Supplement 3. The stability margin

The stability margin characterizes proximity of the current state to the boundaries of the stability zone. Mikhailov's criterion and Mikhailov's hodograph are used to assess the stability margin of discrete systems (Kim, 2007):

$$D(j\nu) = \overline{D}(z)|_{z=\exp(j\nu)}, \quad \nu \in [-\pi, \pi] \qquad \text{(S1)}$$

where $\overline{D}(z)$ is a characteristic polynomial (of the system under study) normalized to the highest degree $z$.
A system with a certain characteristic polynomial is stable if Mikhailov's hodograph (S1), when variable $\nu$ changes from $-\pi$ to $\pi$, sequentially goes around point $z = 0$ counterclockwise starting on the real axis and covering $4n$ quadrants (Gaiduk et al., 2011).
Let us examine an autoregressive equation for the model of the gypsy moth's population density dynamics in Khabarovsk Krai:

$$y(i) = 1.39y(i-1) - 0.94y(i-2) \qquad \text{(S2)}$$

Characteristic polynomial $\overline{D}(z)$ for eq. S2 is given by

$$\overline{D}(z) = z^2 - 1.39z + 0.94 = 0 \qquad \text{(S3)}$$

After switching from variable $z$ to new complex variable $\nu$, $z = \frac{1+\nu}{1-\nu}$, we obtain a characteristic equation for $\nu$:

$$\overline{D}(j\nu) = e^{j2\nu} - 1.39e^{j\nu} + 0.94 \qquad \text{(S4)}$$

To construct Mikhailov's hodograph, real component $P(\nu)$ and imaginary component $Q(\nu)$ are introduced into expression S4:

$$\begin{aligned} P(\nu) &= \cos 2\nu - 1.39\cos\nu + 0.94 \\ Q(\nu) &= \sin 2\nu - 1.39\sin\nu \end{aligned} \qquad \text{(S5)}$$

Next, in the $\{P(\nu), Q(\nu)\}$ plane, for $\nu$ values from $-\pi$ to $\pi$, a hodograph curve is built, and each point on the plane has values $P(\nu_0)$ and $Q(\nu_0)$ at some value $\nu = \nu_0$. According to the Mikhailov criterion, stability margin $\eta$ is the radius of the circle (centered at point $z = 0$) that can be inscribed in the Mikhailov hodograph.
Despite the seemingly complicated calculation procedure, the stability margin is estimated using a simple program in the MATLAB software (Table S1).

Table S1. Settings of a program in the MATLAB software for calculating the stability margin of an autoregressive model (Gaiduk et al., 2011).

| |
|---|
| Dz = [1 a[1] a[2]; |
| Dz = Dz/Dz(1); |
| nu = (-pi: pi/(100*length(Dz)):pi; |
| j = sqrt(-1); |
| z = exp(j*nu); |
| GM = polyval (Dz, z); |
| Eta = min (abs(GM)); |
| disp ([‘stability margin =‘ num2str(eta)]) |

To calculate the estimated stability margin, one has to load the program from Table S1 into the MATLAB package and to perform only one operation: enter into line 1 the values of autoregressive-model coefficients. Mikhailov's hodograph for the model of gypsy moth's population density dynamics in Khabarovsk Krai is shown in Fig. S1.

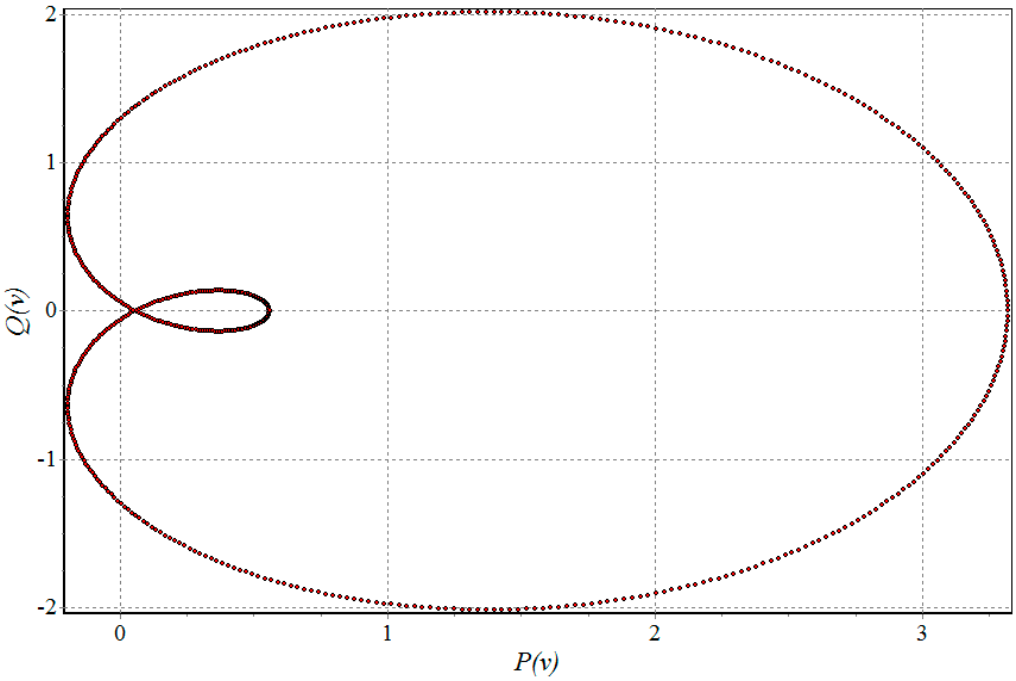


Figure S1. Mikhailov's hodograph for the model of gypsy moth population density dynamics in Khabarovsk Krai (Russia).

Literature

Gaiduk A.R., Belyaev V.E., Pyavchenko T.A. Automatic control theory: examples and problems. St. Petersburg: Lan. 2001. 464 p. (Russian)

Kim D.P. Theory of automatic control. Moscow: Fizmatlit. 2007. V.1. 312 p. (Russian)