

Article

Evaluating the Unconventional Monetary Policy of the Bank of Japan: A DSGE Approach

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Abstract: When the nominal interest rate reaches the zero lower bound (ZLB), a conventional monetary policy, namely, the adjustment of short-term interest rate, may become impractical and ineffective for central banks. Therefore, quantitative easing (QE) is one of the few available policy options of central banks for stimulating the economy and dealing with deflationary pressure. Since February 1999, the Bank of Japan (BoJ) has conducted several unconventional monetary policy programs. Considering the scarce research in this field from a structural macroeconomic model approach, a medium-scale New Keynesian DSGE model with government bonds of different maturities was developed to check the portfolio rebalancing channel of quantitative qualitative easing (QQE) conducted by the BoJ from April 2013 on the basis of the assumption of imperfect asset substitutability. The model was calibrated on the basis of the structure of the Japanese economy in April 2013. The main conclusion is that the BoJ's asset purchase has a real effect on pushing output and inflation higher, and long-term interest rates lower. Sensitivity simulation analysis shows that, given the same size of asset purchase, the persistence of asset purchase determines the peak effect in the short run. A long-lasting asset purchase can push up inflation higher, and long-term interest rates lower for a relatively longer period, but the long-run effect on output and investment does not have much difference. The policy implication for BoJ is just to announce a long-lasting QE program and make it credible to the market.

Keywords: Bank of Japan; unconventional monetary policy; quantitative easing; DSGE; asset purchase

JEL Classification: E43; E44; E52; E58



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1. Introduction

For almost 15 years, from February 1999, when the Bank of Japan (BoJ) announced the commitment to the zero interest rate policy (ZIRP), to April 2013, when it started quantitative qualitative easing (QQE), it implemented unconventional monetary policy. During the recent global financial crisis, many advanced economies had to depart from conventional ways of conducting monetary policy, as they faced the zero lower bound (ZLB) and systemic risk. The importance of an unconventional monetary policy has been realized by macroeconomists and central banks both theoretically and practically. [Krugman et al. \(1998\)](#), [Svensson \(2003\)](#), and [Bernanke et al. \(2004\)](#) are the early contributors in this field. Unconventional monetary policies can take many forms besides those that are generally publicly recognized. For example, during the global financial crisis, the Danish National Bank permitted the use of a negative interest rate policy (NIRP). Generally, as a main option of unconventional monetary policy, quantitative easing (QE) can be defined as the change in the composition and size of the central bank's balance sheet. The change can be the result of the large-scale asset purchase of private assets or government bonds, and it can also occur through direct lending or capital injection from the central bank to the private sector or the financial system.

[Joyce et al. \(2012\)](#) comprehensively introduced the QE conducted by the Federal

Reserve (Fed), the Bank of England (BoE), and the European Central Bank (ECB) with a theoretical background of unconventional monetary policy. In the United States, from December 2008 to the end of 2009, the Fed conducted the first phase of QE (QE1—officially large-scale asset purchases (LSAPs)) by expanding its portfolio assets to provide liquidity to the financial system and reduce the risk premium. Following QE1, QE2 lasted from October 2010 to June 2011 and was conducted by the Fed through the large purchase of US treasury securities. Bernanke also announced the purchase of mortgage-backed securities (MBS) in September 2012, which is known as QE3, with the objective of pushing down the long-term yield curve to support financial system reconstruction and stimulate aggregate demand. During the same period, in the UK, the BoE began a QE program by establishing the Asset Purchase Facility (APF), the operations of which are conducted by purchasing medium- and long-term UK government bonds.

Related studies on unconventional monetary policy mostly focused on empirical evidence for the policy effect of asset purchase on financial markets, especially through event study¹ or time-series regression², to check the effect on domestic yields such as the 10-year treasury yield or 30-year MBS yield³. Compared to empirical analysis focusing on the impact of asset purchase on financial markets, there are relatively few studies that investigate the impact of asset purchase on the macroeconomy. Generally, there exist three approaches in this field. The first approach does not rely on macroeconomic theory, but on statistical causality investigated by regression or nonstructural VAR methodology. Consequently, empirical results obtained from such a nonstructural approach are not stable and robust among the different choices of regressors or identification schemes. The second approach is theoretical evaluation with a dynamic stochastic general equilibrium (DSGE) macroeconomic model that has the specified transmission mechanism of unconventional monetary policy. Typical works⁴ using this approach are [Gertler and Karadi \(2011, 2013\)](#), [Curdia and Woodford \(2011\)](#) and [Chen et al. \(2012\)](#). Most only studied the transmission mechanism of unconventional monetary policy without empirically estimating the model because of the technical difficulties incurred by ZLB. The third approach, which lies somewhere between the two previous approaches, is to follow a two-step procedure. The first step is to measure the policy effect of asset purchase on the nominal interest rate; the second step is to use the interest rate as a proxy to evaluate the policy effect on macroeconomic variables with a traditional macroeconomic model⁵ such as the FRB/US model. It is more complicated to empirically evaluate the effect of forward guidance because forward guidance affects the future expectation of the public and the market. Unlike asset purchase, there is no consensus about the policy effect of forward guidance. The guidance may not be fully understood by market. Even if understood, the guidance may not be fully believed for the longer term given the possibility that the central banks may not be able to guarantee the consistency of future decisions beyond a shorter horizon.

Recently, as a good measure of the stance of monetary policy, shadow interest rate has been used in many works to find the empirical evidence of unconventional monetary policy. [Wu and Xia \(2016\)](#) and [Krippner \(2013\)](#) are two representative works in this field. [Wang \(2019a, 2019b\)](#) showed that the shadow rate can be used in the estimation of structural DSGE models. The advantage of the shadow rate is that it is a consistent measure of the stance of monetary policy, both in zero- and nonzero-interest-rate environments. When using structural macroeconomic models, the advantage of the shadow rate allows for not needing to specify the detailed transmission mechanism. The shadow rate itself contains necessary information about the stance of unconventional monetary policy. As an example, [Huber and Punzi \(2020\)](#) used a time-varying parameter vector autoregression (TVP-VAR) model to investigate the relationship between unconventional monetary policy and housing markets in advanced economies. Their findings suggested that the monetary policy transmission mechanism to the housing market did not change with the implementation of quantitative easing or forward guidance for most economies such as the US, the EU, and Japan.

In Japan, the first phase of QE started 15 years ago, beginning in March 2001 until March 2006. After a pause in operations from April 2006 to September 2010, the QE program, known as the comprehensive easing policy, was restarted from October 2010 and lasted until March 2013. Its purpose was to stimulate the real economy and protect the financial system from the global financial crisis by purchasing a variety of assets, including commercial papers (CP), exchange traded funds (ETF) and Japan real estate investment trusts (J-REITs). With the advent of the BoJ's new president, Haruhiko Kuroda, the new stage of QE, known as quantitative qualitative easing (QQE), began with a more aggressive scale of balance sheet expansion and with more varieties of asset purchases than those in the past. QQE was positioned as one of Abenomics' three arrows. At the same time, BoJ clearly declared a 2% inflation target to shape the formation of expectations. Since the start of QQE in April 2013, two and a half years passed. It is still ongoing, so a comprehensive evaluation and final conclusion about QQE may be inappropriate at this time. However, we still recognize the significance of a temporary evaluation of QQE⁶. Related works about BoJ's QE generally take a nonstructural approach, including VAR analysis or event study to obtain empirical evidence about the effectiveness of QE. Especially in VAR analysis, as surveyed by Ugai (2007), different choices of variables and specifications of models lead to different results. In contrast to the nonstructural econometric approach, the DSGE framework has inherent advantages for policy evaluation. The transmission mechanism of monetary policy can be identified with a clear explanation on the basis of economic theory. In addition, to the best of our knowledge, no trials have been conducted in this area. For these reasons, we conducted an empirical project to evaluate the QQE of BoJ by the calibration and numerical simulation of the DSGE model in this study. Since September 2016, the BoJ started a new framework of monetary policy, which is known as the price stability target of 2 percent, and quantitative and qualitative monetary easing with yield curve control. In this framework, the BoJ also allows for NIRP. Fukuda (2018) investigated the impact of the BoJ's NIRP on Asian financial markets and showed that it might have benefitted Asian economies through the positive effects on Asian stock prices. Angrick and Nemoto (2017) provided an overview of the operational implementation of NIRP in Europe and Japan.

Among existing studies, few used DSGE model methodology to study the empirical effect of the BoJ's QE. In this paper, we develop a model and focus on the portfolio rebalancing mechanism of QE to study the effect of the QE conducted by the BoJ since April 2013. Given the fact that the model was calibrated on the basis of the structure of the Japanese economy in April 2013, there are limitations to the conclusion, which is based on that period. The main conclusion is that, through the portfolio rebalancing mechanism, the BoJ's asset purchase had a stimulative effect on the Japanese economy. However, the policy options for BoJ may be limited because, based on sensitivity analysis, if the central bank wishes to improve the effectiveness of asset purchase, a larger scale and longer period are the only two options. For the situation of the BoJ, it is better to announce a longer period of QE policy to achieve a long-lasting effect.

The remainder of the study is organized as follows. Section 2 describes the derivation of the model. Section 3 is the calibration of the model's parameters and the steady state. Section 4 presents the results of simulation with sensitivity analysis. Section 5 concludes the study.

2. Model

Meier (2009) noted that there are different approaches to unconventional monetary policy, that can be motivated by alternative views of the transmission channels and their effect on the economy. The model developed here has the standard structure and specification of the new Keynesian DSGE model, but the bond trading market proposed by Lars and Sargent (2012, Chapter 13, Section 8) was incorporated to isolate the portfolio rebalancing mechanism of large asset purchases by the central bank. Tobin (1969) initially described this mechanism, whereby variation in the relative supplies of financial assets

with different maturities and liquidities triggered by large asset purchases of the central bank can have a real effect on the yield curve due to imperfect asset substitutability. Tobin and Brainard (1963) define the imperfect substitution assumption as follows:

Assets are assumed to be imperfect substitutes for each other in wealth-owners' portfolios. That is, an increase in the rate of return on any one asset leads to an increase in the fraction of wealth held in that asset, and to a decrease or at most no change in the fraction held in every other asset.

Relating this assumption to unconventional monetary policy, the basic idea is that the central bank's purchase of assets held by the private sector increases the price of these assets. As asset prices increase, yields fall, stimulating aggregate demand. Even when the short-term nominal interest rate faces ZLB, asset purchases can be a practical policy instrument for the central bank. Large-scale purchases of government bonds by BoJ can be evaluated using this approach in a dynamic stochastic general equilibrium framework.

2.1. Household

There is a continuum of representative households existing continuously in $i \in (0, 1)$ where i is the indexation⁷. The representative household derives utility from consumption C_t , and real money balance $\frac{M_t}{P_t}$ and disutility from labor supply L_t . The utility function is additively separable:

$$U_t = \frac{(C_t - \theta C_{t-1})^{1-\sigma}}{1-\sigma} + \frac{1}{1-\xi} \left(\frac{M_t}{P_t} \right)^{1-\xi} - \frac{\eta_L}{1+\chi} L_t^{1+\chi}$$

where σ is the inverse of the elasticity of intertemporal substitution, θ is the degree of habit formation, ξ is the interest rate semielasticity of money demand, and χ is the inverse of the Frisch elasticity of labor supply. η_L is a preference parameter that measures the relative weight of disutility from labor supply. The household maximizes the discounted infinite stream of utility $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_t^u} U_t \left(C_t, \frac{M_t}{P_t}, L_t \right)$ subject to intertemporal budget constraint

$$\frac{B_{S,t}}{P_t R_{S,t}} + \frac{B_{L,t}^H (1 + AC_{B,t})}{P_t R_{L,t}} + \frac{M_t}{P_t} + I_t (1 + AC_{K,t}) = \frac{B_{S,t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_{S,t}} + \frac{M_{t-1}}{P_t} + w_t L_t + q_t K_t - C_t - T_t \quad (1)$$

and the standard law of motion of capital accumulation.

$$K_t = I_t + (1 - \delta) K_{t-1} \quad (2)$$

ε_t^u is a preference shock process following

$$\varepsilon_t^u = \rho_u \varepsilon_{t-1}^u + \mu_t^u \quad (3)$$

and $\mu_t^u \sim N(0, \sigma_u^2)$ is an i.i.d. exogenous shock.

The household allocates wealth among real money holdings $\frac{M_t}{P_t}$, capital K_t with rental rate q_t and two types of government bonds⁸, short-term bonds $B_{S,t}$, of which the maturities are equal to or shorter than 1 year with yield $R_{S,t}$, and long-term bonds $B_{L,t}^H$ ⁹, of which the maturities are equal to or longer than 10 years with yield $R_{L,t}$. The household supplies labor L_t , receives real wages w_t , and pays a real lump-sum tax T_t at general aggregate price level P_t . Investment I_t and capital accumulation processes occur with adjustment cost $AC_{K,t} = \frac{\varphi_K}{2} \left(\frac{I_t}{K_t} \right)^2$ and the portfolio adjustment between two kinds of bonds also accompanies cost $AC_{B,t} = \frac{\varphi_B}{2} \left(\kappa_B \frac{B_{S,t}}{B_{L,t}^H} - 1 \right)^2 Y_t$, where κ_B is the steady state ratio of long-term bond holdings of the household to short term bond holdings $\frac{B_{L,t}^H}{B_{S,t}}$; so, at the steady state, the portfolio is adjusted to its optimal allocation and adjustment cost, which is paid in terms of the household's income of zero.

The first-order conditions of the household's maximization with respect to consumption C_t , labor supply L_t , real money $\frac{M_t}{P_t}$, short-term bond $B_{S,t}$, long-term bond $B_{L,t}$, capital K_t , and investment I_t are given as follows.

$$e^{\varepsilon_t^u} (C_t - \theta C_{t-1})^{-\sigma} - \beta \theta \mathbb{E}_t e^{\varepsilon_{t+1}^u} (C_{t+1} - \theta C_t)^{-\sigma} = \lambda_t \quad (4)$$

$$e^{\varepsilon_t^u} \eta_L L_t^\chi = \lambda_t w_t \quad (5)$$

$$e^{\varepsilon_t^u} (m_t)^{-\xi} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\Pi_{t+1}} = \lambda_t \quad (6)$$

$$\beta \mathbb{E}_t \frac{\lambda_{t+1}}{\Pi_{t+1}} = \frac{\lambda_t}{R_{S,t}} + \frac{\kappa_B \varphi_B \lambda_t Y_t}{R_{L,t}} \left(\kappa_B \frac{b_{S,t}}{b_{L,t}^H} - 1 \right) \quad (7)$$

$$\mathbb{E}_t \frac{\beta \lambda_{t+1}}{\Pi_{t+1} R_{S,t+1}} - \frac{\lambda_t}{R_{L,t}} = \frac{\varphi_B \lambda_t Y_t}{2 R_{L,t}} \left(\kappa_B \frac{b_{S,t}}{b_{L,t}^H} - 1 \right)^2 - \frac{\kappa_B \varphi_B \lambda_t Y_t b_{S,t}}{R_{L,t} b_{L,t}^H} \left(\kappa_B \frac{b_{S,t}}{b_{L,t}^H} - 1 \right) \quad (8)$$

$$\beta (1 - \delta) \mathbb{E}_t \mu_{t+1} = \mu_t - \lambda_t \left[q_t + \varphi_K \left(\frac{I_t}{K_t} \right)^3 \right] \quad (9)$$

$$\beta \mathbb{E}_t \mu_{t+1} = \lambda_t \left[1 + \frac{3 \varphi_K}{2} \left(\frac{I_t}{K_t} \right)^2 \right] \quad (10)$$

λ_t and μ_t are two Lagrangean multipliers corresponding with budget constraints and the law of motion of capital accumulation, respectively. $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the gross inflation rate at the $t + 1$ period. For convenience, bonds and money are rewritten in real terms $b_{L,t}^H = \frac{B_{L,t}^H}{P_t}$, $b_{S,t} = \frac{B_{S,t}}{P_t}$ and $m_t = \frac{M_t}{P_t}$ in lowercase letters.

Now we discuss the adjustment cost of the portfolio introduced above. There are necessary conditions under which the purchase of private sector assets or government securities by the central bank can be effective. As discussed by [Eggertsson and Woodford \(2004\)](#), if representative agents who have rational expectations with an infinite time horizon and face no credit frictions or restrictions consider assets held by the government and by the central bank to be indistinguishable from assets held by themselves, then asset purchases by the central bank change nothing. This proposition is analogous to Ricardian equivalence in fiscal theory. However, if credit or financial frictions and borrowing constraints exist, then this proposition no longer holds. In [Curdia and Woodford \(2011\)](#), an unconventional monetary policy, direct facility lending from the central bank to the private sector (credit easing), affected the aggregate economy. [Kiyotaki and Moore \(2012\)](#) described a monetary economy with the heterogeneous liquidity of financial assets. In their model, when entrepreneurs wanted to undertake new investment projects, they could only finance a limited proportion by issuing new equities. Therefore, purchases of such less-liquid equities by the central bank could change their prices, leading to real effects on investment decisions. This is the credit channel of QE. [Gertler and Kiyotaki \(2010\)](#), and [Gertler and Karadi \(2011\)](#) also contributed to this area. The framework in the above-mentioned research is highly complicated, as it includes the full sketch of financial intermediaries or the banking sector. In this study, we focus only on the portfolio-rebalancing channel of QE. This approach is more appropriate for the QE implemented by the BoJ¹⁰. [Falagiarda and Marzò \(2012\)](#), [Zagaglia \(2013\)](#), [Falagiarda \(2014\)](#), and [Chen et al. \(2012\)](#) took the same approach to evaluating the QE of the Fed and the BoE. The rationale for including portfolio-adjustment frictions was intuitional. As mentioned by [Falagiarda \(2014\)](#), long-term bond holdings have less liquidity. Households realize this risk and hold short-term bonds as precautionary liquidity holdings relative to their longer-term investments. Another justification for this adjustment cost comes from the theory of preferred habit. [Vayanos and Vila \(2009\)](#) emphasised that agents prefer different bond maturities, and any deviation from the preferred portfolio allocation is costly. More simply, the management of the portfolio itself is costly.

2.2. Firm

In the same way as in the standard new Keynesian DSGE models, final-goods firms produce homogeneous final goods by bundling differentiated intermediate goods with CES technology $Y_t = \left(\int_0^1 Y_{f,t}^{\frac{1}{1+\epsilon_t}} df \right)^{1+\epsilon_t}$, so the intermediate-goods market is monopolistic. We used the type of staggered price setting of Calvo (1983) to replicate rigidity of price. As pointed out by Woodford (2003), the output of all intermediate-good firms is equal to the output of all final-goods firms, and the aggregate production function holds at the steady state when the dispersion of price is unity. $f \in (0, 1)$ is the indexation of each intermediate-goods firm and ϵ_t is the time-varying price mark-up that has relationship $\epsilon_t = \frac{1}{\sigma_t - 1} > 0$ with elasticity of substitution $\sigma_t > 1$ between different intermediate goods. After log-linearizing the model, time-varying price mark-up ϵ_t can be represented as a cost-push mark-up shock process that follows

$$\epsilon_t^p = \rho_p \epsilon_{t-1}^p + \mu_t^p \quad (11)$$

where $\mu_t^p \sim N(0, \sigma_p^2)$ is an i.i.d. shock. The cost minimization of final-goods firms leads to the intermediate-goods demand function $Y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\frac{1+\epsilon_t}{\epsilon_t}} Y_t$ and aggregate price index $P_t = \left(\int_0^1 P_{f,t}^{-\frac{1}{\epsilon_t}} df \right)^{-\epsilon_t}$.

Under the Calvo (1983) type price setting, each period of $1 - \eta$ fractions of all intermediate-goods firms can adjust price to their optimal level, and the others just index their prices to a weighted average of the inflation of the last period and steady state with weights $1 - \gamma$ and γ , respectively.

$$P_{f,t+j} = P_{f,t+j-1} \Pi_{t+j-1}^\gamma \Pi^{1-\gamma} = P_{f,t} \left[\prod_{k=1}^j \left(\frac{\Pi_{t+k-1}}{\Pi} \right)^\gamma \Pi \right]$$

Intermediate-goods firms first minimize cost of production $w_t L_{f,t} + q_t K_{f,t}$ subject to its production technology, $Y_{f,t} = e^{\epsilon_t^a} L_{f,t}^{1-\alpha} K_{f,t}^\alpha - \phi$, where ϕ is a fixed cost keeping all intermediate-goods firms' profits zero at the steady state. ϵ_t^a represents the TFP that follows the AR(1) process.

$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \mu_t^a \quad (12)$$

$\mu_t^a \sim N(0, \sigma_a^2)$ is an i.i.d. shock driving the TFP process.

$$\frac{K_{f,t}}{L_{f,t}} = \frac{\alpha w_t}{(1-\alpha)q_t}$$

Aggregating the first-order condition of cost minimization over each intermediate-goods firm by $\int_0^1 K_{f,t} df = K_t$ and $\int_0^1 L_{f,t} df = L_t$ leads to the relationship of aggregate capital stock and labor supply.

$$\frac{K_t}{L_t} = \frac{\alpha w_t}{(1-\alpha)q_t} \quad (13)$$

Marginal cost is identical among all intermediate-goods firms.

$$MC_t = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{q_t}{\alpha} \right)^\alpha \quad (14)$$

Then, intermediate-goods firms set the optimal price to maximize the discounted profits.

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} \eta^j \left(\beta^j \frac{\lambda_{t+j}}{\lambda_t} \right) \left[\frac{P_{f,t}}{P_{t+j}} \left(\prod_{k=1}^j \left(\frac{\Pi_{t+k-1}}{\Pi} \right)^\gamma \Pi \right) - MC_{t+j} \right] Y_{f,t+j}$$

$$s.t. Y_{f,t+j} = \left[\frac{P_{f,t}}{P_{t+j}} \left(\prod_{k=1}^j \left(\frac{\Pi_{t+k-1}}{\Pi} \right)^\gamma \Pi \right) \right]^{-\frac{1+\epsilon_{t+j}}{\epsilon_{t+j}}} Y_{t+j}$$

The first-order condition is given by Equation (15).

$$\mathbb{E}_t \sum_{j=0}^{\infty} \left\{ (\beta\eta)^j \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\epsilon_{t+j}} \left[\frac{P_t^*}{P_t} \left(\prod_{k=1}^j \left(\frac{\Pi_{t+k-1}}{\Pi} \right)^\gamma \frac{\Pi}{\Pi_{t+k}} \right) \right]^{-\frac{1+\epsilon_{t+j}}{\epsilon_{t+j}}} Y_{t+j} \right. \\ \left. \times \left[\frac{P_t^*}{P_t} \left(\prod_{k=1}^j \left(\frac{\Pi_{t+k-1}}{\Pi} \right)^\gamma \frac{\Pi}{\Pi_{t+k}} \right) - (1 + \epsilon_{t+j}) MC_{t+j} \right] \right\} = 0 \quad (15)$$

where P_t^* represents the optimal price set at period t . The law of motion of the general price level is given by aggregating the optimal prices set by all intermediate-goods firms in each period.

$$1 = \left[\int_0^1 \left(\frac{P_{f,t}}{P_t} \right)^{-\frac{1}{\epsilon_t}} df \right]^{-\epsilon_t} \\ = \left[(1 - \eta) \left(\frac{P_t^*}{P_t} \right)^{-\frac{1}{\epsilon_t}} + \eta(1 - \eta) \left(\frac{P_{t-1}^* \Pi_{t-1}^\gamma \Pi^{1-\gamma}}{P_t} \right)^{-\frac{1}{\epsilon_t}} + \dots \right]^{-\epsilon_t} \\ = (1 - \eta) \left\{ \left(\frac{P_t^*}{P_t} \right)^{-\frac{1}{\epsilon_t}} + \sum_{j=1}^{\infty} \eta^j \left[\frac{P_{t-j}^*}{P_{t-j}} \left(\prod_{k=1}^j \left(\frac{\Pi_{t-k}}{\Pi} \right)^\gamma \frac{\Pi}{\Pi_{t-k+1}} \right) \right]^{-\frac{1}{\epsilon_t}} \right\} \quad (16)$$

The log linearization of Equations (15) and (16) leads to the hybrid new Keynesian Phillips curve (NKPC) equation. The final aggregate output with price dispersion¹¹

$\Theta_t = \int_0^1 \left(\frac{P_{f,t}}{P_t} \right)^{-\frac{1+\epsilon_t}{\epsilon_t}} df$ equal the aggregate of all intermediate output.

$$\int_0^1 Y_{f,t} df = \int_0^1 \left(\frac{P_{f,t}}{P_t} \right)^{-\frac{1+\epsilon_t}{\epsilon_t}} Y_t df = \int_0^1 \left(e^{\epsilon_t^a} L_{f,t}^{1-\alpha} K_{f,t}^\alpha - \phi \right) df = e^{\epsilon_t^a} L_t^{1-\alpha} K_t^\alpha - \phi \\ Y_t \Theta_t = e^{\epsilon_t^a} L_t^{1-\alpha} K_t^\alpha - \phi \quad (17)$$

2.3. Fiscal and Monetary Authorities

The joint budget constraint of government and central bank is given by

$$\frac{B_{S,t}}{P_t R_{S,t}} + \frac{B_{L,t}}{P_t R_{L,t}} + \frac{\Delta S_t}{P_t} = \frac{B_{S,t-1}}{P_t} + \frac{B_{L,t-1}}{P_t R_{S,t}} + G_t - T_t$$

where $B_{L,t}$ and $B_{S,t}$ are the total amount of long- and short-term government bonds, respectively. The central bank holds long-term government bonds $B_{L,t}^{CB}$ as an asset, and supplies money as a liability, so its balance sheet variation ΔS_t can be represented as the change of these two parts.

$$\frac{\Delta S_t}{P_t} = \frac{M_t - M_{t-1}}{P_t} - \left(\frac{B_{L,t}^{CB}}{P_t R_{L,t}} - \frac{B_{L,t-1}^{CB}}{P_t R_{S,t}} \right)$$

Central-bank holdings of long-term governments bonds are a fraction x_t of the total amount of long-term bonds. All households hold the remaining long-term bonds¹². The

asset purchase by the central bank can be described by the variation of this fraction variable x_t that we assumed to be an AR (1) process.

$$B_{L,t}^{CB} = x_t B_{L,t} \quad (18)$$

Combining the joint budget constraint of government and central bank, balance sheet variation, and Equation (18) by cancelling $\frac{\Delta S_t}{P_t}$ and $B_{L,t}^{CB}$ and rewriting the nominal terms into real terms leads to the joint budget constraint of government and central bank represented by Equation (19).

$$\frac{b_{S,t}}{R_{S,t}} + \frac{b_{L,t}}{R_{L,t}} + m_t - \frac{m_{t-1}}{\Pi_t} - \left(x_t \frac{b_{L,t}}{R_{L,t}} - x_{t-1} \frac{b_{L,t-1}}{\Pi_t R_{S,t}} \right) = \frac{b_{S,t-1}}{\Pi_t} + \frac{b_{L,t-1}}{\Pi_t R_{S,t}} + G_t - T_t \quad (19)$$

$$B_{L,t}^H = (1 - x_t) B_{L,t} \quad (20)$$

$$\log\left(\frac{x_t}{x}\right) = \rho_x \log\left(\frac{x_{t-1}}{x}\right) + \mu_t^x \quad (21)$$

where x is the fraction of the central bank's long-term bond holdings $\frac{B_{L,t}^{CB}}{B_L}$ at the steady state. $\mu_t^x \sim N(0, \sigma_x^2)$ is an i.i.d shock to drive the asset purchase process. By calibrating the size of μ_t^x and ρ_x , we can simulate the effect of asset purchase by the central bank on aggregate economic activity. ρ_x needs to be carefully calibrated because it represents the exit strategy of the central bank when the central bank stops the QE and returns to the normal amount of government debt holdings.

Government spending is assumed to follow an AR (1) process with shock term $\mu_t^g \sim N(0, \sigma_g^2)$. Long-term bonds supplied by the government are assumed to be an AR (1) process, as in Zagaglia (2013), where $\mu_t^{b_L} \sim N(0, \sigma_{b_L}^2)$.

$$\log\left(\frac{G_t}{G}\right) = \rho_g \log\left(\frac{G_{t-1}}{G}\right) + \mu_t^g \quad (22)$$

$$\log\left(\frac{b_{L,t}}{b_L}\right) = \rho_{b_L} \log\left(\frac{b_{L,t-1}}{b_L}\right) + \mu_t^{b_L} \quad (23)$$

As proposed by Leeper (1991), to prevent inflation triggered by fiscal expansion, a passive fiscal policy rule was introduced by Falagiarda (2014) to characterize tax collection as a function of total government debt:

$$T_t = \tau + \tau_S \left(\frac{b_{S,t-1}}{\Pi_t} - \frac{b_S}{\Pi} \right) + \tau_L \left(\frac{b_{L,t-1}}{R_{S,t} \Pi_t} - \frac{b_L}{R_S \Pi} \right) \quad (24)$$

where τ_S and τ_L are parameters that represent the reaction to bond deviation from the steady state value. Lump-sum tax T_t at the steady state is τ . Because T_t is the real tax income of government, the bonds are also represented in real terms $b_{L,t}$ and $b_{S,t}$. This specification shows that the deviation of government debt from a long-run steady state can be offset or compensated by the lump-sum tax collection from households.

The central bank is assumed to follow a standard Taylor (1993) rule with nominal interest-rate smoothing ρ_R .

$$\log\left(\frac{R_{S,t}}{R_S}\right) = \rho_R \log\left(\frac{R_{S,t-1}}{R_S}\right) + (1 - \rho_R) \left[\varphi_Y \log\left(\frac{Y_t}{Y}\right) + \varphi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \right] + \varepsilon_t^r \quad (25)$$

Monetary policy shock was also assumed to be an AR (1) process with disturbance term $\mu_t^r \sim N(0, \sigma_r^2)$.

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \mu_t^r \quad (26)$$

Lastly, we close the model by imposing aggregate resource constraint $Y_t = C_t + G_t + I_t(1 + AC_{K,t}) + \frac{b_{L,t}^H}{R_{L,t}} AC_{B,t}$. Total output is allocated to consumption, government-investment expenditure, and two types of adjustment cost. This completes the description of the model. Steady state and log linearization are given in Appendixs A and B. We have 25 endogenous variables:

$$\left\{ Y_t, K_t, I_t, C_t, L_t, w_t, MC_t, \mu_t, q_t, \lambda_t, m_t, \Pi_t, R_{L,t}, R_{S,t}, b_{L,t}^H, b_{S,t}, b_{L,t}, b_{L,t}^{CB}, G_t, T_t, x_t, \varepsilon_t^p, \varepsilon_t^a, \varepsilon_t^u, \varepsilon_t^r \right\}$$

and 6 exogenous variables, as follows.

$$\left\{ \mu_t^a, \mu_t^s, \mu_t^p, \mu_t^u, \mu_t^r, \mu_t^{b_L}, \mu_t^x \right\}$$

Generally, the price level is not determined in the new Keynesian DSGE model. The linearization of Equations (15) and (16) leads to the NKPC equation.

2.4. Analysis of Portfolio Rebalancing Mechanism

Before proceeding to numerical simulation, we performed an analytical investigation regarding the asset market to check the transmission mechanism of QE. Log-linearizing first-order condition¹³ Equations (7) and (8) and combining them by cancelling $\tilde{\lambda}_t, \tilde{\lambda}_{t+1}$ and π_{t+1} leads to Equation (27).

$$\tilde{R}_{L,t} = \tilde{R}_{S,t} + \mathbb{E}_t \tilde{R}_{S,t+1} - \left(\frac{\kappa_b \varphi_B Y}{R_S} + \varphi_B Y \right) \tilde{b}_{S,t} + \left(\frac{\kappa_b \varphi_B Y}{R_S} + \varphi_B Y \right) \tilde{b}_{L,t}^H \quad (27)$$

where the parameters in Equations (7) and (8) can be cancelled using steady state values¹⁴ for the steady state of the model. The above result shows that the long-term interest rate is positively related to the short-term interest rate, and the expectation of short-term interest rate and long-term bonds held by private sector, but negatively related to short-term bonds because of the imperfect substitution of two kinds of bond assets. When the central bank purchases a long-term bond from the private sector, the long-term interest rate can be reduced to stimulate the economy. Conversely, when the central bank reduces long-term bond holdings, less liquid asset holdings (long-term bonds) of the private sector increase, leading to an increase in interest-rate spread. This mechanism, the portfolio-rebalancing channel of QE, is summarized below.

$$b_{L,t}^{CB} \uparrow \Rightarrow b_{L,t}^H \downarrow, b_{S,t} \uparrow \Rightarrow \tilde{b}_{L,t}^H < 0, \tilde{b}_{S,t} > 0 \Rightarrow \tilde{R}_{L,t} < 0 \Rightarrow R_{L,t} \downarrow \Rightarrow R_{L,t} - R_{S,t} \downarrow$$

$$b_{L,t}^{CB} \downarrow \Rightarrow b_{L,t}^H \uparrow, b_{S,t} \downarrow \Rightarrow \tilde{b}_{L,t}^H > 0, \tilde{b}_{S,t} < 0 \Rightarrow \tilde{R}_{L,t} > 0 \Rightarrow R_{L,t} \uparrow \Rightarrow R_{L,t} - R_{S,t} \uparrow$$

Parameter φ_B represents the degree of adjustment cost in portfolio management. The existence of adjustment cost invalidates the standard arbitrage condition. When this friction disappears, $\varphi_B = 0$, the first-order condition in the log linearization of Equations (7) and (8), is simplified to the standard Euler equation, arbitrage equation, and the term structure between long- and short-term interest rate, which are familiar in the standard DSGE models without adjustment cost of assets with different maturities.

$$\begin{aligned} \tilde{\lambda}_t &= \tilde{R}_{S,t} + \mathbb{E}_t(\tilde{\lambda}_{t+1} - \pi_{t+1}) \\ \tilde{\lambda}_t &= \tilde{R}_{L,t} + \mathbb{E}_t(\tilde{\lambda}_{t+1} - \pi_{t+1} - \tilde{R}_{S,t+1}) \\ \tilde{R}_{L,t} &= \tilde{R}_{S,t} + \mathbb{E}_t \tilde{R}_{S,t+1} \end{aligned}$$

To check QE's transmission mechanism from the asset market to the real economy, combining the log linearization of Equations (7) and (8) by cancelling bond variables $\tilde{b}_{S,t} - \tilde{b}_{L,t}^H$ yields the Euler equation of consumption.

$$\tilde{\lambda}_t = \mathbb{E}_t(\tilde{\lambda}_{t+1} + \pi_{t+1}) + \frac{R_S}{\kappa_B + R_S} \tilde{R}_{S,t} + \frac{\kappa_B}{\kappa_B + R_S} (\tilde{R}_{L,t} - \mathbb{E}_t \tilde{R}_{S,t+1}) \quad (28)$$

Following analysis of the transmission mechanism inside the asset market, the transmission mechanism from the asset market to the real economy is summarized below.

$$R_{L,t} \downarrow \Rightarrow \tilde{R}_{L,t} < 0 \Rightarrow \tilde{\lambda}_t < 0 \Rightarrow \lambda_t \downarrow \Rightarrow C_t \uparrow \Rightarrow Y_t \uparrow$$

Summarizing the entire above analysis, QE in this model can be described as follows:

Long-term bond purchases by the central bank lead to a change in assets with different maturities, and so asset returns (from Equation (27)). Consequently, the real economy is stimulated through the general equilibrium (from Equation (28)).

The above investigation describes the whole scenario. To check the accurate dynamics triggered by asset purchase by the central bank, we conducted a calibration exercise.

3. Calibration

This model was developed to simulate the effects of QQE conducted by the BoJ from April 2013. The benchmark calibration of the steady state was adjusted to match quarterly data over the most recent periods prior to April 2013. Steady state values could be calculated from the System of National Accounts (SNA) of Japan. GDP at steady state was normalized to a unit. Total government debt $b_S + b_L$, short-term debt¹⁵ b_S and long-term debt¹⁶ b_L , long-term debt held by private sector b_L^H and the central bank b_L^{CB} , were obtained from the OECD Statistical Database, Ministry of Finance in Japan¹⁷ and the BoJ, and calculated as the relative ratio to output. Steady state of model is given in Table 1.

Table 1. Calibration for steady state.

Notation	Description	Steady State Value ¹⁸
Y	Output	1 (normalization)
C	Consumption	0.6114
I	Investment	0.2173
L	Labor supply ¹⁹	0.2308
G	Government Expenditure	0.119
T	Lump-sum Tax	0.1196
R_S	Gross short-term interest rate	1.01
R_L	Gross long-term interest rate	1.0201
Π	Gross inflation rate	1.0039
$b_S + b_L$	Total debt	1.5493
b_S	Total short-term debt	0.0869
b_L	Total long-term debt	1.4624
b_L^{CB}	Long-term debt held by central bank	0.2296
b_L^H	Long-term debt held by private sector	1.2328
κ_B	Steady state ratio of $\frac{b_L^H}{b_S}$	14.1864
x	Steady state ratio of $\frac{b_L^{CB}}{b_L}$	0.1570

3.1. Calibration

Structural and policy parameters are directly obtained from the DSGE literature. Parameters such as discount factor β , capital share α , and depreciation rate δ were set to their general values. Average mark-up rate in the economy was set to 0.2. Calvo type price rigidity set equal to 0.75 implies an average price duration of 4 quarters, a value consistent with much empirical evidence. Parameters in the monetary-policy rule equation take the standard values in a way that is consistent with Taylor's original rule. To reflect a situation similar to ZLB, ρ_R was set at a highly persistent value of 0.995 to prevent the short-term interest rate from responding to inflation and output change, as proposed

by Falagiarda (2014) to avoid the indeterminacy of model's solution. Other structural parameters were calibrated as the values that are generally used in the DSGE literature. Table 2 summarizes the values of all structural parameters.

Table 2. Calibration for structural and policy parameters.

Notation	Description	Value
α	Capital share	0.36
δ	Depreciation rate	0.025
β	Discount factor	0.994
θ	Habit formation	0.7
ϕ	Fixed cost in production	0.2
χ	Inverse of Frisch elasticity of labor supply	5
σ	Inverse of intertemporal substitution (risk aversion)	2
ξ	Interest-rate semielasticity of money demand	4
η	Calvo type price rigidity	0.75
γ	Price indexation	0.5
ϵ	Steady state mark-up Rate	0.2
φ_B	Portfolio Adjustment Friction ²⁰	0.01
φ_K	Investment Adjustment Friction ²¹	770.6056
τ	Steady state lump-sum tax	0.1196
τ_S	Response to short-term debt deviation	0.3
τ_L	Response to long-term debt deviation	0.3
φ_Y	Response to output	0.25
φ_π	Response to inflation	1.5
ρ_R	Monetary-policy smoothing	0.995

Two key parameters, ρ_x and σ_x^2 , were calibrated to replicate QE's persistence and scale²². The BoJ announced on 4 April 2013 that the long-term bond held by BoJ would be increased from JPY 89 trillion to JPY 190 trillion from the end of 2012 to end of 2014, which meant a 113.48% increase in long-term bond holdings. Considering the inaccuracy of calibration, the σ_x was set to be 1 to simulate the effect of the long-term bond purchase by the BoJ. Other exogenous shock parameters were set to the usual values.

4. Results

Under benchmark calibration, we now report the baseline simulation results of long-term bond purchase by the BoJ. We consider a scenario in which the central bank increases its long-term bond holdings 100% and takes 6 years to gradually return to its normal level.

4.1. Baseline Simulation

Figure 1 shows the impulse response function of each variable that is represented in the percentage deviation from its steady state. Figure 1 shows that QE has a strong effect on output and investment. The effect on investment can be identified from Equation (10). From the linearized version of Equation (10),

$$\tilde{I}_t = \left(\frac{6\varphi_K\delta^2}{2 + 3\varphi_K\delta^2} \right)^{-1} (E_t\tilde{\mu}_{t+1} - \tilde{\lambda}_t) + \tilde{K}_t$$

The decrease in $\tilde{\lambda}_t$ and increase in μ_t could lead to increasing of \tilde{I}_t . This transmission mechanism is also confirmed from Figure 1. Peak impact on output and investment was almost 0.51% and 1.29%, respectively. The stimulated effect by QE lasted for almost 5 quarters. As set up in the scenario, the central bank increased its long-term bond holdings (In Figure 1, panel $\tilde{b}_{L,t}^{CB}$) on its balance sheet by 100%, and returned to a normal level 6 years later. During the same period, long-term bonds held by the private sector (In Figure 1, panel $\tilde{b}_{L,t}^H$) decrease 18.62% and returns to a normal level 6 years later. The inflation rate increased by 0.41% from the QE stimulation. The long-term interest rate, which is critical to the investment, was decreased by 0.5%. Considering the low-interest-rate environment

existing in the Japanese economy, 0.5% decreasing the yield curve is not a small number. As long as the QE has its effect, the long-term interest rate is suppressed to a low level. The baseline simulation results show that the mechanism analyzed in Section 2.4 was appropriate.

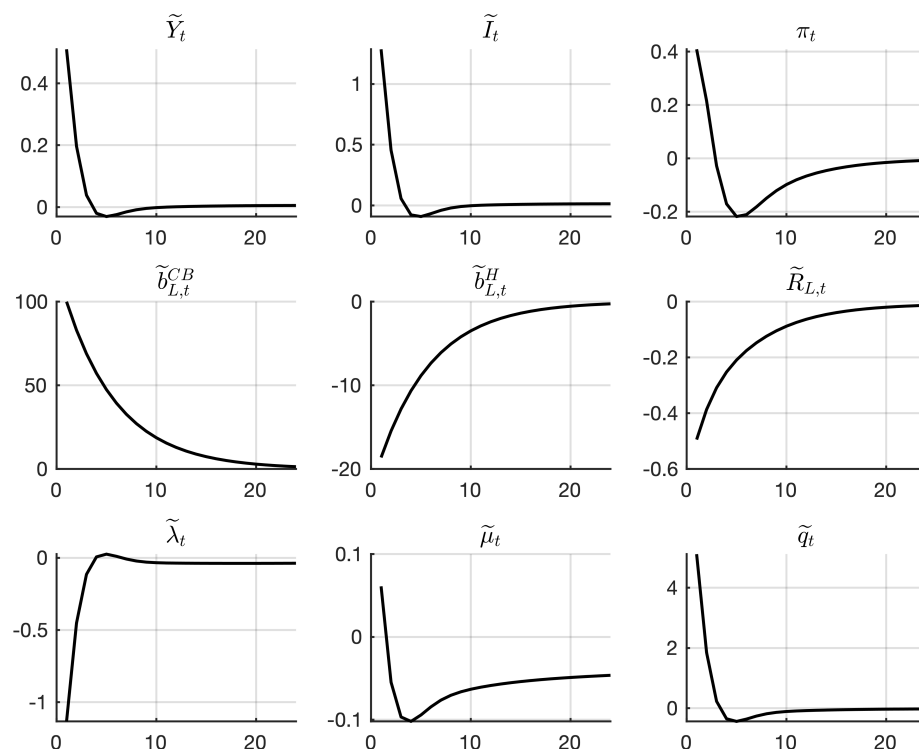


Figure 1. Baseline simulation for BoJ's QE.

In addition, the effect stimulated by asset purchase was limited because it merely lasted for just more than 1 year. In this study, we did not explicitly introduce the balance sheet of the central bank, and BoJ operations are more complicated than what we simulated, but the positive effect of QE on the real economy could be identified with a rigorous structural explanation.

4.2. QE Sensitivity Analysis

We consider different exit strategies of the central bank's QE and its effects. The benchmark simulation was set to be a 6-year QE policy for $\rho_x = 0.83$. As sensitivity analysis in Falagiarda (2014), we ran two more simulations for a long-lasting QE policy (8 years and $\rho_x = 0.88$) and short-lasting QE policy (4 years and $\rho_x = 0.76$). Figure 2 shows that the longer the duration of QE was, the stronger its effect was. Especially for long-term interest rate, the push-down effect of QE to a long-term interest rate lasted longer when the QE policy had high persistence. Table 3 shows the simulated peak impact of QE under different scenarios of QE policy.

As mentioned in Section 3.1, φ_B is also considered to have a critical role in the effect of QE. Similar sensitivity analysis was conducted for two other cases, higher portfolio adjustment cost ($\varphi_B = 0.02$) and lower portfolio adjustment cost ($\varphi_B = 0.005$), and they were compared with the benchmark case ($\varphi_B = 0.01$). Results were similar to those in Figure 2, so we do not report the IRF here again. Sensitivity analysis shows that with a higher portfolio adjustment cost, short- and long-term bonds become less substitutable. The asset purchase conducted by the central bank thereby had macroeconomic effects. The effects were also amplified as φ_B increased. When $\varphi_B = 0$, two kinds of bond were perfectly substitutable, and no effects could be generated by QE.

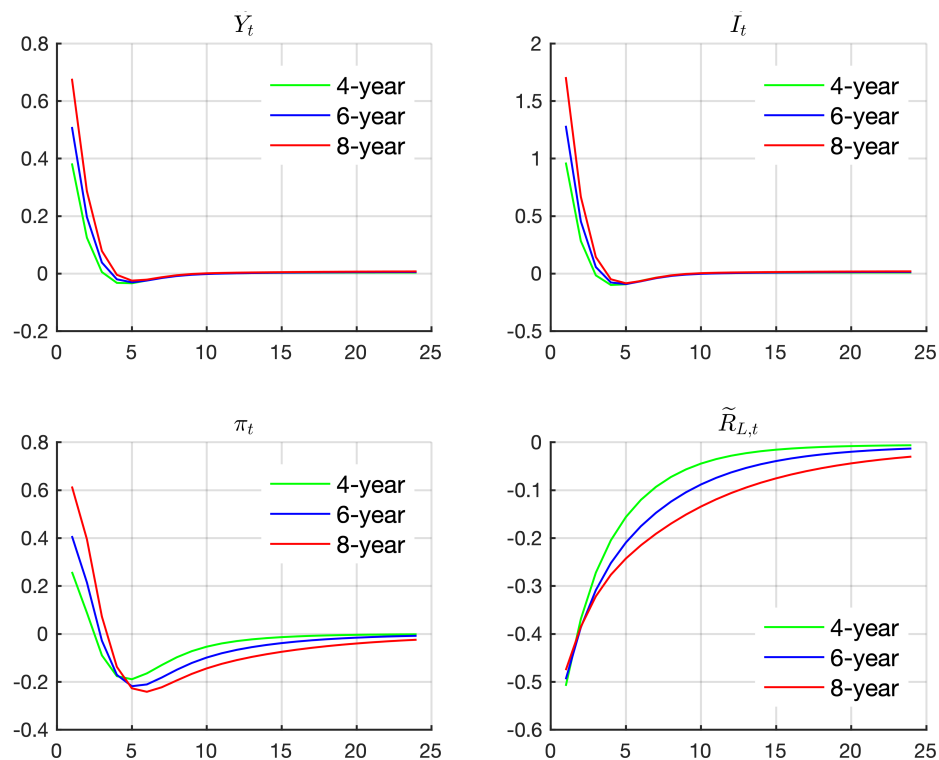


Figure 2. Sensitivity simulation for BoJ's QE.

Table 3. Simulated peak impact of QE.

QE Persistence	Output	Investment	Inflation	Long-Term Interest Rate
$\rho_x = 0.76$ (4 years)	0.38%	0.97%	0.26%	−50.84 bp
$\rho_x = 0.83$ (6 years)	0.51%	1.29%	0.41%	−49.46 bp
$\rho_x = 0.88$ (8 years)	0.68%	1.71%	0.62%	−47.56 bp

5. Concluding Remarks

In this study, a DSGE model was developed to capture the portfolio-rebalancing channel of QE, and the model was calibrated to match the Japanese economy and BoJ's policy in April 2013.

There were two main conclusions from the simulation. First, QE policy that the BoJ introduced had an effect on the real economy, pushing up output and inflation, and pushing down long-term interest rates to stimulate investment. The peak impact on output was moderate for the benchmark case 0.51%, and the pushing-up effect lasted for merely 5 quarters, but the pushing-down effect on long-term rates was persistent, lasting for the whole period when the policy was effective. As the QE period became longer, the effect became larger. Under the same level of asset purchases, the central bank should announce a long-lasting time frame for QE policy. The second conclusion is that the key assumption in this study, the imperfect substitution of different assets and the corresponding cost of portfolio adjustment cost, is critical to the effectiveness of QE. Key parameter φ_B is not a policy-controlled parameter. So, it is not the choice for the central bank. If the central bank wishes to improve the effectiveness of QE, a larger scale and longer period are two options. Considering the huge stock of Japanese government bonds, issuing more bonds is also not a smart choice for the Japanese government. The policy implication for the BoJ is just to announce a long-lasting QE program and make it credible to the market.

Another contribution is that the developed model here can be extended to more rigorous specifications of economic agents, such as the balance sheet of the central bank and the introduction of different assets. Other important channels of QE, the credit channel

and wealth channel, can be verified with the incorporation of financial intermediaries, the housing market, or financial frictions.

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Data Availability Statement: Publicly available dataset was used in this study. This dataset can be downloaded from the homepage of Ministry of Finance in Japan (<https://www.mof.go.jp/jgbs/reference/appendix/index.htm> accessed on 1 May 2021). Dynare and MATLAB codes that generates the figures and simulation results in this paper are available upon request.

Conflicts of Interest: The author declares no conflict of interest.

Appendix A. The Steady State

$$\begin{aligned} I &= \delta K, (1 - \beta\theta)(C - \theta C)^{-\sigma} = \lambda, R_L = R_S^2, \beta R_S = \Pi, m^{-\xi} = \lambda \left(1 - \frac{\beta}{\Pi}\right), \\ \lambda(q + \varphi_K \delta^3) &= \mu[1 - \beta(1 - \delta)], 2\beta\mu = \lambda(2 + 3\delta^2 \varphi_K), \varphi_K = \frac{2(1-\beta+\beta\delta-q\beta)}{\delta^2(3\beta-3-\beta\delta)}, \\ x &= \frac{b_L^{CB}}{b_L} = \frac{b_L - b_L^H}{b_L}, Y = C + I(1 + \frac{\varphi_K}{2}\delta^2) + G, MC = \frac{1}{1+\epsilon}, \\ w &= (1 - \alpha)MC \frac{Y+\phi}{L}, q = \alpha MC \frac{Y+\phi}{K}, \frac{Y}{Y+\phi} = \frac{1}{1+\epsilon}, Y = qK + wL, \\ \frac{b_S}{R_S} + \frac{b_L^H}{R_L} + m + I(1 + \frac{\varphi_K}{2}\delta^2) &= \frac{b_S}{\Pi} + \frac{b_L^H}{\Pi R_S} + \frac{m}{\Pi} + wL + qK - C - T \end{aligned}$$

Appendix B. The Log-Linearized Model

In our log-linearized model, we have 25 endogenous variables for 26 nonlinear equilibrium conditions. Equations (15) and (16) can be log-linearized and written as an NKPC equation, which is denoted as Equation (A15). So, we actually have 25 linear equilibrium conditions. Variables with \sim mean the percentage deviation from its steady state, which is defined as $\tilde{X}_t = \frac{X_t - X}{X}$.

$$\left\{ \tilde{Y}_t, \tilde{K}_t, \tilde{I}_t, \tilde{C}_t, \tilde{L}_t, \tilde{w}_t, \tilde{MC}_t, \tilde{\mu}_t, \tilde{q}_t, \tilde{\lambda}_t, \tilde{m}_t, \tilde{\pi}_t, \tilde{R}_{L,t}, \tilde{R}_{S,t}, \tilde{b}_{L,t}^H, \tilde{b}_{S,t}, \tilde{b}_{L,t}, \tilde{b}_{L,t}^{CB}, \tilde{G}_t, \tilde{T}_t, \tilde{x}_t, \epsilon_t^p, \epsilon_t^a, \epsilon_t^u, \epsilon_t^r \right\}$$

For gross inflation rate Π_t , its log-linearized variable is denoted as π_t .

$$\begin{aligned} &\frac{b_S}{R_S}(\tilde{b}_{S,t} - \tilde{R}_{S,t}) + \frac{b_L^H}{R_L}(\tilde{b}_{L,t}^H - \tilde{R}_{L,t}) + m\tilde{m}_t + I\tilde{I}_t + \varphi_K I \delta^2 \left(\frac{3}{2}\tilde{I}_t - \tilde{K}_t\right) \\ &= \frac{b_S}{\Pi}(\tilde{b}_{S,t-1} - \pi_t) + \frac{b_L^H}{R_S \Pi}(\tilde{b}_{L,t-1}^H - \pi_t - \tilde{R}_{S,t}) + \frac{m}{\Pi}(\tilde{m}_t - \pi_t) + Y\tilde{Y}_t - C\tilde{C}_t - T\tilde{T}_t \end{aligned} \quad (A1)$$

$$\tilde{K}_t = \delta \tilde{I}_t + (1 - \delta)\tilde{K}_{t-1} \quad (A2)$$

$$\epsilon_t^u = \rho_u \epsilon_{t-1}^u + \mu_t^u \quad (A3)$$

$$\tilde{\lambda}_t = \frac{1}{(1 - \beta\theta)(1 - \theta)} \left[\beta\theta\sigma \mathbb{E}_t \tilde{C}_{t+1} - \sigma(\beta\theta^2 + 1)\tilde{C}_t + \sigma\theta\tilde{C}_{t-1} \right] + \frac{1}{1 - \beta\theta}(\epsilon_t^u - \beta\theta \mathbb{E}_t \epsilon_{t+1}^u) \quad (A4)$$

$$\epsilon_t^u + \chi \tilde{L}_t = \tilde{w}_t + \tilde{\lambda}_t \quad (A5)$$

$$\xi \tilde{m}_t = \frac{\beta}{\Pi - \beta} \mathbb{E}_t (\tilde{\lambda}_{t+1} - \pi_{t+1}) - \frac{\Pi}{\Pi - \beta} \tilde{\lambda}_t + \epsilon_t^u \quad (A6)$$

$$\frac{\beta}{\Pi} \mathbb{E}_t (\tilde{\lambda}_{t+1} - \pi_{t+1}) = \frac{1}{R_S}(\tilde{\lambda}_t - \tilde{R}_{S,t}) + \frac{\kappa_B \varphi_B Y}{R_L}(\tilde{b}_{S,t} - \tilde{b}_{L,t}^H) \quad (A7)$$

$$\frac{\beta}{R_S \Pi} \mathbb{E}_t(\tilde{\lambda}_{t+1} - \pi_{t+1} - \tilde{R}_{S,t+1}) = \frac{1}{R_L}(\tilde{\lambda}_t - \tilde{R}_{L,t}) - \frac{\varphi_B Y}{R_L}(\tilde{b}_{S,t} - \tilde{b}_{L,t}^H) \quad (\text{A8})$$

$$\beta(1 - \delta) \mathbb{E}_t \tilde{\mu}_{t+1} = \tilde{\mu}_t - [1 - \beta(1 - \delta)] \tilde{\lambda}_t - \frac{2\beta q}{2 + 3\delta^2 \varphi_K} \tilde{q}_t + \frac{6\varphi_K \delta^3}{2 + 3\varphi_K \delta^2} (\tilde{K}_t - \tilde{I}_t) \quad (\text{A9})$$

$$\mathbb{E}_t \tilde{\mu}_{t+1} = \tilde{\lambda}_t + \frac{6\varphi_K \delta^2}{2 + 3\varphi_K \delta^2} (\tilde{I}_t - \tilde{K}_t) \quad (\text{A10})$$

$$\varepsilon_t^p = \rho_u \varepsilon_{t-1}^p + \mu_t^p \quad (\text{A11})$$

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \mu_t^a \quad (\text{A12})$$

$$\tilde{K}_t - \tilde{w}_t = \tilde{L}_t - \tilde{q}_t \quad (\text{A13})$$

$$\widetilde{MC}_t = (1 - \alpha) \tilde{w}_t + \alpha \tilde{q}_t \quad (\text{A14})$$

$$\pi_t = \frac{\beta}{1 + \beta\gamma} \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1 + \beta\gamma} \pi_{t-1} + \frac{(1 - \eta)(1 - \beta\eta)}{\eta(1 + \beta\gamma)} (\widetilde{MC}_t + \varepsilon_t^p) \quad (\text{A15})$$

$$\tilde{Y}_t = \left(1 + \frac{\phi}{Y}\right) [\varepsilon_t^a + \alpha \tilde{K}_t + (1 - \alpha) \tilde{L}_t] \quad (\text{A16})$$

$$\tilde{b}_t^{CB} = \tilde{x}_t + \tilde{b}_t^L \quad (\text{A17})$$

$$\begin{aligned} & \frac{b_S}{R_S} (\tilde{b}_{S,t} - \tilde{R}_{S,t}) + \frac{b_L}{R_L} (\tilde{b}_{L,t} - \tilde{R}_{L,t}) + m \tilde{m}_t \\ & - \frac{m}{\Pi} (\tilde{m}_{t-1} - \pi_t) - \frac{x b_L}{R_L} (\tilde{b}_{L,t} - \tilde{R}_{L,t}) + \frac{x b_L}{\Pi R_L} (\tilde{b}_{L,t} + \tilde{x}_{t-1} - \pi_t - \tilde{R}_{S,t}) \\ & = \frac{b_S}{\Pi} (\tilde{b}_{S,t-1} - \pi_t) + \frac{b_L}{R_S \Pi} (\tilde{b}_{L,t-1} - \pi_t - \tilde{R}_{S,t}) + G \tilde{G}_t - T \tilde{T}_t \end{aligned} \quad (\text{A18})$$

$$\tilde{b}_{L,t}^H = \tilde{b}_{L,t} + \frac{x}{x - 1} \tilde{x}_t \quad (\text{A19})$$

$$\tilde{x}_t = \rho_x \tilde{x}_{t-1} + \mu_t^x \quad (\text{A20})$$

$$\tilde{G}_t = \rho_g \tilde{G}_{t-1} + \mu_t^g \quad (\text{A21})$$

$$\tilde{b}_{L,t} = \rho_{b_L} \tilde{b}_{L,t-1} + \mu_t^{b_L} \quad (\text{A22})$$

$$T \tilde{T}_t = \tau_S \frac{b_S}{\Pi} (\tilde{b}_{S,t-1} - \pi_t) + \tau_L \frac{b_L}{R_S \Pi} (\tilde{b}_{L,t-1} - \tilde{R}_{S,t} - \pi_t) \quad (\text{A23})$$

$$\tilde{R}_{S,t} = \rho_R \tilde{R}_{S,t-1} + (1 - \rho_R) (\varphi_\pi \pi_t + \varphi_Y \tilde{Y}_t) + \varepsilon_t^r \quad (\text{A24})$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \mu_t^r \quad (\text{A25})$$

Notes

- ¹ Reith (2011) provided a good a comparison of QE in Japan and the US. Gupta and Marfatia (2018), and Gupta et al. (2019) also took the event-study approach to study the impact of unconventional monetary policy on stock markets.
- ² Siranova and Kotlebova (2018) used a structural vector autoregression (SVAR) model to check ECB monetary-policy effects via the banking sector in Slovenia. Caraiiani et al. (2020), and Huber and Punzi (2020) discussed the wealth channel of unconventional monetary policy. Caraiiani et al. (2020) used a quantile structural vector autoregressive (QSVAR) model to analyze whether the impact of monetary-policy shocks on real housing returns in the United States was contingent on the initial state of housing-market sentiment.
- ³ For related studies, please refer to Olmo and Sanso-Navarro (2018), Kiss and Balog (2018), Chebbi and Derbali (2019).
- ⁴ For related studies, please refer to Harrison (2011, 2012), Falagiarda (2014), Cova et al. (2015), Del Negro et al. (2017), McKay et al. (2016) and Priftis and Vogel (2016, 2017).
- ⁵ Socci et al. (2018) used the calibrated dynamic computable general equilibrium (DCGE) model of the Italian economy to check the effects of the unconventional monetary policy of ECB.
- ⁶ This paper was written in 2015, and the conclusion is based on the situation of the Japanese economy in that time.
- ⁷ Indexation of each household is omitted because they are homogenous and identical.
- ⁸ This kind of classification is also used in model calibration, the steady state ratio of two kinds of bonds with different maturities relative to the total amount of government bonds.
- ⁹ $B_{L,t}^H$ means the long-term bonds held by households.
- ¹⁰ BoJ also purchases risky assets such as ETFs and J-REITs from the private sector, but the quantity of these purchases is much less than the purchased quantity of Japanese government bonds is.
- ¹¹ As proved in Gali (2015, Chapter 3), at the steady state, price dispersion Θ_t is approximate to unity at the first order, and zero at the second order, which means that all intermediate-goods firms choose the same price, and price dispersion disappears at the steady state.
- ¹² This is not true for a real economy because other financial institutions can hold government debt. In this model, financial intermediaries are neglected, and all private-sector households hold the remaining long-term bonds.
- ¹³ See Appendix B for log-linearization of the model.
- ¹⁴ See Appendix A.
- ¹⁵ Short-term debt b_s includes bonds held by the central bank as the operation instrument in the interbank market plus bonds with maturity less than or equal to 1 year.
- ¹⁶ Long-term debt b_L is calculated by subtracting its amount from total debt.
- ¹⁷ Data of Japanese government bonds can be obtained from <http://www.mof.go.jp/jgbs/reference/appendix/index.htm> accessed on 1 May 2021.
- ¹⁸ For other steady state values, see Appendix A.
- ¹⁹ The steady state of labor supply is calculated by assuming that the share of representative household's time endowment spent on labor supply $\frac{L}{1-L}$ is equal to 0.3.
- ²⁰ In similar research, this parameter was set to different values such as Chen et al. (2012) (0.015), Andres et al. (0.045), Harrison (2011, 2012) (0.1, 0.09). Following Falagiarda (2014), φ_B was set to 0.01, which means that 1% of household's income is paid for the portfolio adjustment cost. Sensitivity analysis in the next section checks the role of this parameter in the portfolio-rebalancing channel of QE.
- ²¹ φ_K is derived from the steady state of the first-order conditions Equations (9) and (10). See Appendix A.
- ²² This calibration was conducted by checking the impulse response of $\tilde{x}_t = \rho_x \tilde{x}_{t-1} + \mu_t^x$ through trial and error. Just like parameter φ_B , ρ_x and σ_x were also assumed to be important in the portfolio-rebalancing channel of QE. Sensitivity analysis is given in the next section.

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