *Management Science* 26: 79–88.

Chen, Menglong. 2016. Asymmetric dynamic in China's stock markets correlations. *Journal of Financial Research* 9: 41–48.

Chen, Yi-Hsuan, and Anthony H. Tu. 2013. Estimating hedged portfolio value-at-risk using the conditional Copula: An illustration of model risk. *International Review of Economics and Finance* 27: 514–28.

Chen, Shoudong, Xiaowei Yi, and Yang Liu. 2014. Research on asymmetric effects of correlation between China's stock markets and Macro-Economy under uncertain policies. *Journal of Contemporary Finance and Economics* 1: 45–55.

Chen, Cathy W. S., Hong Than-Thi, and Manabu Asai. 2021. On a bivariate hysteretic AR-GARCH model with conditional asymmetry in correlations. *Computational Economics* 58: 413–33.

Cho, Yongbok, and Yongwoong Lee. 2022. Asymmetric asset correlation in credit portfolios. *Finance Research Letters* 49: 1030–37.

Choy, S. T. Boris, Cathy W. S. Chen, and Edward M. H. Lin. 2012. Bivariate asymmetric GARCH models with heavy tails and dynamic conditional correlations. *Quantitative Finance* 14: 1–17.

Christoffersen, Peter, Vihang Errunza, Kris Jacobs, and Hugues Langlois. 2012. Is the potential for international diversification disappearing? A dynamic Copula approach. *Review of Financial Studies* 25: 3711–51.

Chuang, O-Chia, Xiaojun Song, and Abderrahim Taamouti. 2022. Testing for asymmetric comovements. *Oxford Bulletin of Economics and Statistics* 84: 1153–80.

Chung, Y. Peter, and Thomas S. Kim. 2017. Asymmetric correlation as an explanation for the effect of asset skewness on equity returns. *Asia-Pacific Journal of Financial Studies* 46: 686–99.

Chung, Y. Peter, Hyun A. Hong, and S. Thomas Kim. 2019. What causes the asymmetric correlation in stock returns? *Journal of Empirical Finance* 54: 190–212.

Conrad, Jennifer, Mustafa N. Gultekin, and Gautam Kaul. 1991. Asymmetric predictability of conditional variances. *Review of Financial Studies* 4: 597–622.

Dajčman, Silvo. 2013a. A formal test of asymmetric correlation in stock market returns and the relevance of time interval of returns—A case of Eurozone stock markets. *Acta Polytechnica Hungarica* 10: 9–19.

Dajčman, Silvo. 2013b. Asymmetric correlation of sovereign bond yield dynamics in the Eurozone. *Panoeconomicus* 60: 775–89.

DeMiguel, Victor, and Francisco J. Nogales. 2009. Portfolio selection with robust estimation. *Operations Research* 57: 560–77.

Demirer, Riza, and Donald Lien. 2004. Firm-level return dispersion and correlation asymmetry: Challenges for portfolio diversification. *Applied Financial Economics* 14: 447–56.

Dewick, Paul R. 2022. On financial distributions modelling methods: Application on regression models for time series. *Journal of Risk and Financial Management* 15: 461–76.

Dewick, Paul R., and Shuangzhe Liu. 2022. Copula modelling to analyse financial data. *Journal of Risk and Financial Management* 15: 104–15.

Ding, Liang, Hiroyoki Miyake, and Hao Zou. 2011. Asymmetric correlations in equity returns: A fundamental-based explanation. *Applied Financial Economics* 21: 389–99.

El Abed, Riadh. 2016. On the comovements among European exchange rates and stock prices: A multivariate time-varying asymmetric approach. *Journal of Applied Finance and Banking* 6: 53–79.

Embrechts, Paul. 1999. Extreme value theory as a risk management tool. *North American Actuarial Journal* 3: 30–41.

Engle, Robert F. 1982. Autoregressive conditional heteroscedasticity and estimates of UK inflation. *Econometrica* 50: 987–1008.

Engle, Robert F. 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics* 20: 339–50.

Engle, Robert F., and Kenneth F. Kroner. 1995. Multivariate simultaneous generalized ARCH. *Econometric Theory* 11: 122–50.

Erb, Claude B., Campbell R. Harvey, and Tadas E. Viskanta. 1994. Forecasting international equity correlations. *Financial Analysts Journal* 50: 32–45.

Forbes, Kristin, and Roberto Rigobon. 2002. Nontagion, only interdependence: Measuring stock market co-movements. *Journal of Finance* 57: 2223–61.

Garcia, René, and Georges Tsafack. 2011. Dependence structure and extreme comovements in international equity and bond markets. *Journal of Banking and Finance* 35: 1954–70.

Gjika, Dritan, and Roman Horváth. 2013. Stock market comovements in Central Europe: Evidence from the asymmetric DCC model. *Economic Modelling* 33: 55–64.

Hong, Yongmiao, Jun Tu, and Guofu Zhou. 2007. Asymmetries in stock returns: Statistical tests and economic evaluation. *Review of Financial Studies* 20: 1547–81.

Horvath, Roman, and Petr Poldauf. 2012. International stock market comovements: What happened during the financial crisis? *Global Economy Journal* 12: 1–21.

Jiang, Lei, Wu Ke, and Guofu Zhou. 2018a. Asymmetry in stock comovements: An Entropy Approach. *Journal of Financial and Quantitative Analysis* 53: 1479–507.

Jiang, Lei, Esfandiar Maasoumi, Jiening Pan, and Ke Wu. 2018b. A test of general asymmetric dependence. *Journal of Applied Econometrics* 33: 1026–43.

Jondeau, Eric. 2016. Asymmetry in tail dependence in equity portfolios. *Computational Statistics and Data Analysis* 100: 351–68.

Kalotychou, Elena, Sotiris K. Staikouras, and Gang Zhao. 2014. The role of correlation dynamics in sector allocation. *Journal of Banking and Finance* 48: 1–12.

Kang, Byoung Uk, Francis In, Gunky Kim, and Tong Suk Kim. 2010. A longer look at the asymmetric dependence between hedge funds and the equity market. *Journal of Financial and Quantitative Analysis* 45: 763–89.

Kearney, Colm, and Valerio Potì. 2005. Correlation dynamics in European equity markets. *Research in International Business and Finance* 20: 305–21.

Kim, Myeong Hyeon, Seyoung Park, and Jong Mun Yoon. 2021. Industry portfolio allocation with asymmetric correlations. *European Journal of Finance* 27: 178–98.

Kristjanpoller, Werner, Elie Bouri, and Tetsuya Takaishi. 2020. Cryptocurrencies and equity funds: Evidence from an asymmetric multifractal analysis. *Physica A: Statistical Mechanics and Its Applications* 545: 123711.

Kroner, Kenneth F., and Victor K. Ng. 1998. Modeling asymmetric comovements of asset returns. *Review of Financial Studies* 11: 817–44.

Kunovac, Davor. 2012. Asymmetric correlations on the Croatian equity market. *Financial Theory and Practice* 35: 1–24.

Lee, Shih-Cheng, Chien-Ting Lin, and Chih-Kai Yang. 2011. The asymmetric behavior and procyclical impact of asset correlations. *Journal of Banking and Finance* 35: 2559–68.

Li, Fuchun. 2014. Identifying asymmetric comovements of international stock market returns. *Journal of Financial Econometrics* 12: 507–43.

Liu, Shuangzhe, and Chris Heyde. 2008. On estimation in conditional heteroskedastic time series models under non-normal distributions. *Statistical Papers 49:* 455–69.

Liu, Shuangzhe, and Heinz Neudecker. 2009. On pseudo maximum likelihood estimation for multivariate time series models with conditional heteroskedasticity. *Mathematics and Computers in Simulation 79:* 2556–65.

Livan, Giacomo, and Luca Rebecchi. 2012. Asymmetric correlation matrices: An analysis of financial data. *European Physical Journal B* 85: 213–24.

Longin, Francois, and Bruno Solnik. 2001. Extreme correlation of international equity markets. *Journal of Finance* 56: 649–76.

Manner, Hans. 2010. Testing for asymmetric dependence. *Studies in Nonlinear Dynamics and Econometrics* 14: 1–32.

Markowitz, Harry. 1952. Portfolio selection. *Journal of Finance* 7: 77–91.

Mashal, Roy, and Assaf Zeevi. 2002. *Beyond Correlation: Extreme Co-Movements between Financial Assets*. Technical Report. New York: Columbia University.

Mensi, Walid, Ahmet Sensoy, Xuan Vinh Vo, and Sang Hoon Kang. 2020. Impact of COVID-19 outbreak on asymmetric multifractality of gold and oil prices. *Resources Policy* 69: 101829.
Michayluk, David, Patrick J. Wilson, and Ralf Zurbruegg. 2006. Asymmetric volatility, correlation and returns dynamics between the U.S. and U.K. securitized real estate markets. *Real Estate Economics* 34: 109–32.
Mitchell, Mark, and Todd Pulvino. 2001. Characteristics of risk and return in risk arbitrage. *Journal of Finance* 56: 2135–75.
Miyazaki, Tomomi, and Shigeyuki Hamori. 2016. Asymmetric correlations in gold and other financial markets. *Applied Economics* 48: 4419–25.
Ozsoy, Sati Mehmet. 2013. Asymmetric Correlations in Financial Markets. Ph.D. thesis, Department of Economics, Duke University, USA.
Pan, Zhiyuan, Xu Zheng, and Qiang Chen. 2014. Testing asymmetric correlations in stock returns via empirical likelihood method. *China Finance Review International* 4: 42–57.
Pastpipatkul, Pathairat, Woraphon Yamaka, and Songsak Sriboonchitta. 2018. Portfolio selection with stock, gold and bond in Thailand under vine Copulas functions. *Econometrics for Financial Applications* 760: 698–711.
Patton, Andrew J. 2004. On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. *Journal of Financial Econometrics* 2: 130–68.
Patton, Andrew J. 2006. Modelling asymmetric exchange rate dependence. *International Economic Review* 47: 527–56.
Sklar, Abe. 1959. Fonctions derépartitionàn dimensions et leurs marges. *Publication Institute Statistics University Paris* 8: 229–31.
Taamouti, Abderrahim, and Georges Tsafack. 2009. Asymmetric Effects of Return and Volatility on Correlation between International Equity Markets. Available online: https://ssrn.com/abstract=1344416 (accessed on 16 February 2009). http://dx.doi.org/10.2139/ssrn.1344416.
Thampanya, Natthinee, Muhammad Ali Nasir, and Toan Luu Duc Huynhc. 2020. Asymmetric correlation and hedging effectiveness of gold & cryptocurrencies: From pre-industrial to the 4th industrial revolution. *Technological Forecasting and Social Change* 159: 120195.
Toyoshima, Yuki, and Shigeyuki Hamori. 2013. Asymmetric dynamics in stock market correlations: Evidence from Japan and Singapore. *Journal of Asian Economics* 24: 117–23.
Tsafack, Georges. 2009. Asymmetric dependence implications for extreme risk management. *Journal of Derivatives* 17: 7–20.
Tse, Yiu Kuen, and Albert K. C. Tsui. 2002. A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business and Economics Statistics* 20: 351–62.
Uhm, Daiho, Jong-Min Kim, and Yoon-Sung Jung. 2012. Large asymmetry and directional dependence by using copula modeling to currency exchange rates. *Model Assisted Statistics and Applications* 7: 327–40.
Van den Goorbergh, Rob W. J., Christian Genest, and Bas J. M. Werker. 2005. Bivariate option pricing using dynamic copula. *Mathematics and Economics* 37: 101–14.
Vuolteenaho, Tuomo. 2002. What drives firm-level stock returns? *Journal of Finance* 57: 233–64.
Wang, Xiang, and Fuqiang Nie. 2016. Research of asymmetric dynamics in the correlations of the Chinese stock markets. *Journal of Applied Statistics and Management* 5: 907–15.
Wang, Nianling, Lijie Zhang, Zhuo Huang, and Yong Li. 2021. Asymmetric correlations in predicting portfolio returns. *International Review of Finance* 21: 97–120.
Wei, Yanhua, and Shiying Zhang. 2005. Research on asymmetric tail dependence structure in financial markets. *Chinese Journal of Management* 5: 601–5.
Xu, Lin, Xiaoying Lin, and Wan Xiao. 2021. Asymmetric multifractal cross-correlations analysis on global stock market based on modeified MF-ADCCA model. *Statistics and Information Forum* 10: 41–54.
Yang, Jian, Yinggang Zhou, and Wai Kin Leung. 2010. Asymmetric correlation and volatility dynamics among stock, bond, and securitized real estate markets. *Journal of Real Estate Finance and Economics* 45: 491–521.
Yu, Chih-Hsien, and Chunchi Wu. 2001. Economic sources of asymmetric cross-correlation among stock returns. *International Review of Economics and Finance* 10: 19–40.
Yuan, Kathy. 2005. Asymmetric price movements and borrowing constraints: A rational expectations equilibrium model of crises, contagion, and confusion. *Journal of Finance* 60: 379–411.