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Credit Risk, Regulatory Costs and Lending Discrimination in Efficient Residential Mortgage Markets

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Abstract: Significant differences in loan terms between demographically distinct groups of borrowers in the United States are often interpreted as evidence of systematic ethnic, racial or gender discrimination by lenders. The appearance and interpretation of such discrimination has long been a controversial issue in public policy and has significant implications for both the economic efficiency and equity of credit markets. Arising from concern for borrowers disadvantaged by such discrimination, the design and implementation of regulations preventing the disparate treatment of demographically distinct groups by lenders are generally considered to have enhanced the equality of access to credit. Unfortunately, existing research has not examined whether this gain in social equity comes at a cost in efficiency borne by all market participants. The reliance on adverse selection or moral hazard in current models of limited lending and credit rationing poses difficulties in empirical testing for the presence and magnitude of such costs. This paper offers a novel theoretical framework in which lending discrimination can endogenously arise in the presence of value-maximizing lenders competing in an economy with complete markets, common knowledge and arbitrage-free pricing. By avoiding the reliance of current models on the exogenous presence of adverse selection or moral hazard, this framework allows potential efficiency costs to beexamined in a market environment without an ex ante assumption of informational market failure. Owing to the presence of common knowledge among participants, we first show how equilibrium loan terms to borrowers in different demographic classes can diverge in such an efficient environment. We then apply the properties exhibited in market equilibria to measure the potential costs of misallocating credit risk owing to the type of regulations observed in actual credit markets.

Keywords: credit rationing; lending discrimination; market efficiency

JEL Classification: G13; G15; O16

1. Introduction

This paper presents a novel theoretical model of lending discrimination and credit rationing in mortgage loan markets and uses that model to generate numerical measures of the magnitude of such discrimination and to provide a measure of the costs of risk misallocation to participants in the market from a stylized depiction of current regulations against such discrimination in the United States and elsewhere. Our model features complete risk markets, intertemporal loan contracts and strategic behavior by lenders and borrowers. It starkly contrasts with conventional explanations of limited lending and credit rationing, which depend upon systematic demographic bias in lender preferences or the a priori assumption of market inefficiencies owing to adverse selection or moral hazard.

Demographic discrimination in credit markets occurs when borrowers of one type receive different terms or loan balances than other types of borrowers, on the basis of one or more observable demographic qualities distinguishing borrower types.[1] Empirical research into mortgage lending in the United States supports claims that some demographically distinct borrowers appear to encounter significantly different loan terms and higher rates of

loan denials than do other borrowers even when standard statistical underwriting models attribute a similar degree of credit risk to both groups.[2] This evidence of inefficient risk pricing and distributional inequities poses significant issues for both traditional models of asset pricing and for policymakers.

Two different explanations for this evidence currently exist, each depending on implicit or explicit situations of market failure. The first, popular among policymakers and the public, asserts that this disparate treatment of borrowers is the result of lender preferences biased against certain demographic classes which leads to systematic differences in loan terms and access to credit.[3] Conventional economic explanations, in contrast, involve inefficiencies affecting loan underwriting owing to an exogenous asymmetric dispersion of information across lenders and borrowers. Models based upon adverse selection or moral hazard, for example, can produce predictions consistent with evidence of lending discrimination.[4]

We wish to measure the costs of risk misallocation induced by the observed form of actual regulations prohibiting the disparate treatment of borrowers in a model devoid of any presumption of underwriting inefficiency or lender bias in the provision of mortgage credit. Our model features a credit market which generates endogenous discrimination in the equilibrium pricing and allocation of credit to different demographic classes in the framework of an economy with complete capital markets, common knowledge of credit risk and the strategic exercise by lenders and borrowers of the options inherent in the standard form of loan contract used in mortgage and other markets for credit. This framework allows the use of the familiar replication techniques of contingent claims analysis to derive the respective arbitrage-free values of the debt and equity interests in the risky property asset being financed and securing the loan contract negotiated between a lender and borrower.

We use these replication techniques to present solutions for the respective valuations by lender and borrower of the equilibrium terms of the mortgage contract. We subsequently use these valuations to provide numerical examples, conditional on the choice of market parameters, of both the endogenous magnitude of observable discrimination and the costs to both parties of anti-discrimination regulations based only on this magnitude, without consideration of the incentives of lenders and borrowers.

These results also allow us to determine whether the alternative assumptions of lender bias or informational inefficiencies in the extant explanations for discrimination are both necessary and sufficient for observations of disparate borrower treatment or whether these observations can simply result from the economic incentives of lenders and borrowers inherent in actual mortgage lending.

The paper is organized as follows. We describe the historical context of demographic lending discrimination and provide a review of previous research into the causes of such discrimination in Section 2. Our model of endogenous lending discrimination in an economy with complete markets, full information and common knowledge, and the method for the solution of its perfect Markov equilibrium, are described in Section 3. Section 4 presents and discusses our results for the numerical characteristics of such an equilibrium, including the measurement of the extent of equilibrium demographic discrimination and the costs to market participants of the form of current regulations of such discrimination used in the U.S. for different selections of parameter values. Concluding remarks appear in the final section.

2. Discrimination and Regulation: Context and Literature Review

2.1. Historical Context

The history of racial inequities in access to residential mortgage credit in the United States has been extensively documented (Phillips 2016; Taylor 2019; Ammon and Pritchett 2020). Prompted by widespread domestic unrest, political assassinations and the findings of a Presidential Commission, effective federal legislation against such discrimination was first enacted in the middle of the 20th century.[5] Evidence that lending discrimination persists, however, in the form of disparate loan terms offered to borrowers from distinct de-

mographic classes appears in a variety of studies, including those of Ross (1996), Nickerson et al. (2000), Avery et al. (2007) and Bhutta and Hizmo (2021).

Regulations implementing this legislation are primarily based on either subjective or objective evidence of correlation between lending patterns to different demographic groups found by periodic "fair lending" examinations of lender records (Walter 1995).[6] These patterns involve three different forms of prohibited practices. The first, *"disparate treatment,"* occurs when borrowers are found to receive different loan terms or suffer higher probabilities of denial because of their minority status. The second, *"disparate impact,"* occurs when the access to credit for minority borrowers resembles that of other borrowers, but the loan acceptance criteria used by the lender has a relatively adverse impact on minority borrowers. The final practice, *"redlining,"* involves differential loan terms and originations extended to loan applicants residing in certain geographical areas with a predominant minority population. Findings by regulators of a consistent pattern and practice of any of these constitute prima facie evidence of discrimination by lenders.[7]

These regulations are atheoretical and based on making these prohibited practices operational through the direct comparison of the observable outcomes of loan approvals and the terms and balances obtained by demographically distinct borrowers. Regulators are limited in their ability to infer the reasons lenders engage in these practices and, as a consequence, consideration of the incentives of lenders to discriminate among borrowers is largely absent from these regulations and their enforcement. There are three reasons for this. First, existing legislation concerning lending discrimination is largely based on a concern for distributional inequalities in the opportunities of different classes of borrowers to obtain credit on similar terms. Second, any useful economic guidance for the design of regulations is precluded by the reliance of formal models on an assumption of bias in lender preferences or on assumptions about the presence of adverse selection or moral hazard and the implications of these assumptions for market efficiency. Models using either assumption yield predictions that are observationally equivalent, given the means feasible for regulatory examiners to either observe or infer lender incentives which motivate their practices. Finally, there is no practical motivation for lenders to infer such economic incentives since both types of models assume in advance that credit markets are inefficient.

Legislation and regulations based on the requirement of an approximate equality in ex post observations of the magnitude of balances and terms in the mortgage loans actually made to classes of borrowers with contrasting demographic identities has been, to a degree, successful in improving the economic equity of credit markets. The apparent exclusion of any consideration of incentives in the design of regulations to implement fair lending legislation supports the presumption of efficiency losses imposing costs on both mortgage lenders and borrowers which could potentially be avoided by the use of incentive-based regulations. Our objective here is not to design such regulations but rather to provide an alternative framework to model discrimination in ex post observations of mortgage lending which, in contrast to existing models, offers predictions of discrimination based on economic incentives which are readily testable and allow for the measurement of efficiency costs resulting from the nature of current regulations.

2.2. Prior Research

Our analysis can be placed within the context of several earlier strands of literature relevant to loan contracts, limits on access to credit, credit rationing and demographic discrimination in credit markets.

Becker (1971, 1993) is generally credited with developing the first rigorous economic research into discrimination in both labor and loan markets. The foundation of his extensive empirical research was a simple static model of a competitive market, in which, applied to mortgage lending, demographic biases in lender preferences generated divergence in the respective terms offered by lenders to majority and minority borrowers.

Macroeconomic and labor market models by Phelps (1972) and Arrow (1973), based on incomplete information, were extended to explain lending discrimination by assuming that improvements in the estimation of the credit risk posed by different classes of borrowers were limited by the significant costs of obtaining information about actual loan performance required to refine their prior belief in the relative riskiness of borrowers from these respective classes.

Some subsequent theoretical and empirical research into demographic loan discrimination adapted the framework of Stiglitz and Weiss (1981), in which the presence of adverse selection or moral hazard among borrowers precluded the risk pricing of loans to individual borrowers on the basis of their actual but unobservable credit risk. An inability to distinguish the risk of individual borrowers allowed lenders to offer loan terms reflecting the average risk of borrowers. The result of this was that lenders raised the loan rate to compensate for the occurrence of unexpectedly high rates of default among borrowers who obtained loans with terms insufficient to compensate for the risk they posed to the lender. This increase penalized more creditworthy borrowers and caused their exit, leaving a higher proportion of less creditworthy borrowers in the market. This subsequent increase in the riskiness of remaining borrowers led, under certain circumstances, to a relative insensitivity of loan balances to higher loan rates and ultimately to the rationing of credit to these borrowers. Extending the Stiglitz–Weiss framework to groups of borrowers with unobservable individual risk but with observable racial or ethnic differences could, should lenders perceive differences in the average degree of risk of majority and minority groups, lead to systematic discrimination in the loan terms offered to each group by unbiased but value-maximizing lenders. Models loosely based on this approach appear in numerous studies, including those by Lang and Nakamura (1993), Ferguson and Peters (2000), Pryce (2003), Davidoff and Welke (2004), Martin (2010) and Lim (2020).

Our model and results differ in two fundamental ways from this earlier research. First, we avoid relying on exogenous asymmetries of information between lenders and borrowers and instead endogenously derive both the existence and magnitude of equilibrium lending discrimination by incorporating into the complete markets framework of traditional asset-pricing models the capacity for strategic behavior by both lender and borrower in regard to their exercise of the options available to them by the standard form of intertemporal loan contracts. Second, using plausible parameter values, we quantitatively measured the efficiency costs of current regulations through the use of numerical solutions of the underlying differential equations describing the respective valuation of a lender and borrower of their debt and equity interests defined by the equilibrium terms of the loan contract. This allowss us to also prove that lending discrimination can occur in the presence of efficient loan pricing despite the absence of assumptions of informational market failure required by the conventional models of credit rationing.

Our paper is also related to research on the nonstrategic exercise of the default and prepayment options possessed by mortgage borrowers. A number of studies of credit risk and the contingent valuation of mortgage loans, including those by Kau et al. (1995), Keenan and Keenan (1995), Deng et al. (2000), Dunsky and Ho (2007), and Kau et al. (2012), have used the risky-debt framework of Merton (1974) to value both fixed and adjustable rate mortgage contracts in the presence of the default, foreclosure and prepayment options inherent in the standard residential mortgage contract. Our paper differs from these in analyzing the option to default in the strategic context of loan negotiation between lender and borrower. We consequently expanded this traditional option-based literature on mortgage termination to include strategic options, deriving the endogenous response of the lender's offer of loan terms and supply of mortgage credit to the strategic timing of the decision of the borrower to exercise his default or prepayment options as well as to the characteristics of the property which secures his loan.

Finally, our approach to the design and valuation of mortgage contracts parallels that of the "strategic debt service" literature, as exemplified by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Jones and Nickerson (2002) and Acharya et al. (2006). Extending the Merton (1974) model to a game-theoretic context, these authors analysed the effect on debt pricing of strategic renegotiation and bankruptcy costs. However, our model differs from these in focusing on volatility in the value of the asset used to secure the given loan and the timing with which the borrower and lender strategically exercise their respective options embedded in the class of standard mortgage loans as the primary sources of risk and differential levels of volatility in the assets respectively pledged by different demographic groups, all with the same degree of measured credit risk as the cause of endogenous lending discrimination on the basis of demographic differences between borrowers.

3. The Market for Mortgage Credit

Our model of strategic behavior by lender and borrower within the market for mortgage credit consists of five elements: (1) the class of standard contracts which contain the loan terms and covenants pertaining to both lender and borrower, (2) the nature of the options embedded in the standard mortgage contract and held by the borrower and the lender, (3) the institutional and economic aspects of our representative mortgage market, (4) the respective strategies of borrower and lender in negotiating the mortgage contract and the nature of the resulting noncooperative equilibrium in the mortgage market and (5) the numerical solution procedure for this equilibrium. Each such element is sequentially described below.

3.1. The Class of Loan Contracts

We considered a market for mortgage credit in which loans are made between a lender and a representative borrower from one of multiple classes of loan applicants. Each class is distinguished by observable demographic qualities common to its members and borrowers from any two such classes exhibiting similar statistical degrees of credit risk as measured on the basis of their personal financial data used in automated underwriting models.[8] While lenders are unbiased in that their preferences pertain solely to the risk-adjusted values of their loans, lenders do, however, have the discretion to supplement these statistical scores by including other observable characteristics of borrowers in making their final decision to approve a loan application.

The parties negotiate the terms of the loan in the context of a type of contract standard to most mortgage markets, and we model the determination of these terms in the context of a noncooperative game. The lender advances an initial balance at the time of origination, in return for which the borrower agrees to remit a flow of constant coupon payments c and a non-negative terminal payment C at maturity T. Amortization at a continuously compounded annual loan rate y requires that the value $m(t)$ of the loan balance outstanding at date t satisfies

$$1 = c \int_0^t e^{-ys}\, ds + m(t)e^{-yt} \text{ for } 0 \leq t \leq T \tag{1}$$

where $C = m(T)$. The loan finances the purchase of a property of random value $a(t)$ which acts as the collateral securing the loan. Owing to his equity interest in the property, servicing this loan entitles the borrower to receive a flow of services accruing at rate $\pi(a, t)$. If the borrower breaches the contract by failing to service the loan at date t, his default allows the lender to foreclose and sell the property at a cost $b(a,t)$, receiving $a(t) - b(a,t)$ after the sale, with any remaining revenue in excess of the outstanding balance $m(t)$ refunded to the borrower. The parties can also trade continuously and costlessly in complete markets with respect to the risk created by volatility in the value of the property over the term to maturity as well as in riskless bonds with a continuously compounded yield r. Finally, we assume that the form of the contract and its parameters, which are the values of the

property, rate of service flows, foreclosure costs and outstanding balances, are all common knowledge to the parties at all dates t.[9]

3.2. Options and Strategies

Inherent in the standard mortgage contract considered here is a variety of embedded options. Within the noncooperative game played between the parties to the contract, the set of such options available to each party comprises their respective strategy sets. A strategy for each party consists of their choices, at all dates t between loan origination and maturity and conditioned on the elements of $\{\mathcal{F}_t\}_{0\leq t}$ of the timing of their exercise of these options. The strategy space of the borrower consists of his choice over the time of default, should he choose to exercise this option. The principal component of the strategy space of the borrower is his option to breach the contract through default. The space of the lender is composed of the initial balance $C(0)$ he offers the borrower at origination, the option to foreclose in the event of default and the option to call the loan if there is a significant decline in the value of the property at a given date relative to the outstanding loan balance at that date. We measure the significance of the decline in terms of the lender's choice of a value λ, $0 \leq \lambda \leq 1$, for the ratio of property value to unpaid loan balance at any given date.

Conditional on the selection of parameters describing the lending environment, the noncooperative equilibrium is defined by a party's selection of the strategy that maximizes the value of his position in the loan contract at origination, respectively $L(a, 0)$ for the lender and $B(a, 0)$ for the borrower, subject to the strategy chosen by his counterparty. The observable values of the initial balance and terms of a loan are endogenously determined by this equilibrium.

Two aspects of the standard mortgage contract used in our model are particularly relevant to our analysis. First, in the event of default, the value of the past flow of property services accruing to the borrower's equity position cannot be retrieved by the lender. Since the prospect of these flows was capitalized in the initial market value of the property, that value is excluded from the security available to the lender at any time at which default can occur. Both borrower and lender, consequently, are aware that an inverse relationship exists between the conditional value of the loan collateral at any time before the loan matures and the magnitude of property services. Second, each player will choose the timing of his exercise of the options comprising his strategy in his own best interest. Since these choices are endogenous, the risk posed by one party's choice of strategy to his counterparty cannot be hedged.

3.3. The Lending Environment

Our model of the market for mortgage credit is assumed to operate within a continuous time economy which satisfies the conditions of the classical asset valuation environment. In particular, this economy has a complete filtered probability space $[\Omega, \mathcal{F}, P]$, where Ω represents the space of events in this economy, $\mathcal{F}$ is the corresponding filtration of these events and consists of a set of sequential sigma algebras $\{\mathcal{F}_t\}_{0\leq t\leq T}$ representing all information relevant to the participants in our market at time t and P is the actuarial probability measure defined on Ω. The respective decisions taken by lenders and all borrowers at the time t in our market are based on their observations of $\mathcal{F}_t$, with a particular focus on all those events in Ω and observed in $\mathcal{F}_t$ provoking movements in the value of the representative property serving as loan collateral. All observations within $\mathcal{F}$ are also assumed to be common knowledge. The stochastic processes defining the evolution of all asset values in the larger economy, including the values of the representative property being financed by the respective members of each demographic class, are assumed to be well-defined and adapted in this space, and these markets are complete with respect to all sources of risk, including random movements in property values. The financial interests of lenders and borrowers in the properties being financed in this market, measured by their respective date t positions $L(a,t)$ and $B(a,t)$ of the mortgage contract, consequently exhibit arbitrage-free valuation.

The demographic classes of borrowers are indexed by the elements θ of a bounded set of real numbers Θ. Each borrower from a given class θ wishes to finance a property common to all borrowers in his class and the vector of qualities distinguishing the representative property of any given class can be aggregated, in terms relevant to its risk-adjusted value, into a unique real-valued scalar φ. Since the demographic class of a borrower is, by the assumptions regarding the standard measurement of credit risk above, the only information relevant to a lender's final judgement of that borrower's actual credit risk, the values θ distinguishing classes will also serve to index the representative property from each of these classes. The belief by lenders in a correlation between the credit risk of a borrower and his demographic class can then be expressed by a function $\varphi = \varphi(\theta)$. Whether objective or subjective, this common belief of all lenders and the exogenous constant value of this correlation are also common knowledge.[10]

We assume the value at any time t of the property securing a loan, $a(t, \varphi)$, evolves according to the diffusion

$$a(t,\ \varphi) = \alpha(a,t)dt + \sigma(\varphi)a(t)dz(t) \tag{2}$$

where $z(t)$ is a standard Brownian motion, $\alpha(a, t)$ is the expected drift at all t and $\sigma(\varphi)$ measures the conditional volatility of this value. This diffusion and its parameters are taken to be common knowledge and, since they are exogenous and cannot be influenced by either party to a loan, neither moral hazard nor adverse selection are present. Based on our assumptions above, the perceived volatility $\sigma(\varphi)$ in property value is the sole source of credit risk in the mortgage market.[11]

Note that the lenders in our market are assumed to have no independent interest in the demographic identity of any given borrower. However, given common knowledge of the relation between the demographic identity of a borrower and the volatility in the value of his collateral, as expressed by $\varphi = \varphi(\theta)$, the loan terms offered by a competitive lender will still be indirectly affected by demographic considerations through the riskiness of the collateral value in Equation (1):

$$\sigma = \sigma(\varphi(\theta)) \tag{3}$$

We will, for notational simplicity, use only σ in the expression in Equation (1) for the value of the generic property, but we will use the perception of risk by the lender in Equation (2) to solve for the presence and magnitude of lending discrimination in what follows.

While he services the loan, the borrower receives a continuous flow of services, $\pi(a, t)$, from his use of the property. If he instead exercises his option to default, the lender has the option to foreclose. By the preceding description of our model, the exercise of this option by the lender will immediately follow default. Foreclosure, in turn, creates e another choice for the lender. He can choose to liquidate the property at some cost, $b(a(t))$, or, if $b(t)$ is sufficiently large, he can choose to discard it. Foreclosure consequently results in the lender receiving the payoff *max* $[a(t) - b(a,t), 0]$, with the borrower receiving, should the property be sold, any residual funds exceeding the unpaid balance of his loan.

The respective spaces of strategies available to each party include their choice of the timing of any exercise of the options inherent in the mortgage contract. The space of the lender is composed of the initial balance $C(0)$ he offers the borrower at origination, the option to foreclose in the event of default and the option to call the loan if there is a sufficient decline in the value of the property. The strategy space of the borrower consists of the option to default on the loan, including his choice of timing, should he choose to exercise this option. Each party will select his strategy to maximize the value of his contingent claim on the property collateralizing the loan. We denote by $L(a,t)$ the resulting value at time t of the mortgage contract to the lender and by $B(a,t)$ the value of that contract at the same time to the borrower. Equilibrium occurs when each party chooses that strategy which maximizes the value of the contract to him at origination subject to the strategy chosen by his counterparty.

3.4. Credit Market Equilibrium

The observed features of the mortgage contract are determined by the equilibrium achieved in the noncooperative game played between the lender and borrower. This perfect Markov equilibrium is defined, for each selection of market parameters, by the choice of best-reply strategies by both parties. These strategies consist of the decisions by the lender and borrower in regard to the exercise of those options available to them within the setting of our standard form of mortgage contracts and are taken by each party to maximize the arbitrage-free value of their respective positions, $L(a,t)$ and $B(a,t)$, in the mortgage contract. The option comprising the strategy of the borrower is to continue or to cease servicing the loan at any chosen date, while the strategy of the lender includes his decisions, at the same dates, to exercise his options to call the loan, to foreclose and sell the collateral in the event of default and his choice of an initial balance to offer the borrower at the time of loan origination. Since both the diffusion, given in (2), generating the evolution of the value of the property collateralizing the mortgage loan and the sample path of this evolution are common knowledge, the value of the state variable, *(a,t)*, contains all the information relevant to these decisions at each date t in the life of the loan.

Denoting the range of possible property values by $A = [0, \infty)$ and by T the continuum of decision dates from origination to maturity, the Markov nature of the game implies that $A \times T$ is the support of all potential states *(a,t)* relevant to the strategies of each party. Since the loan terminates in the event of a default by the borrower, a call by the lender or reaching the maturity date T, the strategy set for the borrower can be represented by the closed subset D of $A \times T$. Since all strategies available to the lender consist of the choice of initial balance at origination $t = 0$, and the options to call the loan or to foreclose should the borrower default, the strategy set of the lender is the union of three closed subsets of $A \times T$, which we can denote by $A \times \{0\} \cup C \cup F$. A realization of *(a,t)* in D induces default by the borrower and a realization of *(a,t)* in C or F, respectively, causes the lender to call the loan or to foreclose following default. Since, in this version of our model, we do not include the option to renegotiate the loan by borrower and lender, foreclosure will immediately follow default and, as a consequence, the subsets D and F will coincide. Finally, the open subset S of $A \times T$, defined as the complement of $D \cup C$, consists of all values of *(a, t)* in which the loan remains active.

Appealing to the standard arbitrage valuation methods of contingent claims analysis, the respective value functions of the lender and borrower must satisfy the Bellman equations in S,

$$L(a,t) = c\,dt + L^*(a,t), \tag{4}$$

$$B(a,t) = (\pi - c)dt + B^*(a,t). \tag{5}$$

Defining the E_t as the conditional expectations operator under the unique equivalent Martingale measure arising from the presence of complete markets, where $L^*(a,t)$ and $B^*(a,t)$, are defined as

$$L^*(a,t) = E_t\left(e^{r\,dt}L(a+da, t+dt)\right), \tag{6}$$

$$B^*(a,t) = E_t\left(e^{r\,dt}B(a+da, t+dt)\right), \tag{7}$$

and represent the respective risk-adjusted expected values at time t of the future claims of the lender and borrower, each discounted at the riskless interest rate. These conditions, in turn, must satisfy the respective partial differential equations over S,

$$rL = \left(\frac{1}{2}\right)(a\sigma)^2 L_{aa} + (ra - \pi)L_a + c + L_t, \tag{8}$$

$$rB = \left((\tfrac{1}{2})(a\sigma)^2\, B_{aa} + (ra - \pi)B_a + (\pi - c) + B_t \right),\qquad(9)$$

where $(ra - \pi)$ is the risk-adjusted "drift" of the property value, interpretable as the expected rate of appreciation, from the diffusion in (2) under the equivalent Martingale measure used in the valuation by both parties.

Unique solutions for the valuation Equations in (7) and (8) are determined by the relevant boundary conditions delineating the strategy spaces D, F and C, where, in the absence of the opportunity for contract renegotiation between lender and borrower, D coincides with F. Properties of the boundaries of these respective default and call regions can be expressed through use of the value-matching and smooth-pasting conditions (Dixit 1991; Dumas 1991). The former condition requires, at any point on the boundary of the relevant region, continuity in the value function of the party who exercises the option terminating the loan at any time t, and the second requires continuity in the derivative of that party's value function with respect to a at the same points.

Denoting by $\breve{a}$ and $\hat{a}$ the property values which respectively trigger default by the borrower and a call on the loan by the lender at any time t, the application of these conditions yields the respective boundaries of D and C as the continuous curves $\breve{a}(t)$ and $\hat{a}(t)$, where the properties that these curves must satisfy represent the free boundary conditions which allow unique solutions for the value functions in (7) and (8). These boundary conditions are

$$L(\breve{a}, t) = max\,[0,\ \breve{a} - \pi(\breve{a}, t)],\qquad(10)$$

$$\pi(\breve{a}, t) - c + B^*(\breve{a}, t) = 0,\qquad(11)$$

and

$$c\, dt + L^*(\hat{a}(t), t) = \hat{a}(t) - b(\hat{a}(t)\qquad(12)$$

$$B(\hat{a}(t), t) = max[0,\ \hat{a}(t) - c(t)\,]\qquad(13)$$

where the expression in (12) reflects the condition that the borrower receives any excess of the property value over the outstanding balance at the time of foreclosure and liquidation of the property.

The determination of equilibrium in our credit market model proceeds through the generic forms of the valuation equations that solve the paired differential equations in (7) and (8). The best-reply strategies of the parties to the contract correspond to the delineation by the curve $\breve{a}(t)$ of the boundary of the set D. In this set D, any realization of the state variable *(a,t)* induces borrower defaultand, inthe absence of contractual renegotiation, default induces subsequent lender foreclosure. The analogous delineation by $\hat{a}(t)$ of the boundary of the set C consists of those values of property and corresponding times t which cause the lender to call the loan. Conditional on the parameters in the diffusion in (2), the flow $\pi(a, t)$ of property services to equity, the cost *b(a,t)* of foreclosure, the contract maturity *T*, the value of the loan premium $y - r$ and the implicit purchase price of the property at loan origination, the perfect equilibrium in our model of the market for mortgage loans is described by the unique solution of the combined Equations (7)–(12) for the valuations made by the lender and borrower of their respective positions in the mortgage contract at origination, *L(a, 0)* and *B(a, 0)*, with the equilibrium values of the initial loan balance and required "down payment" (initial equity investment) respectively equal to *L(a, 0)* and *B(a, 0)*.

3.5. Numerical Solution Procedure

Our objective in this paper is to use our model of the credit market as a framework in which quantitative measures may be made of the magnitude of lending discrimination, in the form of disparate treatment of demographically distinct classes of borrowers, and of the costs of the current form of regulations against such disparate treatment. We do

this through the derivation of numerical solutions for the market equilibrium valuation equations and corresponding boundaries of the regions *D* and *C* defining the property values and dates at which the respective default and call options of the borrower and lender will be exercised.[12]

We use a finite difference procedure to solve for these components of equilibrium in a sample of representative parametric cases. The set $A \times T$ is represented by a discrete rectangular grid defined by all possible integer values consistent with the distribution of property values given by the diffusion in (2) and the choice of a discrete maturity date *T*, where the vertices of the grid (gridpoints) are the observable values of the state variable (*a*, *t*). Conditional on the selection of parameter values, the numerical solution for equilibrium is a set of *L* and *B* values at these gridpoints, with the corresponding determination of whether each such point is in *D*, *C* or *S*. Consistent with the solution method used in virtually all game-theoretic models in economics and finance, these values are found by recursively iterating from the selected maturity date *T* to the origination date *0* (Myneni 1992).[13]

4. Results and Discussion

Two of the three mortgage lending practices which are used by U.S. regulators to identify prohibited forms of lending discrimination are the disparate treatment of different demographic groups in regard to loan terms and the redlining of neighborhoods or regions hosting a significant proportion of residents from those groups. Representing disparate treatment and redlining through the loan balances and terms offered to demographically distinct borrowers, the numerical results presented in this section show that both phenomena can be explained as the result of the efficient risk-pricing of loans by lenders based on their common knowledge of the credit risk posed by individual borrowers. These results, as a consequence, refute the necessity of credit market inefficiency, whether through biased lender preferences or asymmetric information, as the source of lending discrimination. They demonstrate, contrary to common belief, that such discrimination can also be observed as an equilibrium phenomenon in the presence of complete risk markets and complete information.

Our premise is that it is common knowledge or belief among lenders that the credit risk from a loan to minority borrowers is larger than that of a loan to majority borrowers. We have assumed that the degree of risk posed by a borrower is measured in standard underwriting models on the basis of the financial history of that borrower, and it is this measure of risk that is observed by parties external to the market, including regulators. In contrast, the measure of the actual credit risk of a loan from the perspective of lenders is measured by the volatility in the value of the property securing it, a parameter not used in standard calculations of risk.

Based on this premise, the numerical solutions from our model for loan balances and the subsequent costs of loan regulations, which are presented in Tables 1 and 2 below, have three fundamental implications for the analysis of demographic discrimination. The first is to show that these practices can arise in the equilibrium of a credit market without either of the sources of inefficiency in current explanations of discrimination and in the presence of efficient risk pricing from the perspective of lenders. The second and third are that, conditional on the parametric values of the loan market, our model also offers a means to respectively measure the magnitudes of both the differential access to credit by borrowers on a demographic basis and the potential misallocation of credit risk induced by the current form of lending regulations. It does this without the need for unobservable variables, unlike models with biased preferences or adverse selection and moral hazard, which cannot be used to predict quantitative values for differential credit access and the cost of current financial regulations. Note that inclusion of the lender's option to call the loan affects only the quantitative solutions of our model but not their qualitative character. Since the above implications can, consequently, be shown without the inclusion of the lender's option to call the loan, we omit that option in calcuating the numerical results

in Tables 1 and 2 below. Numerical results inclusive of this option are available from the author on demand.

Table 1. Rate spread, risk and contract value.

Rate Spread	Contract Value	Contract Value	Contract Value	Contract Value
	(σ = 10%)	(σ = 20%)	(σ = 30%)	(σ = 40%)
0.010	0.866	0.775	0.577	0.393
0.020	0.926	0.864	0.708	0.537
0.030	0.952	0.908	0.784	0.633
0.040	0.966	0.932	0.832	0.700
0.050	0.975	0.948	0.864	0.749
0.060	0.982	0.958	0.886	0.784
0.070	0.991	0.974	0.918	0.831
0.080	0.992	0.978	0.927	0.847
0.090	0.994	0.982	0.937	0.865
0.100	0.995	0.985	0.946	0.881

Table 2. Credit risk and the comparative loss in contract value from regulation.

Rate Spread γ	Loss of Contract Value	Loss of Contract Value	Loss of Contract Value
	(σ = 20%)	(σ = 30%)	(σ = 40%)
0.010	−0.105	−0.334	−0.546
0.020	−0.067	−0.235	−0.420
0.030	−0.046	−0.176	−0.335
0.040	−0.035	−0.139	−0.275
0.050	−0.028	−0.114	−0.232
0.060	−0.024	−0.098	−0.202
0.070	−0.017	−0.074	−0.161
0.080	−0.014	−0.066	−0.146
0.090	−0.012	−0.057	−0.130
0.100	−0.010	−0.049	−0.115

A prerequisite for all three purposes is to choose a parameter that distinguishes different demographic classes on the basis of the differential degree of credit risk perceived by lenders from lending to different classes. Since the number of demographic classes in most databases such as HMDA used by regulators is limited, we present our results in terms of two such classes, implying that Φ has only two elements. We respectively refer to borrowers from these two classes as "*majority*" and "*minority.*" We have earlier assumed for simplicity that loan applicants from a given demographic class each seek to finance a representative property, distinguished by certain parametric characteristics, specific to this class.[14] The parameter we use in this paper to distinguish the respective properties for which majority and minority borrowers seek financing is the perceived volatility σ of the representative properties securing the mortgage loans and the costs b of foreclosing on these properties. We have, in each table, normalized the value of the property serving as the collateral to one, so that our results are independent of the actual dollar value of a property.

The degree of annualized volatility in collateral values is the underlying variable that drives the respective results in Tables 1 and 2. Associating different degrees of such volatility with the demographic identities of distinct borrowers is one means of representing the common knowledge among lenders that leads to observable discrimination.

Table 1 presents the arbitrage-free value of the mortgage contract to the lender, equivalent to initial balances offered to borrowers by lenders, for a range of different rate premiums, $c - r$, when majority and minority borrowers are distinguished by lenders on the basis of the annual volatility in the value of the respective properties serving to secure mortgage loans to majority and minority borrowers. Based on our premises above, the results in Table 1 can be directly interpreted as the magnitude of disparate treatment between majority and minority borrowers. We then use the differences in these magnitudes to quantitatively measure, in Table 2, the misallocation of risk, from the current form of regulation prohibiting such disparate treatment. This misallocation is measured in terms of the decline in the value of the mortgage contract to lenders as the credit risk of borrowers increases.[15] In each case, all parameter values, other than the value of the parameter distinguishing majority and minority classes of borrowers, are held constant at a selected set of "benchmark" values. These values are 3% for the riskless interest rate r; 20% for the volatility in collateral value σ; 10% each for a maturity T of five periods, an annual remittance c and a flow of property services to the borrower π; and 0 for foreclosure costs, all relative to property value.

The arbitrage-free values of the mortgage contract to a lender, expressed as a percentage of property value at origination, are displayed for a sequence of rate spreads y-r, for four different values of the annual volatility in the value of the underlying property serving as collateral. Column 1 features an increasing sequence of rate premiums from a minimum value of 1% to a maximum value of 10%. Each of the next four columns display, at each corresponding rate spread, the contract value to the lender and initial balance to the borrower for four different values of volatility, ranging from $\sigma = 10\%$ to $\sigma = 40\%$.

Based on this assumption, the results in Table 1 simultaneously represent an example of the accurate and efficient response by lenders, in terms of initial loan balance per rate spread, to the credit risk they observe but represent disparate treatment from the perspective of regulators.

We can, as a consequence, measure the extent of lending discrimination observed by regulators, in terms of the difference in balances at different rate spreads offered to minority relative to majority borrowers, on the basis of assigning a higher volatility in collateral value to the representative property used as collateral by minority borrowers than that assigned to majority borrowers. These respective values and the validity of the corresponding ranking by the common knowledge or belief shared by lenders of the risk posed by minority relative to majority borrowers, of course, could be empirically ascertained by including the volatility of collateral in either statistical underwriting methods or in any actual lending examination. The significance of the example illustrated by Table 1, however, is that, from any such belief in a difference in representative (average) volatilities in the value of the property being financed by each of these two classes both explains the incentive of demographically unbiased lenders to seemingly engage in lending discrimination and offers the means to quantitatively measure the degree of disparate treatment of minority borrowers observed by parties external to the market, including regulators, and does so without the ex ante presumption of such discrimination, whether from lender bias or asymmetric information, that is inherent in existing explanations of lending discrimination.

Prior to representing these different perceptions of credit risk, we can verify the qualitative properties of our model and illustrate its quantitative predictions by examining the results displayed in Table 1. These results show, in terms of the qualitative accuracy of the model, that, as expected, the contract value for each level of volatility increases as the rate spread increases, while that value decreases as volatility in collateral value increases for each value of the rate spread. Illustrating the quantitative predictions of the model, we see that at a rate spread of 1%, for example, the initial balance on a loan with the lowest collateral volatility, $\sigma = 10\%$, is approximately 86.6% of the initial property value, declining with each ten-percentage-point increase in volatility, with an initial balance of only 39.3% of initial property value. The initial loan balance for the lowest collateral variance ($\sigma = 10\%$) increases with each one-percentage-point increase of rate spread, from the same 86.6% of

collateral value for a spread of 1% to approximately 99.5% at the maximum spread of 10%, while, for the loan with the highest variance in collateral value (σ = 10%*)*, the initial balance ranges from the same 39.3% at a spread of 1% to 88.1% of the initial value of collateral at a spread of 10%.

Equally noteworthy is the appearance of limited lending in the results in Table 1. Each one-percentage-point increase in the rate spread induces an increasingly smaller response of initial loan balances offered by lenders. Consider, as an example, the loan with the lowest volatility in collateral value (σ = 10%). The initial balance for this loan rises from 86.6% to 92.6% of property value at origination for an increase in the rate spread from 1% to 2%, but the response of the loan balance to each successive percentage-point change in spread decreases monotonically, until an increase in the spread from 9% to 10% creates an increase in the initial loan balance, from 99.3% to 99.4% of property value. An alternative and unit-free measure of this same responsiveness, the elasticity of the loan balance to rate spread, goes from a percentage change in initial balance per percentage increase in rate spread of 0.069 when the rate spread rises from 1% to 2% but declines monotonically until this elasticity is only 0.009 as the spread rises from 9% to 10%.

We now use the results in Table 1 to illustrate the model's provision of a method of quantitatively measuring the extent of lending discrimination, in terms of disparate treatment, regardless of the different interpretations for such treatment made by lenders and regulators. Consider a hypothetical case in which the risk σ posed by majority borrowers is 10%, while that posed by minority borrowers is believed by lenders to be a higher value. At a common rate spread of 1%, the difference in perceived risk induces lenders to offer an initial balance of 86.6% of property value at origination but with respective initial balances of only 77.5%, 57.7% and 39.3% of this value offered to minority borrowers with the respective volatilities of 20%, 30% and 40%. Repeating the same comparison for a rate spread of 10%, majority borrowers with the same degree of property volatility would be offered an initial balance of 99.5%, relative to the respective balances of 98.5%, 94.6% and 88.1% for minority borrowers with volatilities of 20%, 30% and 40%.

To make these comparisons more concrete, assume that the representative properties of majority and minority borrowers have a common value at origination of $500,000.00. At a rate spread of 1%, the differences between majority borrowers and minority borrowers with property volatilities ranging from 20% to 40% are respectively $45,500.00, $144,500.00 and $236,500.00. A majority borrower at this spread would receive an initial loan balance of $433,000.00, while, in comparison, a minority borrower with perceived credit risk corresponding to a 20% volatility would receive $387,500.00. A borrower with credit risk corresponding to 30% would receive $288.500.00. A borrower with credit risk of 40% would receive only $196,500.00. Under our assumptions, these differences in, and levels of, the initial balances offered to majority and minority borrowers with identical degrees of credit risk are inferred on the basis of personal credit characteristics, but an unbiased lender would regard these differences as the result of efficient loan terms based on common knowledge or belief in the degrees of credit risk used in Table 1.

Based on the same parameter values as used in Table 1, Table 2 illustrates the cost per dollar of collateral value owing to the misallocation of risk caused by offering the same loan terms to all borrowers. This is intended to measure the effects of a prohibition on offering efficient risk-based terms to borrowers with different levels of collateral risk. The resulting losses in contract value are measured using the risk-based terms offered to a borrower with collateral risk corresponding to an annual volatility in collateral value of 10% and offered to borrowers with collateral of the same value at origination but with successive increases in ten percentage points in their σ value.

Consider, as an illustrative example, the loss in contract values occurring through the offer of the efficient risk-based terms of a loan to finance a property with annual volatility of 10% to all borrowers. The value of the mortgage contract to the lender with a 1% rate spread is worth 99.5% of the collateral value at origination. Offering the same risk spread and initial balance to borrowers with volatilities in property value of 20%, 30% and 40% induces respective percentage losses in contract values of 1%, 4.9% and 11.5%. If the loan terms appropriate to a borrower with a 10% volatility in collateral value are instead a 1% spread and initial balance of 88.6% of collateral value, these same respective losses are 10.5%, 33.4% and 54.6%. Expressed in dollar terms for the same example of a property worth $500,000.00 at origination, the losses on a loan in terms of a 10% rate spread and a loan-to-collateral-value ratio of 99.5% to borrowers with 20%, 30% and 40% in the respective volatilities of their collateral, also worth $500,000.00 at origination, are $45,500.00, $144,500.00 and $236,500.00.

Although these numbers represent losses from the misallocation of risk due to the offer of similar loan terms to all borrowers, the interpretation of these losses in terms of the misallocation of risk caused by a simple and uniform prohibition of disparate treatment is valid only in the presence of common knowledge of an invariant objective correlation between actual collateral risk and the demographic differences distinguishing minority from majority borrowers. Should this correlation be subjective and the actual credit risk of different borrowers have no actual relationship to their demographic identities, lenders who are entirely free of demographic bias would still act on their incentives to offer different terms to borrowers on the basis of demographic considerations. Moreover, regardless of the empirical validity of such a correlation, the presence of such different terms, in the absence of significant consideration of the nature of collateral used to secure mortgage loans, would still be interpreted as disparate treatment on purely demographic grounds by regulators and other external observers.

5. Conclusions

The purpose of this paper is to contribute two results that are novel and significant contributions both to the body of research into the causes of demographic lending discrimination and to policymakers in their assessment of the necessity and design of regulations to mitigate such discrimination and the effects from the implementation of those regulations.

The first contribution involves the conditions that lead to the appearance of such discrimination. We show that lending discrimination, in the form of disparate treatment of borrowers on the basis of their demographic identity, can endogenously emerge in a credit market in which lenders act on the basis of common knowledge of a correlation between the observable demographic characteristics distinguishing different borrowers and the degree of credit risk posed to them by these borrowers This is despite the equality in the measurement of their risk by standard underwriting methods and in the presence of complete risk markets and arbitrage-free asset valuation. This explanation of discrimination contrasts starkly with conventional explanations and demonstrates that the conditions required by these conventional explanations, which are the presence of demographic bias in the preferences of lenders or the existence of adverse selection or moral hazard on the part of borrowers, are not necessary for discrimination, in the form of the disparate treatment of demographically distinct borrowers, to occur.

The second contribution is to offer a framework in which quantitative measurement of both the magnitude of such disparate treatment and the extent of costs from the potential misallocation of credit risk caused by regulations mandating lenders to offer similar loan terms to borrowers who may in reality pose different degrees of such risk. The virtue of such a framework is that these measurements can be made using the straightforward and ubiquitous valuation methods of contingent claims analysis and without the potential for measurement errors and lack of empirical testability created by exogenous assumptions about preferences or asymmetric information. This framework also has the additional virtue of avoiding the presumption of market failure present in both conventional explanations,

allowing an objective assessment to be made of both the need for regulatory measures or other types of public policy in terms of economic efficiency and the costs of achieving social equity in regard to credit access through the current form of financial regulations.

　　　Future research into lending discrimination on the basis of this framework can include a variety of additional features of actual credit markets, such as the effects of foreclosure costs and the implications of endogenous variation in the rate spread of mortgage loans and the addition of opportunities for loan prepayment and refinancing costs to borrowers that will allow more refined measures of demographic discrimination and regulatory costs to be made.

Notes

1　Descriptions of different forms of lending discrimination can be found in ABA (2012) and an overview of current U.S. fair lending regulations and statutes can be found in Federal Reserve Federal Reserve Board (2020)

2　Munnell et al. (1996) is the semi+nal empirical study of demographic lending discrimination. LaCour-Little (1999), Turner and Skidmore (1999), Ross and Yinger (2002) and Dymski (2006) all survey empirical research into mortgage lending discrimination.

3　This explanation has two fundamental drawbacks. First, there is at best a limited capacity in competitive credit markets for the managers of lending instiututions to indulge in demographic discrimination, particularly when it harms the interests of the shareholders of that institution. Second, rigorous econometric tests do not support it, as described in Black (1999) and Black et al. (2003).

4　See Ross and Yinger (2002), Han (2004) and Longhofer and Peters (2005).

5　Established by Executive Order 11365 in 1967 to investigate the causes of urban riots in that same year, the National Advisory Commission on Civil Disorders issued the results of its investigations and recommendations for future housing, mortgage, education, and social-service policies in the 1968 Kerner Report (https://www.ojp.gov/ncjrs/virtual-library/abstracts/national-advisory-commission-civil-disorders-report). Federal legislation concerning discrimination in housing and mortgage markets began with the 1968 Fair Housing Act (42 U.S.C. 3601) and the 1974 Equal Opportunity to Credit Act (15 U.S.C. 169) and related legislation on consumer protection such as the Truth in Lending Act (TILA) of 1968 (P.L. 90-301), the Real Estate Settlement Procedures Act (RESPA) of 1974 (P.L. 93-533) and the Home Mortgage Disclosure Act (HMDA) of 1975 (P.L. 94-200). More recent legislation includes the Home Ownership Equity Protection Act (P.L. 103-325) and provisions from the Dodd-Frank Wall Street Reform and Consumer Protection Act (P.L. 111-203).

6　Note that, while bank examinations routinely yield the greatest proportion of allegations of lending discrimination, two other means by which individual borrowers can provide evidence of discrimination are through filing with the U.S. Justice Department or to one or more of the financial regulatory agencies (which include the Office of the Comptroller of the Currency (OCC), the Federal Deposit Insurance Corporation (FDIC) and the Consumer Finance).

7　Respective grounds for disputing evidence of disparate impact or redlining involve showing the practice is essentially vital to the legitimate business interests and solvency of the lender or, in the case of redlining, compliance with federal directives such as the National Flood Insurance Program. Details of these legally defensible reasons are discussed in Walter (1995).

8　We make the assumption that standard statistical underwriting models assign the same value of credit risk to borrowers from any two demographic classes in order to focus on the existence and magnitude of endogenous lending discrimination without the complication of having to adjust observed loan terms to two demographically distinct borrowers to filter out the effects of their being assigned different scores of credit risk in the automated underwriting process. This assumption, however, can be rationalized if the statistical measures of credit risk of these two loan applicants are based only on their similar individual financial histories and characteristics, excluding characteristics of the properties each applicant wishes to finance that affect volatility in their respective future values and are consequently germane to the actual credit risk incurred by the lender in approving both loan applications.

9   While our modeling of the mortgage market is independent of exogenous developments outside of this market, a referee kindly suggested considering the effects of the COVID-19 pandemic on our model. Most research on this issue, such as Federal Reserve Federal Reserve Board (2020), Deloitte (2020) and Golding et al. (2020), has found that with the expected appreciation rates for single- and multi-family residential properties in the U.S. and globally, the effect on volatility in those rates has been surprisingly small. Although his expected rate appears in the expression in (1) for the evolution of housing value, it does not affect the contingent-claims valuation of options in general nor, specifically, the options embedded in residential mortgage loans which are the focus of our results.

10 Note that lenders in our market are assumed to have no independent interest in the demographic traits of any given borrower. However, given common knowledge of the relation between the demographic identity of a borrower and the volatility in the value of his collateral, as expressed by $\varphi = \varphi(\theta)$, the loan terms offered by a demographically unbiased lender will still be indirectly affected by demographic considerations through the riskiness of the collateral value in the equation.

11 We will, for notational simplicity, suppress the arguments of σ in the expressions below but we will use the perception of risk $\sigma(\varphi(\theta))$ by the lender in deriving the presence of lending discrimination and in measuring its magnitude in Section 4 below.

12 We use expressions for foreclosure costs $b(a,t)$ and service flows $\pi(a, t)$ in this procedure that are linear in property value and, consequently, independent of a direct influence of time, for computational simplicity.

13 Our solutions for the respective L and B values at a given gridpoint are based on corresponding best-reply choices of borrower and lender here and at each previous gridpoint. Calculations are based on the Crank–Nicolson procedure to approximate the relevant partial derivatives of L and B with respect to s and t at each point. A description of the Crank–Nicolson method and its properties can be found in Press et al. (1986), which is the basis of its implementation in our Fortran-based simulation program .

14 The assumption of such a representative property, of course, is a proxy for the average property being financed by a given class of borrowers.

15 Note that this decline in contract value does not represent a nondistortionary redistribution of wealth to borrowers, since loans are assumed to be efficiently risk-priced by lenders.

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