Journal of
Risk and Financial Management

MDPI

Communication

A Balanced Portfolio Can Have a Higher Geometric Return Than the Risky Asset †

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† We thank Jerry Hausman, Scott Cederburg, and Ming-sen Wang for helpful comments.

Abstract: In the U.S., the geometric return on stocks has been higher than the geometric return on bonds over long periods. We study whether balanced portfolios have a larger geometric return (and expected log return) than stock portfolios when the risk premium is low. We use a theoretical model and historical data and find that this is the case. This low-risk premium is often observed in other developed countries. Further, in the past two decades, a balanced portfolio with 70% or 90% invested in the U.S. stock market (with the remainder invested in U.S. government bonds) performed better than a 100% stock or bond portfolio. The reason for this is that a pure stock portfolio loses a large fraction of its value in a downturn. We show that this result is not driven by outliers, and that it occurs even when the returns are log normally distributed. This result has broad policy implications for the construction of pension systems and target-date mutual funds.

Keywords: balanced portfolio; stock return; bond return; recession

JEL Classification: E4; G1; G5

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Citation: Arden, Miriam, and Tiemen Woutersen. 2021. A Balanced Portfolio Can Have a Higher Geometric Return Than the Risky Asset. *Journal of Risk and Financial Management* 14: 409. https://doi.org/10.3390/jrfm14090409

Academic Editor: Waël Louhichi

Received: 21 July 2021
Accepted: 26 August 2021
Published: 1 September 2021

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1. Introduction

In *Stocks for the Long Run*, Jeremy Siegel documents that the geometric return on stocks has been historically higher than the geometric return on bonds.[1] In accordance with this, in the last two decades, a stock portfolio had a higher geometric return than a bond portfolio in the U.S. However, a balanced portfolio with 70% or 90% invested in stocks (with the remainder invested in U.S. government bonds) performed better than a 100% stock or bond portfolio in the last two decades.[2] This paper shows that balanced portfolios have a larger geometric return (and expected log return) than pure stock or bond portfolios when the risk premium is low. The reason for this is that a pure stock portfolio can lose a large fraction of its value in a downturn. We show that this result is not driven by outliers and that it occurs even when the returns are log normally distributed.

Historically, the equity premium in the U.S. has been large (Mehra and Prescott 1985). However, the U.S. return history is short. As such, it is useful to examine the stock returns in other developed countries. For example, the Nikkei 225 Index produced real returns of −21% from 1990 to 2019.[3] This result is not rare; other developed countries have experienced worse stock market performance or even market failure (Jorion and Goetzmann 1999). In fact, evidence from a sample of stock returns from developed countries indicates a 12.1% probably of loss in buying power over a 30-year horizon (Anarkulova et al. 2020). Therefore, a low-risk premium is more likely to occur than the U.S. stock market data might suggest.

2. Literature Review

The literature on stock market returns is large and shows that the equity premium can be low. For example, Fama and French (1998) and Asness et al. (2013) study global equity markets. Schwert (2003), Jorion and Goetzmann (1999); Dimson et al. (2002); McLean and Pontiff (2016); and Linnainmaa and Roberts (2018) explore average stock market returns and equity premiums across countries, while Harvey et al. (2021) and Shirvani et al. (2021) estimate the equity premium using current econometric techniques. This paper contributes that a balanced portfolio can have a larger geometric return (and expected log return) than a pure stock portfolio. This result has broad policy implications for the construction of pension systems (Shiller 2003, 2006; Poterba et al. 2007) and target-date mutual funds (Viceira 2008; Pástor and Stambaugh 2012; Mitchell and Utkus 2021).

3. Portfolio Returns: Theoretical Model

Bodie et al. (2017) argue that stock returns are approximately log normally distributed. In our data, this is particularly true for balanced portfolios. There are several motivations to use geometric returns. First, maximizing geometric returns is equivalent to maximizing expected utility for an individual with a logarithmic utility function. Mutual funds also publish their geometric returns.[4] Finally, it is a simple and historically popular approach.

Let the safe asset, such as government bonds, have expected return ρ. Let stocks have expected return $\rho + \omega$, where ω is the risk premium. Let the portfolio have expected return $E\{R(\kappa)\} = 1 + \rho + \kappa\omega$, where κ is the fraction of the portfolio invested in stocks. The variance of the portfolio when invested in stocks and 10-year government bonds is

$$(1-\kappa)^2 {\sigma^2}_{bond} + \kappa^2 {\sigma^2}_{stock} + 2\kappa(1-\kappa)\sigma_{bond,\, stock}, \tag{1}$$

where ${\sigma^2}_{bond}$ is the variance of the bond return, ${\sigma^2}_{stock}$ is the variance of the stock return, and $\sigma_{bond,\, stock}$ is the covariance of the bond and stock return[5].

We also assume that the log return is normally distributed with the mean and variance implied by the regular return. Let $\mu(\kappa)$ denote the mean of the log normal return as a function of κ, and let $\sigma(\kappa)^2$ denote the variance of the log normal return as a function of κ. This implies that

$$1 + E\{R(\kappa)\} = e^{\mu(\kappa) + \frac{\sigma(\kappa)^2}{2}}. \tag{2}$$

This also implies that

$$var\{R(\kappa)\} = \left(e^{\sigma(\kappa)^2} - 1\right)\left(e^{2\mu(\kappa) + \sigma(\kappa)^2}\right). \tag{3}$$

Combining these terms and the above variance of the portfolio yields,

$$\mu(\kappa) = \ln(1 + E\{R(\kappa)\}) - \frac{1}{2}\ln\left(1 + \frac{(1-\kappa)^2 {\sigma^2}_{bond} + \kappa^2 {\sigma^2}_{stock} + 2\kappa(1-\kappa)\sigma_{bond,\, stock}}{(1 + E\{R(\kappa)\})^2}\right). \tag{4}$$

The last formula shows that the geometric return decreases in the variance of the portfolio. In the next section, we use the historical variation of bond and stock portfolios to show that the geometric return is maximized at a kappa that is smaller than one, i.e., the geometric return is maximized by a balanced portfolio rather than a portfolio that is fully invested in stocks.

4. Portfolio Returns: Empirical Results

We consider yearly returns. For the bond portion of the portfolio, we assume that the investor holds a 10-year government bond for one year at a time (i.e., the return is the one-year return of a 10-year government bond). Using the historical variance and/covariance of the S&P 500 returns and the 10-year government bond returns[6] from the last 50 years (1970–2019), we find that a balanced portfolio of 90% stocks and 10% bonds always has a higher geometric return if the equity premium is 2% or lower. We also find that a balanced

portfolio with 70% stocks and 30% bonds does better than the pure stock portfolio in some cases, i.e., when $\rho = -1\%$ and $\rho = 0\%$, and it never does worse. This includes cases where the geometric return of stocks is larger than the geometric return of bonds, as in Siegel (2014). These low values of the risk-free rate are particularly relevant in the current environment, as the real interest on Treasury Inflation-Protected Securities is negative. The 60% stock and 40% bonds portfolio always had a lower geometric return than the pure stock portfolio. When the equity premium is large, i.e., 3% or higher, then a 100% stock portfolio performs better than a balanced portfolio, see Table 1 and Figure 1 for details.

Table 1. Log Returns, Risk Premiums, and Asset Allocation.

Expected Return Safe Asset	Risk Premium	μ for $\kappa = 0.6$	μ for $\kappa = 0.7$	μ for $\kappa = 0.9$	μ for $\kappa = 1$
−0.01	0	−0.0159	−0.0173	−0.0212	−0.0237
−0.01	0.01	−0.0097	−0.0101	−0.0120	−0.0134
−0.01	0.02	−0.0037	−0.0030	−0.0028	−0.0032
−0.01	0.03	0.0024	0.0040	0.0063	0.0069
−0.01	0.04	0.0084	0.0110	0.0153	0.0169
−0.01	0.05	0.0143	0.0179	0.0242	0.0268
0	0	−0.0057	−0.0071	−0.0110	−0.0134
0	0.01	0.0004	0.0000	−0.0018	−0.0032
0	0.02	0.0064	0.0070	0.0073	0.0069
0	0.03	0.0123	0.0140	0.0163	0.0169
0	0.04	0.0183	0.0209	0.0252	0.0268
0	0.05	0.0242	0.0278	0.0340	0.0366
0.01	0	0.0044	0.0030	−0.0008	−0.0032
0.01	0.01	0.0104	0.0100	0.0083	0.0069
0.01	0.02	0.0163	0.0169	0.0172	0.0169
0.01	0.03	0.0222	0.0238	0.0261	0.0268
0.01	0.04	0.0281	0.0307	0.0350	0.0366
0.01	0.05	0.0340	0.0375	0.0437	0.0463
0.02	0	0.0143	0.0130	0.0093	0.0069
0.02	0.01	0.0203	0.0199	0.0182	0.0169
0.02	0.02	0.0262	0.0268	0.0271	0.0268
0.02	0.03	0.0320	0.0336	0.0359	0.0366
0.02	0.04	0.0378	0.0404	0.0447	0.0463
0.02	0.05	0.0436	0.0472	0.0533	0.0559

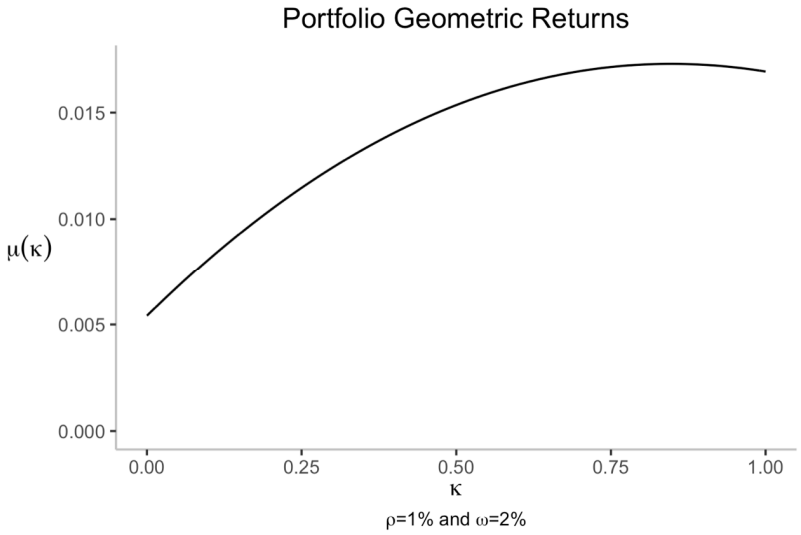


Figure 1. Portfolio Geometric Returns.

The higher geometric return of the 90% and 70% stock (with the remainder invested in U.S. government bonds) portfolios means that a pure stock portfolio cannot first order stochastically dominate these balanced portfolios, not even in the long run. Further, the only reason that these higher geometric returns do not imply first order stochastic dominance of the pure stock portfolio is that the balanced portfolios have lower variances than the pure stock portfolio.

5. Discussion and Conclusions

In *Stocks for the Long Run*, Jeremy Siegel documents that the geometric return on stocks has been historically higher than the geometric return on bonds. However, stocks lose a large fraction of their value in a downturn. This is relevant for the U.S. as stocks lost a lot of value during the Great Financial Crisis. Relatedly, a balanced portfolio, with 70% or 90% invested in stocks (with the remainder invested in bonds), performed better than a 100% stock or bond portfolio in the last two decades. This occurred despite the fact that a stock portfolio had a higher geometric return than a bond portfolio in the last two decades. This paper shows that balanced portfolios have a larger geometric return (and expected log return) than pure stock or bond portfolios when the risk premium is low. The reason for this is that a pure stock portfolio loses a larger fraction of its value in a downturn than the balanced portfolio. We show that this result is not driven by outliers, and that it occurs even when the returns are log normally distributed. The results are relevant to the construction of pension systems and target-date mutual funds. Our results complement those by Mitchell and Utkus (2021), who document the importance of target date funds being low-cost funds.

Author Contributions: Conceptualization, M.A. and T.W.; methodology, M.A. and T.W.; software, M.A.; validation, M.A. and T.W.; formal analysis, M.A. and T.W.; investigation, M.A. and T.W.; resources, M.A. and T.W.; data curation, M.A.; writing—original draft preparation, M.A. and T.W.; writing—review and editing, M.A. and T.W.; visualization, M.A. and T.W.; supervision, M.A. and T.W.; project administration, M.A. and T.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data is public and the source is mentioned in the article.

Conflicts of Interest: The authors declare no conflict of interest.

Notes

1 (Siegel 2014), Figure 5-1 and Figure 5-4.

2 The total (geometric) return of the S&P 500 was 3.73% for 2000–2019. For a 90/10 portfolio with 10-year government bonds, it was 3.92%. For a 70/30 portfolio, it was 4.11%.

3 Bloomberg: https://www.bloomberg.com/quote/NKY:IND (accessed on 10 January 2020).

4 See morningstar.com (accessed on 10 January 2020).

5 In our data, the covariance of the bond and stock return is 0.0005, and the correlation is 0.033.

6 We report the full results in Table 1 and used data from the NYU Stern School of Business, http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html (accessed on 10 January 2020).

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