Sensors **2008**, *8*, 1343-1350

sensors

ISSN 1424-8220

www.mdpi.org/sensors

Full Research Paper

Range and Velocity Estimation of Moving Targets Using Multiple Stepped-frequency Pulse Trains

Gang Li [1,*], Huadong Meng [1], Xiang-Gen Xia [2] and Ying-Ning Peng [1]

1 Department of Electronic Engineering, Tsinghua University, Beijing 100084, China
2 Department of Electrical and Computer Engineering, University of Delaware, Newark, DE 19716, USA

* Author to whom correspondence should be addressed. Email: gangli@tsinghua.edu.cn

Received: 2 February 2008 / Accepted: 20 February 2008 / Published: 22 February 2008

Abstract: Range and velocity estimation of moving targets using conventional stepped-frequency pulse radar may suffer from the range-Doppler coupling and the phase wrapping. To overcome these problems, this paper presents a new radar waveform named multiple stepped-frequency pulse trains and proposes a new algorithm. It is shown that by using multiple stepped-frequency pulse trains and the robust phase unwrapping theorem (RPUT), both of the range-Doppler coupling and the phase wrapping can be robustly resolved, and accordingly, the range and the velocity of a moving target can be accurately estimated.

Keywords: moving target, range and velocity estimation, stepped-frequency, robust phase unwrapping

1. Introduction

Range and velocity estimation of moving targets using high range resolution radar is a topic of great interest. To achieve this goal, there are two problems to be considered. The *first problem* is how to obtain high range resolution. It is known that the range resolution is inversely proportional to the bandwidth of the transmitted signal and directly transmitting and receiving the ultra-wideband signal are difficult to be realized. Stepped-frequency pulse train processing is a well-known technique to obtain high range resolution without the requirement of wide instantaneous bandwidth [1-7], and it is adopted in this paper. The *second problem* is how to correctly measure the range and velocity of the moving target. When the target of interest is stationary, its position can be estimated by inverse discrete Fourier transform (IDFT) on the stepped-frequency pulse train [4-6]. However, when the target is moving, it is necessary to deal with the following difficulties. Firstly, the radial velocity of the target may cause range estimate shifted, which is called range-Doppler coupling [4-6], and both range position and radial velocity can not be correctly retrieved in the IDFT results. Secondly, for high speed target such as airplane or missile, the IDFT results only provide the remainders of the estimate of coupled range and velocity due to the 2π modulo folding of the IDFT, which is called phase wrapping. Thirdly, the remainders may be erroneous due to the possible interference, e.g., noise or clutter. Several effective methods have been proposed to resolve the range-Doppler coupling, such as by randomized stepped frequency modulation [6] or by elaborated design of the interpulse frequency coding [4-5]. However, how to robustly resolve the phase wrapping problem is not considered in these methods.

In this paper, we propose a new algorithm to robustly resolve both of the range-Doppler coupling and the phase wrapping. The radar transmits and receives multiple stepped-frequency pulse trains with different carrier frequencies, and IDFT is operated on every pulse train. It will be shown that, using the difference information among multiple IDFT results and the robust phase unwrapping theorem (RPUT) [8-9], both range and radial velocity of the moving target can be correctly estimated.

This paper is organized as follows. In Section 2, the range-Doppler coupling and the phase wrapping is reviewed in conventional stepped-frequency pulse train processing. In Section 3, the multiple stepped-frequency pulse trains are introduced and an algorithm is proposed to resolve both of the range-Doppler coupling and the phase wrapping. In Section 4, some numerical examples are given to demonstrate the proposed algorithm.

2. Single Stepped-Frequency Pulse Train Processing

In this section, we review the single stepped-frequency pulse train processing and introduce the range-Doppler coupling and the phase wrapping. Like most studies on stepped-frequency pulse train processing, we take the following assumptions: 1) the target moves with constant velocity v relative to the radar during the observation duration; 2) in each pulse the radar transmits the narrowband chirp signal and the pulse bandwidth is Δf; 3) the pulse train to be processed consists of the pulses stepped in frequency with a fixed frequency step and the step is equal to Δf; 4) the observation duration is short so that the target range migration does not exceed the range resolution cell of each pulse (denoted by ΔR and determined by $\Delta R = c/(2\Delta f)$), which is reasonable for most moving targets since Δf is small according to 2).

The received baseband echo can be expressed as

$$x(\mu,n) = \mathrm{rect}\left(\frac{\mu - 2R(n)/c}{T_p}\right) \cdot \exp\left[j\pi\gamma\left(\mu - \frac{2R(n)}{c}\right)^2\right] \cdot \exp\left[-j\frac{4\pi}{c}(f_c + n\Delta f)R(n)\right] \quad , \tag{1}$$

where $\mathrm{rect}(\cdot)$ is the rectangular function, T_p is the pulse width, γ is the chirp rate, f_c is the base carrier frequency, n is the pulse index number, n=1, 2, …N, N is the total number of pulses, μ is the fast-time (or range-time), c is the speed of light, $R(n) = R_0 - vTn$, R_0 is the initial range position of the target, and T is the pulse repetition interval. The goal of this paper is to estimate R_0 and v.

After the range compression in every pulse, (1) becomes

$$x(\mu, n) \approx \delta\left(\mu - \frac{2M\Delta R}{c}\right) \cdot \exp\left[-j\frac{4\pi}{c}(f_c + n\Delta f)(M\Delta R + r - vTn)\right], \quad (2)$$

where $\delta(\cdot)$ is the Delta-Dirac function, M is range cell index number where the target is detected in each pulse, $r = R_0 - (M-1)\Delta R$ and $0 \le r < \Delta R$. Since M has been determined in each pulse, we only need to estimate r and v from (2). Multiplying $\exp\left(j4\pi f_c M\Delta R / c\right)$ to (2), we have

$$x(n) = \exp\left\{-j\frac{4\pi}{c} f_c r - j2\pi\left(\frac{r}{\Delta R} - \frac{2 f_c vT}{c}\right) \cdot n + j2\pi\frac{vT}{\Delta R} \cdot n^2\right\}. \quad (3)$$

When the target is stationary, i.e., v=0, one can directly estimate r by IDFT in terms of n on (3) and finding the peak position. However, when the target is moving, i.e., $v \neq 0$, in IDFT results the peak appears at the position $\alpha' = \mathrm{mod}(\alpha, 1)$, where mod($\cdot$) is the modulus operation and

$$\alpha \triangleq \frac{r}{\Delta R} - \frac{2 f_c vT}{c}. \quad (4)$$

We first consider the case that v is small so that $0 \le \alpha' = \alpha < 1$. It is clear in (4) that r and v cannot be respectively retrieved by using the knowledge of α. This problem is called range-Doppler coupling. It is also noted that the last exponential term on the right-hand side of (3) causes the power spread in IDFT results but it does not affect the peak position α', so we ignore this term in what follows. On the other hand, when v is large so that $\alpha = \alpha' + K$ for an unknown integer K, the value of α can not be uniquely determined from IDFT results. This problem is called phase wrapping. Moreover, the estimate of α' may be erroneous due to the existence of noise and clutter. Several effective methods have been proposed to resolve the range-Doppler coupling [4-6]; however, the robust phase unwrapping is not considered in these methods. In next section we propose a new algorithm to resolve both of the range-Doppler coupling and the phase wrapping.

3. Multiple Stepped-Frequency Pulse Trains Processing

In this section, we present a new waveform named multiple stepped-frequency pulse trains and describe how to resolve both range-Doppler coupling and phase wrapping using this waveform.

We replace the single stepped-frequency pulse train by multiple stepped-frequency pulse trains. This new waveform is shown in Figure 1, where three (or multiple) stepped-frequency pulse trains with different base carrier frequencies are simultaneously used. This can be realized by the following way: three antennas are fixed in the radar platform, and the same stepped-frequency pulse train is modulated on different carrier frequencies f_{c1}, f_{c2} and f_{c3} and transmitted by three antennas, respectively, and then the different echoes on f_{c1}, f_{c2} and f_{c3} are collected and demodulated by three antennas, respectively. Without loss of generality, we assume $f_{c1} < f_{c2} < f_{c3}$ and $(f_{c2} - f_{c1})$ and $(f_{c3} - f_{c1})$ are two distinct positive real numbers and not integer multiple of each other.

Now we perform the processing described in the previous section on three stepped-frequency pulse trains. By performing *P*-point IDFT in terms of *n* in (3), we have

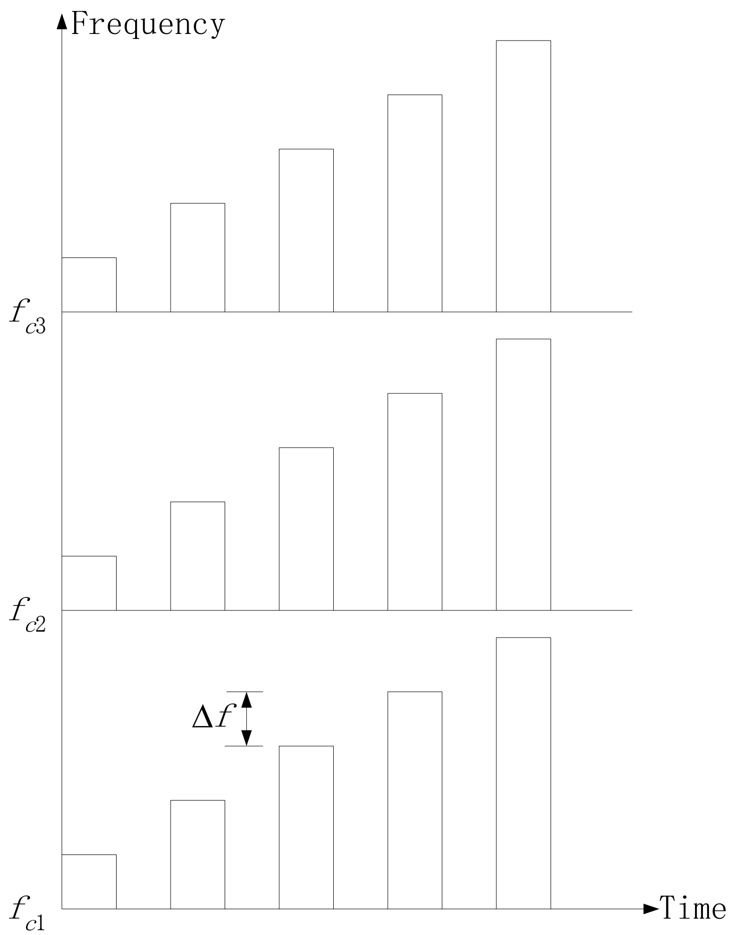
$$\frac{r}{\Delta R}-\frac{2f_{ci}vT}{c}=K_i+\frac{p_i}{P}+\varepsilon_i, \tag{5}$$

where integers p_i are the peak position estimates in the *P*-point IDFT results on f_{ci}, $0\le p_i<P$, ε_i are estimation errors, and K_i are unknown integers, $i=1, 2, 3$. When *P* is larger than the pulse number *N*, the sequence $x(n)$ in (3) should be zero-padded before the IDFT. Assume that the errors ε_i are bounded by

$$|\varepsilon_i|\le\frac{1}{2P}+\frac{\tau}{P}, \tag{6}$$

where $0\le\tau<M$ is the maximal error level in estimation of the remainders p_i. At the right hand side of (6), the first term is the quantization error and the second term is caused by possible interference, e.g., noise or clutter. Herein we are interested in the estimation of *r* and *v* from (5).

Figure 1. Multiple stepped-frequency pulse trains with different base carrier frequencies.



We first estimate *v*. Subtracting the first equation from the other two equations in (5), respectively, we have

$$\begin{aligned} \frac{2vT}{c}(f_{c2}-f_{c1}) &= L_1+\frac{\beta_1}{P}+\xi_1 \\ \frac{2vT}{c}(f_{c3}-f_{c1}) &= L_2+\frac{\beta_2}{P}+\xi_2 \end{aligned}, \tag{7}$$

where

$$\begin{aligned} &\beta_1 = p_1 - p_2 &\quad& \beta_2 = p_1 - p_3 \\ &L_1 = K_1 - K_2 && L_2 = K_1 - K_3\,, \\ &\xi_1 = \varepsilon_1 - \varepsilon_2 && \xi_2 = \varepsilon_1 - \varepsilon_3 \end{aligned} \tag{8}$$

and $|\xi_i| < \frac{1}{P} + \frac{2\tau}{P}$. The problem is to estimate *v* using the reminders β_1 and β_2. In (7), while ξ_i cause small errors and they are difficult to be eliminated, the wrong L_i may cause large folding errors. Thus, to estimate *v*, it is necessary to correctly determine L_i, which follows the robust phase unwrapping problem studied in [8-9] and is what we focus on in the following. Since $0 \le p_i < P$, $\left|\frac{\beta_1}{P}\right| < 1$ and $\left|\frac{\beta_2}{P}\right| < 1$. If $\beta_1 < 0$, the first equation of (7) can be rewritten as

$$\frac{2vT}{c}(f_{c2}-f_{c1}) = (L_1-1) + (\frac{\beta_1}{P}+1) + \xi_1$$

so that $0 \le \left(\frac{\beta_1}{P}+1\right) < 1$. This is not essential for estimating *v* because determining L_1 is replaced by determining (L_1-1). Thus, without loss of generality, we assume $0 \le \frac{\beta_1}{P}, \frac{\beta_2}{P} < 1$. Let Γ be the smallest positive real number such that $\Gamma_1 \triangleq \Gamma/(f_{c2}-f_{c1})$ and $\Gamma_2 \triangleq \Gamma/(f_{c3}-f_{c1})$ are integers and they are coprime. This holds because $(f_{c2}-f_{c1})$ and $(f_{c3}-f_{c1})$ are not integer multiples of each other, and Γ can be known since all f_{ci} are known. Multiplying Γ_1 and Γ_2 to the two equations in (7), respectively, yields

$$\begin{aligned} \frac{2vT}{c}\Gamma &= L_1\Gamma_1 + \frac{\beta_1}{P}\Gamma_1 + \xi_1\Gamma_1 \\ \frac{2vT}{c}\Gamma &= L_2\Gamma_2 + \frac{\beta_2}{P}\Gamma_2 + \xi_2\Gamma_2 \end{aligned}. \tag{9}$$

According to the RPUT obtained in [8], if

$$|v| < \frac{1}{2}\cdot\frac{c}{2T}\cdot\frac{\Gamma_1\Gamma_2}{\Gamma}, \tag{10}$$

where the presence of factor 1/2 is due to two possible motion directions for the target, i.e., toward the radar and away from the radar, and

$$P > 2(1+2\tau)(\Gamma_1+\Gamma_2), \tag{11}$$

then L_1 and L_2 can be correctly determined as

$$(L_1, L_2) = \arg\min_{\substack{l_1=0,1,\cdots\Gamma_2-1 \\ l_2=0,1,\cdots\Gamma_1-1}} \left| l_1\Gamma_1 + \frac{\beta_1}{P}\Gamma_1 - l_2\Gamma_2 - \frac{\beta_2}{P}\Gamma_2 \right| . \tag{12}$$

About the detailed proof of (12), we would like to refer the readers to [8]. Then *v* can be estimated by

$$\hat{v} = \frac{c}{4T}\left[\frac{1}{(f_{c2}-f_{c1})}\left(L_1+\frac{\beta_1}{P}\right)+\frac{1}{(f_{c3}-f_{c1})}\left(L_2+\frac{\beta_2}{P}\right)\right], \tag{13}$$

and the estimation error can be upper bounded by

$$|v-\hat{v}| \le \frac{(1+2\tau)c}{4TP}\left[\frac{1}{(f_{c2}-f_{c1})}+\frac{1}{(f_{c3}-f_{c1})}\right]. \tag{14}$$

Submitting (13) into (5) and using $0 \le \frac{r}{\Delta R} < 1$, r can be estimated by

$$\hat{r} = \frac{\Delta R}{3}\sum_{i=1}^{3}\mathrm{mod}\left(\frac{p_i}{P}+\frac{2f_{ci}\hat{v}T}{c},1\right), \tag{15}$$

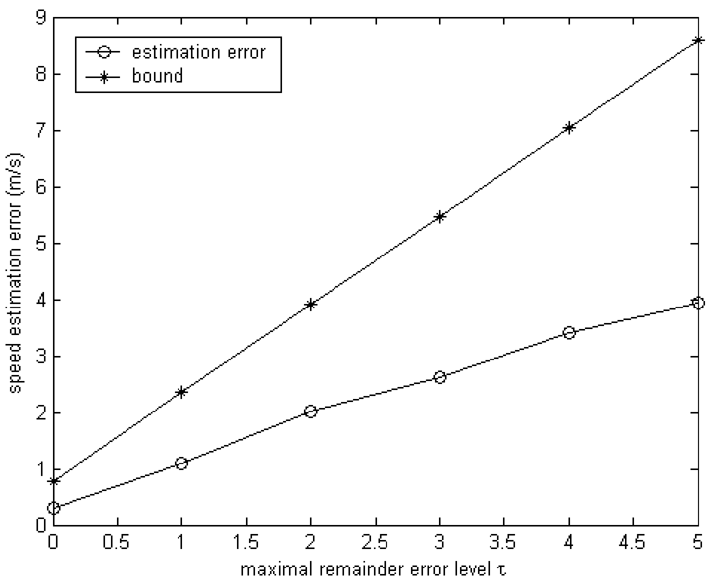
and the target range position can be accordingly estimated by $\hat{R}_0 = (M-1)\Delta R + \hat{r}$. The estimation error of r can be upper bounded by

$$|r-\hat{r}| \le \Delta R \cdot \frac{1+2\tau}{2P} \cdot \left\{\frac{(f_{c1}+f_{c2}+f_{c3})}{3} \cdot \left[\frac{1}{(f_{c2}-f_{c1})}+\frac{1}{(f_{c3}-f_{c1})}\right]+1\right\}. \tag{16}$$

To sufficiently utilize the information of the N pulses, it is better to let $P \ge N$, and accordingly, (11) may be rewritten as

$$P \ge \max\left[N, 2(1+2\tau)(\Gamma_1+\Gamma_2)+1\right]. \tag{17}$$

Figure 2. The estimation error of speed v versus various maximal remainder error level τ.



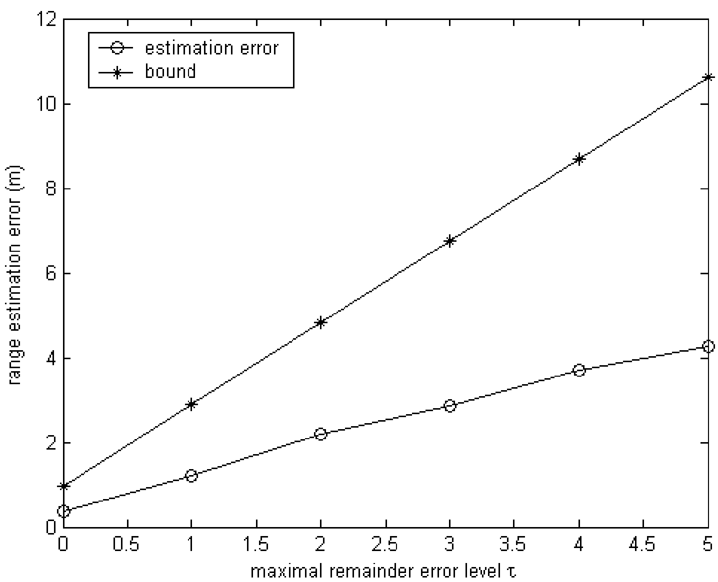
4. Numerical Examples

In this section, some numerical examples are given to show the effectiveness of the proposed algorithm. Assume that the observation range is about 20km, the pulse repetition interval T=0.625ms, the pulse number N=64, the frequency step (it is equal to the pulse bandwidth) Δf =5MHz, the base carrier frequencies f_{c1}, f_{c2} and f_{c3} are 7GHz, 9GHz and 10GHz, respectively, a target is moving toward the radar with speed 100m/s, and the initial distance between the radar and the target is R_0=19885m. In each pulse, the low range resolution is $c/(2\Delta f)$=30m, so after the range compression in every pulse, the target can be detected at 663th low range resolution cell, i.e., M=663, and

$r = R_0 - (M-1)\Delta R$ =25m. Substituting above parameters into (4) yields $|\alpha|$>1, which implies that the phase wrapping problem occurs.

According to the derivation in the previous section, one can calculate that $\Gamma = 6\times 10^9$, Γ_1=3 and Γ_2=2. The maximal determinable speed is $|v_{\max}|$=120m/s according to (10). We consider the maximal remainder error levels τ =0, 1, 2, 3, 4, 5 and let *P*=128 that satisfies (11). In Fig. 2, the error between the estimate $\hat{v}$ in (13) and the true value of *v* and the error upper bound in (14) versus various τ are plotted, and they are marked by ○ and *, respectively. In Fig. 3, the error between the estimate $\hat{r}$ in (15) and the true value of *r* and the error upper bound in (16) versus various τ are plotted, and they are marked by ○ and *, respectively. In Fig. 2 and Fig. 3, τ =0 means high SCNR (signal-to-clutter-noise ratio) case while the larger τ means low SCNR case, and the results are obtained by 100 trials. One can see the proposed algorithm robustly provides small estimation errors, even when the remainders are erroneous.

Figure 3. The estimation error of range r versus various maximal remainder error level τ .



5. Conclusion

In this paper, a new radar waveform is designed and a new algorithm is proposed to estimate the range and velocity of a moving target. In order to resolve both range-Doppler coupling and phase unwrapping, three (or multiple) stepped–frequency pulse trains and the RPUT are used. Theoretical analysis and the simulation results show that the proposed algorithm can accurately estimate the range and the velocity of the moving target. As a remark, the proposed algorithm relies on the detectability of the moving targets. About the moving target detection using stepped-frequency pulse train, we would like to refer the readers to [7] and the corresponding topic will also be under our future investigation.

Acknowledgements

This work was supported by the Chuanxin Foundation from the Department of Electronic Engineering, Tsinghua University. Xia's work was partially supported by the Air Force Office of Scientific Research (AFOSR) under Grant No. FA 9550-05-1-0161 and a DEPSCoR Grant W911NF-07-1-0422 through ARO.

References and Notes

1. Rihaczek, W. *Principles of high resolution radar*; McCraw-Hill, 1969.
2. Farnet, E. C.; Stevens, G. H. Pulse compression radar. In *M. I. Skolnik (Ed.), "Radar handbook" 2nd ed.*; McCraw-Hill, 1990.
3. Brenner, A. R.; Ender, J. H. G. Demonstration of advanced reconnaissance techniques with the airborne SAR/GMTI sensor PAMIR. *IEE Proceedings – Radar, Sonar & Navigation*, **2006**, *153(2)*, 152-162.
4. Levanon, N. Stepped-frequency pulse-train radar signal. *IEE Proceedings – Radar, Sonar & Navigation*, **2002**, *149(6)*, 297-309.
5. Chen, H.-Y.; Liu, Y.-X.; Jiang, W.-D.; Guo, G.-R. A new approach for synthesizing the range profile of moving targets via stepped-frequency waveforms. *IEEE Geoscience and Remote Sensing Letters*, **2006**, *3(3)*, 406-409.
6. Axelsson, S. R. J. Analysis of random step frequency radar and comparison with experiments. *IEEE Transactions on Geoscience and Remote Sensing*, **2007**, *45(4)*, 890-904.
7. Zhang, Q.; Yeo, T. S. ISAR imaging in strong ground clutter using a new stepped-frequency signal format. *IEEE Transactions on Geoscience and Remote Sensing*, **2003**, *41(5)*, 948-952.
8. Xia, X.-G.; Wang, G. Phase unwrapping and a robust Chinese remainder theorem, *IEEE Signal Processing Letters*, **2007**, *14(4)*, 247–250.
9. Li, G.; Xu, J.; Peng, Y.-N.; Xia, X.-G. An efficient implementation of a robust phase-unwrapping algorithm. *IEEE Signal Processing Letters*, **2007**, *14(6)*, 393-396.