



# Article On Combining Evidence from Heteroskedasticity Robust Panel Unit Root Tests in Pooled Regressions

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**Abstract:** Volatility break robust panel unit root tests (PURTs) recently proposed by Herwartz and Siedenburg (Computational Statistics & Data Analysis 2008, 53, 137–150) and Demetrescu and Hanck (Econometrics Letters 2012, 117, 10–13) have different performances under both the null and local alternatives. Common practice in empirical research is to apply multiple tests if none is uniformly superior. We show that this approach tends to produce contradictory evidence for the tests considered, making it unclear whether to reject the null. To address this problem, we advocate a combined testing procedure. Simulation evidence shows that the combined test has good size control and closely tracks the more powerful test. An empirical application reinvestigates whether there is a unit root in OECD inflation rates. We find evidence that inflation is stationary for long observation periods, but we cannot reject nonstationarity in most subsets of countries for the last three decades.

Keywords: panel unit root; inflation; nonstationary volatility; multiple testing

JEL Classification: C12; C23; E31

# 1. Introduction

Empirical applications of unit root testing procedures commonly face uncertainty as to which test to use when there is a large battery of methods to choose from but no a priori information in favour of one procedure is available. Often, there are unknown characteristics of the data generating process (DGP) that favour some testing procedure(s). Examples include uncertainty about the initial condition or about deterministic patterns. Both imply the risk of low power when applying an inferior testing procedure; see Müller and Elliott (2003) and Elliott et al. (1996). It then is common practice to apply the whole set of tests and to jointly interpret the results. Such an approach is, however, problematic when not all/some tests reject the null, i.e., when there are "mixed signals". This is a non-negligible issue in (panel) cointegration testing, in particular when the tests under consideration are from distinct classes (Gregory et al. 2004; Hanck 2012). This problem also occurs in empirical studies that employ multiple panel unit roots tests (PURTs). For example, Hurlin (2010) studied stochastic trends in fourteen macro and financial variables from the seminal paper by Nelson and Plosser (1982) using OECD panel data and partly obtains ambiguous results. Mullineux et al. (2010) analysed financial integration in EU economies using flows of funds data and found mixed results for the convergence of equity and internal financing using a battery of PURTs.

Two nonstationary volatility robust PURTs proposed by Demetrescu and Hanck (2012a) and Herwartz and Siedenburg (2008),  $t_{DH}$  and  $t_{HS}$  henceforth, may confront the researcher with a similar situation. Simulation evidence shows that  $t_{DH}$  has better size-control than  $t_{HS}$  and that power of the tests is different under various forms of cross-sectional dependence and heteroskedasticity. The extent of these differences depends on the degree and type of both cross-sectional dependence and variance breaks—characteristics which are generally unknown in applications. Moreover, although both

statistics are nonstationary volatility robust versions of the test in Breitung and Das (2005), different size and power characteristics suggest that mixed signals may occur.<sup>1</sup> This might be especially problematic under the alternative when the tests are applied in small to medium-sized panels because the power differential may then be large.

As mixed signals imply imperfect correlation among  $t_{DH}$  and  $t_{HS}$ , a strategy to "reject at level  $\alpha$  if the most significant test outcome implies rejection" is invalid since the actual size of the strategy will generally be larger than  $\alpha$ . These considerations demand a joint test decision by means of a multiple testing procedure which controls size.

The aim of this paper is to analyse the prevalence of mixed signals from  $t_{DH}$  and  $t_{HS}$  and to suggest a simple but effective multiple testing method (cf. Simes 1986) which tackles the issues outlined above. The test is a modification of the Bonferroni correction and has been shown to be more powerful when the underlying test statistics are positively dependent under the null; see Sarkar (1998). An empirical application investigates the trend behaviour of inflation rates of OECD countries by means of the individual and the combined test. We thus reinvestigate some of the results presented in Culver and Papell (1997) by using the original dataset from February 1957 to September 1994 as well as for a longer observation period which also covers the recent three decades. Cross-dependence and heteroskedasticity are marked features of the data, stressing the need to apply robust PURTs.

Section 2 introduces the econometric framework and briefly discusses both test statistics. Section 3 provides Monte Carlo results which investigate the prevalence of mixed signals among  $t_{DH}$  and  $t_{HS}$ . We then present the multiple testing strategy and compare the performance of both tests with the combined test. Section 4 discusses the application to inflation rates. Section 5 concludes.

## 2. Econometric Framework and Robust PURTs

#### 2.1. Panel Model and Assumptions

We consider a first-order autoregressive panel model with time-varying second moments,

$$y_t = \mu + \rho y_{t-1} + e_t, \quad t = 1, \dots, T,$$
 (1)

where  $\mu$ ,  $y_t$ ,  $y_{t-1}$ , and the errors  $e_t$  are  $N \times 1$  and  $e_t \sim (0, \Omega_t)$ . The PURTs (cf. Section 2.2) are designed for testing the unit root null hypothesis  $H_0$ :  $\rho = 1$  against the homogenous alternative  $H_1$ :  $|\rho| < 1$ using a test statistic from a pooled regression.

Following Herwartz et al. (2016), we impose the following assumptions on  $e_t$  which basically mirror the framework used in Breitung and Das (2005) but allow for the time-dependent variability of  $\Omega_t$  in addition to weak cross-sectional dependence.

#### Assumption $\mathcal{A}$

- (i) The  $e_t$  are serially uncorrelated with  $E(e_t) = 0$  and variance-covariance matrices  $\Omega_t$  for all t.
- (ii) The  $\Omega_t$  are positive definite with positively bounded eigenvalues  $\lambda_t^{(1)} \leq \lambda_t^{(2)} \leq \cdots \leq \lambda_t^{(N)}$  for all *t*.
- (iii)  $E(u_{it}u_{jt}u_{kt}u_{lt}) < \infty$  for all i, j, k, l where the u are elements of  $u_t = (u_{1t}, u_{2t}, \dots, u_{nt})' = \Omega_t^{-1/2} e_t$ .

Assumption  $A_{(i)}$  implies that the innovations are uncorrelated over time. Assumption  $A_{(ii)}$  allows for weak forms of cross-dependence between the series in Equation (1). Regarding heteroskedasticity,  $A_{(ii)}$  covers both discrete shifts and patterns of trending (co)variances which are consistent with the

<sup>&</sup>lt;sup>1</sup> See, e.g., Hanck and Czudaj (2015) for simulation evidence on the implications of heteroskedasticity for the PURT statistic proposed in Breitung and Das (2005).

assumption that  $\Omega_t$  has bounded eigenvalues.<sup>2</sup> Assumption  $\mathcal{A}_{(iii)}$  is a common assumption in the panel literature and demands finite fourth moments of the  $e_t$  (and thus the  $u_t$ ).

#### 2.2. Volatility-Break Robust Panel Unit Root Tests

This section briefly reviews the two PURTs under study here.

# 2.2.1. The White-Type Test with Cauchy Instrumenting

The robust homogenous PURT proposed by Demetrescu and Hanck (2012a) builds on the work of So and Shin (1999), who found that the "Cauchy" estimator of the autoregressive coefficient in a univariate AR(1) model has a Gaussian limit even under the unit root null. The statistic

$$\bar{t}_{IV} = (NT)^{-1/2} \sum_{t=1}^{T} \operatorname{sign}(\widetilde{\boldsymbol{y}}_{t-1})' \boldsymbol{\varepsilon}_{t}^{*}$$
(2)

is an adaption to dependent panels that has been proposed by Shin and Kang (2006). They account for cross-dependence by using orthogonalised residuals  $\varepsilon_t^*$ ;  $\tilde{y}_{t-1}$  denotes recursively mean-adjusted lagged levels.<sup>3</sup> Demetrescu and Hanck (2012b) show that  $\bar{t}_{IV}$  retains its asymptotic properties under unconditional heteroskedasticity thanks the transformation of the information in the individual series by the sign function sign(·). A shortcoming of  $\bar{t}_{IV}$  is the orthogonalisation procedure, as it requires  $T \gg N$  for reasonable size (Demetrescu and Hanck 2012b). To circumvent this, Demetrescu and Hanck (2012a) suggest a modification of the Shin and Kang (2006) test which involves a White (1980) covariance estimator. The test statistic is computed as

$$t_{DH} = \frac{\sum_{t=1}^{T} \operatorname{sign}(\boldsymbol{y}_{t-1})' \Delta \boldsymbol{y}_t}{\sqrt{\sum_{t=1}^{T} \operatorname{sign}(\boldsymbol{y}_{t-1})' \bar{\boldsymbol{e}}_t \bar{\boldsymbol{e}}_t' \operatorname{sign}(\boldsymbol{y}_{t-1})}},$$
(3)

where  $\bar{e}_t = \Delta y_t = e_t$ . Motivated by the Gaussian limiting null distribution of  $\bar{t}_{IV}$  and Monte Carlo evidence,  $t_{DH}$  is used with standard normal critical values. The test is reported to perform well under factor-type cross-dependence as well as nonstationary volatility.

# 2.2.2. The White-Type Test

Herwartz and Siedenburg (2008) proposed the test statistic

$$t_{HS} = \frac{\sum_{t=1}^{T} \mathbf{y}_{t-1}^{\prime} \Delta \mathbf{y}_{t}}{\sqrt{\sum_{t=1}^{T} \mathbf{y}_{t-1}^{\prime} \overline{\mathbf{e}}_{t} \overline{\mathbf{e}}_{t}^{\prime} \mathbf{y}_{t-1}}},$$
(4)

related to  $t_{DH}$  in also using a panel generalisation of the heteroskedasticity-consistent White (1980) covariance estimator.

Herwartz and Siedenburg (2008) conjecture that the White correction renders  $t_{HS}$  robust to general forms of heteroskedasticity. Herwartz et al. (2016) indeed proved that  $t_{HS}$  is asymptotically Gaussian under variance shifts. They suggest that, hence, using Cauchy instruments in Equation (3) is not necessary for asymptotic standard normality. This, however, does not render  $t_{DH}$  superfluous as it turns out that the tests have distinct characteristics both under the null and the alternative: Simulation evidence in Section 3.3.1 confirms that  $t_{DH}$  has more reliable size control and that the power of the

<sup>&</sup>lt;sup>2</sup> As pointed out in Herwartz et al. (2016),  $A_{(ii)}$  does not require synchronous covariance switching and thus permits that, e.g., only a fraction of the series exhibits distinct variance regimes.

<sup>&</sup>lt;sup>3</sup> Here,  $\tilde{y}_{i,t-1} = y_{i,t-1} - (t-1)^{-1} \sum_{j=1}^{t-1} y_{i,t-1}$ ;  $\varepsilon_t^* = \hat{\Gamma}' \varepsilon_t$  with  $\hat{\Gamma}$  from LU decomposing  $\hat{\Sigma}^{-1} = \left(1/(T-1) \sum_{t=3}^{T} \bar{\varepsilon}_t \overline{\varepsilon}_t'\right)^{-1}$  and the  $\bar{\varepsilon}_{i,t}$  are consistent residuals obtained from estimation under the null.

tests is different in small to medium-sized panels under cross-dependence and various scenarios of breaks in the innovation variance.

#### 2.2.3. Data Preprocessing

In order for  $t_{HS}$  and  $t_{DH}$  to be valid in applications where Equation (1) is unlikely to hold given (possibly distinct) patterns of short-run dynamics in the series, it is necessary to prewhiten the data. This can be done as proposed in Breitung and Das (2005).

$$\widehat{\Delta y}_{i,t} = \Delta y_{i,t} - \sum_{j=1}^{p_i} \widehat{b}_{i,j} \Delta y_{i,t-j} \text{ and } \widehat{y}_{i,t} = y_{i,t} - \sum_{j=1}^{p_i} \widehat{b}_{i,j} y_{i,t-j}$$
(5)

where the  $\hat{b}_{i,j}$  are ordinary least squares (OLS) estimates from individual regressions  $\Delta y_{i,t} = \sum_{j=1}^{p_i} b_{i,j} \Delta y_{i,t-j} + e_{i,t}$ . The  $p_i$  are chosen using a consistent order selection criterion.

Similarly, the test statistics are not pivotal if the data exhibit deterministic patterns such as nonzero intercepts or linear time trends under the alternative. While it is nontrivial to account for the latter, individual specific intercepts may be efficiently removed by recursive demeaning (So and Shin 2001, see footnote 3) or by subtracting the first observations  $y_0$  from the level series (Breitung and Das 2005).<sup>4</sup> Since Herwartz et al. (2016) reported  $t_{DH}$  and  $t_{HS}$  to be slightly less powerful when the data are recursively demeaned, we use the latter approach and prewhiten the data beforehand as in Equation (5), if necessary.

# 3. Mixed Signals and a Combined Testing Procedure

This section examines the prevalence of mixed signals when  $t_{DH}$  and  $t_{HS}$  are applied to the same data for various scenarios of cross-sectional dependence and variance, described in Section 3.1. Section 3.2 discusses the results. We then propose our alternative testing procedure based on the multiple test by Simes (1986) and compare its performance with that of  $t_{DH}$  and  $t_{HS}$  in Section 3.3.

## 3.1. Simulation Setup

The setup follows the design in Herwartz et al. (2016), which considers the following DGPs:

DGP A: 
$$y_t = (\iota - \rho) \odot \mu + \rho \odot y_{t-1} + e_t$$
, (6)

DGP B: 
$$y_t = (\iota - \rho) \odot \mu + \rho \odot y_{t-1} + \nu_t, \quad \nu_t = \theta \odot \nu_{t-1} + e_t,$$
 (7)

where bold symbols are  $N \times 1$ ,  $\iota$  is a vector of ones, and  $\odot$  denotes elementwise products. DGP *A* generates panels with uncorrelated errors. DGP *B* allows to investigate the performance under serial correlation for  $\theta_i \sim iid \mathcal{U}(0.2, 0.4)$  with  $\mathcal{U}(a, b)$  denoting the uniform distribution on [a, b]. Under the null,  $\rho = \iota$  so that both DGPs generate driftless panel random walks. Under  $\mathcal{H}_1$ ,  $\rho = \gamma$  with  $\gamma_i \sim iid \mathcal{U}(0.9, 1)$ , yielding processes with heterogenous degrees of persistence and individual effects  $\mu_i \sim iid \mathcal{U}(0, 0.02)$ .

Under cross-sectional independence, we set  $\Omega_t = \text{diag}(\sigma_t^2)$ . We further consider the three following scenarios for simulating cross-sectional dependence.

1. *Spatial Correlation*: Contemporaneous correlation is introduced by a first-order spatial autoregressive (SAR) process in the following innovations:

$$\boldsymbol{e}_t = (\mathbf{I}_N - \Theta \mathbf{W}_N)^{-1} \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \operatorname{diag}(\sigma_t^2))$$
(8)

<sup>&</sup>lt;sup>4</sup> Detrending procedures as proposed by Chang (2002) lead to a nonzero expectation of the numerators in (3) and (4) under forms of heteroskedasticity allowed by  $A_{(ii)}$  such that approximations by a Gaussian distribution are invalid. See Herwartz and Walle (2018) for a more detailed argument and an alternative bootstrap procedure.

where the scalar spatial AR coefficient  $\Theta$  governs strength of the dependence.  $\mathbf{W}_N$  is a symmetric row-normalised  $N \times N$  spatial weights matrix of the one-ahead-and-one-behind type. Elements of  $\mathbf{W}_N$  one ahead and one behind the main diagonal are set to 0.5 and zeros elsewhere; see Baltagi et al. (2007).<sup>5</sup>  $\Theta$  is set to 0.8 throughout.

2. Equicorrelation: Dependence between the series in each panel arises from equicorrelated innovations

$$\boldsymbol{e}_t = \boldsymbol{\Sigma}^{1/2} \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\boldsymbol{0}, \operatorname{diag}(\sigma_t^2)) \tag{9}$$

as in O'Connell (1998). Here,  $\Sigma = \mathbf{1}_N + \omega(\mathbb{I}_N - \mathbf{1}_N)$  with  $\mathbf{1}_N$  as the  $N \times N$  identity and  $\mathbb{I}_N$  as the  $N \times N$  all-ones matrix such that  $(\Sigma)_{kl} = \omega \neq 0 \ \forall \ k \neq l$ . We set  $\omega = 0.5$ .

3. *Single Common Factor*: We let

$$\boldsymbol{e}_t = \boldsymbol{v}_t \cdot \boldsymbol{\eta} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \operatorname{diag}(\sigma_t^2))$$
 (10)

with common factor  $v_t \sim iid \mathcal{N}(0, 1)$ .  $\eta$  is a  $N \times 1$  vector of loadings such that  $\eta_i \sim iid \mathcal{U}(0, 0.02)$ ; see Pesaran (2007).

Both a one-factor structure and equicorrelation imply strong cross-sectional dependence in the terminology of Breitung and Pesaran (2008) and thus violate assumption  $\mathcal{A}_{(ii)}$ .<sup>6</sup> They hence serve to study the robustness properties of the tests considered here.

To incorporate unconditional heteroskedasticity, we introduce variance breaks. Breaks can occur at time  $\lfloor \tau_j T \rfloor$ , j = 1, ..., k so that

$$\operatorname{Var}(\epsilon_{i,t}) = \sum_{j=1}^{k} \sigma_{j}^{2} \mathcal{I}\left\{ \lfloor \tau_{j-1} T \rfloor \leq t < \lfloor \tau_{j} T \rfloor \right\}$$
(11)

where  $\tau_0 = 0$ ,  $\tau_k = 1$ , and  $\mathcal{I}(\cdot)$  is the indicator function. The setup allows for *k* different variance regimes, where  $\sigma_1^2 = 1$  for convenience such that  $r_j := \sigma_j / \sigma_1$  can be interpreted as the scaling factor for the *j*th regime's standard deviation relative to the first regime. We consider break scenarios from Cavaliere and Taylor (2007) who found that in cases with two variance regimes, size and power of common time series unit roots tests deteriorate considerably when there are early negative ( $\tau_1 = 0.2$ ,  $r_1 = 1/3$ ) or late positive ( $\tau_1 = 0.8$ ,  $r_1 = 3$ ) volatility shifts. Thus, it is interesting to investigate the performance of a combined test under these circumstances.

We consider all combinations of  $N \in \{10, 50\}$  and  $T \in \{25, 50, 100, 250\}$  and use a burn-in period of 50 observations in order to avoid dependence on initial conditions under the alternative. If not stated otherwise, we use M = 25,000 replications and test at the 5% level using the corresponding Gaussian critical value. Size-adjusted critical values are used for power comparisons.

## 3.2. Evidence of Mixed Signals

As shown in Herwartz et al. (2016),  $t_{DH}$  and  $t_{HS}$  have distinct performances when applied in small to medium-sized samples. While  $t_{DH}$  has better size control,  $t_{HS}$  is often but not uniformly more powerful. The extent of these differences depends on both the type of cross-sectional dependence, time or degree of a structural break in the unconditional variance, and serial dependence. As these characteristics of the data are generally unknown, it is nontrivial to draw conclusions when one test rejects the null while the other does not—a so-called "mixed signal".

<sup>&</sup>lt;sup>5</sup> A spatial arrangement of economic entities that could be mapped by this choice of  $W_N$  is when the correlation between units depends on their economic distance.

<sup>&</sup>lt;sup>6</sup> Breitung and Pesaran (2008) distinguished between weak dependence, where  $\lambda_t^{(N)} = O(1)$  and strong dependence where,  $\lambda_t^{(N)} = O(N)$ , as  $N \to \infty$ .

Clearly, the relevance of this issue is linked to its likelihood of occurrence. Therefore, we conduct a simulation study along the lines of Gregory et al. (2004).<sup>7</sup> We first use the simulated tests statistics to compute empirical *p*-values by taking rank order across the empirical distribution and dividing by *M*. For these pairs of empirical significance levels, empirical correlations are computed in order to gauge the degree of similarity in test outcomes under the null. Then, we investigate rejections both under the null and the alternative when the tests are applied to the same data. More specifically, we compute the proportions of joint rejection using size-adjusted critical values. At last, and probably most interestingly, the proportion of cases where one test rejects and the other one retains the null is assessed. This is our measure for the likelihood of obtaining ambiguous test outcomes.

## 3.2.1. Results

Figure 1 gives indications for mixed signals under the null. It displays probits of empirical p-values obtained for four scenarios of cross-dependence under an early variance break. While the plots demonstrate positive correlation for  $t_{DH}$  and  $t_{HS}$ , we find evidence for ambiguous test decisions in all four scenarios of cross-dependence. Table 1 presents empirical correlations of p-values and proportions of joint rejections and mixed signals under the null and under the alternative.



**Figure 1.** Pairs of probits for empirical *p*-values under an early negative variance break: N = 10 and T = 100. M = 5000 replications. (a) Cross-sectional independence; (b) spatial correlation,  $\Theta = 0.8$ ; (c) equicorrelation,  $\omega = 0.5$ ; and (d) single common factor,  $v_t \sim iid \mathcal{N}(0, 1)$ ,  $\eta_i \sim iid \mathcal{U}(0, 0.02)$ .

Under the null, we confirm moderate to high correlations of empirical *p*-values. Furthermore, rates of joint rejections (mostly ranging below 3%) are uniformly smaller than rates of mixed outcomes, i.e., the tests are likely to produce contradictory outcomes. These results show little variation across

<sup>&</sup>lt;sup>7</sup> Gregory et al. (2004) found that common time series cointegration tests tend to produce conflicting test decisions. Hanck (2012) confirmed that the problem persists for panel cointegration tests and does not alleviate with growing sample size.

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*N* and *T* for all scenarios of cross-dependence and variance regimes. This suggests that the findings are relevant for datasets that are commonly used in empirical application and that the issue does not alleviate quickly with larger panels.

				CS Indep	endence					SAR	R(1)				
			$\mathcal{H}_0$			$\mathcal{H}_1$			$\mathcal{H}_0$			$\mathcal{H}_1$			
N	Т	$\widehat{\text{Cor}}$	Joint	Mixed	Joint	$t_{DH}$	$t_{HS}$	Cor	Joint	Mixed	Joint	$t_{DH}$	$t_{HS}$		
Hon	ioskedasticity														
10	25	75.7	2.3	5.4	13.0	5.8	12.2	87.8	3.1	3.9	7.6	3.5	4.4		
10	50	75.7	2.3	5.7	34.6	6.1	21.5	87.7	3.0	4.3	17.1	4.5	8.6		
10	100	76.0	2.4	5.6	72.8	4.1	14.4	87.6	3.0	4.5	39.0	5.5	12.4		
10	250	75.7	2.4	5.8	97.6	1.3	0.9	87.5	3.0	4.6	78.0	3.4	7.9		
50	25	74.9	2.2	5.4	54.8	4.8	20.2	85.0	3.0	3.9	23.4	6.3	9.7		
50	50	74.6	2.3	5.2	95.7	0.5	3.4	84.6	2.8	4.2	61.5	5.4	11.5		
50	100	73.8	2.2	5.3	100	0	0	84.5	3.0	4.0	95.6	1.1	2.1		
50	250	74.4	2.3	5.4	100	0	0	84.8	2.9	4.3	100	0	0		
Earl	y negative shift														
10	25	76.0	2.3	5.3	4.3	3.4	4.6	89.2	3.1	3.8	3.7	2.3	2.2		
10	50	75.7	2.4	5.3	10.5	5.0	10.7	88.5	3.2	4.1	6.6	3.1	4.6		
10	100	75.7	2.4	5.5	30.6	5.8	22.7	87.6	2.9	4.8	16.2	4.0	9.0		
10	250	75.9	2.4	5.8	77.0	2.4	16.1	87.6	2.9	5.0	44.7	4.2	14.5		
50	25	75.4	2.3	5.2	8.7	5.5	7.7	85.6	3.0	3.9	5.5	3.2	3.4		
50	50	74.4	2.3	5.3	41.2	5.8	21.7	85.5	3.0	4.3	19.0	5.4	9.9		
50	100	74.2	2.2	5.5	92.8	0	0	85.0	3.1	4.1	58.2	4.3	15.2		
50	250	73.5	2.3	5.4	100	0	0	84.6	2.9	4.4	97.4	0.1	2.1		
Late	positive shift														
10	25	74.9	2.2	5.5	9.1	6.3	8.2	87.3	2.9	4.2	6.4	4.1	3.9		
10	50	74.9	2.2	5.8	24.2	9.4	13.5	86.9	3.0	4.4	13.4	5.6	6.3		
10	100	75.0	2.3	5.7	85.4	10.1	11.5	87.0	2.8	4.8	29.5	8.6	9.0		
10	250	75.5	2.3	5.8	94.4	4.1	0.7	87.0	2.9	5.2	67.9	11.1	5.0		
50	25	73.1	2.1	5.3	33.2	11.2	14.0	84.4	2.8	4.1	15.7	8.2	6.0		
50	50	73.3	2.2	5.8	81.7	6.6	5.6	84.3	2.8	4.3	42.8	13.1	6.6		
50	100	73.6	2.1	5.9	99.7	0.2	0.0	84.6	2.9	4.5	82.6	9.7	1.6		
50	250	74.3	74.3 2.3 5.8		100	0	0	84.6	2.7	5.0	99.7	0.3	0		
				Equicor	elation					Common	on Factor				
			$\mathcal{H}_0$		$\mathcal{H}_1$				$\mathcal{H}_0$		$\mathcal{H}_1$				
N	Т	$\widehat{\text{Cor}}$	joint	mixed	joint	$t_{DH}$	$t_{HS}$	Cor	joint	mixed	joint	$t_{DH}$	$t_{HS}$		

**Table 1.** Analysis of mixed signals: Columns  $t_{DH}$  and  $t_{HS}$  report proportions of only that test rejecting.  $\gamma_i \sim iid \ \mathcal{U}(0.9, 1), \ \mu_i \sim iid \ \mathcal{U}(0, 0.02)$ . SAR(1):  $\Theta = 0.8$ . Equicorrelation:  $\omega = 0.5$ ; Common Factor:  $v_t \sim \mathcal{N}(0, 1); \ \lambda_i \sim iid \ \mathcal{U}(0, 0.02); 25,000$  replications.

				Equicon	ciation										
			$\mathcal{H}_0$			$\mathcal{H}_1$			$\mathcal{H}_0$		_	$\mathcal{H}_1$			
N	Т	$\widehat{\text{Cor}}$	joint	mixed	joint	$t_{DH}$	$t_{HS}$	Cor	joint	mixed	joint	$t_{DH}$	$t_{HS}$		
Hon	ioskedasticity														
10	25	86.5	2.8	4.5	9.7	3.9	6.2	76.1	2.3	5.4	13.6	6.3	11.6		
10	50	86.1	2.7	5.0	21.4	4.5	10.7	75.7	2.4	5.6	35.6	6.5	20.2		
10	100	85.6	2.7	5.3	45.8	4.5	11.9	75.1	2.4	5.6	73.6	4.5	13.4		
10	250	85.8	2.6	5.4	79.8	2.9	7.5	75.7	2.3	5.6	97.9	1.2	0.8		
50	25	91.7	3.0	4.1	16.5	3.7	5.3	74.7	2.3	5.4	55.1	5.4	19.3		
50	50	91.6	3.0	4.5	33.7	4.3	6.8	74.0	2.2	5.5	95.8	0.5	3.2		
50	100	91.2	3.0	4.8	59.8	3.8	6.0	74.2	2.3	5.5	100	0	0		
50	250	91.2	2.8	5.2	87.8	1.1	4.6	74.2	2.2	5.5	100	0	0		
Earl	y negative shift														
10	25	87.3	2.6	4.7	3.6	2.6	2.9	76.1	2.3	5.3	4.5	3.5	5.0		
10	50	86.8	2.8	4.8	7.9	3.4	5.5	76.0	2.3	5.7	11.1	5.0	11.4		
10	100	86.4	2.6	5.7	20.0	4.3	10.4	76.2	2.4	5.6	31.6	5.7	22.6		
10	250	85.8	2.6	5.6	50.0	3.5	13.0	75.8	2.4	5.6	78.2	2.1	15.7		
50	25	92.8	3.3	3.3	4.8	2.3	2.0	75.7	2.2	5.6	9.5	5.5	7.9		
50	50	92.4	3.2	4.1	13.1	3.0	4.6	75.5	2.2	5.5	42.9	5.7	20.9		
50	100	91.8	3.1	4.4	31.7	3.2	7.3	74.7	2.3	5.4	93.6	0.4	5.2		
50	250	91.4	2.9	5.0	64.5	2.0	7.3	74.6	2.3	5.4	100	0	0		
Late	positive shift														
10	25	85.2	2.7	4.6	7.5	4.4	5.0	74.9	2.2	5.6	9.8	6.7	7.8		
10	50	85.3	2.5	5.1	15.9	5.9	8.2	75.2	2.4	5.5	23.9	9.8	12.9		
10	100	85.6	2.6	5.7	34.9	8.5	9.0	75.0	2.4	5.6	58.2	11.0	10.9		
10	250	85.7	2.6	5.8	71.5	9.4	4.3	75.5	2.4	5.9	94.4	4.2	0.6		
50	25	90.8	2.9	4.3	12.2	4.8	4.3	73.5	2.2	5.6	34.0	11.9	13.1		
50	50	90.7	2.9	4.7	26.2	6.5	5.9	73.6	2.1	5.9	81.8	7.0	5.3		
50	100	90.7	3.0	4.9	48.2	8.7	4.9	74.2	2.1	5.9	99.6	0.3	0		
50	250	90.8	3.0	5.2	80.2	8.9	1.8	74.2	2.3	5.7	100	0	0		

A somewhat different picture emerges under the alternative where joint rejection rates naturally improve in N and T due to consistency of both tests. However, proportions of mixed outcomes differ: They are linked to the difference in power between  $t_{DH}$  and  $t_{HS}$ , which it turn depends on N and T as well as the time and the degree of variance breaks; see Herwartz et al. (2016).<sup>8</sup> Our results indicate that mixed signals occur in all scenarios of cross-dependence and heteroskedasticity where the pattern is often similar: The probability of observing contradicting results is low to moderate when T is small relative to N, increases for panels of moderate dimensions, and then decreases towards zero as N and T get large. The proportion of mixed signals may be higher than 20%. An explanation is that one of the tests is often more powerful than the other such that the dissimilarity between test outcomes may be high. This is in contrast to larger panels where the additional amount of information against the null drives both tests towards rejection, viz. coinciding results are more likely. Furthermore, Table 1 indicates that mixed signals under the alternative cannot always be attributed to the less powerful test not rejecting. This is observed in all scenarios of cross-dependence under a late positive variance shift.

In summary, the results show that, despite the similar design of  $t_{DH}$  and  $t_{HS}$ , a researcher is likely to encounter mixed signals when both tests are applied to the same data.

#### 3.3. A Combined Testing Procedure

Consider the *p*-values  $p_1, \ldots, p_n$  obtained from suitably testing the individual null hypotheses  $\mathcal{H}_{0,1}, \ldots, \mathcal{H}_{0,n}$  such that  $p_j \sim \mathcal{U}(0, 1)$  under  $H_{0,j}$ . The so-called Bonferroni correction for testing the global or intersection null hypothesis  $\mathcal{H}_0 = \bigcap_{i=1}^n \mathcal{H}_{0,j}$  rejects  $\mathcal{H}_0$  when

$$p_j \le \alpha/n$$
 for at least one  $j = 1, \dots, n$ . (12)

Equation (12) allows to test the global null, controlling the family-wise error rate at  $\alpha$ . A particular merit of Equation (12) is that it does not require assumptions about the joint distribution of the underlying test statistics or their dependence. However, the procedure may be quite conservative and may have low power.

As an alternative, we consider the following modified Bonferroni procedure proposed by Simes (1986).

# Simes' Test

- 1. Obtain ordered *p*-values  $p_{(1)} \leq \cdots \leq p_{(n)}$  of *n* tests
- 2. Reject the global null if

$$p_{(j)} < j\alpha/n$$
 for at least one  $j = 1, \dots, n$ . (13)

Consider the case n = 2. Simes' test rejects  $\mathcal{H}_0 = \mathcal{H}_{0,1} \cap \mathcal{H}_{0,2}$  at  $\alpha = 0.05$  if  $p_{(1)} \le 0.025$  or  $p_{(2)} \le 0.05$ .

Simes (1986) proves that Equation (13) yields a conservative test provided the test statistics are independent. Simulation evidence in Simes (1986) also suggests that Equation (13) dominates the classical Bonferroni procedure if the individual tests statistics are highly positively correlated. Sarkar (1998) established conservativeness of Equation (13) for test statistics of which the joint null distribution is "multivariate totally positive of order 2" (MTP<sub>2</sub>), a substantially weaker assumption than that of independence which covers a rather large family of multivariate distributions.

Given the various positive correlations of  $t_{DH}$  and  $t_{HS}$  in the scenarios of cross-dependence and variance breaks, the simplicity and robustness to (certain forms of) positive dependence render

<sup>&</sup>lt;sup>8</sup> See Tables A1–A4 for a comparison of empirical size and size-adjusted power of  $t_{DH}$  and  $t_{HS}$  in various scenarios of cross-dependence and variance breaks.

Equation (13), henceforth S, a promising candidate for a combined test.<sup>9</sup> It is unclear whether the joint distribution of the tests is MTP<sub>2</sub>, and we do not give a proof. It has, however, been shown that the MTP<sub>2</sub> condition can be relaxed even further; see Sarkar (2008). To what extent S is of use in our setting is discussed by means of the simulation results discussed next.

## 3.3.1. Results

Tables A1–A4 report empirical rejection frequencies. The left panels correspond to DGP A while the right panels refer to DGP B. We confirm the finite sample results for  $t_{DH}$  and  $t_{HS}$  presented in Herwartz et al. (2016), i.e., distinct small-sample properties both under the null and under the alternative. Performance is also mostly satisfying under serial correlation. However,  $t_{DH}$  then is somewhat conservative for small *T*. Both tests have lower power than under uncorrelated disturbances. The results do not deteriorate much when weak-form cross-dependence imposed by Assumption  $A_{(ii)}$ is violated as is the case under equicorrelation and the common factor structure.

Turning to the *S* test, we find it does well in combining information from  $t_{DH}$  and  $t_{HS}$ . Under the null, *S* is often closer to the nominal level of 5% than  $t_{HS}$  and does compete well with the empirical size of  $t_{DH}$ . Under independence and spatial correlation, the size of *S* is best when *N* and *T* are large and when *T* is large relative to *N*. This is also the case under serially correlated disturbances, a setting that highlights a slight downside of *S*'s property to track the performance of  $t_{DH}$  under the null: *S* is undersized when  $t_{DH}$  produces too few rejections. While this behaviour is not overly pronounced, it stresses the need for larger samples when adjusting the data for short-run dynamics in applications. Largely, the same conclusions are obtained for equicorrelated panels and under a common factor, except that *S* is slightly conservative.

In terms of power, *S* typically tracks the more powerful of the individual tests. Nonetheless, *S* seems superior to simultaneously applying both tests, let alone application of only one test. *S* is especially beneficial in small to medium-sized panels where  $t_{HS}$  is often more powerful than  $t_{DH}$  as mixed signals are more likely; see Section 3.2 and Table 1.

We conclude that combining information from  $t_{HS}$  and  $t_{DH}$  may be beneficial for the following reasons: First, *S* avoids the choice either to risk size-distortions or low power in applications of only one test. Second, *S* circumvents the risk of mixed signals when using both tests. Both scenarios have been confirmed to be relevant in our simulation studies. Third, application of *S* is attractive in terms of simplicity because it comes with virtually no additional expense in computation. This is in contrast to other prominent methods used in meta-analysis such as, e.g., the *p*-value combination tests in Fisher (1932) and Stouffer et al. (1949) and modifications thereof as in Hartung (1999) and more recent contributions to the unit root literature like the union of rejections tests in Harvey et al. (2009). All these depend on the particular correlation structure of the test statistics and therefore require, e.g., use of resampling techniques.

#### 4. Application to Inflation Rates

Since, at least, Nelson and Plosser (1982), it has been argued that inflation may follow a stochastic trend. The persistence of inflation has far-reaching implications for the analysis and interpretation of fundamental economic relationships. This includes structural models of, e.g., different Phillips curve concepts and numerous models that rely upon price rigidities or the trending behaviour of the real interest rate, a major determinant of savings and investment (Culver and Papell 1997; Rose 1988). Further, the behaviour of inflation is important for central banks' monetary policy rules.

<sup>&</sup>lt;sup>9</sup> Other applications to panel unit root testing we are aware of are Hanck (2013), who suggested a PURT based on combining the significance of augmented Dickey-Fuller (ADF) tests by Equation (13) and provides simulation evidence for the procedure to work well in cross-dependent panels and extensions by Hanck and Czudaj (2015) which additionally accommodate for nonstationary volatility.

Empirical research arrives at different conclusions. There is pioneering work reporting stationarity, for example, Rose (1988), who investigated the stability of the U.S. real interest rate, and Barsky (1987), who analysed U.S. inflation and reported stationarity for subperiods. On the other hand, some authors found evidence that inflation is I(1), e.g., Johansen (1992), Ball and Cecchetti (1990), and Johansen and Juselius (2001).

More recent contributions reinvestigate the issue, e.g., Culver and Papell (1997), who studied monthly panel data for thirteen OECD countries ranging from February 1957 to September 1994. They found evidence for nonstationarity using ADF and KPSS tests but acknowledged the shortcomings of these tests and thus resorted to PURTs.<sup>10</sup> Their panel approach is based on the test by Levin et al. (2002) and rejects the unit root null even for various 4-element subsets of countries. A follow-up contribution by Basher and Westerlund (2008) revisits these conclusions as the employed testing procedure is not robust to cross-dependence and heteroskedasticity. Hence, one should be cautious in attributing different outcomes of time series and panel approaches to potential power gains from exploiting the additional variation in the cross-section dimension. As noted by Basher and Westerlund (2008), this is problematic as structural changes such as variance breaks tend to be more likely in macro series when long data spans are considered and comovements are strong. Basher and Westerlund (2008) reported results from second-generation PURTs which suggest that stationarity of inflation holds after accounting for general forms of cross-sectional dependence and structural breaks.

Next, we reconsider some of the conclusions in Culver and Papell (1997) by applying  $t_{DH}$ ,  $t_{HS}$ , and *S* to several panels of OECD inflation rates.<sup>11</sup> We consider two datasets and a total of four observation periods. First, the original monthly consumer price index (CPI) data from Culver and Papell (1997) from February 1957 to September 1994. Second, we use a larger set of monthly CPI data from April 1961 to March 2019 obtained from the OECD database for which we also consider two subperiods of roughly 30 years for robustness checks. These range from April 1961 to December 1989 and from January 1990 to March 2019. It is interesting to study evidence for these subsamples as they reflect different patterns of macroeconomic activities that led to structural shifts in volatility. This is particularly the case for the significantly less volatile last three decades which are marked by the advent of inflation targeting.

#### 4.1. Preliminary Analysis

Figure 2 plots year-on-year inflation rates of all countries for the longest observation span. Two characteristics shared by most of the series which are crucial for conducting inference based on PURTs are rather obvious: Eyeballing shows considerable comovement suggesting cross-dependence to be accounted for. Additionally, the moderate period up to the mid-1970s lapses into a highly volatile period which is then followed by a marked downward shift in volatility at the beginning of the 1990s. Reduced volatility persists for the following decades up to a kink situated around the financial crisis in 2008. These findings are supported by means of Figure 3 displaying quartiles for the sample distribution of inflation rates across the economies. The distribution's spread corresponds to previously identified regimes, i.e., moderate variance at the beginning of the sample, followed by a quickly extending interquartile range which diminishes again beyond 1990 and emerges into a more stable period characterised by low volatility. The findings thus suggest three distinct variance regimes.

<sup>&</sup>lt;sup>10</sup> ADF and KPSS tests struggle to distinguish between a unit root and stationarity for highly persistent but stationary time series. See, e.g., Maddala and Kim (1999) or Caner and Kilian (2001).

<sup>&</sup>lt;sup>11</sup> See the notes in Table 2 for groupings.



**Figure 2.** Inflation rates of OECD countries: Inflation rates are computed as year-on-year differences in the logarithm of monthly CPIs. The CPI base year is 2015. Data are obtained from the OECD database. The observation period is April 1961 to March 2019.





**Figure 3.** Summary statistics of OECD inflation rates: Inflation rates are computed as year-on-year differences in the logarithm of monthly CPIs. The CPI base year is 2015. Data are obtained from the OECD database. The observation period is April 1961 to March 2019.

We may further assess second moment variability via estimated variance profiles, i.e.,

$$\widehat{\eta}_{i}(s) := \frac{\sum_{t=1}^{\lfloor sT \rfloor} \widehat{\ell}_{i,t}^{2} + (sT - \lfloor sT \rfloor) \widehat{\ell}_{i,\lfloor sT \rfloor + 1}^{2}}{\sum_{t=1}^{T} \widehat{\ell}_{i,t}^{2}}, \quad 0 < s < 1$$
(14)

as proposed by Cavaliere and Taylor (2007).<sup>12</sup> Figure 4 shows that, for all four subsamples, none of the estimated variance profiles are close to the dashed red line, the theoretical profile of a homoskedastic series. The profiles display patterns suggesting that the volatility of most series experienced a positive break followed by a downward trend. Consider, e.g., Spanish inflation, which shows a sharp upward shift after about 20% and 30% of the sample that is followed by a moderation. Further, the profiles of some economies show very similar patterns, for instance, those of Italy and the United Kingdom. In this regard, the profiles for the recent three decades are particularly interesting as they display a global decrease in volatility and a considerable degree of convergence at the beginning of the recent financial crisis. This tendency of synchronous variance regime switching may also be seen as an indicator for economic comovement.

Culver and Papell (1997) used a simple dynamic panel approach allowing for country-specific fixed effects. The autoregressive coefficients are restricted to be equal for all cross sections. Thus, inflation  $\pi_{i,t}$  is modelled by a panel autoregressive process

$$\Delta \pi_{i,t} = \mu_i + \rho \pi_{i,t-1} + \sum_{j=1}^{p_i} \theta_{i,j} \Delta \pi_{i,t-j} + e_{i,t}.$$
(15)

Since it is plausible that the data exhibits short-run dynamics as well as nonzero fixed effects, the individual tests are conducted on prewhitened data that are subsequently centered by means of first observations as we expect inflation rates to have nonzero means under stationarity. The Schwarz information criterion (SIC) is used to determine the maximum optimal lag order  $p_i$  which is then used for the prewhitening procedure of Section 2.2.3.

<sup>&</sup>lt;sup>12</sup> The  $\hat{e}_{i,t}$  are OLS residuals from fitting AR(1) models to the series.



- BEL - CAN - DEU - ESP - FIN - FRA - GBR - ITA - JPN - LUX - NLD - NOR - USA

**Figure 4.** Estimated variance profiles for year-on-year inflation rates of selected OECD countries: (a) February 1957–September 1994, monthly CPI data from Culver and Papell (1997). Other panels: monthly CPI data from the OECD database for (b) April 1961–March 2019 (c) April 1961–December 1989 (d) January 1990–March 2019.

#### 4.2. Results from Robust PURTs

Table 2 presents the results of the analysis for eleven groups of countries. We first compare results for the data from Culver and Papell (1997). We find that  $t_{DH}$ ,  $t_{HS}$ , and S coincide and strongly reject the null of a panel unit root at the 1% level, except for the group of G7 countries where S rejects only at 5%. Virtually the same results are obtained for year-on-year inflation rates from April 1961 to March 2019 where all tests find strong evidence against the null with the exception of S, which rejects only at 5% for the group of non-G7 economies.

For the subsample from April 1961 to Dec. 1989, we find weaker evidence against the null as *S* rejects mostly at 5%. A distinctive feature here is that  $t_{DH}$  and  $t_{HS}$  produce mixed signals at 5% for all groupings except the G7 and the "unit root seven" where *S* retains the null at 10%.

Turning to the less volatile period from January 1990 to March 2019, mixed signals are found for all groups since  $t_{DH}$  does not reject once but  $t_{HS}$  finds evidence at 10%, except for group k. However, combining significance by *S* does not reject the null even at 10% as *p*-values for  $t_{HS}$  do not fall below the lower cutoff of 0.05 for any of the groupings. This is a rather peculiar finding as nonstationarity of inflation rates implies no mean reversion which we presume to be an effect of inflation targeting, alongside reduced volatility. A possible explanation is that structural change due to increased integration of monetary policy implies the downward shift in volatility and stronger cross-dependence. Both are observed characteristics of the subsample which were shown to result in lower power of the tests.

Culver and Papell (1997) further assess how much cross-section variation is required for rejection of the unit root null by computing all possible combinations of countries in panels of sizes k = 2, ..., 13. They report that, strikingly, any combination of five countries is already sufficient for rejection at

5% in all  $\binom{13}{5} = 1287$  panels. We reinvestigate this issue for both datasets using *S* and also report rejection rates for  $t_{DH}$  and  $t_{HS}$  to gauge for the potential of observing mixed evidence. Table 3 presents the results.

**Table 2.** Robust testing for unit roots in inflation panels of OECD countries. The groupings are as in Culver and Papell (1997): (a) Belgium, Canada, Germany, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxemburg, the Netherlands, Norway, and the United States; (b) excludes France; (c) excludes Japan (d); excludes the Netherlands; (e) excludes France and Japan; (f) excludes France and the Netherlands; (g) excludes Japan and the Netherlands; (h) excludes France, Japan, and the Netherlands; (i) G7: Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States; (j) the other six: Finland, Japan, Luxemburg, the Netherlands, Norway, and Spain; and (k) "Unit Root Seven": Canada, Finland, Germany, Italy, Luxemburg, the Netherlands, and the United States. <sup>*a*</sup> Rejection at the 1% level. <sup>*b*</sup> Rejection at the 5% level. <sup>*c*</sup> Rejection at the 10% level. Entries for the *S* test are "1" when the underlying *p*-values imply mixed signals and are "0" for all else.

	Culve	er & Papel	l									
	1957-	02–1994-09	)	1961-	04-2019-0	3	1961-	04–1989-12	2	1990-0	1–2019-03	
Grouping	t <sub>DH</sub>	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	s
(a)	-3.39 <sup><i>a</i></sup> (0.0003)	-3.44 <sup><i>a</i></sup> (0.0003)	0 <i>a</i>	-3.63 <sup><i>a</i></sup> (0.0004)	-3.16 <sup><i>a</i></sup> (0.0008)	0 <sup>a</sup>	-1.43 <sup><i>c</i></sup> (0.0764)	-2.31 <sup><i>a</i></sup> (0.0104)	1 <sup>b</sup>	-0.49 (0.3121)	-1.62 <sup>c</sup> (0.0526)	1
(b)	-3.46 <sup><i>a</i></sup> (0.0003)	-3.43 <sup><i>a</i></sup> (0.0003)	0 <i>a</i>	-3.50 <sup><i>a</i></sup> (0.0002)	-3.15 <sup><i>a</i></sup> (0.0008)	0 <sup>a</sup>	-1.40 <sup>c</sup> (0.0808)	-2.30 <sup>b</sup> (0.0107)	1 <sup>b</sup>	-0.51 (0.3050)	-1.62 <sup>c</sup> (0.0526)	1
(c)	-3.48 <sup><i>a</i></sup> (0.0003)	-3.46 <sup><i>a</i></sup> (0.0003)	0 <i>a</i>	-3.51 <sup><i>a</i></sup> (0.0004)	-3.08 <sup><i>a</i></sup> (0.0010)	0 <i>a</i>	-1.20 (0.1151)	-2.19 <sup>b</sup> (0.0143)	1 <sup>b</sup>	-0.55 (0.2912)	-1.54 <sup>c</sup> (0.0618)	1
(d)	$-3.17^{a}$ (0.0008)	$-3.19^{a}$ (0.0007)	0 <i>a</i>	-3.68 <sup><i>a</i></sup> (0.0004)	-2.99 <sup><i>a</i></sup> (0.0014)	0 <i>a</i>	-1.48 <sup><i>c</i></sup> (0.0694)	-2.14 <sup>b</sup> (0.0162)	1 <sup>b</sup>	-0.26 (0.3974)	-1.54 <sup>c</sup> (0.0618)	1
(e)	$-3.58^{a}$ (0.0002)	-3.46 <sup><i>a</i></sup> (0.0003)	0 <i>a</i>	-3.35 <sup><i>a</i></sup> (0.0004)	-3.04 <sup><i>a</i></sup> (0.0012)	0 <i>a</i>	-1.15 (0.1251)	-2.16 <sup>b</sup> (0.0154)	1 <sup>b</sup>	-0.57 (0.2843)	-1.53 <sup>c</sup> (0.063)	1
(f)	-3.24 <sup><i>a</i></sup> (0.0001)	$-3.18^{a}$ (0.0023)	0 <i>a</i>	-3.53 <sup><i>a</i></sup> (0.0004)	-2.98 <sup><i>a</i></sup> (0.0014)	0 <i>a</i>	-1.45 <sup>c</sup> (0.0735)	-2.12 <sup>b</sup> (0.017)	1 <sup>b</sup>	-0.26 (0.3974)	-1.53 <sup>c</sup> (0.063)	1
(g)	$-3.27^{a}$ (0.0005)	$-3.19^{a}$ (0.0007)	0 <i>a</i>	-3.56 <sup><i>a</i></sup> (0.0002)	-2.89 <sup><i>a</i></sup> (0.0019)	0 <i>a</i>	-1.25 (0.1056)	-1.99 <sup>b</sup> (0.0233)	1 <sup>b</sup>	-0.31 (0.3783)	-1.45 <sup>c</sup> (0.0735)	1
(h)	$-3.37^{a}$ (0.0004)	$-3.19^{a}$ (0.0007)	0 <i>a</i>	-3.39 <sup><i>a</i></sup> (0.0003)	-2.85 <sup><i>a</i></sup> (0.0022)	0 <i>a</i>	-1.20 (0.1151)	-1.95 <sup>b</sup> (0.0256)	1 <sup>c</sup>	-0.31 (0.3783)	-1.44 <sup>c</sup> (0.0749)	1
(i)	$-1.87^{\ b}$ (0.0307)	$-1.76^{\ b}$ (0.0392)	0 <sup>b</sup>	-3.85 <sup><i>a</i></sup> (0.0001)	-2.44 <sup><i>a</i></sup> (0.0073)	0 <sup><i>a</i></sup>	-1.67 <sup>b</sup> (0.0475)	-1.71 <sup>b</sup> (0.0436)	0 <sup>b</sup>	-0.02 (0.4920)	-1.41 <sup>c</sup> (0.0793)	1
(j)	$-3.00^{a}$ (0.0013)	$-3.13^{a}$ (0.0009)	0 <sup><i>a</i></sup>	$-2.09^{b}$ (0.0183)	$-2.60^{a}$ (0.0047)	1 <sup>b</sup>	-0.62 (0.2676)	$-1.92^{b}$ (0.0274)	1 <sup>c</sup>	-0.93 (0.1762)	-1.54 <sup>c</sup> (0.0618)	1
(k)	$-2.99^{a}$ (0.0014)	$-2.82^{a}$ (0.0024)	0 a	-2.97 <sup><i>a</i></sup> (0.0015)	-2.28 <sup>b</sup> (0.0113)	0 <i>a</i>	-0.67 (0.2514)	-1.50 <sup>c</sup> (0.0668)	0	0.14 (0.5557)	-1.07 (0.1423)	1

As expected, our results for the original data are somewhat weaker but confirm that the null is rarely accepted in panels of five economies. Evidence against the null increases markedly as we turn to combinations of six and more countries. *S* rejects nonstationarity at 1% in almost 80% of all combinations of seven economies. We find similar results for the extended observation period of year-on-year inflation rates where *S* rejects at 1% in 87.1% of all possible combinations for k = 7. The results for April 1961 to Dec. 1989 also show considerable evidence against the null for  $k \ge 6$ , although the proportion of strong rejections is noticeably lower and most rejections occur at 5%. Finally, the results presented in the bottom panel underline that the recent three decades provide conspicuously little evidence in favour of the alternative since the proportion of combinations in which the null is not rejected ranges above 75%, even in panels of five or more economies. Another result is that the tendency for mixed signals is high, especially when testing at 5% based on year-on-year rates from April 1961 to Dec. 1989. For this subsample,  $t_{HS}$  retains the null far more often than  $t_{DH}$ , such that it appears worthwhile to rely on the combined evidence of *S*.

k = 1, m = 13			k = 2, m = 78			k = 3, m = 286			k = 4, m = 715			k = 5, m = 1287			k = 6, m = 1716			k = 7, m = 1716		1716	
Level	$t_{DH}$	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S
Culver & Pappel (19	997) dati	a																			
1%	23.1	30.8	23.1	11.5	28.2	23.1	26.9	30.1	26.6	42.1	30.8	36.4	57.9	40.6	48.3	72.8	57.6	64.9	83.9	74.7	79.7
5%	15.4	30.8	30.8	47.4	32.1	35.9	41.6	43.4	42.7	39.2	55.0	43.8	33.3	54.8	44	23.5	41.4	31.8	15.3	25.3	19.9
10%	23.1	7.7	7.7	12.8	19.2	15.4	13.3	17.1	14.3	11.9	11.9	14.4	6	4.6	6.1	3.5	1	3.3	0.8	0	0.4
> 10% (no rej.)	38.5	30.8	38.5	28.2	20.5	25.6	18.2	9.4	16.4	6.9	2.4	5.5	2.8	0	1.6	0.2	0	0	0	0	0
1961-04 to 2019-03																					
1%	7.7	0	0	0.3	6.4	6.4	22.7	19.9	17.5	37.1	38.9	33.8	52.1	62.3	53.6	69.4	79.8	72.6	83.7	92.5	87.1
5%	15.4	38.5	30.8	30.8	48.7	33.3	40.6	59.4	49.3	44.1	55.9	52.6	40.4	36.8	42.4	28.4	20.2	26.6	16.0	7.5	12.9
10%	7.7	15.4	15.4	20.5	26.9	25.6	20.3	19.2	22.7	14.1	5.2	11.9	6.5	0.9	3.8	2.2	0	0.8	0.3	0	0
> 10% (no rej.)	69.2	46.2	53.8	38.5	17.9	34.6	16.4	1.4	10.5	4.8	0	1.7	1.0	0	0.2	0.1	0	0	0	0	0
1961-04 to 1989-12																					
1%	7.7	0	0	7.7	0	3.8	11.9	0.3	6.6	19.3	1.8	10.6	27.3	6.3	16.3	37.3	18.9	25.0	48.3	38.7	37.4
5%	15.4	7.7	15.4	20.5	17.9	19.2	32.9	37.4	29.0	40.6	66.4	44.3	45.3	80.8	56.3	46.7	76.6	62.6	43.5	60.4	58.0
10%	7.7	23.1	7.7	19.2	38.5	17.9	19.9	44.1	28.0	20.6	27.1	30.6	18.3	12.2	23.9	12.3	4.5	11.7	7.3	0.9	4.6
> 10% (no rej.)	100	84.6	92.3	52.6	43.6	59.0	35.3	18.2	36.4	19.6	4.6	14.4	9.1	0.7	3.5	3.7	0	0.8	1.0	0	0
1990-01 to 2019-03																					
1%	0	0	0	0	3.8	1.3	0.3	4.9	2.4	0.1	4.8	1.7	0	3.9	1.2	0	3.1	0.6	0	2.0	0.3
5%	0	7.7	0	13.8	12.8	9.0	4.5	14.7	9.4	2.9	18.3	10.8	2.5	20.0	11.9	1.3	20.5	10.8	0.7	20.6	9.5
10%	0	7.7	7.7	3.8	12.8	6.4	5.9	16.1	9.1	6.2	15.5	11.0	4.8	18.9	11.2	4.5	22.0	12.6	3.4	27.0	13.0
> 10% (no rej.)	100	84.6	92.3	92.3	70.5	83.3	89.2	64.3	79.0	90.8	61.4	76.5	92.7	57.3	75.8	94.1	54.4	75.9	95.9	50.3	77.2

**Table 3.** Robust testing for unit roots in inflation rates of OECD countries—all combinations. Economies: Belgium, Canada, Germany, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxemburg, the Netherlands, Norway, and the United States. Inflation rates are computed as differences in logarithms of CPIs. Entries are proportions of rejection of  $m = \binom{13}{k}$  subsets of k countries. For prewhitening, the maximum optimal lag order is determined using the SIC.

Altogether, evidence from the *S* test suggests robustness of the finding of stationarity of inflation in Culver and Papell (1997) when we account for variance breaks and weak cross-dependence for both the original dataset and the extended sample of year-on-year inflation rates. The results for the recent three decades point towards nonstationarity, which contradicts the notion that inflation targeting establishes a stable regime of low inflation rates. This finding should be considered with caution as it might be due to structural breaks in the intercept which may not be well-handled by the tests.

## 5. Conclusions

This article investigates the performance of two recently proposed homogenous panel unit root tests, the White-type tests proposed by Herwartz and Siedenburg (2008) and its Cauchy counterpart suggested by Demetrescu and Hanck (2012a). Both are tailored for applications to weakly cross-dependent data that exhibit nonstationary volatility such as breaks in the unconditional variance. Our Monte Carlo studies confirm that the Cauchy counterpart has better size-control but is less powerful than the White-type test when applied in small to medium-sized samples. A common practice in such a situation is to apply both tests and to interpret the results jointly. This is shown to be problematic: We find that the tests are imperfectly correlated under the null and have a tendency to produce contradicting test outcomes which are not straightforward to interpret. The extent of this issue is found to depend on the type and degree of the cross-sectional dependence in the data. Prevalence of mixed signals is shown to be most pronounced under the alternative in small to medium-sized samples when the power differential of the tests is large. This is problematic for the applied researcher, as it is inadmissible to simply follow the decision of the rejecting test or some related rejection rule which ignores the multiple testing nature of the problem. As a remedy, we suggest combining information from both tests using the improved Benferroni procedure proposed by Simes (1986). Simulation evidence reveals that this simple approach does well in controlling size and has power close to the more powerful individual test.

An empirical application reinvestigates whether there is a unit root in year-on-year inflation rates using panels of monthly data for selected OECD countries. Our findings are twofold. We establish robustness of the results by Culver and Papell (1997), who reported stationarity for monthly inflation rates. The combined test also rejects the null for year-on-year inflation rates observed over an extended sample span which also covers the recent three decades. On the other hand, we find little evidence for stationarity when applying the combined test to a subsample of the last thirty years—a result which may be attributed to ongoing structural change and thus should be treated with caution as the tests may lack power under such conditions. This issue could be investigated using PURTs that allow for breaks both in the variance and in the mean which is, however, beyond the scope of this paper.

Future research could extend the multiple testing procedure towards models that include a linear trend under the alternative. It could also be worthwhile to investigate the benefits of applying the procedure to other PURTs as it is not restricted to the volatility-break robust tests considered here. Furthermore, it would be interesting to systematically compare the performance of Simes' test with other meta tests that are less reliable if the information to be combined is not independent. A feasible path to be followed is resampling, e.g., using the wild bootstrap procedure suggested by Herwartz and Walle (2018).

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# Abbreviations

The following abbreviations are used in this manuscript:

ADF	augmented Dickey-Fuller
CPI	consumer price index
DGP	data generating process
I(1)	integrated of order one
EU	European Union
KPSS	Kwiatkowski-Phillips-Schmidt-Shin
MTP <sub>2</sub>	multivariate totally positive of order 2
OECD	Organisation for Economic Co-operation and Development
PURT	panel unit root test
SIC	Schwarz information criterion

# Appendix A

**Table A1.** Rejection frequencies in cross-sectionally independent panels.  $\gamma_i \sim iid \ U(0.9,1)$ ,  $\mu_i \sim iid \ U(0,0.02)$ . DGP B:  $\theta_i \sim iid \ U(0.2,0.4)$ . Power is computed using size-adjusted 5% critical values; 25,000 replications were used.

				D	GP A			DGP B							
			Size			Power			Size			Power			
N	Т	$t_{DH}$	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S		
Hon	noskedasticit	y													
10	25	5.1	6.4	4.9	18.9	25.2	23.1	3.5	6.2	4.0	15.0	18.0	17.6		
10	50	5.4	6.6	5.2	39.8	54.4	51.5	4.0	6.4	4.4	35.4	47.0	45.0		
10	100	5.2	7.0	5.3	76.7	85.6	86.6	4.4	6.6	4.6	72.1	82.3	83.2		
10	250	5.2	7.2	5.6	98.8	98.2	99.5	4.5	6.6	4.7	98.7	98.2	99.5		
50	25	5.2	5.5	4.5	59.0	77.0	73.8	3.1	8.8	4.9	44.5	55.5	55.5		
50	50	4.9	5.8	4.8	96.5	99.1	99.1	3.7	7.0	4.5	91.8	97.4	97.5		
50	100	4.9	5.5	4.6	100.0	100.0	100.0	4.3	6.3	4.6	100.0	100.0	100.0		
50	250	5.0	6.0	4.9	100.0	100.0	100.0	4.3	5.6	4.3	100.0	100.0	100.0		
Ear	ly negative sh	ıift													
10	25	5.0	6.0	4.3	7.7	8.9	8.7	3.7	6.7	4.1	5.9	6.0	6.0		
10	50	4.8	6.4	4.8	15.7	20.2	18.8	3.1	6.3	3.8	13.2	15.4	14.8		
10	100	5.1	6.5	4.9	36.0	50.8	48.2	3.6	6.4	4.1	32.3	43.6	42.6		
10	250	5.1	6.9	5.3	79.0	91.7	91.4	4.1	6.5	4.7	76.2	89.8	89.6		
50	25	5.1	5.3	4.3	13.8	18.0	16.7	3.9	10.5	6.1	6.2	3.7	4.4		
50	50	5.0	5.4	4.4	46.9	64.8	61.2	2.6	9.1	5.0	35.0	34.5	35.9		
50	100	5.0	5.5	4.5	93.2	98.9	98.7	3.0	7.4	4.5	89.2	95.8	96.7		
50	250	4.9	5.8	4.7	100.0	100.0	100.0	3.9	6.5	4.6	100.0	100.0	100.0		
Late	e positive shif	t													
10	25	5.2	5.4	4.0	15.7	17.3	17.0	4.1	4.8	3.4	11.3	13.1	12.5		
10	50	5.1	5.8	4.5	32.9	35.9	34.8	4.3	5.8	4.0	26.5	27.8	29.3		
10	100	4.9	6.3	4.7	69.6	66.9	71.8	4.9	6.5	4.9	59.1	58.8	62.8		
10	250	4.8	6.6	5.0	98.7	94.2	98.5	4.9	6.4	4.9	97.7	92.9	97.7		
50	25	4.8	4.9	3.7	46.2	49.4	50.0	4.0	6.6	3.8	27.9	29.9	31.7		
50	50	5.2	5.6	4.4	88.4	87.1	90.1	5.2	7.8	5.2	72.8	75.3	78.8		
50	100	5.1	5.6	4.6	100.0	99.6	100.0	5.1	6.9	5.1	99.6	99.2	99.8		
50	250	5.3	5.9	5.1	100.0	100.0	100.0	5.5	6.8	5.2	100.0	100.0	100.0		

**Table A2.** Rejection frequencies in cross-dependent panels—spatial correlation.  $\gamma_i \sim iid \mathcal{U}(0.9, 1)$ ,  $\mu_i \sim iid \mathcal{U}(0, 0.02)$ ,  $\Theta = 0.8$ . DGP B:  $\theta_i \sim iid \mathcal{U}(0.2, 0.4)$ . Power is computed using size-adjusted 5% critical values; 25,000 replications were used.

		DGP A									D	GP B		
			Size			Power	i			Size			Power	•
N	Т	t <sub>DH</sub>	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S		t <sub>DH</sub>	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S
Hor	noskedasticity													
10	25	5.5	7.3	5.1	11.1	12.0	11.7		3.3	5.4	3.1	10.6	11.2	11.5
10	50	5.8	7.8	5.6	20.8	23.8	23.2		4.3	6.6	4.3	19.2	22.0	21.4
10	100	5.8	8.1	5.9	43.2	48.2	47.4		5.2	7.7	5.4	39.2	44.2	43.4
10	250	5.9	8.4	6.0	80.6	83.6	84.8		5.3	8.0	5.5	78.5	81.7	82.6
50	25	5.4	6.1	4.6	30.3	36.3	34.8		3.2	6.3	3.6	24.1	26.6	26.7
50	50	5.4	6.4	5.0	67.3	74.4	73.5		3.9	6.3	4.2	60.2	67.1	66.4
50	100	5.3	6.5	5.0	96.8	97.8	98.2		4.5	6.7	4.6	94.9	96.4	96.9
50	250	5.5	6.9	5.2	100	100	100		4.8	6.5	4.9	100	100	100
Earl	ly negative shift													
10	25	5.1	6.3	4.5	6.0	5.9	5.9		3.4	5.4	3.2	5.0	4.7	4.9
10	50	5.6	7.1	5.0	8.6	9.6	9.4		3.3	5.4	3.1	8.6	9.2	9.2
10	100	5.5	7.7	5.5	18.8	21.1	20.5		4.0	6.6	4.2	17.7	19.5	19.3
10	250	5.5	7.9	5.6	47.3	53.5	52.4		4.6	7.5	5.0	44.9	50.3	50.0
50	25	5.1	5.9	4.4	8.8	9.3	9.3		3.4	7.1	4.1	5.7	4.5	4.8
50	50	5.2	6.4	4.8	23.8	28.0	26.8		2.8	6.6	3.6	20.0	19.3	19.5
50	100	5.3	6.5	5.0	61.5	72.2	69.5		3.3	6.2	3.8	57.0	65.0	64.4
50	250	5.3	6.5	4.9	97.4	99.5	99.3		4.2	6.5	4.5	96.3	99.2	99.1
Late	e positive shift													
10	25	5.3	6.1	4.4	10.5	10.3	10.5		3.4	4.0	2.6	8.3	9.0	8.8
10	50	5.4	6.6	4.9	18.8	18.4	18.8		4.5	5.4	3.8	14.6	15.8	15.7
10	100	5.5	7.3	5.2	37.3	34.1	37.4		4.9	6.4	4.5	32.5	31.3	33.3
10	250	5.7	8.1	5.8	78.0	67.5	76.2		5.6	7.7	5.4	72.6	64.0	71.5
50	25	5.0	5.6	4.2	24.8	22.6	24.6		3.7	4.9	3.0	16.9	17.4	17.8
50	50	5.1	6.0	4.4	56.7	49.1	55.9		4.6	6.2	4.3	43.0	39.6	43.1
50	100	5.4	6.4	5.0	92.0	82.9	90.6		5.2	6.6	4.9	86.1	77.8	84.8
50	250	5.5	6.8	5.1	100	99.6	100		5.4	6.6	5.0	100	99.5	100

**Table A3.** Rejection frequencies in cross-dependent panels—equicorrelation.  $\gamma_i \sim iid \mathcal{U}(0.9, 1)$ ,  $\mu_i \sim iid \mathcal{U}(0, 0.02)$ ,  $\omega = 0.5$ . DGP B:  $\theta_i \sim iid \mathcal{U}(0.2, 0.4)$ . Power is computed using size-adjusted

				DC	GP A			DGP B							
			Size			Power			Size			Power			
N	Т	$t_{DH}$	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S		
Hon	noskedasticity														
10	25	5.4	6.6	4.9	13.6	15.9	15.1	3.3	5.1	3.0	11.6	13.3	13.2		
10	50	5.7	7.4	5.3	25.2	30.4	28.9	4.4	6.4	4.3	23.1	27.1	26.4		
10	100	5.8	7.7	5.8	48.9	54.0	53.2	4.8	7.3	5.0	46.1	50.6	50.3		
10	250	5.9	7.8	5.8	81.8	85.3	85.5	5.3	7.4	5.3	80.3	83.7	84.3		
50	25	6.1	6.9	5.1	18.7	21.0	20.6	3.3	5.5	3.2	16.1	17.5	17.2		
50	50	6.4	7.8	5.8	35.3	37.4	37.4	4.6	6.7	4.2	32.8	35.4	34.9		
50	100	6.8	8.0	6.2	60.4	62.3	61.8	5.3	7.4	5.0	58.1	59.6	60.0		
50	250	7.0	8.5	6.5	87.2	90.5	89.8	6.4	8.2	6.1	85.9	89.3	88.4		
Earl	ly negative shift														
10	25	5.1	6.3	4.4	6.2	6.6	6.6	3.4	5.6	3.3	5.3	4.8	5.1		
10	50	5.4	6.7	4.9	10.7	12.3	11.9	3.3	5.3	3.3	9.7	10.8	10.5		
10	100	5.8	7.6	5.6	22.5	26.6	26.0	3.8	6.2	3.9	20.7	24.3	24.4		
10	250	5.7	7.6	5.6	51.8	58.7	57.9	4.7	7.1	4.8	50.3	57.5	56.4		
50	25	5.1	5.9	4.2	7.2	7.2	7.3	3.3	5.6	3.2	4.7	3.8	4.1		
50	50	5.8	7.0	5.0	14.4	16.5	16.0	2.8	5.5	3.0	13.3	12.5	13.0		
50	100	6.3	7.3	5.6	31.5	35.7	34.4	3.8	6.6	3.9	31.0	32.6	32.7		
50	250	6.4	7.9	5.7	63.4	68.0	67.1	5.3	7.7	5.2	61.2	65.1	64.0		
Late	e positive shift														
10	25	5.3	5.7	4.3	11.8	12.5	12.6	3.6	3.7	2.6	8.9	10.5	10.1		
10	50	5.3	6.2	4.5	21.9	22.7	23.4	4.4	5.4	3.8	17.1	18.2	18.2		
10	100	5.8	7.3	5.4	42.0	38.7	42.3	5.1	6.5	4.6	36.1	36.0	38.1		
10	250	5.9	7.6	5.8	79.8	71.1	78.3	5.7	7.3	5.4	76.2	68.4	75.2		
50	25	5.7	5.6	4.2	16.3	16.9	17.0	3.5	4.2	2.7	11.9	13.1	12.7		
50	50	6.3	6.5	5.0	30.1	30.1	31.0	4.6	5.5	3.8	24.3	25.6	25.6		
50	100	6.4	7.3	5.6	53.6	48.5	52.6	5.7	6.7	4.9	47.4	44.5	47.5		
50	250	6.8	8.0	6.2	87.5	78.0	85.5	6.4	7.4	5.6	84.2	75.8	82.8		

				D	GP A					D	GP B				
			Size			Power			Size			Power			
N	Т	t <sub>DH</sub>	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	$t_{DH}$	$t_{HS}$	S	t <sub>DH</sub>	$t_{HS}$	S		
Hor	noskedasticity														
10	25	4.8	6.3	4.6	20.0	25.2	24.1	3.4	6.3	3.8	15.2	18.0	18.2		
10	50	5.0	6.8	5.1	41.2	53.6	52.0	4.0	6.4	4.3	36.1	46.5	45.2		
10	100	4.9	6.7	5.1	77.7	85.9	86.9	4.3	6.4	4.7	72.9	83.1	83.5		
10	250	5.1	6.8	5.3	99.0	98.5	99.6	4.7	6.9	5.2	98.6	98.2	99.5		
50	25	5.1	5.6	4.5	59.5	76.4	73.4	3.1	8.9	5.1	45.2	54.3	54.2		
50	50	5.0	5.8	4.5	96.2	99.1	99.2	3.8	7.1	4.5	91.8	97.5	97.5		
50	100	5.2	5.8	4.8	100	100	100	3.9	6.1	4.2	100	100	100		
50	250	4.8	6.0	4.9	100	100	100	4.7	6.2	4.8	100	100	100		
Ear	ly negative shift														
10	25	5.0	6.0	4.4	8.0	9.5	9.1	3.5	6.7	3.8	6.5	6.1	6.4		
10	50	5.1	6.6	4.9	15.8	20.7	19.5	3.2	6.5	3.9	13.6	15.4	15.9		
10	100	4.9	6.4	4.8	37.6	52.4	49.4	3.7	6.4	4.2	32.7	45.0	43.0		
10	250	4.8	6.7	5.1	81.0	92.8	92.8	4.4	6.8	4.8	77.5	90.3	90.4		
50	25	5.1	5.6	4.3	14.7	18.0	17.5	3.7	10.6	5.9	7.2	4.4	5.3		
50	50	5.0	5.7	4.6	48.7	64.3	61.6	2.9	8.8	5.1	35.1	37.0	38.1		
50	100	5.0	5.7	4.7	94.0	98.9	98.7	3.2	7.5	4.6	89.1	95.8	96.0		
50	250	4.9	5.8	4.8	100	100	100	3.9	6.7	4.6	100	100	100		
Late	e positive shift														
10	25	4.9	5.5	4.0	16.5	17.6	18.3	3.7	4.6	3.1	11.9	13.2	13.1		
10	50	4.9	6.0	4.6	33.9	35.4	37.0	4.6	5.9	4.3	24.8	27.7	28.5		
10	100	4.9	6.1	4.7	69.5	67.0	72.5	4.6	6.1	4.5	60.7	60.4	64.7		
10	250	5.0	6.9	5.1	98.6	94.0	98.6	4.9	6.8	5.1	97.6	93.1	97.7		
50	25	5.0	5.2	3.9	45.7	48.4	51.0	4.3	6.4	4.0	26.2	30.9	30.8		
50	50	5.1	5.5	4.4	88.4	87.0	90.9	4.9	7.5	5.1	74.2	76.1	79.3		
50	100	5.1	5.7	4.7	99.9	99.7	100	4.9	6.9	4.9	99.6	99.2	99.8		
50	250	5.1	5.8	4.8	100	100	100	5.2	6.7	5.3	100	100	100		

**Table A4.** Rejection frequencies in cross-dependent panels—common factor.  $\gamma_i \sim iid \mathcal{U}(0.9, 1)$ ,  $\mu_i \sim iid \mathcal{U}(0, 0.02)$ ,  $v_t \sim \mathcal{N}(0, 1)$ ,  $\lambda_i \sim iid \mathcal{U}(0, 0.02)$ . DGP B:  $\theta_i \sim iid \mathcal{U}(0.2, 0.4)$ . Power is computed using size-adjusted 5% critical values; 25,000 replications were used.

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