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Uncertainty and Risk in the Cryptocurrency Market

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Abstract: Cryptocurrency investments are often perceived as uncertain and risky. In this study, we assessed if this is indeed the case, using a sample of seven cryptocurrencies and considered a period that encompassed the first real global shock in the life of these relatively new financial assets, the COVID-19 pandemic. Uncertainty was evaluated using Shannon's symbolic entropy. To measure risk, we use value-at-risk and conditional value-at-risk. The results indicate that, except for Tether, the analyzed cryptocurrencies' returns exhibited similar patterns of uncertainty and risk. Levels of uncertainty were close to the maximum values, but high uncertainty is not always associated with high risk. During the pandemic crisis, uncertainty increased while risk decreased, suggesting that the considered assets may have safe haven properties.

Keywords: risk; uncertainty; cryptocurrencies; symbolic entropy; value-at-risk; conditional value-at-risk

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1. Introduction

In the last decade, the cryptocurrency market experienced notable growth. Created as an alternative to fiat monies, cryptocurrencies quickly became a new asset class (Katsiampa 2017; Corbet et al. 2018, 2019), displaying higher volatility, risk and returns than more traditional assets (Ji et al. 2019; Chaim and Laurini 2019). Globalization and financial liberalization have boosted the integration of financial markets but have also reduced diversification opportunities (Mensi et al. 2019). Thus, investors increasingly turn to this new market in search of diversification and hedging options.

Investment in cryptocurrencies is subject to uncertainty and several specific and systematic risks, assessed inter alia by Fry and Cheah (2016), Ardia et al. (2019), Borri (2019), Wei (2018) and Syuhada and Hakim (2020). Uncertainty and risk, though conceptually distinct, are often used as synonyms, perhaps because they are associated with imperfect knowledge (Knight 1921). In this study, we assess, first, uncertainty using Shannon's symbolic entropy, and then risk, with value-at-risk (VaR) and conditional value-at-risk (CVaR), two statistics often used in the context of the cryptocurrency market (see, for instance, Likitratcharoen et al. 2018; Trucíos et al. 2020).

The contribution of our study is twofold. We add to current knowledge by developing a complementary analysis of uncertainty and risk using data for a set of cryptocurrencies and adopting a robust methodology that considers these assets' complex dynamic behavior. Similar studies are rare, and the few that exist are solely focused on the Bitcoin (BTC) (a specificity that also applies to other analyses of the cryptocurrency market-see, inter alia Katsiampa 2017; Ardia et al. 2019; Bouri et al. 2018).

The remainder of the paper is organized as follows: after this introduction, Section 2 briefly reviews the relevant literature; Section 3 presents the data and methodology; results are shown and discussed in Section 4; Section 5 concludes.

2. Literature Review

Several studies concluded that the cryptocurrency market is prone to speculative bubbles (Cheah and Fry 2015; Fry 2018; Agosto and Cafferata 2020; Goodell and Goutte 2021). Analyses of volatility and its predictability are thus useful to assess market risk, also contributing to reducing speculation and speculative bubbles. When uncertainty dominates, significant changes in returns' volatility can significantly and negatively affect risk-averse investors (Bentes and Menezes 2012).

Since the advent of probabilistic risk assessment, risk has been defined as the mathematical product of the probability of an event and some measure of its negative consequences. In financial analyses, risk is often measured using variance, i.e., the higher the variance, the greater the risk. However, historically, variance has also been considered a measure of dispersion, uncertainty, and a means to assess the adjustment of a model (Dionísio et al. 2006). Variance has been used to evaluate risk and uncertainty, justifying the distinction between the two concepts. Risk was defined by Knight (1921) as a situation where the result of a decision is unknown, but the probability distribution of each potential result is known. Uncertainty exists when the probability distribution of the results is unknown.

In empirical studies of the cryptocurrency market, generalized autoregressive conditional heteroskedasticity (GARCH) models, including asymmetric GARCH ones, have been used to model volatility (see Ardia et al. 2019; Chu et al. 2017; Lahmiri et al. 2018; Maciel 2020, among many others) and to estimate VaR and CVaR (Balcilar et al. 2017). There is no consensus about the most appropriate model. However, GARCH models solely consider the second moment of the series of returns and thus can only capture a small fraction of the distribution's informative content. The same occurs with the variance, frequently used to assess volatility, financial risk and uncertainty. Therefore, the use of more robust techniques is justified.

Entropy is a more general measure of uncertainty than variance or standard deviation, as it may be related to higher-order moments of the distributions (Dionísio et al. 2006). Many financial studies apply entropy to estimate risk, but few are focused on the cryptocurrency market. For example, Lahmiri et al. (2018) applied Shannon's entropy to assess the fractality and randomness of the BTC estimated volatilities and found evidence of a high degree of randomness for the analyzed series. Symbolic time series analysis (STSA) was used by Pele and Mazurencu-Marinescu-Pele (2019) and Takada et al. (2019) to compute Shannon's entropy of intraday Bitcoin returns as a measure of uncertainty and to predict VaR and CVaR.

VaR is one of the most common risk measures (Nadarajah et al. 2014). It indicates a possible loss (worst loss) in a given time horizon (h), with a probability related to a significance level (α). Thus, it is a "prognostic tool" to avoid exceeding some previously defined risk tolerance threshold. The VaR is relevant when the objective is to minimize the probability of extreme losses (Mensi et al. 2018), but it has been criticized as a risk measure for not satisfying the subadditivity axiom for all distributions. However, it satisfies this axiom at the tail of heavy-tailed asset return distributions with well-defined means (Danielsson et al. 2013). It is thus subadditive in the most relevant region for risk management. As cryptocurrency returns have fat-tailed distributions (Chu et al. 2017; Phillip et al. 2018), VaR is a suitable risk measure for the cryptocurrency market. CVaR (introduced by Artzner et al. 1999) also identifies the expected loss if the VaR threshold has been crossed, i.e., it provides information about the expected loss when significant losses occur.

The referred studies assessing uncertainty or risk in the cryptocurrency market were developed using data solely for the BTC. They provided mixed evidence concerning the dynamics of the volatility of the examined returns. The simultaneous assessment of uncertainty and risk was also only focused on the BTC (Pele and Mazurencu-Marinescu-Pele 2019; Takada et al. 2019). In what follows, we extend the joint analyses of the two concepts using a sample of seven cryptocurrencies with distinct characteristics to improve knowledge of this diverse market.

3. Materials and Methods

The sample of data comprises daily closing prices for seven cryptocurrencies (with data available since the 7th of August 2015) with more than a USD billion-dollar market capitalization on the 7th of March 2020, retrieved from an open-access source (https://coinmarketcap.com, accessed on 13 October 2021). The sample ended on the 13th of October 2021 (see Table 1).

Table 1. Sample description.

	Cryptocurrency		MC	Launch Year	Properties
1	Bitcoin	BTC	90804613601	2009	Mined; pure financial asset
2	Ethereum	ETH	12366138225	2015	Mined; service platform
3	Ripple	XRP	6118533337	2012	Unmined; service platform
4	Tether	USDT	4891126961	2014	Unmined; stable cryptocurrency
5	Litecoin	LTC	1988180694	2011	Mined; pure financial asset
6	Stellar	XLM	677492669	2015	Unmined; service platform
7	Monero	XMR	577030303	2014	Mined; pure financial asset

Notes: (i) MC is the market capitalization in USD on 03/12/2020.; (ii) Start and end dates are 08/07/2015 and 10/13/2021, respectively, with a total of 2260 observations.

Daily returns for the cryptocurrencies are calculated as $r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$, where $r_{i,t}$ is the return of cryptocurrency i at period t, $P_{i,t}$ and $P_{i,t-1}$, are the prices at times t and $t-1$, respectively.

Investors are interested in the expected returns and in the risk and uncertainty of their investments. Our objectives are thus to evaluate the risk of the cryptocurrencies and then to complement such assessment with an analysis of uncertainty. As mentioned above, this complementary assessment is still rare in the literature and was previously solely performed for the BTC. Risk is assessed using VaR and CVaR; for uncertainty, we use Shannon's symbolic entropy. All estimates have a time horizon of one year, as detailed in Table 2.

Table 2. Periods of analysis.

	Period		Returns
t_1	8 August 2015	7 August 2016	366
t_2	8 August 2016	7 August 2017	365
t_3	8 August 2017	7 August 2018	365
t_4	8 August 2018	7 August 2019	365
t_5	8 August 2019	7 August 2020	366
t_6	8 August 2020	7 August 2021	365
t_7	8 August 2021	13 October 2021	67
	Total		2259

Entropy estimates provide information concerning the level of disorder and uncertainty of a given market or asset. Entropy may be used to assess uncertainty (but not risk), thus allowing a quantitative distinction between both concepts, in line with Knight (1921). In the case of a perfect positive correlation (correlation equal to 1), the risk is at a maximum, and uncertainty is at a minimum. We used symbolic time series analysis to define the entropy of the distribution of each cryptocurrency's daily returns. As we are interested in combinations of cryptocurrencies' positive and negative returns, the zero value was used to partition the time series. There are thus two states: zero and one—the former for returns equal to or higher than zero and the latter for negative returns. The symbolic time-series representation can be carried out by transforming the sequence of real numbers into a binary sequence, i.e., $r_{i,t} \overset{STSA}{\rightarrow} s_{i,t}$, where:

$$s_{i,t} = \begin{cases} 1, \ se\ r_{i,t} < 0 \\ 0, \ se\ r_{i,t} \geq 0 \end{cases} \tag{1}$$

The binary sequence obtained is a sequence of zeros (price increases) and ones (price declines). Considering the definition of entropy provided by Shannon (1948), the symbolic entropy of the series of returns is:

$$H_y = -(\pi_y log_2(\pi_y) + (1-\pi_y)log_2(1-\pi_y)) \tag{2}$$

with $\pi_y = p(s_{i,t} = 1)$ and $1 - \pi_y = p(s_{i,t} = 0)$, and $t = 1, \ldots, T$ days.

VaR allows for the measurement of extreme downsize risk, which may reflect extreme price movements in the cryptocurrency market (Zhang et al. 2021). VaR estimation requires the definition of a significance level (α), with values usually smaller than 0.05, and of a time horizon (h), usually measured in trading days. A $100\alpha\% h - days\ VaR$ corresponds to the maximum loss exceeded with α probability if the initial position (P_0) is maintained for $h - days$, i.e., $p(\Delta P_0 < -VaR) = \alpha$. Negative VaR values correspond to gains and positive ones to losses. In addition, it is also necessary to assume a distribution probability $F(.)$, which may be empirical or parametric. Historical simulation (non-parametric method), parametric methods and semi-parametric methods are frequently used to estimate the VaR. We use historical simulation and parametric methods (normal and Student's *t*) to estimate VaR with two significance levels ($\alpha = 0.05$ and $\alpha = 0.01$). Each VaR is defined as follows:

$$\text{Historical simulation}: \ VaR_\alpha(zero) = -P_0 r^* \tag{3}$$

$$\text{Parametric (Normal distribution)}: \ VaR_\alpha(zero) = -P_0(\alpha\sigma + \mu) \tag{4}$$

$$\text{Parametric (Student's } t\text{-distribution)}: \ VaR_{\nu,\alpha}(zero) = -P_0\left(\sqrt{\nu^{-1}(\nu-2)}\, t_\nu^{-1}(1-\alpha)\sigma + \mu\right) \tag{5}$$

where α is the significance level; P_0 is the initial value of the risk position (we considered a unitary value); r^* is the cut-off return for the smallest of the $\alpha \times n$ returns (in ascending order); μ is the mean of the series of returns; σ is the standard deviation of the series of returns; ν are the degrees of freedom (given by $\nu = \frac{6}{k} + 4$, where k is the kurtosis excess of the empirical distribution); and $t_\nu^{-1}(\alpha)$ is the Student's *t*-distribution $\alpha - quantile$ (they satisfy the condition $-t_\nu^{-1}(\alpha) = t_\nu^{-1}(1-\alpha)$).

The historical simulation is used because it allows linear and non-linear dependence between each cryptocurrency and the underlying risk factors. Parametric methods, with normal and Student's *t*-distributions, are chosen to capture the dynamics of volatility, which is not captured by the non-parametric method. The cryptocurrencies' returns display leptokurtic distributions (as is usually the case for financial returns). The effect of leptokurtosis in the estimation of VaR should not be disregarded. On the one hand, it can produce underestimation at higher significance levels (e.g., $\alpha \leq 0.01$) because $F_{\mathcal{N}}^{-1}(\alpha) \leq F_{t_\nu}^{-1}(\alpha)$ (Mandelbrot 2003), which means that $VaR_{leptokurtic\ distribution;\ \alpha \leq 0.01} > VaR_{norml\ distribution;\ \alpha \leq 0.01}$. On the other hand, for lower significance levels, VaR can be overestimated with a normal distribution, meaning that $VaR_{leptokurtic\ distribution;\ \alpha \leq 0.05} < VaR_{norml\ distribution,\ \alpha \leq 0.05}$.

In risk assessments, it is also useful to have information about the size of a loss when the VaR limits are exceeded. CVaR is a more restrictive risk measure than VaR; it provides information about the expected size of a large loss (when the VaR limits are exceeded). It provides an answer to the following question: "Given that we will have a bad day, how bad should we expect it to be?", i.e., what is the expected loss when it exceeds *Va*R? Considering X as a random variable that represents a loss, with $E(|X|) < \infty$, the CVaR is defined as:

$$CVaR_\alpha(X) = -P_0 E[X | X < -VaR_\alpha] \tag{6}$$

To obtain information about the loss size that exceeds the VaR, CVaR is also estimated for two different significance levels ($\alpha = 0.05$ and $\alpha = 0.01$).

4. Results

4.1. Preliminary Results

The descriptive statistics of the cryptocurrencies' returns are presented in Table 3. Using Stata SE15®, a standard and augmented Dickey–Fuller test for stationarity, was performed (for the sake of brevity, results are not shown but are available upon request). All the series of returns are stationarity (H_0 of the augmented Dickey–Fuller test was rejected). The cryptocurrencies display non-negative and near-zero average returns for almost all periods, which means that cryptocurrencies have generally increased in value. However, in period t_5, most cryptocurrencies (except USDT and XLM) had negative average returns, maybe reflecting the turmoil in these markets following the emergence of the COVID-19 pandemic.

The standard deviation (σ) was higher in t_2 and t_3. It is thus expected that these are periods with which greater risk is associated, given the frequent association between volatility (measured by the standard deviation) and risk. USDT was the less risky cryptocurrency in the sample (the only stable cryptocurrency), followed by the BTC. All cryptocurrency returns (except those of the BTC) display mixed skewness patterns, with positive (higher likelihood of large positive price changes than large negative ones) and negative (higher likelihood of extreme negative events) skewness, the latter especially after the last quarter of 2019. The returns do not follow a normal distribution, displaying fat tails (a stylized fact in financial markets).

Higher volatility levels are often associated with greater risk (if returns have a normal distribution). However, the standard deviation (σ) is commonly used as a risk and uncertainty measure. In view of the above, a preliminary analysis is thus based on the computation of returns (average returns for each period) and risk (calculated using the standard deviation of returns). Figure 1 displays these values for the last period (t_5) (values for the other periods are available upon request). We present this information as a preliminary step. Although volatility does not accurately measure the risk of an investment, it may be a good starting point for its evaluation (Poon and Granger 2003).

Table 3. Cryptocurrency returns and their descriptive statistics.

Cryptocurrency	BTC				ETH				XRP				USDT			
Period \ D.Stat.	Mean	Stdev.	Kurtosis	Skewness	Mean	Stdev.	Kurtosis	Skewness	Mean	Stdev.	Kurtosis	Skewness	Mean	Stdev.	Kurtosis	Skewness
t_1	0.0021	0.0324	8.1478	−0.9954	0.0037	0.1095	55.7676	−4.3573	−0.0008	0.0392	8.6502	1.3647	0.0000	0.0000	39.8160	−2.2803
t_2	0.0051	0.0357	6.1916	−0.0410	0.0099	0.0673	7.0218	1.3669	0.0098	0.0970	39.7127	3.3631	0.0000	0.0072	12.7282	0.1475
t_3	0.0019	0.0517	2.2617	−0.0623	0.0009	0.0574	2.6579	−0.2936	0.0021	0.0838	12.8829	1.8047	0.0000	0.0079	12.3461	0.4981
t_4	0.0016	0.0373	3.6743	−0.1559	−0.0014	0.0502	2.5965	−0.4071	−0.0005	0.0509	7.6885	1.1777	0.0000	0.0057	1.4470	0.1360
t_5	−0.0001	0.0403	49.2708	−3.7816	0.0014	0.0503	40.7387	−3.5042	−0.0002	0.0412	24.8958	−2.3456	0.0000	0.0068	15.5926	0.1184
t_6	0.0037	0.0403	2.1242	−0.1639	0.0058	0.0563	4.2358	−0.5622	0.0028	0.0861	9.5673	0.0279	0.0000	0.0017	75.6404	−3.5385
t_7	0.0038	0.0372	1.3994	−0.3432	0.0020	0.0469	0.8544	−0.2065	0.0049	0.0626	2.0132	−0.0666	0.0000	0.0001	7.9432	0.5887
Cryptocurrency	LTC				XLM				XMR							
Period \ D.Stat.	Mean	Stdev.	Kurtosis	Skewness	Mean	Stdev.	Kurtosis	Skewness	Mean	Stdev.	Kurtosis	Skewness				
t_1	−0.0003	0.0357	7.5704	−0.3116	−0.0007	0.0514	5.1586	0.9217	0.0026	0.0619	3.6733	0.3502				
t_2	0.0074	0.0621	15.9106	2.4310	0.0075	0.1006	16.5463	2.5942	0.0097	0.0811	10.6655	2.0154				
t_3	0.0011	0.0720	6.9620	0.6624	0.0064	0.1004	6.7239	1.1823	0.0021	0.0751	2.6975	0.2480				
t_4	0.0008	0.0518	3.2326	0.5418	−0.0029	0.0472	2.1164	0.0582	−0.0003	0.0501	2.4050	−0.3511				
t_5	−0.0013	0.0474	23.1362	−2.2304	0.0007	0.0485	16.3698	−1.0093	−0.0001	0.0472	32.8282	−3.1450				
t_6	0.0027	0.0627	7.9882	−1.1750	0.0030	0.0755	11.9551	1.2866	0.0029	0.0626	17.6864	−1.4901				
t_7	0.0020	0.0529	3.5610	−0.5790	0.0027	0.0584	2.9431	−0.9180	0.0002	0.0458	3.4543	−0.8862				

Note: D.Stat and Stdev represent the descriptive statistics and the standard deviation, respectively.

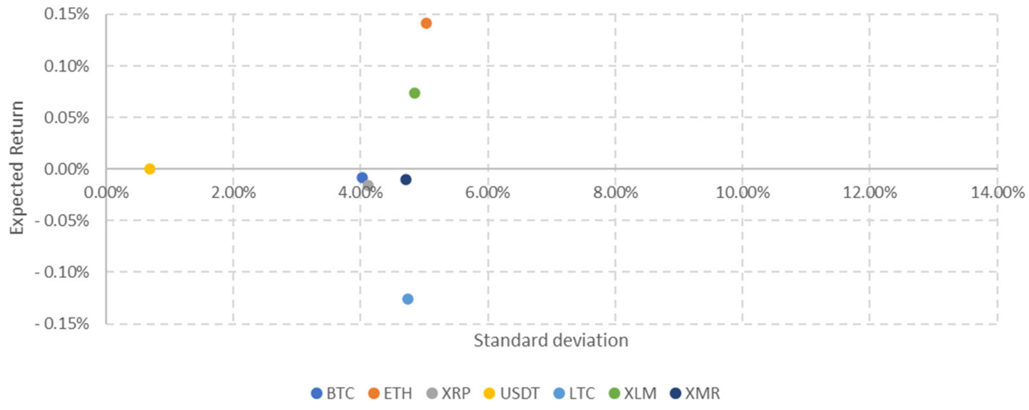


Figure 1. Expected return vs. risk (σ) in period t_5. Note: $expected\ return = \frac{\sum_{t=1}^{t=n} ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)}{n}$.

Figure 1 shows that USDT is the cryptocurrency with the lowest risk (close to 0.682%). It also has a return of zero (with similar values for the remaining periods). If we consider the capital asset pricing model, investors in this cryptocurrency will not be compensated with a risk premium. In general, the cryptocurrencies with the highest risk produce the highest returns. However, this does not happen in the other (not shown) periods. In the period marked by the beginning of the COVID-19 pandemic, four of the seven cryptocurrencies (including the cryptocurrencies considered as purely financial assets and cryptocurrencies that share characteristics of service platforms) produced negative returns, despite also displaying high standard deviations (BTC, XRP, LTC and XMR), thus not allowing the definition of an efficiency frontier. During this period, most cryptocurrencies displayed both low volatility and low returns.

Standard deviations are an accurate measure of risk solely if returns are normally distributed. Since this is not the case for our data, standard deviations may underestimate the underlying risk and provide a flawed assessment of uncertainty. Therefore, other statistics are required, and we used Shannon's symbolic entropy to assess uncertainty and the VaR and CVaR for risk. These are both reliable risk measures for cryptocurrency returns (Kajtazi and Moro 2019).

4.2. Entropy: Uncertainty Assessment

Figure 2 displays the entropy associated with the cryptocurrencies' returns (measured in bits), with all the series displaying similar entropy levels (uncertainty), with the exception of the USDT, for which such levels are always smaller.

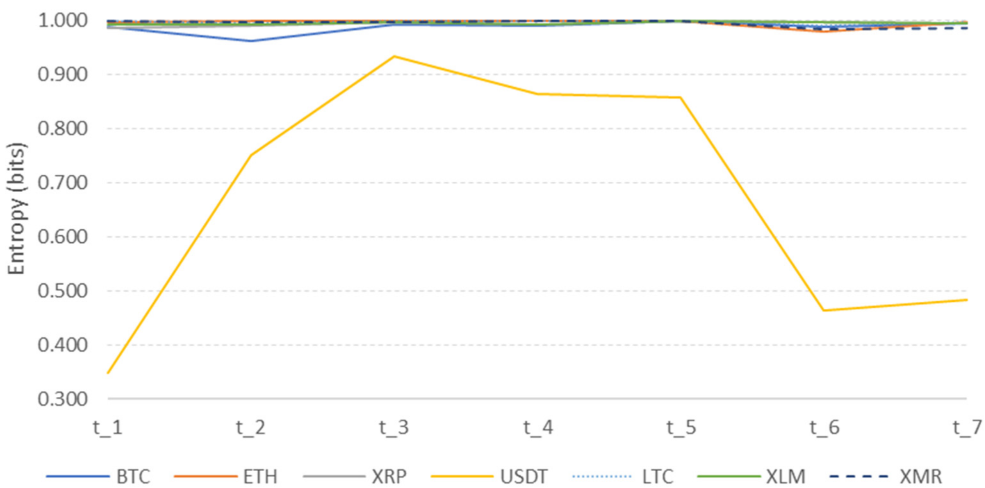


Figure 2. Symbolic entropy to cryptocurrency returns.

USDT is designated as a stable cryptocurrency, i.e., each USDT unit is guaranteed on a one-to-one basis by the current fiat currency (i.e., 1 USDT = 1 USD), held in legal deposit by Tether Limited (Griffin and Shams 2020). This could explain the lower entropy values obtained for this cryptocurrency.

Figure 3 presents the same information as Figure 2 but removes the values obtained for the USDT. In this case, entropy values are between 0.963 (for BTC in t_2) and 1.000 (its maximum possible value, given that it is measured in bits) for various cryptocurrencies, which means that such investments are highly uncertain. BTC was the first cryptocurrency and had the higher market capitalization, which can explain its lower uncertainty levels. The COVID-19 pandemic is a non-financial event, but it impacted most financial markets, including that of cryptocurrencies (Aslam et al. 2020). Thus, the beginning of the COVID-19 pandemic may be a possible explanation for the maximum entropy values (near one) observed in t_5.

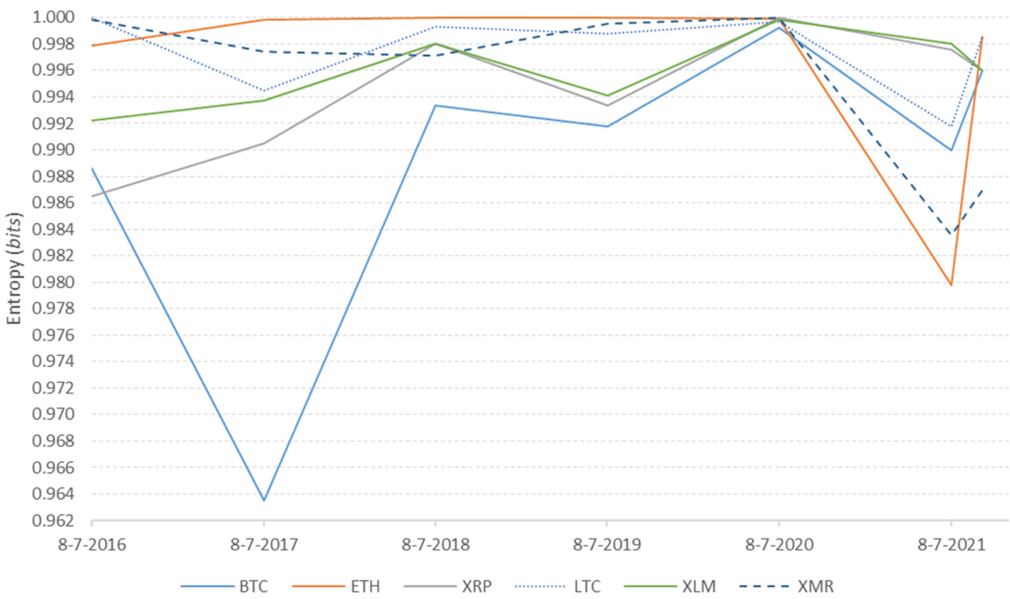


Figure 3. Symbolic entropy to cryptocurrency returns (except for USDT).

Entropy values, as well as their evolution, are similar for all cryptocurrencies. However, the results show that: (i) when USDT is not accounted for, the BTC displays a lower uncertainty in all but the two last periods; (ii) ETH is the cryptocurrency with the highest level of uncertainty until the end of t_5; after that, its pattern changes; and (iii) XRP and XLM (both unmined cryptocurrencies and sharing characteristics of service platforms) follow a similar behavior. Both display an increase in uncertainty in t_2 (the remaining cryptocurrencies, except ETH, display a decrease) and a reduction in uncertainty in t_7 (all the other cryptocurrencies register an increase). This similarity in behavior may reflect the fact that both cryptocurrencies have a more centralized structure, with XLM originally based on the Ripple Labs protocol (Hsieh et al. 2017). Furthermore, they are both "unmined" cryptocurrencies and have validation procedures that are distinct from those of the other mined cryptocurrencies in our sample (BTC, ETH, LTC and XMR) (Cagli 2019).

To assess the relationship between returns and uncertainty in cryptocurrency investments, both are compared. The results are displayed only for t_5 in Figure 4 (information for the remaining periods is available upon request). Figure 4 displays different patterns despite that uncertainty is similar for these cryptocurrencies. Not considering USDT, BTC displays the lowest uncertainty but not the lowest average return. Although ETH, XLM, XRP, LTC and XMR exhibit the same uncertainty levels, they have different average returns. Thus, investing in ETH will potentially provide a higher return than investing in XLM. This may inform selection strategies for cryptocurrency portfolios.

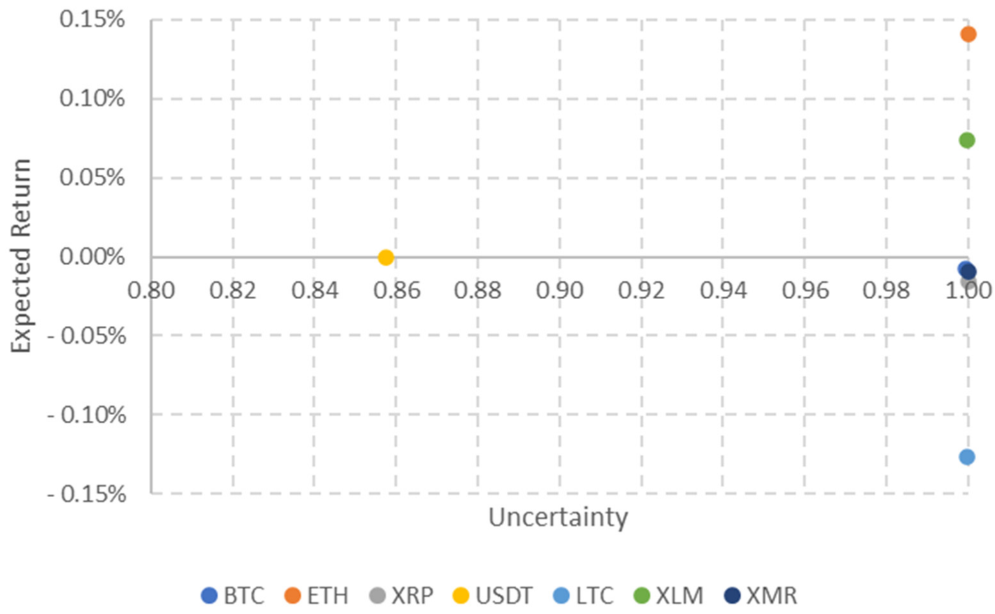


Figure 4. Expected return vs. uncertainty in period t_5. Note: $expected\ return = \frac{\sum_{t=1}^{t=n} ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)}{n}$.

Considering all the periods under analysis, USDT was the cryptocurrency with the lowest uncertainty, though it also produced average null returns. Investors are interested in higher returns with lower uncertainty. However, the results show that higher uncertainty is not necessarily associated with higher returns (except in the cases of BTC and XRP), which suggests that investors in these cryptocurrencies are not uncertainty averse. In the first period of the COVID-19 pandemic, period t_5, all cryptocurrencies (except USDT) display the maximum entropy value, but only XLM and ETH have positive returns. Thus, XLM and ETH may have safe haven properties in periods of turmoil in financial markets. However, this statement should be confirmed with the experience in further crisis episodes with non-financial origin.

4.3. Value-At-Risk (VaR) and Conditional Value-At-Risk (CVaR): Risk Assessment

As cryptocurrencies have continuous trading, all days of each year were considered for the VaR and CVaR estimations. Risk measures were computed considering periods of one year and two significance levels (α = 0.05 and α = 0.01). All VaR and CVaR values were positive, thus corresponding to losses and indicating that investing in the assessed cryptocurrencies always involves the possibility of losses.

VaR estimation involves the consideration of an empirical or parametric probability distribution. We estimate nonparametric VaR, Normal VaR and Student-t VaR (considering normal and Student's t-distributions, respectively) and the obtained values are displayed in Figures 5 and 6i–iii. The results indicate that the normal distribution overestimates VaR (on VaR(95)) while underestimating it (on VaR(99)). This happens because $VaR_{leptokurtic\ distribution;\ \alpha \leq 0.01} > VaR_{normal\ distribution;\ \alpha \leq 0.01}$ and $VaR_{leptokurtic\ distribution;\ \alpha \leq 0.05} < VaR_{normal\ distribution;\ \alpha \leq 0.05}$ (see Alexander 2008 for further details), and indicates that the normal distribution does not capture the most extreme events. The results also show that the leptokurtosis effect is non-negligible and that the Student's t-distribution allows for more accurate VaR estimations. Considering VaR(95) estimations, most cryptocurrencies (exceptions are XRP and ETH) revealed maximum possible losses in t_3. USDT is the less risky cryptocurrency in the sample, with a maximum loss of nearly 2%. This may be related to the USDT properties as a stable cryptocurrency. Although the other cryptocurrencies exhibit a similar pattern of maximum possible losses, they display different risk levels. During 2017 and early 2018, multiple bubbles were detected in the cryptocurrency market (Enoksen et al. 2020), which may explain our results.

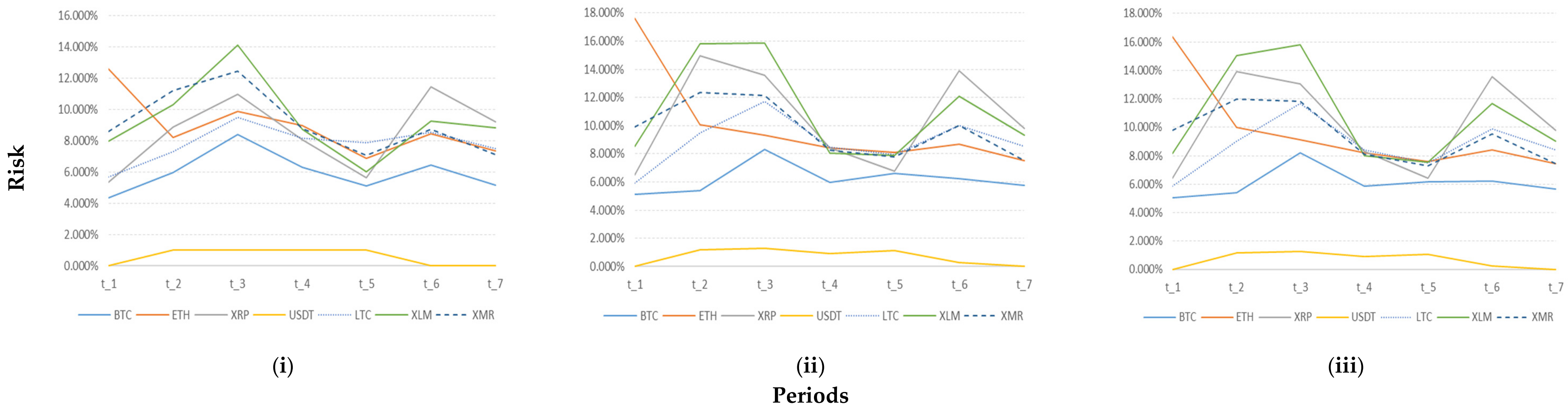


Figure 5. Cryptocurrency VaR(95) evolution. Notes: (**i**) corresponds to nonparametric VaR(95); (**ii**) and (**iii**) correspond to parametric VaR(95), with Normal and Student's *t*-distributions, respectively.

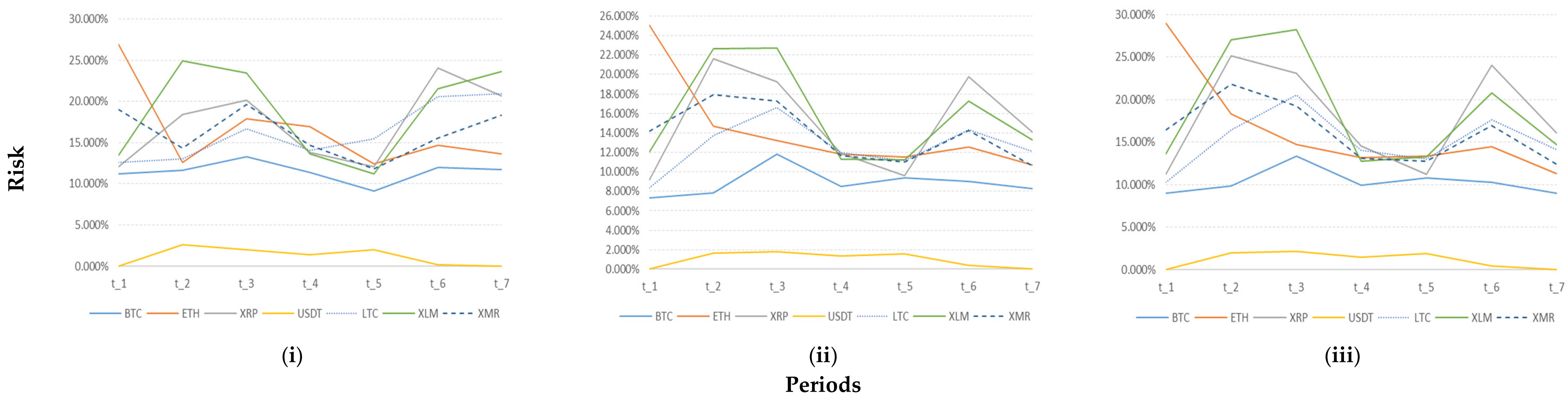


Figure 6. Cryptocurrencies VaR(99) evolution. Notes: (**i**) corresponds to nonparametric VaR(99); (**ii**) and (**iii**) correspond to parametric VaR(99), with normal and Student's *t*-distributions, respectively.

Furthermore, in 2017, the BTC recorded one of its highest gains, followed by a strong devaluation. As this is the oldest cryptocurrency with the highest market capitalization, its inherent high risk could also mean greater risk for investment in altcoins. ETH, the youngest cryptocurrency in the sample (launched at the end of July 2015), displayed the maximum possible losses among the analysed cryptocurrencies. Its maximum possible loss was observed in t_1, the period immediately following its launch, which could mean that investors perceive newer cryptocurrencies as riskier.

In 2020, when the COVID-19 pandemic began, the risk of most cryptocurrencies (except BTC and USDT) decreased. This could indicate they may have safe haven properties when real global shocks occur. Excluding USDT, BTC investment was the least risky, with a worst possible loss between 5.04% and 8.22% (considering parametric volatility modelling and the leptokurtosis effect). VaR(99) was also estimated and, as expected, is higher than VaR(95). However, its pattern is similar to that described above for the VaR(95).

CVaR provides information about the expected average size of a large loss, which is useful for risk managers. CVaR(95) and CVaR(99) estimates (displayed in Figure 7) allow for the quantification of expected average losses in the worst 5% and 1% returns. Figure 7 shows that high expected average losses can occur when they exceed the VaR limits (i.e., losses located in the distribution tail). This evidence is clearer at the tail of the distribution, as is evident when comparing, for example, VaR(99) with CVaR(99) (the risk almost doubles).

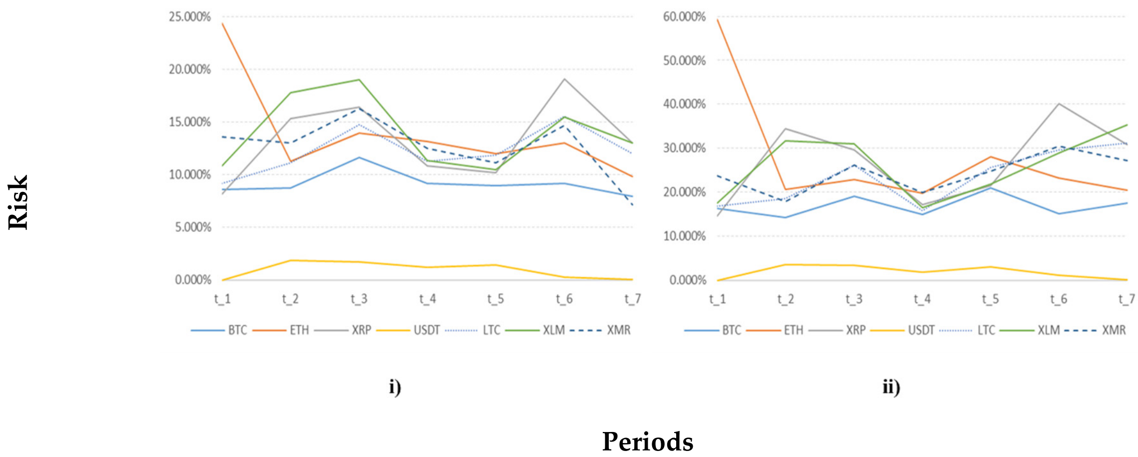
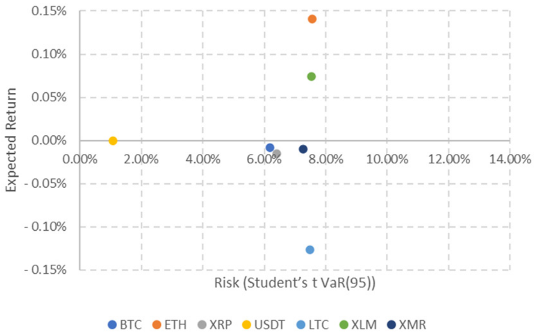


Figure 7. The evolution of the CVaR(95) and CVaR(99) cryptocurrencies. Notes: (**i**) and (**ii**) correspond to CVaR(95) and CVaR(99), respectively.

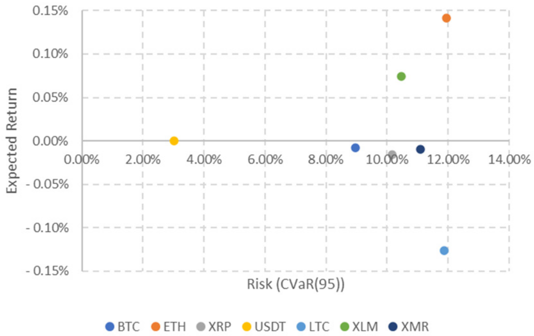
In general, the risk measured by CVaR is similar to that of VaR for the cryptocurrencies in the sample. USDT displayed the lowest average expected loss (less risk), reaching its maximum values in t_2. Excluding USDT, BTC had the lowest average expected loss, while ETH had the highest in the period when it was launched (t_1).

It is also interesting that, considering CVaR(99), the risk of XRP, LTC, XLM, and XMR increases in t_6, while that of BTC and ETH (the cryptocurrencies with the highest trading volume and market capitalization) and USDT (a stable cryptocurrency) decreased in the same period.

In addition to higher returns with lower uncertainty, investors seek higher returns with lower risk. Thus, an analysis of return and risk was carried out. Results are shown in Figure 8 for period t_5 (again, information for the remaining periods is available upon request). In this analysis, the risk is represented by Student's t-VaR(95) in Figure 8i and by CVaR(95) in Figure 8ii. This assessment is of interest because incorrect volatility estimates can significantly impact financial decisions. Underestimating volatility may cause greater risk exposure while overestimating it may lead to missing opportunities. The Student's *t*-VaR(95) was chosen as it is the more accurate risk estimation of those considered. CVaR(95) was chosen because it provides information on the expected average size of a large loss, which is useful for risk management.



(i)



(ii)

Figure 8. Expected return vs. risk (VaR(95) and CVaR(95) in period t_5. Note: $expected\ return = \frac{\sum_{t=1}^{t=n} ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)}{n}$; **(i)** and **(ii)** correspond to the relationship between return vis-à-vis Student-t VaR(95) and CVaR(95), respectively, in period t_5.

In the preliminary results section, a similar analysis was performed, but in that case, the entire distribution was considered. Now, we only consider the tail of the distribution to obtain higher risk levels. Considering the results displayed in Figure 8i, ETH and XLM have the same potential risk level, but ETH has higher expected returns. Thus, the ETH investment seems more attractive as it allows higher expected returns for a similar risk level. ETH exhibits higher returns with the highest level of risk. Considering the results displayed in Figure 8ii, BTC and XMR produce similar returns but with less risk for the former. Thus, the BTC investment is more attractive as it produces similar returns with lower risk.

Considering all the analyzed periods, the relationship between return and risk is similar to that obtained when using the standard deviation as a measure of risk. However, as expected, higher risk levels were now found as extreme events are located in the distributions' tails. This indicates insufficient risk quantification when the standard deviation is used as a risk measure for asymmetric and fat-tailed distributions. For almost all cryptocurrencies, the increased risk is not necessarily associated with increased returns, hinting that cryptocurrency investors are not risk averse (LTC is the exception). USDT was, once again, the least risky cryptocurrency, but with zero average return. BTC is the cryptocurrency that could provide higher returns with lower risk (excluding USDT). Compared to previous periods, in t_5 (a period marked by the beginning of the COVID-19 pandemic), almost all the cryptocurrencies exhibited lower levels of risk (but only XLM and ETH produced positive returns), which could mean they may have possible safe haven properties in periods of turmoil in the financial markets (in line with Corbet et al. 2021). The assessment of the relation between return and risk, in line with that between return and uncertainty, indicates that, although some cryptocurrencies display similar risk levels, they have different average returns. This may also be of interest to inform cryptocurrency portfolio selection strategies.

Uncertainty and risk are, as mentioned before, distinct concepts and are assessed using different techniques. However, they are both of interest to investors. Table 4 summarizes the results of our analysis, to show the potential variation of risk and uncertainty. In Table 4, each Δt corresponds to each indicator variation from one period to the next: $\Delta 1 = t\_2 - t\_1$, … $\Delta 6 = t\_7 - t\_6$.

Table 4. Uncertainty and risk evaluation synthesis.

			Cryptocurrency																							
			BTC						ETH						XRP						USDT					
			D1	D2	D3	D4	D5	D6	D1	D2	D3	D4	D5	D6	D1	D2	D3	D4	D5	D6	D1	D2	D3	D4	D5	D6
Uncertainty	**Entropy**		▼	▲	▼	▲	▼	▲	▲	▲	▲	▼	▼	▲	▲	▲	▼	▲	▼	▼	▲	▲	▼	▼	▼	▲
Risk	**VaR(95)**	**Empirical**	▲	▲	▼	▼	▲	▼	▼	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▲	▲	▲	▼	▲
		Normal	▲	▲	▼	▲	▼	▼	▼	▼	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼	▲	▲	▼	▲	▼	▼
		Student's t	▲	▲	▼	▲	▼	▼	▼	▼	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼	▲	▲	▼	▲	▼	▼
	Var(99)	**Empirical**	▲	▲	▼	▼	▲	▼	▼	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▼	▼	▲	▼	▼
		Normal	▲	▲	▼	▲	▼	▼	▼	▼	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼	▲	▲	▼	▲	▼	▼
		Student's t	▲	▲	▼	▲	▼	▼	▼	▼	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼	▲	▲	▼	▲	▼	▼
	CVaR(95)		▲	▲	▼	▼	▲	▼	▼	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▼	▼	▲	▼	▼
	CVaR(99)		▼	▲	▼	▲	▼	▲	▼	▲	▼	▲	▼	▼	▲	▼	▼	▲	▲	▼	▲	▼	▼	▲	▼	▼

			Cryptocurrency																	
			LTC						XLM						XMR					
			D1	D2	D3	D4	D5	D6	D1	D2	D3	D4	D5	D6	D1	D2	D3	D4	D5	D6
Uncertainty	**Entropy**		▼	▲	▼	▲	▼	▲	▲	▲	▼	▲	▼	▼	▼	▼	▲	▲	▼	▲
Risk	**VaR(95)**	**Empirical**	▲	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼
		Normal	▲	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼
		Student's t	▲	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼
	Var(99)	**Empirical**	▲	▲	▼	▲	▲	▲	▲	▼	▼	▼	▲	▲	▼	▲	▼	▼	▲	▲
		Normal	▲	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼
		Student's t	▲	▲	▼	▼	▲	▼	▲	▲	▼	▼	▲	▼	▲	▼	▼	▼	▲	▼
	CVaR(95)		▲	▲	▼	▲	▲	▼	▲	▲	▼	▼	▲	▼	▼	▲	▼	▼	▲	▼
	CVaR(99)		▲	▲	▼	▲	▲	▲	▲	▼	▼	▲	▲	▲	▼	▲	▼	▲	▲	▼

The main results are summarized as follows: (i) when the COVID-19 pandemic began ($\Delta 5$), most cryptocurrencies displayed a negative variation of uncertainty (entropy) and positive variations of the risk metrics (VaR and CVaR): (ii) BTC and LTC, and XRP and XLM exhibited similar potential uncertainty variations, although with some differences between the two pairs. Such differences may be because the cryptocurrencies in the first pair are considered pure financial assets, and those in the second share features of the service platforms.

5. Conclusions

This study assessed uncertainty and risk associated with cryptocurrency investments and related both with the respective returns. The analysis considered seven cryptocurrencies from August 2015 to October 2021. Our main contributions are a broader and methodologically more robust analysis of volatility, uncertainty and risk. By focusing attention on diverse cryptocurrencies, we developed a novel analysis of the cryptocurrency market and produced results that add to current knowledge about the dynamic behaviour of these assets.

The cryptocurrency market is increasingly relevant due to the growing importance of cryptocurrencies as new investment options. The fact that our analysis covers a period of great turmoil in financial markets, encompassing the first major real crisis since the creation of cryptocurrencies, provides new insights about their behavior in periods of instability with non-financial origin.

Uncertainty and risk are distinct concepts. Although there may be periods in which higher levels of uncertainty are associated with higher levels of risk (and vice-versa), such an association does not always prevail in the cryptocurrency market, thus highlighting the importance of a joint analysis like the one performed in this study. BTC and LTC, the oldest cryptocurrencies in the sample, revealed similar patterns regarding uncertainty and risk, suggesting that maturity may be a relevant factor in this context. XRP and XLM also behave similarly concerning uncertainty and risk, which may be explained by a common process of price formation. USDT displayed a different behaviour, which may be justified by the fact that each USDT is guaranteed by one USD kept in a legal deposit by Tether Limited (Griffin and Shams 2020). However, there is no clear evidence of that (Vukovic et al. 2021). In line with Mokni et al. (2022), all the assessed cryptocurrencies reached maximum uncertainty levels between August 2019 and August 2020 (except USDT, which reached it between August 2017 and August 2018), probably a consequence of the financial instability that followed the onset of the COVID-19 pandemic. Until 2020, and not considering USDT, BTC displayed the lowest level of uncertainty, while ETH exhibited the highest. In this case, BTC investments were the least uncertain and risky and the best option for uncertainty and risk-averse investors.

Regarding the analysis of return and uncertainty, USDT was the cryptocurrency with the least uncertainty, although with zero returns. From a portfolio diversification perspective, USDT does not provide any advantage vis-à-vis the risk-free asset. Despite their similar uncertainty patterns, XRP, XMR and ETH provided the highest returns, suggesting that they are attractive to investors that are not uncertainty averse.

Regarding risk, both metrics considered (VaR and CVaR) produced positive values, which means that investing in cryptocurrencies entailed the possibility of losses in the considered time frame. As expected, CVaR estimations were more conservative for all the cryptocurrencies than the VaR ones. All the cryptocurrencies (except USDT) revealed similar behaviour regarding the evolution of the maximum possible loss, despite having different risk levels.

Considering the return vs. risk analysis, ETH, XLM, and XRP may be an option for non-risk-averse investors. In contrast, USDT was the least risky cryptocurrency, although generating null returns. Thus, this could indicate that there are diversification possibilities between cryptocurrencies, and that, in real contexts of crises, USDT may be a safe haven for investors.

For simultaneously non-uncertainty-averse and non-risk-averse investors, investing in ETH and XRP could be the best options.

Our results have relevant implications for investors, risk managers, policymakers and regulators. The first two can use the information for fund allocation, portfolio management strategies and diversification purposes. Policymakers and regulators can use these results to help inform policy design and contribute to this market's stabilization, reducing its volatility and increasing investors' confidence.

Future research could complement this analysis with an evaluation of the impact of another global non-financial crisis—the Russian invasion of Ukraine—on the cryptocurrency market. This event was not considered in our study because the still-scarce number of observations available would compromise the robustness of the estimates. Nevertheless, the occurrence of two large real shocks (a pandemic and a war in Europe) in a relatively short period justifies further impact assessments in this and other financial markets.

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