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A Novel Analytical Design Technique for a Wideband Wilkinson Power Divider Using Dual-Band Topology

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Abstract: In this paper, a novel analytical design technique is presented to implement a coupled-line wideband Wilkinson power divider (WPD). The configuration of the WPD is comprised of three distinct coupled-line and three isolation resistors. A comprehensive theoretical analysis is conducted to arrive at a set of completely new and rigorous design equations utilizing the dual-band behavior of commensurate transmission lines. Further, the corresponding S-parameters equations are also derived, which determine the wideband capability of the proposed WPD. To validate the proposed design concept, a prototype working at the resonance frequencies of 0.9 GHz and 1.8 GHz is designed and fabricated using 60 mils thick Rogers' RO4003C substrate. The measured result of the fabricated prototype exhibits an excellent input return loss > 16.4 dB, output return loss > 15 dB, insertion loss < 3.30 dB and a remarkable isolation > 22 dB within the band and with a 15 dB and 10 dB references provide a fractional bandwidth of 110% and 141%, respectively.

Keywords: coupled-line; coupler; dual-band; S-parameter; Wilkinson power divider

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1. Introduction

Power divider/splitter circuits are one of the essential building blocks in all modern microwave and wireless communication systems [1]. The three-port Wilkinson power divider (WPD) is considered a paragon among such passive components. WPD was first published in 1960 by Ernest J Wilkinson and used to provide electrically isolated output branches with the same phase and equal power division, with all terminated ports matched, in a very simple layout [2]. However, the presence of the intrinsically narrow bandwidth of the conventional WPD has turned out to be one of the major design challenges. A variety of modified WPD configurations have emerged over the years to cater to the needs of wideband, harmonic-suppressed, and multi-standard topologies [3–27]. Recently, the wideband WPDs have gained paramount importance in various applications such as power amplifiers [28–31], Hilbert transformer-based dual-band single sideband modulator [32,33], antenna arrays [34], phase shifters [35], and other RF front-end systems [36]. This is mainly due to the recent progress in high data-rate 5G/6G wireless systems that has refueled significant research interest in developing the next generation of wideband power dividers [37–48].

To broaden the operational bandwidth, the fundamental idea has been to incorporate a multi-section topology as proposed by Cohn [3]. Many different varieties of broadband power dividers ensued [4–12] following Cohn's idea. The conventional multi-section WPD design [2] is a fully analytical technique but only for a two-section WPD. For three and more sections, the reported technique is a table-based design and is, therefore, less computer friendly. Furthermore, among the multiple other demonstrated techniques to

extend the bandwidth, the notable ones are those based on the stepped-impedance matching network [13], complementary conducting-strip transmission line (CCSTL) [14], port extension [15–17], substrate-integrated waveguide (SIW) [18], series/parallel RLC networks [19–22], composite right-/left-handed transmission (CRLT) lines [23], and optimized isolation network [24–27]. While CCSTL and SIW suggest alternate structure to enhance the bandwidth, on the contrary the lumped RLC networks, CRLT lines as well as the isolation network add complexity in theoretical analysis with an increase in length to serve the same purpose. Most of these works only focus on improving the bandwidth performance for either the input/output reflection parameters or the insertion loss parameter or the isolation parameter or their combinations, but not all together. In addition, for all these conventional designs for a $BW = f_2 - f_1$ the reflection profile appears like the one shown in Figure 1a with f_0 being the mid-band frequency and $-|S_{11m}|$ being the minimum return loss. Therefore, in the event of ubiquitous component/process variations, the bandwidth target is never achieved, necessitating post-design optimization.

Figure 1. The conventional vs. the proposed design methodologies.

In this paper, a new and rigorous analytical design methodology for the classic tri-section cascaded coupled-line-based topology is presented to maximize the bandwidth performance for all the S-parameters simultaneously. Furthermore, as depicted in Figure 1b, the proposed design methodology utilizes a dual-band design concept [37] to arrive at a wideband design. This dual-band design approach guarantees that the resonance frequencies are always located at f_1 and f_2 , and therefore, the achieved bandwidth is $BW = f_H - f_L = (f_2 - f_1) + 2f_{ex}$, where $2f_{ex}$ is the extra bandwidth that provides a margin for process/component variations. This approach is the usual choice for a commercial development setup, where the minimum bandwidth requirement $f_2 - f_1$ will always be met, and $2f_{ex}$ will allow flexibility to meet design goal requirements.

The remainder of this paper is organized as follows. In Section 2, the theory and design equations of the proposed WPD are developed. The derivation of the corresponding S-parameters and bandwidth performance is presented in Section 3. Then, the complete circuit of the proposed WPD along with its step-by-step design procedure is explained in Section 4 and few examples are demonstrated to support the theoretical design capability. Afterward, the fabricated prototype and its EM simulated and measured results are demonstrated in Section 5. Finally, the performance of this work is compared with some of the state-of-the-art techniques followed by a conclusion in Section 6.

2. The Proposed Analytical Design Equations of the WPD

A tri-section coupled-line 3-dB Wilkinson power divider (WPD) configuration is depicted in Figure 2. P1 denotes the input port of the WPD whereas P2 and P3 denote the two output ports. Each of the ports is terminated with a real impedance Z_0 (normally equal to 50 Ω). Further, Z_{ke} and Z_{ko} refer to the even and odd mode impedances of the k th section coupled-line while θ refers to their electrical length with $k \in \{1, 2, 3\}$. As all the transmission lines are coupled, the physical space between the two arms of the divider will normally be very close, resulting in a miniaturized structure.

Figure 2. The tri-section coupled-line WPD.

The coupling coefficient, C_k of the k th section coupled-line is defined as [1]:

$$C_k = 20 \log \frac{Z_{ke} - Z_{ko}}{Z_{ke} + Z_{ko}} \quad (1)$$

As explained in [37], θ at f_1 for a dual-band design is chosen as:

$$\theta = \frac{\pi}{1+r} \quad (2)$$

where f_1 and f_2 refer to the minimum band edge frequencies for the bandwidth requirement and $r = f_2/f_1$ is the band ratio.

Moreover, the three resistors $2R_3$, $2R_2$, and $2R_1$ shown in Figure 2 ensure isolation between port 2 and port 3. Since the WPD shown in Figure 2 is symmetric about a horizontal axis passing through P1, the even/odd mode analysis technique can be utilized for its analysis. The resulting equivalent circuits are depicted in Figure 3.

Figure 3. (a) The even-mode equivalent circuit, (b). The odd-mode equivalent circuit.

2.1. Even-Mode Analysis

The even-mode equivalent circuit is shown in Figure 3a. This circuit has been obtained from Figure 2 by assuming an open circuit along the axis of symmetry. As shown in the figure, the parameters Z_a , Z_b , and Z_c are the input impedances looking towards the right beginning at Section 3. Using the formula of the input impedance of a transmitted transmission line [1], the following expressions can be written:

$$Z_c = Z_{1e} \frac{Z_0 + jaZ_{1e}}{Z_{1e} + jaZ_0} \quad (3)$$

$$Z_b = Z_{2e} \frac{Z_c + jaZ_{2e}}{Z_{2e} + jaZ_c} \quad (4)$$

$$Z_a = Z_{3e} \frac{Z_b + jaZ_{3e}}{Z_{3e} + jaZ_b} \quad (5)$$

where,

$$a = \tan \theta \quad (6)$$

Now, substituting the expressions of Z_c into that of Z_b and subsequently the expression of Z_b into that of Z_a , and finally invoking $Z_a = 2Z_0$ for ideal input matching condition, the following equations are obtained [37].

$$Z_{2e}^2 + \frac{Z_{2e}}{aZ_{3e}} [aZ_{3e}^2 + Z_{3e}X_{1e} - 4aZ_0^2(1-b)] - [4Z_0^2(1-b) + aZ_{3e}X_{1e}] = 0 \quad (7)$$

$$a^2(1-b)Z_{3e}^2 + Z_{3e}(bZ_{2e} - aX_{1e}) - aZ_{2e}(X_{1e} + aZ_{2e}) = 0 \quad (8)$$

where,

$$b = 1 - \frac{R_{1e}}{2Z_0} \quad (9)$$

$$R_{1e} = \operatorname{Re}\{Z_c\} = \frac{Z_0 Z_{1e}^2 (1 + a^2)}{Z_{1e}^2 + Z_0^2 a^2} \quad (10)$$

$$X_{1e} = \operatorname{Im}\{Z_c\} = \frac{aZ_{1e}(Z_{1e}^2 - Z_0^2)}{Z_{1e}^2 + Z_0^2 a^2} \quad (11)$$

Now eliminating Z_{2e} from (7) and (8) results in a 4th-order equation in Z_{3e} :

$$AZ_{3e}^4 + BZ_{3e}^3 + CZ_{3e}^2 + DZ_{3e} + E = 0 \quad (12)$$

where,

$$A = a^2 b (1 - b) \quad (13)$$

$$B = 2a^3 (1 - b) X_{1e} \quad (14)$$

$$C = 2b^2 R_{1e} Z_0 - X_{1e}^2 (a^4 + 2a^2 + b) \quad (15)$$

$$D = -4Z_0 R_{1e} X_{1e} a^3 \quad (16)$$

$$E = 2Z_0 R_{1e} a^2 (X_{1e}^2 - 2b R_{1e} Z_0) \quad (17)$$

Subsequently, Z_{2e} is obtained from (7) and (8) as follows.

$$Z_{2e} = \frac{aZ_{3e}[a(1-b)(4Z_0^2 - Z_{3e}^2) + (1+a^2)X_{1e}Z_{3e}]}{(b+a^2)Z_{3e}^2 - 4Z_0^2(1-b)a^2} \quad (18)$$

The even-mode design is completed by choosing a suitable value of Z_{1e} —thus those of R_{1e} and X_{1e} —and then finding Z_{3e} from (12) and Z_{2e} from (18). Out of the four roots of Z_{3e} obtained from (12) using MATLAB, only a positive real value for Z_{3e} and Z_{2e} lying between 20Ω to 120Ω is considered for modeling and implementation in the microstrip technology. The criteria for choosing Z_{1e} will be outlined later in Sections 3 and 4.

2.2. Odd-Mode Analysis

The odd-mode equivalent circuit is shown in Figure 3b. This circuit has been obtained from the one shown in Figure 2 by assuming a short-circuit along the axis of symmetry. Due to the presence of shunt resistors, it is easier to work in terms of admittances, therefore, G_i and Y_{io} are defined as $G_i = 1/R_i$ and $Y_{io} = 1/Z_{io}$, $i \in \{1, 2, 3\}$.

As shown in Figure 3b, the parameters Y_f , Y_e , Y_d are the input admittances looking towards the left at the i th section including the conductance $G_i = 1/R_i$ at that node. Applying the formula of input admittance for a terminated transmission line section [1], we have:

$$Y_d = G_3 - j \frac{Y_{3o}}{a} \quad (19)$$

$$Y_e = G_2 + Y_{2o} \frac{Y_d + jaY_{2o}}{Y_{2o} + jaY_d} \quad (20)$$

$$Y_f = G_1 + Y_{10} \frac{Y_e + jaY_{10}}{Y_{10} + jaY_e} \quad (21)$$

Now, substituting the expression of Y_d into that of Y_e and subsequently the expression of Y_e into that of Y_f , and finally invoking $Y_f = 1/Z_0 = Y_0$ for the ideal output matching condition, the following equations are obtained.

$$P_1 G_2 + Q_1 G_3 + S_1 G_2 G_3 + T_1 = 0 \quad (22)$$

$$P_2 G_2 + Q_2 G_3 + S_2 G_2 G_3 + T_2 = 0 \quad (23)$$

where,

$$P_1 = a(Y_{20} + Y_{30})(G_1 - Y_0) \quad (24)$$

$$Q_1 = a(Y_{10} + Y_{20})(G_1 - Y_0) \quad (25)$$

$$S_1 = a(Y_{10}) \quad (26)$$

$$T_1 = aY_{10}Y_{20}(Y_{10} + Y_{20}) + Y_{10}Y_{30}\left(aY_{10} - \frac{Y_2}{a}\right) \quad (27)$$

$$P_2 = Y_{10}(Y_{20} + Y_{30}) \quad (28)$$

$$Q_2 = Y_{10}Y_{20} - a^2Y_{10}^2 \quad (29)$$

$$S_2 = a^2(Y_0 - G_1) \quad (30)$$

$$T_2 = (G_1 - Y_0)(Y_{10}Y_{20} + Y_{20}Y_{30} + Y_{10}Y_{30} - a^2Y_{20}^2) \quad (31)$$

Now eliminating G_3 from (22) and (23) results in a second-order equation in G_2 :

$$MG_2^2 + NG_2 + L = 0 \quad (32)$$

where,

$$M = P_2S_1 - P_1S_2 \quad (33)$$

$$N = P_2Q_1 - P_1Q_2 + T_2S_1 - T_1S_2 \quad (34)$$

$$L = T_2Q_1 - T_1Q_2 \quad (35)$$

From (32), G_2 can be found as follows:

$$G_2 = \frac{-N \pm \sqrt{N^2 - 4ML}}{2M} \quad (36)$$

Putting the values of (36) in (22), G_3 can be found as follows:

$$G_3 = \frac{-(T_1 + G_2P_1)}{Q_1 + G_2S_1} \quad (37)$$

Choosing suitable values of G_1 , Y_{10} , Y_{20} , and Y_{30} , and subsequently finding the values G_2 from (36) and that of G_3 from (37) completes the WPD design process. The criteria for choosing the free variables G_1 , Y_{10} , Y_{20} , Y_{30} will be outlined later in Sections 3 and 4.

3. The Scattering Parameters of the WPD and BW Determination

The expression developed in Section 2 can be used to analytically design the three-section coupled-line WPD. However, due to the presence of many free variables, it is pertinent to find the expression of the S-parameters of the WPD. Due to the symmetry of the circuit shown in Figure 1, the expressions of the S-parameters for the WPD are related to the S-parameters of equivalent even-odd mode circuits as follows [16].

$$S_{11} = S_{11e} \quad (38)$$

$$S_{12} = S_{21} = S_{31} = S_{13} = \frac{1}{\sqrt{2}} S_{21e} \quad (39)$$

$$S_{22} = S_{33} = \frac{1}{2} (S_{22e} + S_{22o}) \quad (40)$$

$$S_{23} = S_{32} = \frac{1}{2} (S_{22e} - S_{22o}) \quad (41)$$

From (38) and (39), it is observed that the input matching and transmission performance of the proposed power divider in Figure 1 is only determined by the even-mode parameters S_{11e} and S_{21e} . However, the output matching and isolation performances are affected by both the even as well as the odd-mode parameters S_{22e} and S_{22o} as seen from (40) and (41).

3.1. Expressions of Even-Mode S-Parameters

From the even-mode circuit shown in Figure 3a, S_{11e} , S_{21e} , and S_{22e} are obtained as follows [1]:

$$S_{11e} = \frac{Z_a - 2Z_0}{Z_a + 2Z_0} = \frac{X + Y}{A_e + \frac{B_e}{Z_0} + 2C_e Z_0 + 2D_e} \quad (42)$$

$$S_{21e} = \frac{2\sqrt{2}S_{11e}}{W(X + Y)} \quad (43)$$

$$S_{22e} = \frac{S_{11e}(X - Y)}{(X + Y)} \quad (44)$$

where, Z_a can be calculated utilizing (3), (4), and (5) after finding Z_{1e} , Z_{2e} , and Z_{3e} . Further, the parameters W , X , Y , A_e , B_e , C_e , and D_e can be calculated from the conversion between ABCD-parameters and S-parameters as follows [1]:

$$X = \frac{B_e}{Z_0} - 2C_e Z_0 \quad (45)$$

$$Y = A_e - 2D_e \quad (46)$$

$$W = \left(1 + a^2\right)^{\frac{-3}{2}} \quad (47)$$

$$A_e = \left[1 - a^2\left(u + v + \frac{v}{u}\right)\right] \quad (48)$$

$$B_e = jaZ_{1e} \left[1 + u + v - a^2\left(\frac{v}{u}\right)\right] \quad (49)$$

$$C_e = ja\frac{1}{Z_{1e}} \left[1 + \frac{1}{u} + \frac{1}{v} - a^2\left(\frac{u}{v}\right)\right] \quad (50)$$

$$D_e = \left[1 - a^2\left(\frac{1}{u} + \frac{1}{v} + \frac{u}{v}\right)\right] \quad (51)$$

$$u = \frac{Z_{2e}}{Z_{1e}} \quad (52)$$

$$v = \frac{Z_{3e}}{Z_{1e}} \quad (53)$$

3.2. Expression of Odd-Mode S-Parameters

From the odd-mode circuit in Figure 3b, S_{22o} is obtained as follows:

$$S_{22o} = \frac{\left(\frac{1}{Y_f}\right) - Z_0}{\left(\frac{1}{Y_f}\right) + Z_0} \quad (54)$$

where Y_f can be obtained from (21) after finding Z_{1o} , Z_{2o} , Z_{3o} , G_1 , G_2 , and G_3 .

3.3. Maximum Input Return loss (RL) between f_1 and f_2

To achieve the maximum possible BW between f_L and f_H (as shown in Figure 1), it is desirable to minimize the value of return loss between f_1 and f_2 . To that end, using (38), (42), (45), (46), and (48)–(51) we obtain the following expression for S_{11} :

$$S_{11} = \frac{Z_0(k_1 a^2 - 1) + j(k_2 a - k_3 a^3)}{Z_0(3 - k_4 a^2) + j(k_5 a - k_6 a^3)} \quad (55)$$

where,

$$k_1 = 2\left(\frac{1}{u} + \frac{1}{v} + \frac{u}{v}\right) - (u + v + \frac{v}{u}) \quad (56)$$

$$k_2 = Z_{1e}(1 + u + v) - \frac{2Z_0^2}{Z_{1e}}\left(1 + \frac{1}{u} + \frac{1}{v}\right) \quad (57)$$

$$k_3 = Z_{1e}\left(\frac{v}{u}\right) - \frac{2Z_0^2}{Z_{1e}}\left(\frac{u}{v}\right) \quad (58)$$

$$k_4 = 2\left(\frac{1}{u} + \frac{1}{v} + \frac{u}{v}\right) + (u + v + \frac{v}{u}) \quad (59)$$

$$k_5 = Z_{1e}(1 + u + v) + \frac{2Z_0^2}{Z_{1e}}\left(1 + \frac{1}{u} + \frac{1}{v}\right) \quad (60)$$

$$k_6 = Z_{1e}\left(\frac{v}{u}\right) + \frac{2Z_0^2}{Z_{1e}}\left(\frac{u}{v}\right) \quad (61)$$

Now expressing $|S_{11}|$ in dB, (55) can be written as

$$|S_{11}|_{dB} = 10 \log \frac{Z_0^2(k_1 a^2 - 1)^2 + a^2(k_2 - k_3 a^2)^2}{Z_0^2(3 - k_4 a^2)^2 + a^2(k_5 - k_6 a^2)^2} \quad (62)$$

To find the maxima/minima of $|S_{11}|$ dB between f_1 and f_2 ,

$$\begin{aligned} \frac{d}{da}|S_{11}|_{dB} &= 0 \\ \Rightarrow c_1(a^2)^4 + c_2(a^2)^3 + c_3(a^2)^2 + c_4 a^2 + c_5 &= 0 \end{aligned} \quad (63)$$

where,

$$c_1 = Z_0^2(k_3^2 k_4^2 - k_1^2 k_6^2) - 2k_3^2 k_5^2 + 2k_3 k_6(2k_2 k_6 - k_3 k_5) \quad (64)$$

$$c_2 = 4Z_0^2(k_6^2 k_1 - 3k_3^2 k_4) + 2(k_3^2 k_5^2 - k_2^2 k_6^2) \quad (65)$$

$$c_3 = 2Z_0^4 k_1 k_4(k_4 - 3k_1) + Z_0^2(k_1^2 k_5^2 - k_2^2 k_4^2 + 27k_3^2 + 12k_2 k_3 k_4 - 4k_1 k_5 k_6 - 3k_6^2) + 2k_5(k_2^2 k_6 - k_2 k_3 k_5) \quad (66)$$

$$c_4 = 2Z_0^4(9k_1^2 - k_4^2) + 4Z_0^2(k_5 k_6 - 9k_2 k_3) \quad (67)$$

$$c_5 = 6Z_0^4(k_4 - 3k_1) + Z_0^2(9k_2^2 - k_5^2) \quad (68)$$

Evaluating c_1 , c_2 , c_3 , c_4 , and c_5 from (64) to (68) and subsequently solving (63), we get the values of parameter a , which, upon substitution into (62), gives the maxima/minima of $|S_{11}|_{dB}$ between f_1 and f_2 . It must be noted that earlier we defined the expression for a in (6), which is a particular solution of (63) and indicates the dependence of $|S_{11}|_{dB}$ only at f_1 and f_2 . In contrast, (63) yields a more general solution set of a that provides information about all the maxima/minima between f_1 and f_2 as the even mode impedances (Z_{1e} , Z_{2e} , Z_{3e}) vary.

Some Special Cases for Input RL

The following important points can be observed regarding the behavior of $|S_{11}|_{dB}$.

- (i) Using (55) and (63), the common realizable solutions of a , irrespective of the value of r , can be found as:

$$a^2 = \frac{k_2}{k_3} = \frac{1}{k_1} = \tan^2 \theta \quad (69)$$

These values of a correspond to the values of $|S_{11}|_{dB}$ at f_1 and f_2 where we will always get two minima, which is also evident from (6) owing to the dual-band principle. Now applying any suitable numerical analysis tool, we can calculate two other real values of a (a_{m1} and a_{m2}). These values of a indicate two maxima of $|S_{11}|_{dB}$ at f_{m1} and f_{m2} where $f_1 < f_{m1} < f_c$ and $f_c < f_{m2} < f_2$ and the center frequency, $f_c = 0.5(f_1 + f_2) = 0.5(f_L + f_H)$. These frequency levels are graphically defined in Figure 4.

Figure 4. Specific frequency terms in S-Parameters.

Furthermore, since $\theta \propto f$, the following equations can be derived with the help of (2) for which $|S_{11}|_{dB}$ becomes the maximum.

$$f_{m1} = \frac{f_1(1+r)}{\pi} \tan^{-1} a_{m1} \quad (70)$$

$$f_{m2} = f_1(1+r) - f_{m1} \quad (71)$$

Some examples are shown in Table 1 where different solutions of a from (63) have been tabulated.

Table 1. Calculated real values of a from (63).

Band Ratio, r	Z_{ke} Values, $k \in \{1, 2, 3\}$			Solution of a	Remarks on $ S_{11} _{dB}$ from the Value of a
	Z_{1e} (Ω)	Z_{2e} (Ω)	Z_{3e} (Ω)		
2	56.1520	70.7107	89.0442	$\pm\sqrt{3}$	Minima at f_1 & f_2
				$\pm\sqrt{11}$	Maxima at f_{m1} & f_{m2}
3	59.5380	70.7107	83.9807	± 1	Minima at f_1 & f_2
				$\pm\sqrt{5}$	Maxima at f_{m1} & f_{m2}
4	64.3670	70.7106	77.6800	$\pm\sqrt{5-2\sqrt{5}}$	Minima at f_1 & f_2
				$\pm 8\sqrt{7}/5\sqrt{5}$	Maxima at f_{m1} & f_{m2}

- (ii) From (62), it can be further deduced that, for the minimum band-limited region ($f_1 < f < f_2$), we have the following cases.

$$\text{Case - 1 : } \frac{k_3}{k_6} \geq |S_{11}(f)| \quad (72)$$

For this case, we can obtain two minima at f_1, f_2 , and one maximum at f_c . Thus, the achievable BW can be found at the $|S_{11}(f_c)|_{dB}$ level which is also visible from Figure 5 ($Z_{1e} = 53 \Omega$ and 54Ω as examples where $|S_{11}(f_c)| = -25 \text{ dB}$ and -28 dB , respectively).

$$\text{Case - 2 : } \frac{k_3}{k_6} < |S_{11}(f)| \quad (73)$$

Figure 5. The behavior of $|S_{11}|$ between f_1 and f_2 for different Z_{1e} values.

For this case, we can obtain two minima at f_1, f_2 , and two equal maxima at f_{m1}, f_{m2} . Thus, the achievable BW can be found at the $|S_{11}(f_{m1})|_{dB}$ level which is also visible from Figure 5 ($Z_{1e} = 55 \Omega$ as an example where $|S_{11}(f_{m1})| = -31.5 \text{ dB}$).

$$\text{Case - 3 : } k_3 < k_6 \text{ and } k_3 \approx 0 \quad (74)$$

For the above case, we will get the minimum possible value at f_c , and this can be visualized from Figure 5 ($Z_{1e} = 56 \Omega$ as an example). Also in this case, we will get three minima at f_1, f_c and f_2 , and two equal maxima at f_{m1} and f_{m2} . This is also the point beyond which the maxima of S_{11} cannot be lowered further by choosing any value of Z_{1e}, Z_{2e} , or Z_{3e} . So, it will give the best possible BW with the lowest possible $|S_{11}|_{dB}$ and the achievable BW can be found at the $|S_{11}(f_{m1})|_{dB}$ level where $|S_{11}(f_{m1})| = -33 \text{ dB}$ as seen for $Z_{1e} = 56 \Omega$. Hence, from the value of k_3/k_6 , we can decide which design parameters to choose to arrive at either of the above three cases. In a special case when $k_3 = 0$, it can be inferred from (55) that

$$Z_{3e}Z_{1e} = \sqrt{2}Z_0Z_{2e} \quad (75)$$

Consequently, choosing $Z_{2e} = \sqrt{2}Z_0$ as a particular solution of (18) and using (75), we conclude the following.

$$Z_{2e} = \sqrt{Z_{3e}Z_{1e}} \quad (76)$$

Thus, after finding Z_{1e} that ensures $S_{11} = 0$, Equation (76) can be used to find Z_{3e} which gives the best possible maxima of S_{11} between f_1 and f_2 . This equation will not be applicable unless the best possible maxima of S_{11} is targeted.

- (iii) Based on the variation of maxima of S_{11} versus Z_{1e} shown in Figure 6, it is apparent that the higher bandwidth level can be achieved from the lower values of r . In fact, for $r = 5$, it is not possible to achieve 10 dB return loss as depicted in Figure 6. Further, the best possible maxima can be achieved for a certain value of Z_{1e} . From that value of Z_{1e} shown in Figure 6, we can calculate Z_{2e} and Z_{3e} from (18) and (12) respectively. It can be verified that these values of Z_{1e} , Z_{2e} , and Z_{3e} also satisfy (76).

Figure 6. Variation of Maxima values of $|S_{11}(f_1 < f < f_2)|$ with Z_{1e} .

3.4. Maximum Output Return Loss (RL) and Isolation between f_1 and f_2

A similar procedure is also followed to minimize $|S_{22}|_{dB}$ and $|S_{23}|_{dB}$ between f_1 and f_2 . Since there are more free variables involved compared with $|S_{11}|_{dB}$ case, one can easily perform this analysis directly in MATLAB with the help of (40), (41), (44) and (54).

4. The Proposed Design Methodology

To be physically realizable, coupled-line microstrip technology must have a certain specific value of coupling coefficient. We consider C_d to be the limit of the coupling coefficient such that the actual coupling coefficient of each coupled-line section $|C_k| < |C_d|$. In addition, S_d is the desired S-parameter value in dB to ensure the bandwidth requirement for wideband performance such that $(|S_{11}|_{dB(max)}, |S_{22}|_{dB(max)} \text{ and } |S_{23}|_{dB(max)}) < S_d$ between f_1 and f_2 . With these limits set, Figure 7 depicts the flowchart for the proposed analytical design methodology to find R_k , Z_{ke} and Z_{ko} values ($k = 1, 2, 3$) that will ensure a wideband profile.

Following the above design methodology, some design examples for $r = 1.5, 2, 3, 3.5, 4$, and 4.5 are shown in Table 2. The ideal S-parameters corresponding to $r = 1.5, 3$, and 4.5 are plotted in Figure 8a–c.

We can observe the wideband performance of the proposed WPD from the graphical representations of the ideal S-parameters. From Figure 8a–c, it is inferred that with the increased value of r , the wideband performance deteriorates. Specifically, it is apparent that when $r = 4.5$, the return loss and isolation just touch the -10 dB level between f_1 and f_2 while resulting in a transmission ripple of less than 0.4 dB. Thus, it is theoretically possible to utilize the proposed design methodology to design the wideband WPD for $r < 4.5$.

Figure 7. Flowchart of design steps.

Table 2. Circuit Parameters with the variation of r .

r	$Z_{1e} (\Omega)$	$Z_{1o} (\Omega)$	$Z_{2e} (\Omega)$	$Z_{2o} (\Omega)$	$Z_{3e} (\Omega)$	$Z_{3o} (\Omega)$	$R_1 (\Omega)$	$R_2 (\Omega)$	$R_3 (\Omega)$
1.5	60.00	37.00	81.26	42.00	99.04	50.00	929.00	31.26	5.01
2	58.30	34.80	73.38	40.00	92.45	48.00	500.00	34.52	5.00
3	59.00	36.00	70.72	37.00	83.22	41.00	910.00	84.20	23.45
3.5	60.00	36.00	71.45	38.00	78.64	44.00	1000.00	81.60	31.15
4	63.00	35.00	71.66	42.00	76.04	43.00	355.00	76.25	37.42
4.5	64.50	34.00	73.26	40.00	71.17	41.00	151.00	109.49	39.42

Now, from the examples considered in Table 2 and the simulation results shown in Figure 8a–c, we can easily deduce the cases listed in Table 3 and the corresponding plot depicted in Figure 8d. They reveal the wideband operability for $r < 4.5$ and variation of % FBW, respectively, at different dB levels. We can also assess the associated % FBW for any set of calculated $|S_{11}|_{dB(max)}$, $|S_{22}|_{dB(max)}$ and $|S_{23}|_{dB(max)}$. It must be emphasized that $r = f_2/f_1$ is the frequency ratio that is used only during the design process, since the design process utilizes the dual-band concept. Whereas, the actual bandwidth is determined by attainable band ratio $r_u = f_H/f_L$, as depicted in Figure 1b. For instance, as illustrated in Table 3, for $r = 3$ we have the resonance frequencies at 1 GHz and 3 GHz with the attainable band ratio of 4.00 to 6.70 from 19 dB to 10 dB level (minimum of return loss and isolation) and 3.05 dB to 3.21 dB level (maximum insertion loss [1]), respectively. Since the operability in the wideband region is entirely based on the concept of the dual-band technique, it is also observed from Figure 8d that our proposed WPD can be designed theoretically with

r_u in the range of 1–8.2. Furthermore, it is observed that we can even theoretically achieve a maximum 157% fractional BW from this newly designed WPD.

Figure 8. Ideal S-parameters with the variation of r , r_u , and %FBW.

It is apparent that for $r > 4.5$, the isolation becomes lesser than 10 dB between f_1 and f_2 , and therefore, the proposed WPD exhibits only the dual-band performance.

Table 3. Limitation on Wideband Performance.

Minimum Band Ratio, R	Minimum of Return loss & Isolation (dB)	Maximum Insertion loss (dB)	Operating Band Range, f_L to f_H (GHz)	Extra BW, f_{ex} (GHz)	Attainable Band Ratio, r_u
1.50	28	3.01	0.89–1.61	0.11	1.81
	10	3.20	0.48–2.02	0.52	4.21
2.00	20	3.02	0.80–2.20	0.20	2.75
	10	3.20	0.50–2.50	0.50	5.00
3.00	19	3.05	0.80–3.20	0.20	4.00
	10	3.21	0.52–3.48	0.48	6.69
3.50	16	3.11	0.75–3.75	0.25	6.35
	10	3.22	0.55–3.95	0.45	7.77
4.00	13	3.20	0.68–4.32	0.32	5.00
	10	3.22	0.57–4.43	0.43	7.18
4.50	10	3.35	0.60–4.90	0.40	8.17

5. The Fabricated Prototype and Measurement Results

The proposed analytical design methodology is used to design a wideband WPD with $S_d < -20$ dB and $C_d = -12$ dB as a demonstration. If the resonance frequencies are chosen as $f_1 = 0.9$ GHz and $f_2 = 1.8$ GHz, then the calculated ideal design parameters using MATLAB are found to be as: $Z_{1e} = 54.00 \Omega$, $Z_{2e} = 68.04 \Omega$, $Z_{3e} = 85.63 \Omega$, $Z_{1o} = 35.00 \Omega$, $Z_{2o} = 40.00 \Omega$, $Z_{3o} = 45.00 \Omega$, $R_1 = 599.00 \Omega$, $R_2 = 37.87 \Omega$, and $R_3 = 8.05 \Omega$.

Based on the analytical design, these values are subsequently used to implement and optimize the WPD in microstrip technology using the ADS design environment. The optimized (to tackle junction discontinuities) physical dimensions of the designed prototype are listed in Table 4. Based on these dimensions, the designed WPD electromagnetic (EM) model is shown in Figure 9 whereas the fabricated prototype is depicted in Figure 10. The Rogers' RO4003C laminate is used with the following parameters: dielectric constant, $\epsilon_r = 3.55$, substrate height = 1.524 mm, thickness = 35 μm and loss tangent, $\tan\delta = 0.002$. The final resistors for this prototype WPD are: $2R_1 = 634 \Omega$, $2R_2 = 95.3 \Omega$, and $2R_3 = 24.9 \Omega$ which correspond to commercially available values.

The prototype is measured using the Tektronix TTR506A vector network analyzer (VNA) with a full two-port SOLT calibration as shown in Figure 11. The EM simulated and the measured results in the range of 0.3 GHz–2.7 GHz are compared in Figures 12 and 13.

Table 4. Physical Dimension of the Coupled T-lines.

	3rd Line (n = 3)	2nd Line (n = 2)	1st Line (n = 1)
W_n (in mm)	1.918	2.823	3.987
S_n (in mm)	0.854	0.919	0.999
L_n (in mm)	29.66	33.62	33.82

Figure 9. WPD EM modeling using ADS momentum.

Figure 10. The fabricated prototype of the WPD.

Figure 11. The measurement of the fabricated prototype using the Tektronix VNA.

Figure 12. The EM simulated (subscript s) vs. the measured (subscript m) magnitudes of S-parameters.

Figure 13. Simulated (subscript s) vs. measured (subscript m) phase imbalance between output ports.

It is apparent from Figure 12a,b that the designed prototype has excellent reflection parameters maintaining an input return loss greater than 16.4 dB and an output return loss greater than 15 dB throughout the band from 0.595 GHz to 2.05 GHz. Besides, the transmission parameters are also found to demonstrate decent performance in Figure 12c ensuring an insertion loss of lesser than 3.30 dB, not far from the ideal 3 dB value in the aforesaid band. Moreover, as shown in Figures 12d and 13, the measured isolation and phase imbalance of 22 dB (minimum) and value of 0.02 degrees (maximum), respectively, are remarkable. From the aforesaid band, the measured % FBW is 110% considering the -15 dB reference line whereas it becomes 141% considering a -10 dB reference level from 0.41 GHz to 2.37 GHz, which is outstanding.

Though there is a slight mismatch observed between the EM simulated and the measured values, which normally occurs in any high-frequency design, possibly it is mostly due to the Panasonic isolation resistor's non-idealities at high frequencies. Unfortunately, the model of these resistors is not available, so it was not possible to include their impact in the simulation. Nevertheless, the measured results show the outstanding performance of the WPD design which meets or exceeds all the goals that a practical WPD will require during deployment in the field.

Although the main advantage of the proposed design is its fully analytical and computer-friendly design methodology, to highlight the novelty of this work further, a comparison table is presented in Table 5 where some recent state-of-the-art works have been listed. It is conspicuous from the Table that the proposed WPD proves to be the best fit in terms of fulfilling the desired values of all the S-parameters in the wideband zone and maximizing the % FBW at the same time compared to all these previous works.

Table 5. Comparison of performance among this WPD and some other recent works.

Reference	Techniques/Topology	Band Ratio ($r_u = f_H/f_L$)	Min. Input RL (dB)	Min. Output RL (dB)	Min. Isolation level (dB)	Max. Insertion level (dB)	Max. Phase Imbalance of Output Ports (Degree)	FBW (%)
[24]	RLCT Isolation network	3.55	10	10	20	4.00	-	112.00
[25]	Frequency-selecting coupling structure	2.13	20	-	12	3.40	4.00	62.07
[26]	Parallel coupled filters	1.66	15	14	22	4.00	-	49.50
[27]	Single section optimized isolation network	2.20	19	20	19	3.20	-	75.00
[35]	Two open-circuited stubs and three coupled lines	1.90	10	10	10	-	2.50	62.00
[39]	Isolation stage shifting	2.08	-	-	17	3.90	-	70.20
[40]	Multilayer slot line structure	3.22	10	10	10	5.00	2.00	105.30
[41]	Hybrid Wilkinson and Gysel structure	1.78	16	17	20	4.40	1.70	56.00
[42]	LC ladder	3.08	18	17	18	3.90	-	102.00
[43]	Port matching using three coupled-line, one T-line and one isolation resistor	2.00	15	17	15	3.20	0.50	66.67
[44]	Quasi-coupled line with one open stub and two shorted stubs	3.19	15	15	15	3.66	6.80	104.50
[45]	Transversal filters with RLC network	2.57	15	15	17	-	1.60	87.80
[46]	Four T-lines, two stubs & five resistors	3.42	17.8	20	19.8	4.00	-	109.50
[47]	Port-to-Port isolation structure	2.28	10	10	17.5	3.70	1.50	78.00
[48]	Synthesis theory and optimization algorithm	1.92	21.1	19.2	18.5	3.10	1.20	63.00
This Work	Three Coupled T-Lines & three isolation resistors	3.45	16.4	15	22	3.30	0.017	110.11
		5.78	15.9	12	10	3.50	0.021	141.00

6. Conclusions

A completely new equations-based design approach for a three-section WPD has been discussed in this paper. The methodology started with deriving the required equations for the coupled transmission lines and then utilized the dual-band design technique to arrive at a robust design for a wide-band performance. The design procedure has been outlined through a compact flowchart which is followed to demonstrate some examples highlighting the capabilities of the proposed technique. Afterward, the fabricated prototype shows good agreement with the simulated data from the measurement results and substantiates the theoretical aspect. The biggest challenge of this design was to incorporate the junction discontinuities due to the use of coupled line with different widths, which needed an EM optimization. Nonetheless, the presented design equations will be the ideal choice for a computer-aided design of a miniaturized wideband WPD consisting of microstrip coupled-line.

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