

## Article

# Time Value of Money Application for the Asymmetric Distribution of Payments and Facts of Economic Life

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**Abstract:** This article is devoted to the applied aspects of using the concept of the time value of money for the purpose of determining the present value of cash flows in conditions of asymmetric distribution of payments and facts of economic life over time. Currently, such situation is standard when doing business and should be thoroughly studied. The purpose of the study is to prove that the method of distribution of payments affects the result of discounting, and that this information is essential when making management decisions and should be disclosed to the user of the information. Based on the basic provisions of the theory of the time value of money and analyzing the specifics of the asymmetric distribution of the described events, the authors come to the conclusion that it is necessary to supplement the cost discounting methodology by including in it a description of the basic approaches to distribution. As such approaches, the use of distribution methods that were called First Payment First Sale (FPFS), First Payment Last Sale (FPLS), and Current Payment Current Sale (CPCS) are proposed. Use of these methods in certain calculations is the main novelty of this article. The difference that arises as a result of the use of different approaches to assessment in the conditions of asymmetric distribution is illustrated with the simulated data. Taking into account a specific approach to the distribution of cash flows leads to a better understanding of the basis for discounting indicators, improves the quality of information and the validity of management decisions based on it, and reduces the risks of choosing the wrong financing strategy.

**Keywords:** time value of money; financial calculus; discounting; asymmetrical schedule of deliveries and payments



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## 1. Introduction

The time value of money is one of the most popular and important concepts in economics, financial management, and one of its most famous branches—financial mathematics. It is a widely accepted theory that there is greater benefit to receiving a sum of money now rather than an identical sum later.

The time value of money is among the factors considered when weighing the opportunity costs of spending rather than saving or investing money. As such, it is among the reasons why interest is paid or earned: interest, whether it is on a bank deposit or debt, compensates the depositor or lender for the loss of their use of their money. The increased value to be available later is sufficiently high to offset both the preference to spending money now and inflation.

Some ideas of the concept of the time value of money were formulated in ancient times. For example, in the works of the Ancient Greek philosopher Thales, you can find a statement of the basic principles of modern call options, which give the right to purchase a certain product at a certain point in time. Later, the relevant ideas were founded in the works of Aristotle ([Aristotle 1991](#)).

There is a well-known description of the situation in the Talmud, which concerned the determination of the amount to be paid to false witnesses who claimed that a loan of real duration of 10 years had been taken for only 30 days. The notion was later described by Martín de Azpilcueta and Halil MeteSoner ([Akyildirim and Soner 2014](#)).

In 1202, Fibonacci wrote the first book containing elements of financial mathematics—the Book of Abaca. He calculated the present value of alternative cash flows in addition to developing a general method for expressing investments and solved a wide range of problems related to interest rates ([Sigler 2002](#)).

Since that time, hundreds of works have been published on the issues of the time value of money, the theory of discounting, and the determination of present and future value, the discount rate. You can find sections on these issues in most fundamental works on financial management. In the second half of the twentieth century, complex works devoted to financial mathematics began to appear in large numbers ([Brigham and Ehrhardt 2001](#); [Cissel et al. 1990](#); [Baxter and Rennie 2001](#); [Shreve 2004](#); [Kremer 2005](#)). In this study, we do not discuss in detail the theoretical component of this topic because it is well known. Anyone can read the theoretical foundations of discounting theory in the works of, for example, Eugene Brigham and Michael Ehrhardt ([Brigham and Ehrhardt 2001](#)). This is a classic of financial management, which does not require re-presentation in this publication. However, we'll discuss its applied aspect. After all, it is obvious that the developed theoretical basis began to be applied actively in practice. They are the basis of the functioning of the banking system, financial management, investment business, and mathematical modeling ([Atkins and Dyl 1995](#); [Protter 2008](#); [Dunbar 2019](#)).

In the 1970s, the relevant methodological developments began to be introduced into the practice of accounting. In particular, the elements of the discounting theory have found their application in the valuation of the different elements of financial reporting (assets, liabilities, incomes, and expenses). Their use is provided for by International Financial Reporting Standards. In addition, it was the beginning of the widespread use of these methods in accounting that led to the need to adapt theoretical calculations for practical purposes ([Weil 1990](#); [Crosson and Needles 2008](#); [Baehaqi et al. 2020](#)). At the same time, some problems have not yet been resolved.

In particular, in theory, these methods have been widely developed to bring the value of future cash flow to any point in time at which a certain asset is accounted in the organization accounting. The opposite situation is also possible, associated with the valuation of an asset acquired by the organization on the terms of prepayment. However, there is a problem in case when a certain set of assets consisting of  $m$  parts is acquired using a payment schedule consisting of  $n$  payments. We call this situation an asymmetric payment/delivery schedule. These questions are considered in a number of publications on logistics and in the theory of inventory management ([Dib et al. 2018](#); [Hariga 1995](#); [Brodetskiy 2017](#); [Zhi et al. 2019](#)). However, a comprehensive solution to the issue was not proposed. The development of an accounting methodology based on financial mathematics for the specified schedule of obtaining assets and the payment schedule led to the need to clarify the discounting methodology itself, to include additional parameters in it that allow for distributing payments over time and comparing them with the schedule for receiving values.

The main objective of this study is to illustrate that there are several approaches to the discounting with different distribution of payments. Conditionally, we propose to call these approaches First Payment First Sale (FPFS), First Payment Last Sale (FPLS), and Current Payment Current Sale (CPCS). The proposed methods are novel contributions by the authors and are explained in the Results section.

The purpose is to prove that the method of distribution of payments affects the result of discounting, and that this information is essential when making management decisions and should be disclosed to the user of the information.

## 2. Materials and Methods

The study is based on the concept of the time value of money, which is widespread in financial management, which implies the relationship between the present and future value of cash flow. The basic formula that links the present and future value of money is as follows:

$$PV = \frac{FV}{(1+r)^n} \quad (1)$$

*PV*—the present value of Cash Flow;

*FV*—Future Value of Cash Flow;

*R*—discount rate;

*n*—the duration of the period of recalculating the value of future cash flow to the present level.

The study is based on the processing of conditional data, so no special tolerances are required to obtain the necessary information. All data are given in the publication itself.

As the main methods used in obtaining the results of the study, the methods of financial mathematics are used. As a justification for its own methodology, the study is based on the FIFO methods and the average cost, which are used in accounting when estimating inventories in accordance with the requirements of IAS 2. The authors do not see any legislative or other restrictions on the use of appropriate methods in solving the task.

This issue is a model study, but not an empirical data study. The data for the study were generated by the authors themselves. However, it is obvious to us that such situations with an asymmetric distribution of payments between sales are widespread in practice. However, the goal was not to demonstrate the significance of the approach for any organization, but to emphasize that different approaches to distribution give the researcher different results. Accordingly, information about the methods used is important and should be disclosed to the user.

As a starting point for this research, we accept as a fact that it does not require additional justification that the general relationship between present and future value is described by Formula (1). Obviously, the greatest difficulties in applying this technique may arise when the cash flow occurs both in the periods preceding any fact of economic life, and in the periods after the implementation of this fact. In such case, transforming Formula (1), the formula for valuation of the fact of economic life (the accrual method; not the cash flow method) can be represented as follows:

$$\text{Value of the fact of economic life} = \sum_{t=0}^n PV_t * r^{n-t} + \sum_{T=n}^k \frac{FV_T}{r^{T-n}}$$

*PV<sub>t</sub>*—cash flow in advance for the fact, which happened in *t* periods after the moment of the payment;

*FV<sub>T</sub>*—cash flow for the fact that is in *T* period after the moment of fact of economic life;

*t*—index of the period in which the relevant advance was received;

*T*—index of the period in which the subsequent payment of the shipped of goods was made;

*n*—index of the period in which the corresponding amount of accounts payable on advances received was repaid by non-cash assets;

*k*—index of the period in which the buyer finally repaid the amount of receivables for payment of the non-cash asset shipped to him;

*r*—the value of the discount rate per unit time interval.

For example, an advance payment in the amount of USD 500,000 for products was received in January 2021, the products were shipped in March 2021, and the final settlement—USD 300,000—was made in June 2021. The monthly level of the discount rate is 1.01. Let us determine the value of the fact of economic life:

$$\text{Value of the fact of economic life} = \sum_{t=0}^n PV_t * r^{n-t} + \sum_{T=n}^k \frac{FV_T}{r^{T-n}} = 500 * 1.01^2 + \frac{300}{1.01^3} = 801.23 \text{ thousand \$} \quad (2)$$

The use of the appropriate method for the calculation is due to the fact that the index  $t = 0$  has January 2021, respectively, March has an index  $n = 2$ , and June  $k = 5$ . Then, the difference in indices  $(n - t) = 2$ , and the difference in indices  $(T - n) = 3$ .

In this case, the amount of interest expense is USD 1230, while the difference between the nominal and discounted (accrued) payment amount represents interest expenses, since the amount of fact exceeds the nominal value of the payment. That is, the amount of interest expenses calculated from the amount of advance received before the shipment of products is greater than the amount of income on interest accrued during the deferral of payment:

$$\text{Interest expenses} = 500 * 1.01^2 - 500 = 10.05 \text{ thousand}$$

$$\text{Interest incomes} = 300 - \frac{300}{1.01^3} = 8.82 \text{ thousand}$$

Thus, the research is based on the basic principles of the theory of the time value of money, which are supplemented by the author's approach to the application of these methods, which is described in the next section.

### 3. Results

In a similar manner, it is possible to determine the value of the fact of economic life in the general case, that is, when payments are not made at a time, but periodically, and the very fact of economic life is carried out not simultaneously, but several times. That is, it has an asymmetric distribution of payments in time and a different asymmetric distribution of the facts of economic life in time, but, at the same time, the entire set of payments and the entire set of facts of economic life can be recognized as one business transaction. At the same time, it is necessary to know only the nominal amount of each fact, each payment and the periods of implementation of the corresponding facts and payments.

However, for a correct value, it is not enough just to use the Formula (2), since payments and facts of economic life must be compared with each other—that is, to establish a sequence in which the economic events that have occurred are paid.

For example, to distribute payments between facts, it is proposed to use a scheme called *FPFS*—First Payment First Sale, the meaning of which is that payments are distributed between facts as they are made in chronological order, that is, until the cost of the first fact is repaid, the cost of the next facts should not be repaid. An example of such calculations is given in Table 1. Let us assume a discount rate of 1.01 per month.

- From the USD 1200 thousand payment received in February in accordance with the *FPFS* method, USD 600 thousand fall on the payment of the fact in January, and accordingly they must be discounted with the rate 1.01, since the payment was made in the next month, and USD 600 thousand account for the payment of products in February, and, accordingly, they should not be discounted (payment was made without a deferred payment);
- From the USD 300 thousand payment received in March, USD 300 thousand refers to the payment of the fact made in February, and accordingly they must be discounted also with a rate of 1.01;
- From the USD 1500 thousand of the payment received in April, USD 600 thousand refers to the fact made in March (respectively, the discount rate of 1.01 is applied), USD 300 thousand—to the fact made in April (discount rate—1), USD 300 thousand—to the fact made in May (the rate of increase is 1.01) and USD 300 thousand to the fact made in June (the rate of increase is  $1.01^2$ );

**Table 1.** Distribution of payments between the facts of economic life in accordance with the FPFS method.

Indicator	January	February	March	April	May	June	Total
The nominal (accrued) value of the fact of economic life, falling on the period, thousand \$	600	900	600	300	300	300	3000
The amount of payment, thousand \$	0	1200	300	1500	0	0	3000
Distribution of the nominal value of payments between the facts of economic life, thousand \$	600	600	300	900	300	300	3000
Discounted (accrued) payment amount involved in the valuation of certain fact of economic life, thousand \$	594.06	600.00	297.03	894.06	303.00	306.03	2994.18

Source: own study.

The total amount of the fact of economic life is USD 2994.18 thousand and the total amount of interest incomes is USD 5.82 thousand.

For calculations, it is convenient to use the tabular form for calculating the relevant indicators by using this method (Tables 2–4).

Table 2 shows the distribution of payments between facts made by the organization. The columns in this table show the months in which the respective payments were made, and the rows show the months of facts, which are covered by the corresponding payments.

**Table 2.** Matrix of facts and payments, thousand \$.

Month of the Fact	Month of the Payment						Total Value of the Fact
	January	February	March	April	May	June	
January	0	600	0	0	0	0	600
February	0	600	300	0	0	0	900
March	0	0	0	600	0	0	600
April	0	0	0	300	0	0	300
May	0	0	0	300	0	0	300
June	0	0	0	300	0	0	300
Total payments	0	1200	300	1500	0	0	3000

Source: own study.

Table 3 shows the matrix of discount rates to be used when discounting the amounts of payments received in a given month but relating to different periods.

**Table 3.** Shows the matrix of discount rates.

Month of the Fact/Month of Payment	January	February	March	April	May	June
January	1	1010	1020	1030	1041	1051
February	0.990	1	1010	1020	1030	1041
March	0.980	0.990	1	1010	1020	1030
April	0.971	0.980	0.990	1	1010	1020
May	0.961	0.971	0.980	0.990	1	1010
June	0.951	0.961	0.971	0.980	0.990	1

Source: own study.

**Table 4.** Discounted (accrued) value of payments involved in determining value of the fact (FPFS), thousand \$.

Month of Fact of Economic Life	Month of the Payment						Total Value of the Fact
	January	February	March	April	May	June	
January	0.00	59.406	0.00	0.00	0.00	0.00	59.406
February	0.00	60.000	29.703	0.00	0.00	0.00	89.703
March	0.00	0.00	0.00	59.406	0.00	0.00	59.406
April	0.00	0.00	0.00	30.000	0.00	0.00	30.000
May	0.00	0.00	0.00	30.300	0.00	0.00	30.300
June	0.00	0.00	0.00	30.603	0.00	0.00	30.603
Total	0.00	119.406	29.703	150.309	0.00	0.00	299.418

Source: own study.

Accordingly, Table 4 shows the discounted (increased) amount of cash flow, which forms the amount of revenue, determined by the rate of the values presented in the corresponding cells of Table 2 to the values of the discount coefficient presented in the corresponding cells of Table 3.

If the amounts of payments received in a period are distributed differently between the fact periods the total result of the discount–accrual may be different. However, this difference is maximized when payments received in the initial periods are conditionally considered payments covering the facts of later periods and, vice versa, payments received later are correlated with facts of the initial periods. This method can be indicated as *FPLS*—First Payment Last Sale.

Matrix of facts and payments is in Table 5. Table 6 shows the results of discounting (increasing) the corresponding amounts presented in Table 5.

**Table 5.** Matrix of facts and payments, USD thousand.

Month of the Fact	Month of the Payment						Total Value of the Fact
	January	February	March	April	May	June	
January	0	0	0	600	0	0	600
February	0	0	0	900	0	0	900
March	0	300	300	0	0	0	600
April	0	300	0	0	0	0	300
May	0	300	0	0	0	0	300
June	0	300	0	0	0	0	300
Total	0	1200	300	1500	0	0	3000

Source: own study.

The differences in the results of calculations (despite the slight difference between the final estimation of the fact) are obvious and summarized in Table 7.

**Table 6.** Discounted (accrued) value of payments involved in determining value of the fact (FPLS), thousand \$.

Month of the Fact	Month of the Payment						Total Value of the Fact
	January	February	March	April	May	June	
January	0.00	0.00	0.00	58.235	0.00	0.00	58.235
February	0.00	0.00	0.00	88.227	0.00	0.00	88.227
March	0.00	30.300	30.000	0.00	0.00	0.00	60.300
April	0.00	30.603	0.00	0.00	0.00	0.00	30.603
May	0.00	30.909	0.00	0.00	0.00	0.00	30.909
June	0.00	31.218	0.00	0.00	0.00	0.00	31.218
Total	0.00	123.030	30.000	146.462	0.00	0.00	299.492

Source: own study.

**Table 7.** Estimation of the differences in the discounted value of the fact of economic life by various ways of distributing nominal amounts between periods.

Month of the Fact	FPFS Estimation Amount	FPLS Estimation Amount	$\Delta$
January	59.406	58.235	−1171
February	89.703	88.227	−1476
March	59.406	60.300	894
April	300	30.603	603
May	303	30.909	609
June	30.603	31.218	615
Total	299.418	299.492	0.74

Source: own study.

Thus, when using the FPLS method, the amount of revenue of the initial fact periods decreases, since it is formed when payments are received in the last settlement periods; as a result, the payment amount is discounted with a large rate.

Conversely, the amount of value in recent periods determined using the FPLS method is greater than the amount of value of the fact using the FPFS allocation method.

In addition, we should not pay attention to the final insignificant difference in the total amounts (+0.74). For example, according to the results of the first two months of making payments, the difference between the results is about 1.8%. Furthermore, such difference in a certain time interval is more than significant. It may lead to a revision of the entire payment schedule, a change in financial strategy.

As an alternative to using the FPFS method, we can offer to use the CPCS—Current Payment Current Sale method—that is, the next fact of the current month will be paid first, and then the facts of the previous months in chronological sequence, starting from the earliest (if there is a balance after the payment of the reporting month is repaid). When calculating by using this method, the result of discounted revenue value is 2994.27 thousand \$, that is, slightly more than when using the FPFS method.

Obviously, using the proposed method with the disclosure of relevant information eliminates the risk of bias in the interpretation of relevant information. However, each party should understand what methods were used in the processing of information. The approach is extremely formalized and excludes the subjective factor.

We consider it necessary to focus attention on the fact that the author's ideas are not only a solution to a practical problem. This should certainly be considered a development of the methodology of discounting theory. Even though the differences between the results of calculations using different methods for the distribution of payments may be insignificant, we still insist that the method of distribution of flows during discounting is an essential

element of the methodology, and ignoring this leads to unjustified simplification and elimination of significant events. The use of FIFO and average cost methods also provides results that may differ slightly, and with the full use of inventory, they are generally the same. However, their inclusion in the methodology with mandatory disclosure of relevant information is not denied by experts. Disclosure of information about the applied method of inventory valuation is one of the key provisions of accounting policy, and, in the situation under consideration, the meaning is exactly the same—the user is informed about the approach to the distribution of payments.

Additionally, it is necessary to note that this publication does not consider the influence of other factors on the described processes. We assume that other factors, such as transaction costs, affect all alternatives equally.

#### 4. Discussion

Despite the seemingly insignificant difference in the results of the calculations, the authors insist that the option of distributing payments between the facts of economic life must necessarily be included in the methodology for determining the present value. The methodology should be universal. It should be applicable to any source data. At the same time, it is far from a fact that the FPFS method described by the authors will definitely have an advantage in comparison with other described approaches. For example, drawing a parallel with the inventory valuation, it should be noted that, until now, along with the FIFO method, the average cost method is also used, and for many years, the LIFO method has been used to carry out such procedures.

We also consider it necessary to turn to the main beneficiary from the use of information that can be generated in accordance with the proposed approach. First of all, it seems that they can be specialists in the field of managerial accounting, financial management, internal analysis, and employees of the department of settlements. After all, they are the points of responsibility for decision-making in the field of cash flow management. Even in conditions of low interest rates, there are differences in the amounts obtained by using different approaches to discounting. With an increase in the rate of inflation and, accordingly, an increase in interest rates in some economic systems in 2022, the differences in the results obtained will be even more significant.

It is possible that, with the receipt of new data, it will be necessary to make changes to the content of contracts with partners, a clear establishment of the order of payments made, which will allow organizations to optimize cash flows. It is also important to take into account the described approaches to discounting when compiling payment calendars and cash flow budget.

However, the most important thing is that there is a clarification of the economic valuation, which is the basis for making managerial decisions in any case.

Moreover, we believe that there is a further development for the developed proposals, which involves the use of an option in the distribution, in which asymmetrically distributed payments are associated with specific facts of economic life, regardless of their chronological sequence. This distribution method can be called APAS—actual payment actual sale. At the same time, we believe that organizations should strive for a situation where each payment is linked to a specific fact of economic life. However, this does not negate the authors' proposals, which focus the attention of the user of financial information on the need to include the appropriate parameter in calculations focused on the use of the concept of the time value of money.

Obviously, any research can be continued. As potential directions for the continuation of the research, consideration of the issues of the application of author's approaches in the conditions of different levels of discount rates can be considered. Actually, in this paper, we did not set the task of considering the different degrees of impatience that different economic agents might have. Perhaps, this will be the subject of future publications.



## 5. Conclusions

The information presented using the proposed approach satisfies the principle of usefulness. The costs of obtaining it in the conditions of automation of such procedures are insignificant. The principle of rationality is also fulfilled. Such data generated using appropriate approaches are more accurate from the point of view of reflecting the economic content of the economic transactions performed.

We believe that the proposed options for the distribution of payments between the facts of economic life in case of their asymmetric distribution over time (time interval between payments and paid events) should be considered precisely as part of the methodology for determining the present value of cash flows. Without knowledge of the distribution approach used, it is simply impossible to determine the duration of the time gap between the events.

The use of the concept of the time value of money is an important component of the methodology for the formation of accounting and analytical information in the financial management system. It involves a set of methods that allows for obtaining a more accurate assessment of the fact of economic life. Accordingly, this improves the quality of information financial management and contributes to the adoption of informed management decisions.

We do not see any limitations of this and no special features of implications of our results.

The main idea of the paper was to prove that the existence of different approaches can lead to asymmetric interpretation of information by different users. Accordingly, the main conclusion that follows from the above calculations is the need for certain steps to minimize asymmetry. These may be measures taken by various regulatory authorities, which represent the establishment of only one approach to the relevant calculations. However, this path does not seem promising, since it is impossible to prohibit the user from using methods that have a logically correct basis for its internal needs. Accordingly, the only way is to disclose the relevant information when submitting information. That is, the person processing the information and the person making decisions based on it should interpret it taking into account the known initial conditions used in its processing. The decisions that can be made using this information are obvious—they relate to the time and conditions for granting deferred payment or requesting prepayment. These are the most important decisions that are made today by the financial management of companies. Therefore, the relevance of the authors' conclusions is obvious.

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