Article

Sensorless LC Filter Implementation for Permanent Magnet Machine Drive Using Observer-Based Voltage and Current Estimation

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Abstract: For pulse width modulation (PWM) inverter drives, an LC filter can cascade to a permanent magnet (PM) machine at inverter output to reduce PWM-reflected current harmonics. Because the LC filter causes resonance, the filter output current and voltage are required for the sensorless field-oriented control (FOC) drive. However, existing sensors and inverters are typically integrated inside commercial closed-form drives; it is not possible for these drives to obtain additional filter output signals. To resolve this integration issue, this paper proposes a sensorless LC filter state estimation using only the drive inside current sensors. The design principle of the LC filter is first introduced to remove PWM current harmonics. A dual-observer is then proposed to estimate the filter output current and voltage for the sensorless FOC drive. Compared to conventional model-based estimation, the proposed dual-observer demonstrates robust estimation performance under parameter error. The capacitor parameter error shows a negligible influence on the proposed observer estimation. The filter inductance error only affects the capacitor current estimation at high speed. The performance of the sensorless FOC drive using the proposed dual-observer is comparable to the same drive using external sensors for filter voltage and current measurement. All experiments are verified by a PM machine with only 130 μH phase inductance.

Keywords: low inductance PM machine; PWM inverter filter; observer-based estimation

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1. Introduction

Compared to conventional industrial PM machines, low-inductance machines have a lower inductive voltage and lower back electromotive force (EMF) voltage drop. These machines are suitable for high-speed applications under volume limitation; e.g., drones, spindles and compressors [1–3]. However, considering PWM voltage control, visible switching-reflected current harmonics occur on these low-inductance machines [4]. Although the bus voltage usage is improved at high speed, PWM inverter-reflected current harmonics increase the machine iron loss and cause rotor thermal issues [5,6].

LC filters can be installed in PM machine drives for the reduction of PWM current harmonics. Because LC filters do not include resistances, there is a resulting voltage and current distortion, especially for operation near filter resonance. Circulating currents are induced between the inverter and LC filter. In this case, active damping compensation is used to minimize the filter-reflected current/voltage distortion under the FOC drive. In [7–10], a cascaded controller with both an external current loop and an internal voltage loop is proposed in an induction machine for active damping compensation. However, three voltage sensors and two current sensors are required, with considerable cost. The authors of [11–14] simplify this cascaded control topology by using a simple current controller. In contrast to the voltage and current control in [7–9], only either the capacitor voltage or the current is closed-loop regulated. In this case, the LC filter parameters are required to

maintain the active damping performance. Furthermore, for high-speed machine operation, the LC filter causes additional phase delay on machine currents. In [15,16], a discretized voltage model is proposed for digital delay compensation. A PWM inverter can achieve the current regulation on an RL load with 400 Hz frequency. Among these active damping compensations, it is noted that capacitor currents/voltages and inductor output machine currents are required for the FOC drive and active damping compensation.

For a sensorless machine drive, the inductor output machine current and capacitor voltage must be known for machine current regulation and rotor position estimation. The filter output voltage and current can be obtained by external sensors or estimated by state observers. In general, current/voltage sensors and inverters are typically integrated inside commercial closed-form drives. Additional inverter hardware modification is necessary if external sensors are selected for an FOC drive with an LC filter [17,18]. In contrast, sensorless observer-based estimation is preferred because only the inverter inside the current sensors is used for filter state estimation. In [19], the capacitor current is estimated using an LC filter observer based on the filter output current measurement. Although no voltage sensor is present, three current sensors outside the inverter are added for the observer estimation. To position sensorless FOC drives, the LC filter is also attached for current regulation [5]. However, similar to [19], filter output currents are measured in order to estimate the machine EMF voltage. In [20], with an LC filter, the direct torque and flux control is implemented for the machine drive. Considering the filter effect on the drive, several corrections are added to correct flux and torque commands. More importantly, this direct torque control still requires filter output currents to estimate the machine flux. From the review of these LC filter-related works, the integration of a filter on closed-form drives without external current sensors is desired for high-performance machine control.

This paper proposes a sensorless LC filter implementation for an FOC drive using only the inverter inside current sensors for the filter current and voltage estimation. In [21], a sensorless LC filter state estimation using only the closed-form drive inside current sensors is developed; however, only the preliminary observer topology is illustrated, neglecting various LC filter cross-coupling effects. These coupling issues including the parameter sensitivity on the filter state estimation are fully investigated here. In this paper, a dual-observer topology is proposed to estimate the filter inductor current and capacitor voltage for both current regulation and position estimation, respectively. Compared to conventional model-based filter voltage and current estimation, the proposed dual-observer can reduce the filter parameter sensitivity, resulting in better state estimation performance. Based on experimental results, a comparable FOC drive is developed and compared to the same drive using external sensors for filter voltage and current measurement. All the tests are verified on a 105 W PM machine with only 130 μ H phase inductance.

2. LC Filter Design Principle

This section explains the selection of the LC filter on the PM machine FOC drive. Figure 1 illustrates the overall PWM inverter including an output LC filter. For the PWM machine drive, the LC filter along with the machine stator windings can be viewed as an LCL filter. To remove PWM-reflected current harmonics on the machine, the equivalent resonant frequency f_{filter} of the LCL filter can be selected based on the critical resonance frequency f_{crit} . As reported in [22], f_{crit} is suggested by

$$f_{\text{crit}} = \frac{f_{\text{sample}}}{6} = \frac{f_{\text{PWM}}}{6} \quad \left(\text{assuming } f_{\text{sample}} = f_{\text{PWM}} \right) \quad (1)$$

where f_{sample} is the controller sample frequency synchronized to PWM frequency f_{PWM} . To maintain the dynamic response of machine control, the resonant frequency f_{filter} should be designed higher than f_{crit} . On the other hand, f_{filter} must be lower than half of f_{sample} for the purpose of reducing the PWM-reflected current harmonics. In this paper, f_{sample} and f_{crit} are 10 kHz and 1.667 kHz, respectively. However, the high f_{crit} might cause a challenge regarding the filter state estimation for the proposed senseless integration of the LC filter

and FOC drive. Considering the available specification of inductances and capacitors, the resonant frequency f_{filter} is selected at 2.9 kHz based on the above criteria.

Figure 1. Machine drive with output LC filter [21].

After the determination of f_{filter} , the individual filter inductance and capacitor value are then selected according to the requirements of the machine phase current ripples. In [23,24], the relationship between the inductor current ripple ΔI_L and filter inductance can be shown to be

$$\Delta I_L = \frac{1}{8} \frac{V_C}{L f_{\text{sample}}} \quad (2)$$

where V_C is the corresponding capacitor voltage. To reach a tolerable current ripple of around 5%, the filter inductance is designed at 1.2 mH, where the resulting capacitor is 25.8 μF . Considering the accessibility of LC components in the market, the filter inductance is re-selected at 1 mH, giving a resonant frequency of 2.92 kHz and current ripple of around 6.31%.

3. Machine Drive with LC Filter

This section explains the improvement of the low-inductance PM machine using the LC filter to reduce PWM current harmonics. For variable-frequency drive applications, the positioning of the senseless FOC drive is applied. As seen for the proposed machine drive with an LC filter in Figure 1, $I_{LA}/I_{LB}/I_{LC}$ are the inverter A/B/C-phase currents across three filter inductances, respectively, $I_{CA}/I_{CB}/I_{CC}$ are the phase currents across filter capacitors, and $V_{CA}/V_{CB}/V_{CC}$ are the phase voltages on filter capacitors. Besides, $I_A/I_B/I_C$ are the inductor output currents flowing across the machine phase windings.

It is noteworthy that, for standard machine drives, only the currents of $I_{LA}/I_{LB}/I_{LC}$ output from the inverter can be measured using existing current sensors. Considering the influence of the LC filter, machine phase voltages and currents can be affected by the capacitor voltages $V_{CA}/V_{CB}/V_{CC}$ and currents $I_{CA}/I_{CB}/I_{CC}$. In this case, the PM machine model can be modified by

$$\begin{aligned} V_{CA} &= R_s(I_{LA} - I_{CA}) + L_s \frac{d(I_{LA} - I_{CA})}{dt} + \omega_e \lambda_{pm\_A} \\ V_{CB} &= R_s(I_{LB} - I_{CB}) + L_s \frac{d(I_{LB} - I_{CB})}{dt} + \omega_e \lambda_{pm\_B} \\ V_{CC} &= R_s(I_{LC} - I_{CC}) + L_s \frac{d(I_{LC} - I_{CC})}{dt} + \omega_e \lambda_{pm\_C} \end{aligned} \quad (3)$$

where R_s and L_s are the PM machine phase resistance and inductance, respectively, assuming no saliency for simplicity, and $\lambda_{pm\_A}/\lambda_{pm\_B}/\lambda_{pm\_C}$ are the corresponding phase magnet flux linkages. For the senseless FOC drive, the knowledge of capacitor currents $I_{CA}/I_{CB}/I_{CC}$ and voltages $V_{CA}/V_{CB}/V_{CC}$ is required for both the current regulation and EMF-based position estimation. Table 1 lists the test PM machine specification. The rated speed is 42 krpm with 700 Hz electric frequency. It is noted that the phase inductance is only 130 μH . The visible PWM current harmonics are expected during the operation of the FOC drive. In the next section, the observer-based estimation of capacitor voltages

and currents is explained using only the available measurable current signals for the drive: $I_{LA}/I_{LB}/I_{LC}$.

Table 1. Test PM machine characteristics.

Test Machine	Values
Rotor poles	2-pole
Rated torque	25 mNm
Rated current	2 A
Rated speed	42 krpm (700 Hz elec. frequency)
Phase resistance	0.85 Ω
Phase inductance	130 μ H
DC bus voltage	40 V
LC Filter	Values
Inductance	1 mH
Capacitor	25.8 μ F
Resonant frequency	2.92 kHz

4. Filter Voltage and Current Estimation

This section explains the estimation of the capacitor currents $I_{CA}/I_{CB}/I_{CC}$ and voltages $V_{CA}/V_{CB}/V_{CC}$ using the proposed dual-observer. Figure 2 demonstrates the signal process for the proposed senseless FOC drive with an LC filter. Considering the operation of the FOC drive at high speed, the proposed dual-observer is developed in a dq machine rotor-referred synchronous frame. In this case, all voltages/currents are transferred to DC signals, which are suited for closed-loop current regulation. This dual-observer consists of both a capacitor voltage observer and capacitor current observer for the estimation of $\hat{V}_{Cdq}$, $\hat{I}_{Cdq}$ and $\hat{I}_{dq}$. For the senseless FOC drive, the estimated machine dq currents $\hat{I}_{dq}$ are used for the current regulation. Besides, the estimated capacitor voltages $\hat{V}_{Cdq}$ should be applied for machine EMF estimation and thus rotor position estimation.

Figure 2. Illustration of FOC machine drive with LC filter using proposed dual-observer for the filter voltage and current estimation.

4.1. Capacitor Voltage Estimation

Figure 2 (bottom-left) denotes the proposed capacitor voltage observer. Assuming that the inverter deadtime is negligible, the inverter voltage commands V_{dq}^* can contribute

to both the capacitor voltages V_{Cdq} and inductor voltages V_{Ldq} . Under this effect, the dynamic model of capacitor voltage is illustrated by

$$\begin{bmatrix} V_{Cd} \\ V_{Cq} \end{bmatrix} = \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} - \begin{bmatrix} V_{Ld} \\ V_{Lq} \end{bmatrix} = \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} - \begin{bmatrix} R_f I_{Ld} + L_f \frac{dI_{Ld}}{dt} - L_f \omega_e I_{Lq} \\ R_f I_{Lq} + L_f \frac{dI_{Lq}}{dt} + L_f \omega_e I_{Ld} \end{bmatrix} \quad (4)$$

Based on (4), the capacitor voltage V_{Cdq} can be directly estimated by

$$\begin{bmatrix} \hat{V}_{Cd} \\ \hat{V}_{Cq} \end{bmatrix} = \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} - \begin{bmatrix} \hat{R}_f I_{Ld} + \hat{L}_f \frac{dI_{Ld}}{dt} - \hat{L}_f \hat{\omega}_e I_{Lq} \\ \hat{R}_f I_{Lq} + \hat{L}_f \frac{dI_{Lq}}{dt} + \hat{L}_f \hat{\omega}_e I_{Ld} \end{bmatrix} \quad (5)$$

where $\hat{V}_{Ldq}$ is the estimated dq inductance voltage, $\hat{L}_f$ and $\hat{R}_f$ are the estimated filter inductance and parasitic resistance, and $\hat{\omega}_e$ is the estimated rotor speed. Equation (6) denotes the estimation accuracy of $\hat{V}_{Cdq}$ in the S-domain for the following parameter sensitivity analysis comparing to the proposed observer estimation.

$$\begin{bmatrix} \hat{V}_{Cd\_err}(s) \\ \hat{V}_{Cq\_err}(s) \end{bmatrix} = \begin{bmatrix} V_{Cd}(s) - \hat{V}_{Cd}(s) \\ V_{Cq}(s) - \hat{V}_{Cq}(s) \end{bmatrix} = - \begin{bmatrix} (R_f - \hat{R}_f) - (L_f - \hat{L}_f)s \\ (R_f - \hat{R}_f) - (L_f - \hat{L}_f)s \end{bmatrix} \begin{bmatrix} I_{Ld}(s) \\ I_{Lq}(s) \end{bmatrix} + \begin{bmatrix} -(L_f \omega_e - \hat{L}_f \hat{\omega}_e) \\ (L_f \omega_e - \hat{L}_f \hat{\omega}_e) \end{bmatrix} \begin{bmatrix} I_{Lq}(s) \\ I_{Ld}(s) \end{bmatrix} \quad (6)$$

It is noted that the error of $V_{Cdq\_err} = V_{Cdq} - \hat{V}_{Cdq}$ in Equation (6) is designed to analyze the accuracy of $\hat{V}_{Cdq}$. Considering perfect parameters of $\hat{R}_f = R_f$ and $\hat{L}_f = L_f$, V_{Cdq_err} should be zero for all operation conditions. Figure 3 simulates the estimation accuracy of $|V_{Cdq\_err}(j\omega)/I_{Ldp}(j\omega)|$. In this figure, different absolute magnitudes under different operating frequencies are calculated. The parameter errors of $\hat{R}_f$ and $\hat{L}_f$ are calculated based on Equation (6). As seen in Figure 3a, V_{Cdq_err} is strongly affected by the $\hat{R}_f$ error independent of the operating frequencies. In contrast, in (b), the influence of the $\hat{L}_f$ error on V_{Cdq_err} increases as frequency increases. Besides, in (c), the inductor cross-coupling voltage also results in a constant error of V_{Cdq_err} independent of the operating frequency. Considering the direct capacitor voltage estimation in Equation (5), it is concluded that $\hat{V}_{Cdq}$ estimation accuracy is strongly dependent on the inductor parameters $\hat{R}_f$ and $\hat{L}_f$.

In this paper, the closed-loop observer is developed to reduce the parameter sensitivity through the feedback control of I_{Ldq} . Figure 4 illustrates the first closed-loop observer in the S-domain for the capacitor voltage estimation. In this observer, the inputs are the inductor-measured dq currents I_{Ldq} while the observer outputs are the estimated dq capacitor voltages $\hat{V}_{Cdq}$. Besides, the observer controller $C_V(s)$ and feedforward voltage inputs $V_{CFF\_V}^*$ are designed based on

$$C_V(s) = \frac{K_{p\_V}s + K_{i\_V}}{s} \quad (7)$$

$$\begin{bmatrix} V_{CFF\_Vd}^* \\ V_{CFF\_Vq}^* \end{bmatrix} = \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} + \begin{bmatrix} \hat{V}_{cross\_d} \\ \hat{V}_{cross\_q} \end{bmatrix} = \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} + \begin{bmatrix} \hat{L}_f \hat{\omega}_e I_{Lq} \\ -\hat{L}_f \hat{\omega}_e I_{Ld} \end{bmatrix} \quad (8)$$

where K_{p_V} and K_{i_V} are the proportional and integral gain of the closed-loop observer, respectively. Based on closed-loop regulation, $\hat{V}_{Cdq}$ can be modeled by the external voltage disturbance. Under this effect, the capacitor voltages V_{Cdq} can be estimated by

$$\begin{bmatrix} \hat{V}_{Cd}(s) \\ \hat{V}_{Cq}(s) \end{bmatrix} = K_{p\_V} \begin{bmatrix} I_{Ld}(s) - \hat{I}_{Ld}(s) \\ I_{Lq}(s) - \hat{I}_{Lq}(s) \end{bmatrix} + \frac{K_{i\_V}}{s} \begin{bmatrix} I_{Ld}(s) - \hat{I}_{Ld}(s) \\ I_{Ld}(s) - \hat{I}_{Ld}(s) \end{bmatrix} \quad (9)$$

As seen from the signal process in Figure 4, the overall dynamic model of $\hat{V}_{Cdq}$ can be formulated by

$$\begin{bmatrix} \hat{V}_{Cd}(s) \\ \hat{V}_{Cq}(s) \end{bmatrix} = \left(\frac{K_{p\_v}s + K_{i\_v}}{s} \right) \left(\frac{L_f s^2 + (\hat{R}_f + K_{p\_v})s + K_{i\_v}}{\hat{L}_f s^2 + (\hat{R}_f + K_{p\_v})s + K_{i\_v}} - 1 \right) \begin{bmatrix} I_{Ld}(s) \\ I_{Lq}(s) \end{bmatrix} \\ + \frac{K_{p\_v}s + K_{i\_v}}{\hat{L}_f s^2 + (\hat{R}_f + K_{p\_v})s + K_{i\_v}} (\hat{L}_f \hat{\omega}_e - L_f \omega_e) \begin{bmatrix} I_{Lq}(s) \\ -I_{Ld}(s) \end{bmatrix} \\ + \frac{K_{p\_v}s + K_{i\_v}}{\hat{L}_f s^2 + (\hat{R}_f + j\omega_e \hat{L}_f + K_{p\_v})s + K_{i\_v}} \begin{bmatrix} V_{Cd}(s) \\ V_{Cq}(s) \end{bmatrix} \quad (10)$$

Figure 3. Estimation accuracy evaluation of $\hat{V}_{Cdq}$ between direct estimation in (4) and proposed observer in Equation (9): (a) $\hat{R}_f$ error on $|V_{Cd\_err}(j\omega)/I_{Ld}(j\omega)|$, (b) $\hat{L}_f$ error on $|V_{Cd\_err}(j\omega)/I_{Ld}(j\omega)|$, and (c) $\hat{L}_f$ error on $|V_{Cd\_err}(j\omega)/I_{Lp}(j\omega)|$.

Figure 4. Proposed first closed-loop observer in Figure 2 for capacitor voltage estimation (capacitor voltage observer).

In Equation (10), the estimated $\hat{V}_{Cdq}$ consists of three terms: the first two terms are proportional to I_{Ldq} . If both $\hat{R}_f = R_f$ and $\hat{L}_f = L_f$, these two terms disappear. In contrast, the third term is proportional to the actual capacitor voltages V_{Cdq} . This term is independent of the parameter errors at low frequency. Considering the filter parameter errors, the estimation accuracy of proposed observer can be represented by

$$\begin{bmatrix} V_{Cd\_err}(s) \\ V_{Cq\_err}(s) \end{bmatrix} = \left(\frac{K_{p\_v}s + K_{i\_v}}{s} \right) \left(\frac{L_f s^2 + (R_f + K_{p\_v})s + K_{i\_v}}{\hat{L}_f s^2 + (\hat{R}_f + K_{p\_v})s + K_{i\_v}} - 1 \right) \begin{bmatrix} I_{Ld}(s) \\ I_{Lq}(s) \end{bmatrix} + \frac{K_{p\_v}s + K_{i\_v}}{\hat{L}_f s^2 + (\hat{R}_f + K_{p\_v})s + K_{i\_v}} (\hat{L}_f \hat{\omega}_e - L_f \omega_e) \begin{bmatrix} I_{Lq}(s) \\ -I_{Ld}(s) \end{bmatrix} \quad (11)$$

Comparing the proposed estimation error in Equation (11) to open-loop direct estimation in Equation (6), the estimation accuracy of $\hat{V}_{Cdq}$ is related to both the parameters $\hat{R}_f/\hat{L}_f$ and controller gains K_{p_v}/K_{i_v} . By properly designing K_{p_v} and K_{i_v} , it is clear that the parameter sensitivity on $\hat{V}_{Cdq}$ can be improved.

Figure 3 also illustrates the same estimation accuracy of $|V_{Cdq\_err}(j\omega)/I_{Ldp}(j\omega)|$ based on the proposed closed-loop voltage observer. In this simulation, K_{p_v} and K_{i_v} are selected to achieve a 100 Hz bandwidth considering high-frequency noise reduction. Considering the $\hat{R}_f$ parameter error in Figure 3a, V_{Cdq_err} is reduced when the operating frequency is beyond the observer bandwidth, as presented by the dashed lines. Besides, for the $\hat{L}_f$ error resulting from self-inductance in (b) and cross-inductance in (c), V_{Cd_err} also maintains constant values or decreases beyond the observer bandwidth. By applying the proposed voltage observer in Figure 4, it is shown that the influence of filter parameter errors on $\hat{V}_{Cdq}$ is reduced due to the closed-loop control regulation.

4.2. Capacitor Current Estimation

This section explains the capacitor current I_{Cdq} and machine current I_{dq} estimation using the proposed capacitor current observer in Figure 2. On the basis, the capacitor current I_{Cdq} can be estimated through the capacitor model. It is shown by

$$\begin{bmatrix} I_{Cd} \\ I_{Cq} \end{bmatrix} = \begin{bmatrix} I_{Ld} \\ I_{Lq} \end{bmatrix} - \begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} C_p \frac{dV_{Cd}}{dt} - C_p \omega_e V_{Cq} \\ C_p \frac{dV_{Cq}}{dt} + C_p \omega_e V_{Cd} \end{bmatrix} \quad (12)$$

In Equation (12), the machine currents can also be obtained once capacitor currents are estimated, whereby $\hat{I}_{dq} = I_{Ldq} - \hat{I}_{Cdq}$. Thus, the proposed current observer is focused on $\hat{I}_{Cdq}$ for simplicity. A straightforward way of achieving $\hat{I}_{Cdq}$ estimation can be developed based on Equation (13).

$$\begin{bmatrix} \hat{I}_{Cd} \\ \hat{I}_{Cq} \end{bmatrix} = \begin{bmatrix} \hat{C}_p \frac{d\hat{V}_{Cd}}{dt} - \hat{C}_p \hat{\omega}_e \hat{V}_{Cq} \\ \hat{C}_p \frac{d\hat{V}_{Cq}}{dt} + \hat{C}_p \hat{\omega}_e \hat{V}_{Cd} \end{bmatrix} \quad (13)$$

where $\hat{C}_p$ is the estimated capacitor value. In Equation (13), $\hat{I}_{Cdq}$ estimation is dependent on both $\hat{C}_p$ and $\hat{V}_{Cdq}$, which are estimated from the prior voltage observer. As mentioned in Figure 3, the accuracy of $\hat{V}_{Cdq}$ is dependent on the inductor parameters $\hat{R}_f$ and $\hat{L}_f$. In order to evaluate the influence of $\hat{R}_f$ and $\hat{L}_f$ errors on $\hat{I}_{Cdq}$, $\hat{V}_{Cdq}$ in Equation (13) is replaced by Equation (5), as derived by

$$\begin{bmatrix} \hat{I}_{Cd} \\ \hat{I}_{Cq} \end{bmatrix} = \hat{C}_p \begin{bmatrix} \frac{dV_d^*}{dt} - \hat{\omega}_e V_q^* \\ \frac{dV_q^*}{dt} + \hat{\omega}_e V_d^* \end{bmatrix} + \hat{C}_p \hat{R}_f \begin{bmatrix} -\frac{dI_{Ld}}{dt} \\ -\frac{dI_{Lq}}{dt} \end{bmatrix} + \hat{C}_p \hat{L}_f \begin{bmatrix} -\frac{d^2 I_{Ld}}{dt^2} + \hat{\omega}_e \frac{dI_{Lq}}{dt} \\ -\frac{d^2 I_{Lq}}{dt^2} - \hat{\omega}_e \frac{dI_{Ld}}{dt} \end{bmatrix} + \hat{C}_p \hat{R}_f \begin{bmatrix} \hat{\omega}_e I_{Lq} \\ -\hat{\omega}_e I_{Ld} \end{bmatrix} + \hat{C}_p \hat{L}_f \begin{bmatrix} \hat{\omega}_e \frac{dI_{Lq}}{dt} + \hat{\omega}_e^2 I_{Ld} \\ -\hat{\omega}_e \frac{dI_{Ld}}{dt} - \hat{\omega}_e^2 I_{Lq} \end{bmatrix} \quad (14)$$

Similar to Equation (6), the $\hat{I}_{Cdq}$ estimation accuracy can be analyzed in the S-domain based on the definition of the capacitor current error, $\hat{I}_{Cdq\_err} = I_{Cdq} - \hat{I}_{Cdq}$. It is shown to be

$$\begin{bmatrix} I_{Cd\_err}(s) \\ I_{Cq\_err}(s) \end{bmatrix} = \begin{bmatrix} I_{Cd}(s) - \hat{I}_{Cd}(s) \\ I_{Cq}(s) - \hat{I}_{Cq}(s) \end{bmatrix} = (C_p - \hat{C}_p) \begin{bmatrix} sV_d^* - \omega_e V_q^* \\ sV_q^* + \omega_e V_d^* \end{bmatrix} \\ + (C_p R_f - \hat{C}_p \hat{R}_f) \begin{bmatrix} -sI_{Ld} \\ -sI_{Lq} \end{bmatrix} + (C_p L_f - \hat{C}_p \hat{L}_f) \begin{bmatrix} -s^2 I_{Ld} + s\omega_e I_{Lq} \\ -s^2 I_{Lq} - s\omega_e I_{Ld} \end{bmatrix} \\ + (C_p R_f - \hat{C}_p \hat{R}_f) \begin{bmatrix} \omega_e I_{Lq} \\ -\omega_e I_{Ld} \end{bmatrix} + (C_p L_f - \hat{C}_p \hat{L}_f) \begin{bmatrix} s\omega_e I_{Lq} + \omega_e^2 I_{Ld} \\ -s\omega_e I_{Ld} - \omega_e^2 I_{Lq} \end{bmatrix} \quad (15)$$

Here, there is no speed estimation error, where $\hat{\omega}_e = \omega_e$ is assumed for simplicity. It is found that I_{Cdq_err} is proportional to both the measured inductor current I_{Ldq} and inverter voltage command V_{dq}^* . In this case, the estimation accuracy of I_{Cdq_err} with respect to either I_{Ldq} and V_{dq}^* is investigated. Figure 5 demonstrates the estimation accuracy of $|I_{Cdq\_err}(j\omega)/I_{Ldq}(j\omega)|$ and $|I_{Cdq\_err}(j\omega)/V_{dq}^*(j\omega)|$ based on Equation (15) considering parameter errors on $\hat{L}_f$ and $\hat{C}_p$. In this sensitivity analysis, the $\hat{R}_f$ error is excluded for simplicity. As seen in Equation (15), the $\hat{R}_f$ error also affects $\hat{I}_{Cdq}$ estimation dependent on the magnitude of I_{Ldq} . However, considering the laminated core inductance, the parasitic resistance is sufficiently low and can be neglected.

Figure 5. Estimation accuracy evaluation of $\hat{I}_{Cdq}$ between direct estimation in Equation (13) and proposed observer in Figure 6: (a) $\hat{L}_f$ error on $|I_{Cd\_err}(j\omega)/I_{Ld}(j\omega)|$, (b) $\hat{L}_f$ error on $|I_{Cd\_err}(j\omega)/I_{Lq}(j\omega)|$, and (c) $\hat{C}_p$ error on $|I_{Cd\_err}(j\omega)/V_d^*(j\omega)|$.

Figure 6. Proposed second observer in Figure 2 for capacitor current estimation and machine current estimation.

Figure 5 illustrates the $\hat{L}_f$ error regarding the d-axis capacitor current estimation error $I_{Cd_{err}}$ resulting from the self-coupled d-axis inductor current I_{Ld} and cross-coupled q-axis current I_{Lq} , respectively. As seen from $|I_{Cd_{err}}(j\omega)/I_{Ld}(j\omega)|$ in (a), a constant error occurs at low frequency. In contrast, for $|I_{Cd_{err}}(j\omega)/I_{Lq}(j\omega)|$ in (b), the estimation error increases as frequency increases. Besides, Figure 5c shows the $\hat{C}_p$ error on $I_{Cd_{err}}$ caused by the self-coupled d-axis voltage command V_d^* . As seen from Equation (15), the estimation error also increases as frequency increases. A similar result can be found for the $\hat{L}_f$ error on cross-coupled $|I_{Cd_{err}}(j\omega)/I_{Lq}(j\omega)|$. Based on the sensitivity analysis on the direct $\hat{I}_{Cdq}$ estimation in Equation (13), the $\hat{L}_f$ error results in a constant estimation error on $\hat{I}_{Cdq}$ at low frequency, and more importantly, the estimation error increases as the operating frequency increases. Below, a closed-loop observer-based current estimation is proposed to reduce this parameter sensitivity.

Similar to the capacitor voltage observer, the capacitor current I_{Cdq} can be estimated through another closed-loop observer to improve the estimation performance. Figure 6 proposes another current observer for both I_{Cdq} and machine current I_{dq} estimation. In this observer, the input is the estimated capacitor voltage $\hat{V}_{Cdq}$ from the first observer in Figure 4, and the outputs are the estimated capacitor current $\hat{I}_{Cdq}$ and machine current $\hat{I}_{dq}$. It is noted that $\hat{I}_{dq}$ is useful for the machine current regulation of the FOC drive, while $\hat{I}_{Cdq}$ can be applied for the active damping compensation of the LC filter.

For the proposed current observer in Figure 6, the dynamic model is related to the estimated voltage $\hat{V}_{Cdq}$ and voltage feedforward $V_{CFF\_Idq}^*$. First, the capacitor voltage model in dq frame can be derived in the S-domain by

$$\begin{bmatrix} \hat{I}_{Cd} \\ \hat{I}_{Cq} \end{bmatrix} = \begin{bmatrix} I_{Ld} - \hat{I}_d \\ I_{Lq} - \hat{I}_q \end{bmatrix} = \begin{bmatrix} \hat{C}_p s \hat{V}_{Cd} - \hat{C}_p \omega_e \hat{V}_{Cq} \\ \hat{C}_p s \hat{V}_{Cq} + \hat{C}_p \omega_e \hat{V}_{Cd} \end{bmatrix} \quad (16)$$

After that, the inductor currents I_{Ldq} in Equation (16) can be replaced from both V_{dq}^* and $\hat{V}_{Cdq}$ in Equation (5). In this case, the resulting dynamic model is shown by

$$\begin{bmatrix} [\hat{L}_f \hat{C}_p s^2 + \hat{R}_f \hat{C}_p s + (1 - \omega_e \hat{L}_f \hat{C}_p)] \hat{V}_{Cd} \\ [\hat{L}_f \hat{C}_p s^2 + \hat{R}_f \hat{C}_p s + (1 - \omega_e \hat{L}_f \hat{C}_p)] \hat{V}_{Cq} \end{bmatrix} = \begin{bmatrix} V_d^* + (2\hat{L}_f \hat{C}_p \omega_e s + \hat{R}_f \hat{C}_p \omega_e) \hat{V}_{Cq} - (\hat{R}_f + \hat{L}_f s) \hat{I}_d + \hat{L}_f \omega_e \hat{I}_q \\ V_q^* - (2\hat{L}_f \hat{C}_p \omega_e s + \hat{R}_f \hat{C}_p \omega_e) \hat{V}_{Cd} - (\hat{R}_f + \hat{L}_f s) \hat{I}_q - \hat{L}_f \omega_e \hat{I}_d \end{bmatrix} \quad (17)$$

It should be noted in Equation (17) that $\hat{I}_{dq}$ is the desired estimator. In order to formulate the current observer in Figure 6 with voltage regulation, Equation (17) is organized as

$$\begin{bmatrix} \hat{V}_{dis\_d} \\ \hat{V}_{dis\_q} \end{bmatrix} + \begin{bmatrix} V_{CFF\_Id}^* \\ V_{CFF\_Iq}^* \end{bmatrix} = \begin{bmatrix} \hat{L}_f \hat{C}_p s^2 + \hat{R}_f \hat{C}_p s + (1 - \omega_e^2 \hat{L}_f \hat{C}_p) \\ \hat{L}_f \hat{C}_p s^2 + \hat{R}_f \hat{C}_p s + (1 - \omega_e^2 \hat{L}_f \hat{C}_p) \end{bmatrix} \begin{bmatrix} \hat{V}_{Cd} \\ \hat{V}_{Cq} \end{bmatrix} \quad (18)$$

where the observer internal disturbances $\hat{V}_{dis\_dq}$ and total feedforward $V_{CFF\_Idq}^*$ are equivalent to

$$\begin{bmatrix} \hat{V}_{dis\_d} \\ \hat{V}_{dis\_q} \end{bmatrix} = \begin{bmatrix} -\hat{R}_f \hat{I}_d - \hat{L}_f s \hat{I}_d + \hat{L}_f \hat{\omega}_e \hat{I}_q \\ -\hat{R}_f \hat{I}_q - \hat{L}_f s \hat{I}_q - \hat{L}_f \hat{\omega}_e \hat{I}_d \end{bmatrix} \approx \begin{bmatrix} -\hat{R}_f \hat{I}_d + \hat{L}_f \hat{\omega}_e \hat{I}_q \\ -\hat{R}_f \hat{I}_q - \hat{L}_f \hat{\omega}_e \hat{I}_d \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} V_{CFF\_Id}^* \\ V_{CFF\_Iq}^* \end{bmatrix} = \begin{bmatrix} V_d^* + (2\hat{L}_f \hat{C}_p \hat{\omega}_e s + \hat{R}_f \hat{C}_p \hat{\omega}_e) \hat{V}_{Cq} \\ V_q^* - (2\hat{L}_f \hat{C}_p \hat{\omega}_e s + \hat{R}_f \hat{C}_p \hat{\omega}_e) \hat{V}_{Cd} \end{bmatrix} \approx \begin{bmatrix} V_d^* + \hat{R}_f \hat{C}_p \hat{\omega}_e \hat{V}_{Cq} \\ V_q^* - \hat{R}_f \hat{C}_p \hat{\omega}_e \hat{V}_{Cd} \end{bmatrix} = \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} + \begin{bmatrix} \hat{V}_{Cd\_cross} \\ \hat{V}_{Cq\_cross} \end{bmatrix} \quad (20)$$

In Equation (19) and Equation (20), two differential terms are neglected for simplicity, assuming nearly DC values for both $\hat{V}_{Cdq}$ and $\hat{I}_{dq}$ at steady state. Under this effect, the observer system plant $G_{sys}(s)$ in Figure 6 is obtained by

$$\begin{bmatrix} G_{sys\_d}(s) \\ G_{sys\_q}(s) \end{bmatrix} = \begin{bmatrix} \frac{\hat{V}_{Cd2}}{V_{CFF\_Id}^* + \hat{V}_{dis\_d}} \\ \frac{\hat{V}_{Cq2}}{V_{CFF\_Iq}^* + \hat{V}_{dis\_q}} \end{bmatrix} = \frac{1}{\hat{L}_f \hat{C}_p s^2 + \hat{R}_f \hat{C}_p s + (1 - \hat{\omega}_e^2 \hat{L}_f \hat{C}_p)} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (21)$$

Considering the design of the observer controller $C_{Io}(s)$ in Figure 6, the purpose is to achieve no steady state error for the closed-loop voltage regulation. Thus, this is given by

$$C_{Io}(s) = \begin{bmatrix} \hat{V}_{dis\_d} / [\hat{V}_{Cd} - \hat{V}_{Cd2}] \\ \hat{V}_{dis\_q} / [\hat{V}_{Cq} - \hat{V}_{Cq2}] \end{bmatrix} = K_{p\_I} \frac{\hat{L}_f \hat{C}_p s^2 + \hat{R}_f \hat{C}_p s + (1 - \hat{\omega}_e^2 \hat{L}_f \hat{C}_p)}{s} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (22)$$

where K_{p_I} is the proportional gain used to determine the overall observer estimation bandwidth. By substituting $G_{sys}(s)$ in Equation (21) and $C_{Io}(s)$ in Equation (22) into Figure 6, this current observer can be simplified as shown in Figure 7.

Figure 7. Simplification of capacitor current observer in Figure 6.

As seen in Figure 7, the overall observer dynamic property is equivalent to a first-order low-pass filter where the bandwidth is dependent on the proportional gain K_{p_I} . Considering the estimation errors from the PWM inverter, K_{p_I} is set at 100 Hz to remove these high-frequency noises.

Similar to the capacitor voltage observer mentioned in part A, $\hat{V}_{dis\_dq}$ in Equation (19) represents the internal disturbances for the proposed current observer. Considering the ideal observer regulation without steady state error, $\hat{V}_{dis\_dq}$ should appear at the signal node illustrated in Figure 6. It is important that $\hat{V}_{dis\_dq}$ contains the desired estimator of machine current $\hat{I}_{dq}$. In this case, $\hat{I}_{dq}$ can be eventually obtained from Equation (23). This is shown by

$$\begin{bmatrix} \hat{I}_d \\ \hat{I}_q \end{bmatrix} = \frac{-1}{\hat{R}_f^2 + (\hat{L}_f \hat{\omega}_e)^2} \begin{bmatrix} \hat{R}_f & \hat{L}_f \hat{\omega}_e \\ -\hat{L}_f \hat{\omega}_e & \hat{R}_f \end{bmatrix} \begin{bmatrix} \hat{V}_{dis\_d} \\ \hat{V}_{dis\_q} \end{bmatrix} = \begin{bmatrix} G_{cal\_d} \\ G_{cal\_q} \end{bmatrix} \begin{bmatrix} \hat{V}_{dis\_d} \\ \hat{V}_{dis\_q} \end{bmatrix} \quad (23)$$

where C_{cal} in Figure 6 is the ratio between $\hat{V}_{dis\_dq}$ and $\hat{I}_{dq}$. Besides, the capacitor current $\hat{I}_{Cdq}$ can be simply calculated by $\hat{I}_{Cdq} = I_{Ldq} - \hat{I}_{dq}$. In contrast to the direct capacitor current estimation in Equation (14), it is interesting to evaluate the estimation accuracy for the proposed closed-loop current observer. Based on Equation (17), $\hat{I}_{dq}$ and $\hat{I}_{Cdq}$ are both estimated using the estimated capacitor voltage $\hat{V}_{Cdq}$ and LC filter parameters. To easily compare the current estimation error based on the direct estimation in Equation (15), only the capacitor current estimation error I_{Cdq_err} is calculated based on the proposed

voltage observer in Equation (10) and current observer model in Equation (17). Through the mathematical organization, the estimation accuracy of the proposed current observer in Figure 6 is given by

$$\begin{bmatrix} I_{Cd\_err} \\ I_{Cq\_err} \end{bmatrix} = \frac{K_{p,I}}{s+K_{p,I}} \frac{-1}{\hat{R}_f^2 + (\hat{L}_f \hat{\omega}_e)^2} \left\{ \mathbf{C} \begin{bmatrix} V_d^* \\ V_q^* \end{bmatrix} + \mathbf{D} \begin{bmatrix} V_q^* \\ -V_d^* \end{bmatrix} \right. \\ \left. + (-\mathbf{C}(\mathbf{R}_f + \mathbf{L}_f \mathbf{s}) + \mathbf{D}\mathbf{L}_f \hat{\omega}_e) \begin{bmatrix} I_{Ld} \\ I_{Lq} \end{bmatrix} + (-\mathbf{D}(\mathbf{R}_f + \mathbf{L}_f \mathbf{s}) - \mathbf{C}\mathbf{L}_f \hat{\omega}_e) \begin{bmatrix} I_{Lq} \\ -I_{Ld} \end{bmatrix} \right\} \quad (24)$$

For simplification, two variables $\mathbf{C}$ and $\mathbf{D}$ in Equation (24) are defined by

$$\begin{bmatrix} \mathbf{C} \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} \hat{R}_f(\mathbf{A} - \hat{\mathbf{A}}) - \hat{L}_f \hat{\omega}_e(\mathbf{B} - \hat{\mathbf{B}}) \\ \hat{R}_f(\mathbf{B} - \hat{\mathbf{B}}) + \hat{L}_f \hat{\omega}_e(\mathbf{A} - \hat{\mathbf{A}}) \end{bmatrix} \quad (25)$$

where $\mathbf{A}$ and $\mathbf{B}$ are

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} L_f C_p s^2 + R_f C_p s + (1 - \omega_e^2 L_f C_p) \\ R_f C_p \omega_e \end{bmatrix} \quad (26)$$

In Equation (25), $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are defined in the same manner as $\mathbf{A}$ and $\mathbf{B}$; however, the estimated parameters $\hat{L}_f$, $\hat{R}_f$, $\hat{C}_p$ and $\hat{\omega}_e$ are substituted. Similar to the open-loop estimation error of capacitor current I_{Cdq_err} in Equation (15), the current estimation error based on the proposed observer is investigated with respect to I_{Ldq} and V_{dq}^* . Figure 5 illustrates the estimation accuracy of the proposed current observer under the parameter variation of $\hat{L}_f$ and $\hat{C}_p$. In Figure 5a,b, $|I_{Cdq\_err}(j\omega)/I_{Ld}(j\omega)|$ and $|I_{Cdq\_err}(j\omega)/I_{Lq}(j\omega)|$ with respect to the inductance $\hat{L}_f$ are analyzed. Compared to the direct current estimation, the proposed current observer is able to reduce the sensitivity error of $\hat{L}_f$ in the whole frequency range. Besides, in Figure 5c, the variation of $|I_{Cdq\_err}(j\omega)/V_d^*(j\omega)|$ with respect to $\hat{C}_p$ is investigated. In contrast to direct current estimation, better estimation performance with respect to $\hat{C}_p$ error is achieved due to the closed-loop observer estimation.

5. Experiment Results

This section experimentally verifies the proposed dual-observer estimation on a low-inductance PM machine drive with an LC filter. Figure 8 shows the test setup of the PM machine with an LC filter. A 105 W PM machine with only 130 μH phase inductance was tested. Detailed motor and filter specifications are listed in Table 1. Considering the high-speed limitation, different fans were attached on the motor shaft instead of servo machines for the torque load operation.

Figure 8. Test setup of PM motor drive with LC filter.

5.1. LC Filter on Current Harmonics

This part verifies the improvements of the PWM reflected current harmonics using the LC filter. Figure 9 compares the machine phase currents and the corresponding spectrum for the drive (a) without and (b) with the LC filter. The machine speed was operated at 40 krpm (666.7 Hz electric frequency) through the FOC with external current sensors. The phase currents $I_A/I_B/I_C$ in Figure 1 were measured using current probes. For this low-inductance machine

drive without an LC filter in (a), the PWM switch caused considerable current harmonics with 65.1% total harmonic distortion (THD). PWM-reflected rotor loss and a high rotor temperature were also observed. In contrast, in (b) with the LC filter, these current harmonics were reduced with only 20% THD. Based on this test, it is concluded that the LC filter effectively improves the PWM-reflected current harmonics on the FOC drive.

Figure 9. Machine three-phase currents and the corresponding spectrum for the drive (a) without and (b) with LC filter (40 krpm and fan load).

5.2. LC Filter Effect on FOC Drive

This part demonstrates the influence of the LC filter on the FOC machine drive. Figure 10 compares three different A-phase currents—the inverter output current, I_{LA} , capacitor current I_{CA} and machine actual current I_A in Figure 1, respectively—at (a) a low speed of 1 krpm and (b) high speed of 40 krpm. As seen in Figure 10a, the inverter current I_{LA} and machine current I_A were nearly the same, while the capacitor current I_{CA} contained some PWM harmonics. Besides, current waveform distortion was observed on both I_{LA} and I_A . In general, the PWM inverter contained the deadtime effect, affecting the current regulation, especially at low speed. In this paper, the dead-time compensation was applied based on the direct voltage compensation dependent on the phase current polarity [25]. In contrast, at high speed, as shown in Figure 10b, a visible magnitude and phase difference between I_{LA} and I_A were observed due to the high magnitude of I_{CA} . Based on this experiment, it is concluded that, at high speed, the actual machine current I_A must be obtained to stabilize the FOC drive.

Figure 10. Experimental comparison of A-phase inverter output current I_{LA} , capacitor current I_{CA} and machine actual current I_A at (a) a low speed of 1 krpm and (b) a high speed of 40 krpm (FOC drive and fan load).

5.3. Capacitor Voltage Estimation

This section evaluates the capacitor voltage estimation using the proposed capacitor voltage observer in Figure 4. Figure 11 shows the comparison of the actual dq capacitor voltages V_{Cdq} and estimated capacitor voltages $\hat{V}_{Cdq}$. Here, V_{Cdq} values were measured using external voltage sensors. In this test, the motor was accelerated from 6 krpm to 42 krpm within 4 s. The FOC drive was applied with external current sensors to clearly evaluate the estimation performance. In this test, the sensorless FOC was implemented where the rotor position was obtained from the EMF estimation. Because the EMF is insufficient at very low speed, the proposed FOC drive with an LC filter was implemented at 6 krpm with accurate EMF estimation. As seen in Figure 11, $\hat{V}_{Cdq}$ was close to V_{Cdq} in the range of 6~42 krpm under different voltage magnitudes. It is concluded that the proposed voltage observer can obtain actual capacitor voltages at different speeds and magnitudes using only the existing drive current sensors.

Figure 11. Experiment of capacitor voltage estimation performance (acceleration from 6 krpm to 42 krpm within 4 s and fan load).

5.4. Motor Current Estimation

This section shows the machine current estimation using the proposed capacitor current observer in Figure 6. It is noted that both the machine current I_{dq} and capacitor current I_{Cdq} can be obtained from this observer. In this context, only the motor $\hat{I}_{dq}$ estimation performance is evaluated for simplicity.

Figure 12 compares the actual machine I_{dq} and estimated $\hat{I}_{dq}$ using the proposed current observer. The test condition was the same as Figure 11 in terms of the acceleration with the FOC drive. As shown in Figure 8, an external fan was coupled to the machine shaft to easily manipulate the torque load. This machine drive test setup was similar to [26]. In general, the torque load is a quadratic function dependent on the machine speed. At 6 krpm, as shown in Figure 12, relatively high current noises were observed. However, the current noises were negligible as the speed increased since the magnitude of noise remained the same irrespective of the speed.

More importantly, $\hat{I}_{dq}$ was close to I_{dq} irrespective of the machine speed and current magnitude. As seen in Figure 12, q-axis current estimation error was around 2.56% at 6 krpm. This increased to 4.54% as the speed increased to 42 krpm. In contrast, the d-axis current estimation error was negligible for the speeds between 6 krpm and 42 krpm. As a result, the proposed current observer was also able to estimate $\hat{I}_{dq}$ accurately at different speeds once the current magnitude was higher than a certain value.

Figure 13 further illustrates the time-domain waveforms of machine AB-line voltage $\hat{V}_{AB}$ and machine A-phase current $\hat{I}_A$ at a low speed of 1 krpm (16.67 Hz) with respect to the externally measured V_{AB} and I_A . Both (a) a 25% load and (b) a 50% load were compared based on different fan load sizes. For the voltage estimation, $\hat{V}_{AB}$ was close to V_{AB} at different loads. It is noted that the filter cutoff frequency was designed at 2.92 kHz.

In this case, the PWM voltage could be removed in both V_{AB} and $\hat{V}_{AB}$, leading to nearly sinusoidal voltage waveforms. In contrast, for current estimation, a relatively low signal-to-noise ratio was observed in $\hat{I}_A$. As mentioned in Equation (18), the machine current was estimated from voltage disturbances $\hat{V}_{dis\_dq}$ through the RLC circuit model. This voltage was relatively small at low speed, resulting in estimation noises. However, the magnitudes between $\hat{I}_A$ and I_A were almost the same. It is concluded that the proposed machine current estimation can still be used for FOC current regulation.

Figure 12. Experiment of machine current estimation performance (acceleration from 6 krpm to 42 krpm within 4 s and fan load).

Figure 13. Experiment of estimated filter output capacitor voltage and machine current at (a) 25% and (b) 50% load (1 krpm rotor speed).

5.5. Sensitivity Analysis of Filter State Estimation

This section further verifies the influence of filter parameter errors on the proposed observer-based estimation. As mentioned in Figure 5, the estimation of the filter output machine current $\hat{I}_{dq}$ requires a filter capacitor and inductance parameter. More importantly, these parameter errors can be reduced using the proposed observer with the closed-loop feedback regulation.

Figure 14 shows the time-domain waveforms of the measured speed, measured and estimated machine I_q and $\hat{I}_q$ and the estimation error $I_q - \hat{I}_q$. The test condition was the same as in Figures 11 and 12 in terms of the FOC acceleration. In (a), a 50% capacitor parameter error was intentionally added in the proposed current observer. The current estimation error was negligible as the speed accelerated from 6 krpm to 42 krpm. Thus, the proposed current observer demonstrated negligible sensitivity under the filter capacitor variation. This test result is consistent with the analytical model in Figure 5c.

Figure 14. Sensitivity analysis of proposed observer-based current estimation under (a) filter capacitor error and (b) inductance error (6 krpm to 42 krpm and fan load).

In contrast, in Figure 14b, the same 50% filter inductance error was included to evaluate the same $\hat{I}_q$ estimation performance. In contrast to the capacitor parameter, the estimation error increased as the speed increased. At 6 krpm, the current estimation error was negligible. However, this error increased to 26.3% as the speed increased to 42 krpm. This result was the same as the simulation in Figure 5a. Nevertheless, the filter inductance parameter should be known as it is selected by the drive designer.

5.6. FOC Performance Comparison

The machine speed closed-loop control for the drive with an LC filter is compared in this section. Figure 15 compares the current regulation using the feedback of (a) the inverter current I_{Ldq} in Figure 1, (b) the estimated machine current $\hat{I}_{dq}$ with direct estimation in Equation (14) and (c) the estimated machine current $\hat{I}_{dq}$ with proposed observer-based estimation in Figure 6. Due to the encoder installation issue on the test machine, the EMF-based position estimation was applied where the estimated capacitor voltage $\hat{V}_{Cdq}$ was used for the position estimation, as illustrated in Figure 2.

As shown in Figure 15a, if the inverter current I_{Ldq} was directly used for current regulation ignoring the resonant dynamics of the LC filter, the speed control ultimately failed when the speed reached 40 krpm (666.7 Hz frequency). The difference between I_{Ldq} and I_{dq} at high speed was the primary issue. In contrast, in (b) when the directly estimated $\hat{I}_{dq}$ was used, the speed control was able to stably operate at the rated speed under fan load. However, current noises were observed. These current noises resulted in speed estimation noises with 5.5% error in the high-speed region. More importantly, in Figure 15c, better current regulation performance compared to (b) was achieved using the capacitor current observer for $\hat{I}_{dq}$ estimation. In particular, at a high-speed of 42 krpm, there was no evident noise in the speed estimation error. For both direct estimation and proposed dual-observer, there was a 5.5% error in speed estimation at 6 krpm due to the low noise-to-signal ratio of back-EMF voltage. Thus, the proposed dual-observer could accurately obtain $\hat{V}_{Cdq}$ and $\hat{I}_{dq}$ for the position sensorless FOC drives under the PWM drive with an LC filter.

Figure 15. Comparison of current regulation using (a) inverter current I_{Ldq} , (b) estimated machine current $\hat{I}_{dq}$ with direct estimation in (14) and (c) estimated machine current $\hat{I}_{dq}$ with proposed observer-based estimation in Figure 6.

6. Conclusions

This paper proposes a dual-observer to estimate the capacitor voltage/current and machine current for a sensorless FOC drive using the drive inside current sensors. The observer estimation performance considering the filter parameter sensitivity is fully investigated. Table 2 lists the performance comparison of the conventional model-based estimation and proposed dual-observer estimation. Compared to conventional model-based estimation, the proposed dual-observer demonstrates robust estimation performance under parameter errors. The capacitor parameter error results in a negligible influence on the proposed observer estimation. The filter inductance error only affects the capacitor current estimation at high speed. By using commercial PWM drives, the FOC can be applied to PM machines where an LC filter is added to reduce the PWM harmonics.

Table 2. Comparison between conventional model-based estimation and proposed dual-observer estimation.

	Model-Based Estimation	Proposed Dual-Observer
Parameter sensitivity	Capacitor and inductor sensitive (drive is unstable for experimental test)	Capacitor (negligible error) Inductor (negligible error at 6 krpm and 26.3% error at 42 krpm)
FOC speed control	5.5% error between 6~42 krpm	5.5% error at 6 krpm and negligible error at 42 krpm
Memory usage	14.2 Kb	18.5 Kb

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