Entropy **2004,** *6* 133–152

Entropy
ISSN 1099-4300
www.mdpi.org/entropy

Delocalization and Sensitivity of Quantum Wavepacket in Coherently Perturbed Kicked Anderson Model

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Received: 2 July 2003/ Accepted: 20 December 2003 / Published: 21 March 2004

Abstract: We consider quantum diffusion of the initially localized wavepacket in one-dimensional kicked disordered system with classical coherent perturbation. The wavepacket localizes in the unperturbed kicked Anderson model. However, the wavepacket gets delocalized even by coupling with monochromatic perturbation. We call the state "dynamically delocalized state". It is numerically shown that the delocalized wavepacket spreads obeying diffusion law, and the perturbation strength dependence of the diffusion rate is given. The sensitivity of the delocalized state is also shown by the time-reversal experiment after random change in phase of the wavepacket. Moreover, it is found that the diffusion strongly depend on the initial phase of the perturbation. We discuss a relation between the "classicalization" of the quantum wavepacket the initial phase dependence. The complex structure of the initial phase dependence is related to the entropy production in the quantum system.

Keywords: Localization, Delocalization, Quantum Diffusion, Scaling, Time-irreversibility, Phase sensitivity, Dissipation

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1 Introduction

Generally, quantum system is classicalized by decoherence and dephasing, and the system becomes dissipative [1, 2, 3, 4, 5, 6]. The classical pictures for the dissipation in the quantum system are treated by heat-bath theory [1, 2, 3], linear response theory [4, 5] and Landauer formula [6] and so on. In the standard pictures the coupling with external infinite number of degrees of freedom (DOF) and/or stochastization mechanism are implicitly or explicitly assumed in the system itself. We consider a question: is the coupling with macroscopic number of DOF and/or external stochastization mechanism really necessary for destruction of the quantum coherence in the quantum system [7, 8]? In the present paper, we investigate the localization and delocalization in 1DDS as a typical quantum interference effect and the destruction by the coupling with coherent perturbation.

Recently, we have found an interesting property in Anderson model with coherently time-dependent perturbation [8, 9]. In the unperturbed Anderson model, it is well-known that almost all eigenstates are localized and the quantum diffusion of initially localized wavepacket is suppressed at the localization length, as a result of the interference between scattering waves [10]. When classical coherent perturbation consisting of some frequency components makes the localized state delocalized and we called the state "dynamically delocalized state". The similar models have been investigated by some groups [11, 12, 13]. Moreover, when we couple an oscillator in the ground state with the system in the delocalized state, the energy of the system in the excited state has been irreversibly transferred to the oscillator [8]. Although it is a very interesting feature of the dynamically delocalized state because the phenomenon hints a kind of the potential dissipative property in the quantum system, I do not repeat the results here. For the details, see Ref.[8, 9].

The similar delocalization phenomena to Anderson model can be also observed in *kicked* Anderson model [15]. The spread of the initially localized wave packet are suppressed and the packets are exponentially localized in the kicked Anderson model too. Moreover, the localization can be also delocalized by the coherent perturbation. Accordingly, instead of Anderson model we can use the kicked Anderson model to investigate the property of the localization and the delocalization phenomena. The kicked system is convenient to save CPU time for long-time numerical simulation. How does the localization change as perturbations from other DOF is introduced in a kicked 1DDS? In the present paper, we give some more extended numerical results than the preliminary reports in Ref.[15].

The outline of the present paper is as follows. In Sec.2 model systems investigated are introduced. In Sec.3, we numerically investigate the wave packet dynamics in unperturbed and coherently perturbed kicked Anderson model. First we show the localization in the unperturbed kicked Anderson model, and estimate the localization length as a function of the disorder strength.

Secondly, we investigate the delocalization in monochromatically perturbed case. It is shown that when the perturbation strength is small the spread of the packet obeys subdiffusion law characterizing by an index α, and the spatial decay of the wave packet obeys stretched Gaussian form characterizing by an index β. We give the numerical result of the relation between α and β with comparing to an analytical result given by Zhong *et al.* [14]. Moreover, the wave packet spreads obeying a normal diffusion law in the polychromatically perturbed kicked Anderson model. In Sec.4, we give numerical result for time-reversal experiments after random phase-change of the wave packet, in comparison to cases in periodic system. In Sec.5, furthermore, we show that the delocalization is sensitive to change of the initial phase of the classical perturbation. It is briefly discussed about the relation between the phase sensitivity and the classicalization of the quantum wave packet. The last section is devoted to summary and discussion.

2 Model

The tight binding Hamiltonian for the kicked 1DDS is given by,

$$H_0(t) \;=\; T(\hat{p}) + \sum_n |n > V(n) < n| \sum_m \delta(t-m), \tag{1}$$

where $\hat{p}(= -i\frac{\partial}{\partial x})$ is a shift operator and $V(n)$ is uniformly distributed on-site energy in the range $[-W, W]$ and $T(\hat{p})(= 2(\cos\hat{p} - 1))$ is hopping term between nearest neighbor sites. The dynamics of kicked Anderson model is given by Floquet operator,

$$U = \exp(-\frac{i}{\hbar}\frac{T(\hat{p})}{2})\exp(-\frac{i}{\hbar}V(n))\exp(-\frac{i}{\hbar}\frac{T(\hat{p})}{2}). \tag{2}$$

The value of the wave function is determined in the middle of the two successive kicks, and the periodic boundary conditions are assumed. It is instructive to recall the relation Harper model and kicked Harper model [16, 17, 18]. Note the form is equivalent to second order symplectic integration method for time evolution of the general time-dependent system, and the potential strength W is which is equivalent to strength of the kick.

Furthermore, we consider parametrically perturbed kicked Anderson model to investigate the delocalization phenomena, which Hamiltonian is given as follows:

$$H^{osc}(t) \;=\; H_0(t) + \sum_n |n > V(n,t) < n| \sum_m \delta(t-m), \tag{3}$$

$$V(n,t) \;=\; \frac{\epsilon V(n)}{\sqrt{L}} \sum_j^L \cos(\omega_j t + \phi_j), \tag{4}$$

where the frequency components of the classical coherent perturbation $\{\omega_j\}$ are taken to be mutually incommensurate numbers and the order is $O(1)$. The time evolution operator $U(s)$ for $s$steps is given by,

$$U(s) = \prod_{k=1}^{s} \exp(-\frac{i}{\hbar}\frac{T(\hat{p})}{2}) \exp(-\frac{i}{\hbar}V(n,k)) \exp(-\frac{i}{\hbar}\frac{T(\hat{p})}{2}). \tag{5}$$

We set $\phi_j = 0$ until Sec.5.

3 Dynamical delocalization

In this section, we show the diffusion property in the kicked Anderson model with increasing of the number of the frequency components of the perturbation.

3.1 Unperturbed case (L=0)

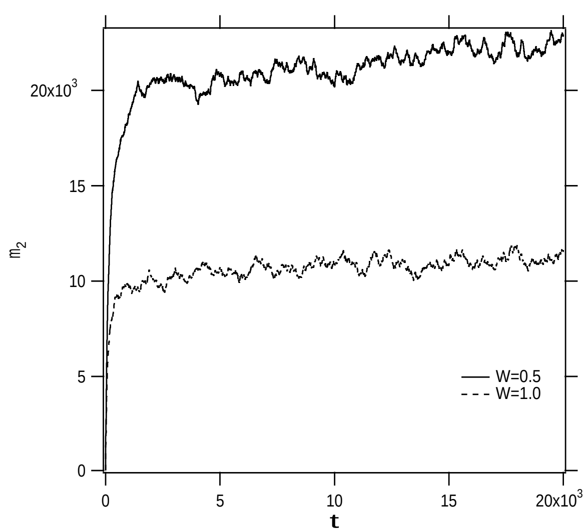

Figure 1: Time dependence of MSD of kicked Anderson model with some potential strength W. The system and ensemble size are 2^{14} and 30 respectively. $\hbar$ =0.125.

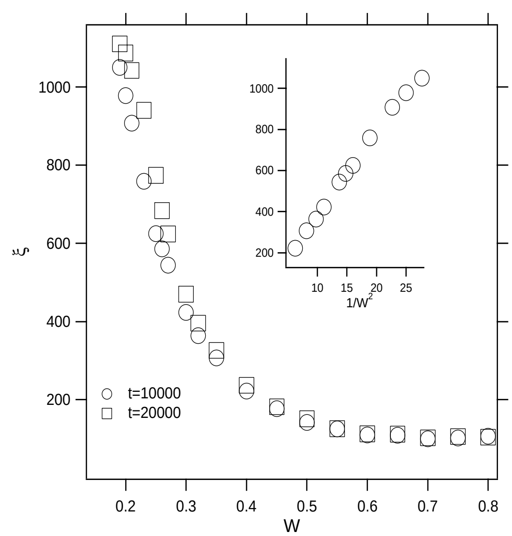

Figure 2: Localization length ξ as a function of potential strength W in the kicked Anderson model. The ξ is estimated by MSD as $m_2(t) \sim \xi^2(t)$ at t =10000 and 20000 denoted by open circles and squares. The insert is a plot for ξ versus W^{-2}. $\hbar$ =0.125.

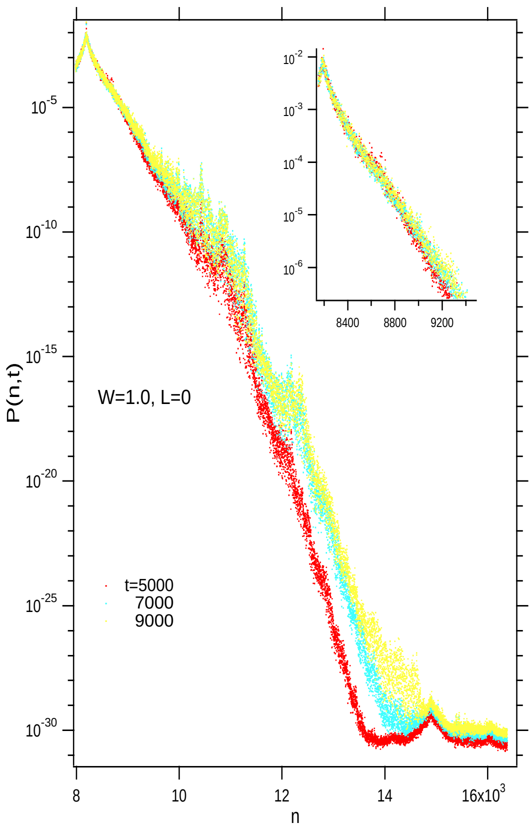

Figure 3: Some snapshots of the ensemble averaged probability distribution function $P(n,t)$ of the kicked Anderson model without perturbation (t =5000,7000,9000). Only the right half of the distribution is shown. The insert is a expansion of the vicinity of the center of the distribution. $n_0 = 2^{13}$, and W =1.0.

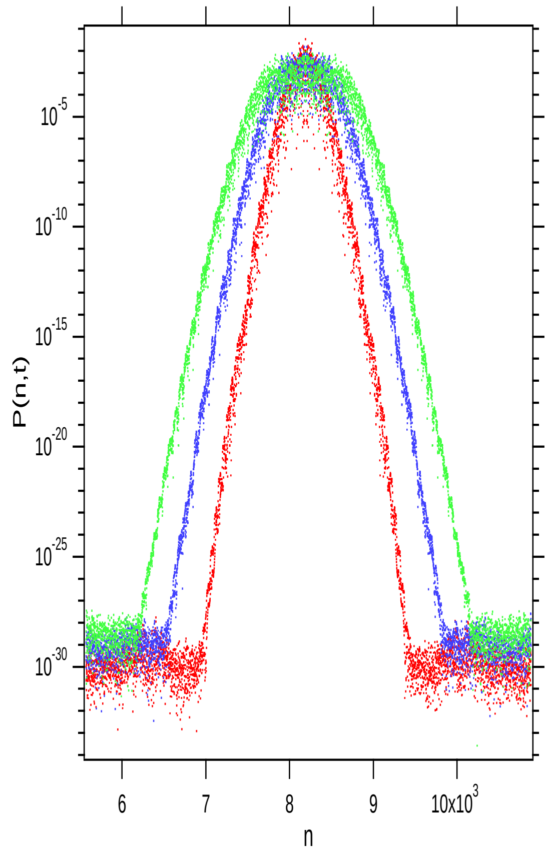

Figure 4: Some snapshots of the ensemble averaged probability distribution function of binary periodic system with monochromatic perturbation $L = 1, \epsilon = 0.1$. (t =500,700,900.)

First we numerically show time dependence of the mean square displacement (MSD),

$$m_2(t) =< \Psi(t)|(n-n_0)^2|\Psi(t)>, \tag{6}$$

of initially localized wave packet $\Psi(n,t=0) = \delta_{n,n_0}$ in the kicked Anderson model. Diffusion of the packets are suppressed and localized as shown in the Fig.1. Figure 2 shows the localization length ξ as a function of the fluctuation strength W numerically estimated by $\xi(t)^2 = m_2(t)$ at several time. The localization length ξ decreases as the potential strength W increases as

$\xi \propto W^{-2}$. The W dependence saturates around $W \sim 3.5$ given by $W/\hbar \simeq \pi$. Figure 3 shows time-dependence of the ensemble-averaged probabilistic function $P(n,t) =< |\Psi(n,t)|^2 >$. We can see clear exponential localization in the unperturbed kicked Anderson model. On the other hand, the ensemble-averaged probablistic functions in a binary periodic system show Gaussian process like behavior. (See Fig.4.)

3.2 Monochromatically perturbed case (L=1)

Next, we use the strongly localized case $W = 1.0$ in the unperturbed kicked Anderson model to show the delocalization phenomena. As seen in Fig.5 the wave packet is delocalized even in the monochromatically perturbed case ($L = 1$), and we call the state "dynamically delocalized state" in a sense that any stochastic perturbation is not imposed on the system [9]. Note that the axes of Fig.5(b) are in log-scale. When the perturbation strength ϵ is small, it seems that the diffusion is not a normal diffusion: a clear subdiffusion is well observed. As increase of the ϵ, the diffusion process approaches normal diffusion. However, we can not judge whether the critical value ϵ_c dividing the subdiffusion and normal diffusion exists or not by the data. We can formally characterize the diffusive behavior as the subdiffusion by following index α in a form, $m_2 \sim t^{\alpha}$. Figure 6(b) shows the index α as a function of the perturbation strength ϵ. The exponent α approaches unity from below with increasing the ϵ. The diffusion rate D estimated formally by $m_2 \simeq Dt$ in Fig.5(a) is also given in Fig.6(a). The ϵ dependence changes around "certain value" $\epsilon_c (\simeq 0.1)$, and the D begins to increase rapidly. For $\epsilon \geq 0.3$ the α saturates near unity. Accordingly, the way of the spread of the wave packet gradually transits from subdiffusive behavior to normal diffusive one with increasing ϵ through the transition range ($\epsilon \sim 0.1 - 0.3$).

3.3 Polychromatically perturbed cases (L=2,3)

In the polychromatically perturbed cases, we can observe clear normal diffusion even for the small perturbation strength as seen in Fig.7. The perturbation strength dependence of the diffusion rate D estimated by the MSD is shown in Fig. 6(a). It seems that the ϵ dependence of the D rapidly grows without "threshold" compared with the case of $L = 1$ in Fig.6(a). We can regard appearance of the normal diffusion as a kind of "classicalization" of the quantum wave packet, which is caused by the coupling with the other DOF.

3.4 Spatio-temporal distribution function (L=1)

Moreover, in the monochromatically perturbed cases the space-time dependence of the ensemble-averaged probabilistic function $P(n,t)$ are shown in Fig.8 [9, 19]. The functional form of the distribution function approaches Gaussian function as increase of the strength ϵ, which corresponds

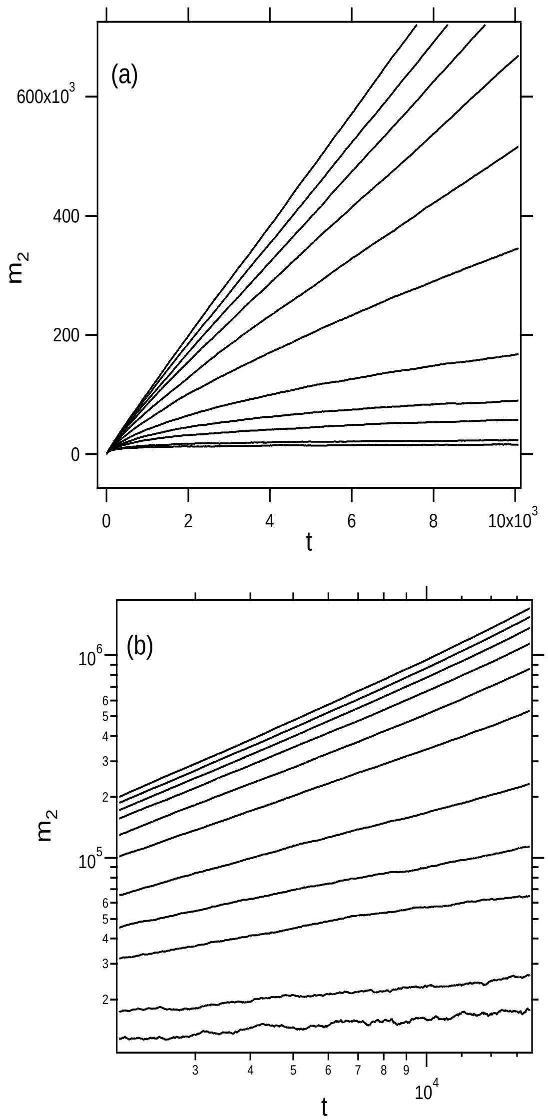

Figure 5: (a) Time dependence of MSD in kicked Anderson model with monochromatic perturbation ($L = 1$) at some perturbation strength ϵ(0.01-0.4), and (b)the log-log plots. The other parameters are same to Fig.1. We set $\phi_i = 0$.

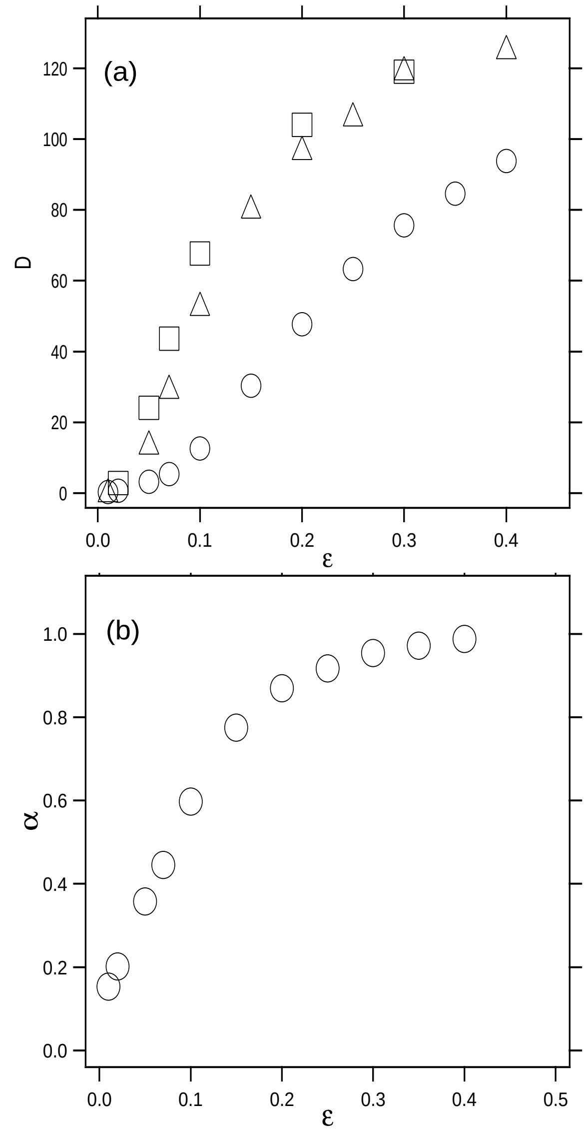

Figure 6: Perturbation strength dependence of (a)diffusion rate D and (b)power index α eatimaed by data in Fig.5 based on scaling form $m_2 \sim Dt$ and $m_2 \sim t^\alpha$ respectively. The open triangle and square in the (a) represent diffusion rate in $L = 2$ and $L = 3$, respectively.

to a solution in diffusion equation in the stochastic process [19]. The peaks around the center of the distribution come from a remain of the localization in the unperturbed case [20, 21, 24]. In

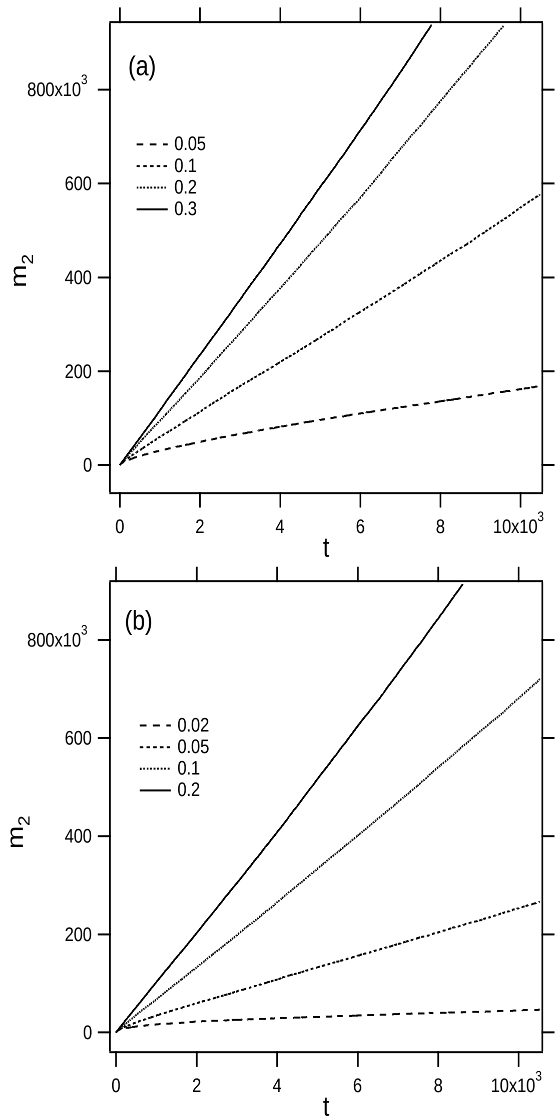

Figure 7: Time dependence of MSD of kicked Anderson model with polychromatic perturbation at some perturbation strength ϵ. (a)$L = 2$ and (b)$L = 3$. The other parameters are same to Fig.1.

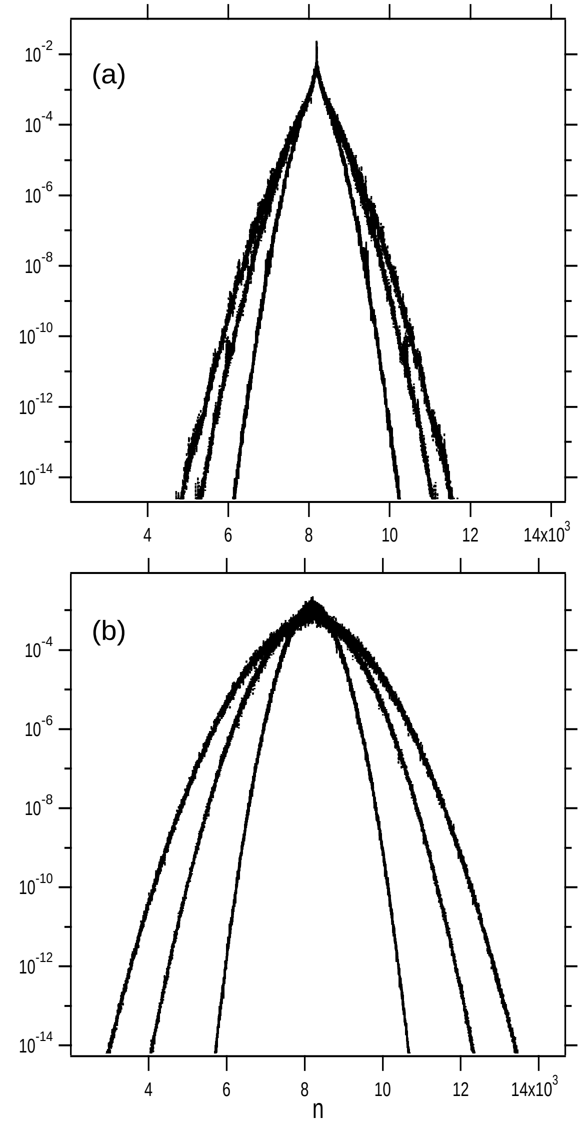

Figure 8: Some snapshots of the ensemble averaged probability distribution function of the kicked Anderson model with perturbation, (a)$L = 1$, $\epsilon = 0.05$ and (b)$L = 1$, $\epsilon = 0.4$. (t =1000,3000,5000.)

our pervious paper, it is numerically shown that the quantum diffusion of the wave packet obeys a scaled form in the perturbed Anderson model [9]. The scaled form of the distribution function is given by the stretched Gaussian distribution,

$$P(n,t) \propto \exp\{-\text{const.}(\frac{|n|}{t^{\alpha/2}})^{\beta}\}, \tag{7}$$

except for the range close to the center of the distribution.

Thus the distribution function is specified by the two exponents, i.e., α characterizing the temporal growth of the wave packet, and β characterizing its spatial decay. The distribution function is of a unified form, which contains the two extreme limits, i.e., the exponential localization ($\alpha = 0, \beta = 1$) and the normal diffusion ($\alpha = 1, \beta = 2$) as special cases, and in general interpolates them [19]. Here we estimate the (α, β) for the subdiffusive behavior in the monochromatically perturbed kicked Anderson model. The same procedure employed in Ref.[9] is used to determine the index β characterizing the spatial decay. Figure 9 shows the plot of (α, β) obtained for various value ϵ.

We consider the relation between α and β. Under some assumptions Zhong *et al.* analytically derived following relation between the two indexes in the quantum diffusion,

$$\beta = \frac{2}{2-\alpha}, \tag{8}$$

in our notation [14]. The curve is also overwritten in the Fig.9. Some data in the prtuerbed Anderson model are also overploted [9]. In the normal diffusion side ($\alpha \sim 1, \beta \sim 2$) the numerical data coincide with the universal relation, while in the localization side ($\alpha \sim 0, \beta \sim 1$) it deviate from the curve. The deviation might be caused by one of the assumptions they used in the derivation. Although they used a generalized master equation with memory function as the start point, the equation does not describe exponential localization. Accordingly it seems that the universal relation is true for the diffusive side, while is not always true for the localization side.

Recently, we have some mathematical tools in order to treat with the power-law behaviors with long-term memory effects such as fractional calculus [22], Tsallis statistics [23] and so on. However the derivation of the power-law indeces α, β and the relations from the dynamical precess is a remaining and important problem.

4 Time irreversibility

In this section, we show the result for time-reversal experiments after random phase-change of the wave packet in the unperturbed and the monochromatically perturbed kicked Anderson model, comparing with the results in periodic system. The same idea based on the time irreversibility have been used to investigate the "classicalization" for quantum chaos systems [25].

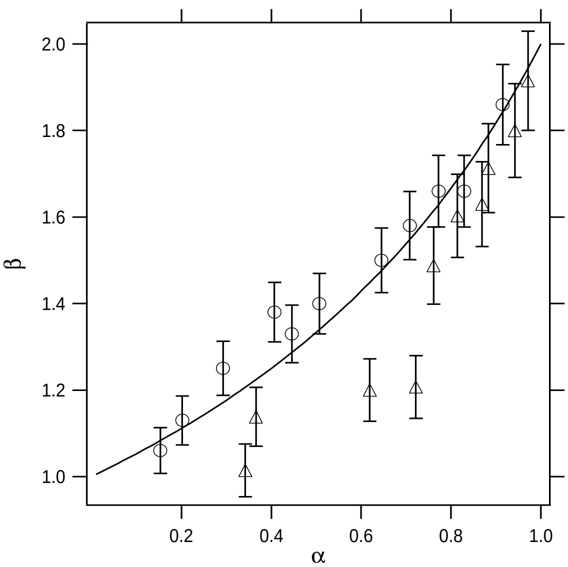

Figure 9: Plot of the (α, β) obtained at various subdiffusive cases. The relation $\beta = 2/(2-\alpha)$ is overwritten. Some data ($\triangle$) in Anderson model are also overploted. The error bars denote 50 percent error.

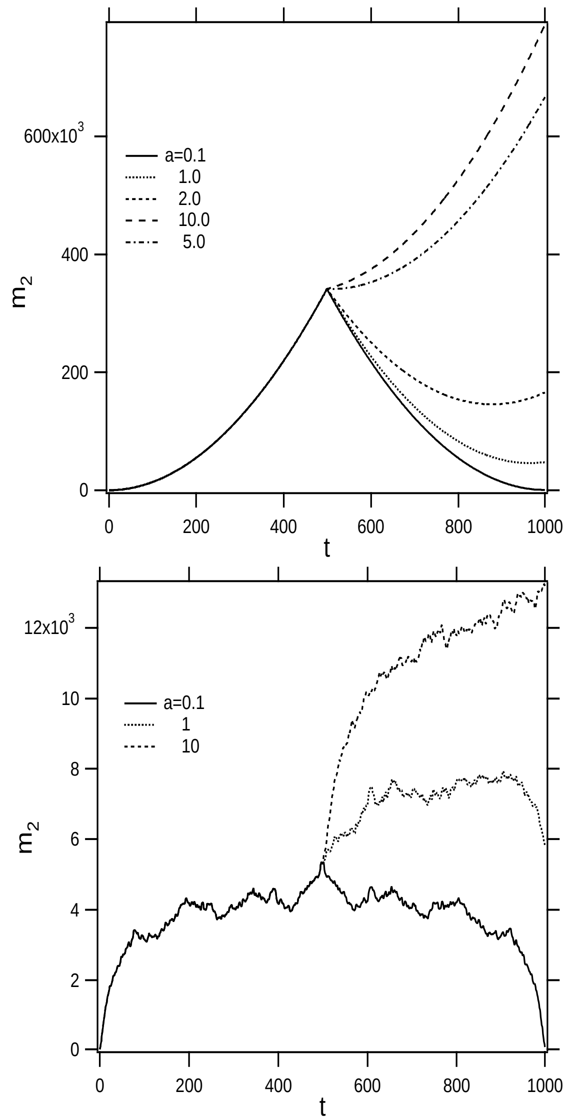

Figure 10: Time reversal experiments for various phase-change strength a in (a) binary periodic and (b) disordered cases without perturbation($L = 0$). The system size are 2^{14} and we used a sample. $\hbar$ =0.125.

4.1 Unperturbed case (L=0)

We give the method for the time-reversal experiments used in the present paper. First the system evolves by the time-evolution operator U^T given in eq.(2), until $t = T$. At $t = T$ a perturbation

$\hat{P}$ is applied for the wave packet. We used the random phase-change of the amplitude $\Psi(n,T)$ at each site n, accordingly the perturbation does not change the probability amplitude $|\Psi(n,T)|^2$ of the quantum state. After applying the perturbation $\hat{P}$, we evolves the state by the unitary operator U^{T-s} until $s=T$ as follows.

$$\Psi(s+T) = U^{T-s}\hat{P}U^{T}\Psi(0). \tag{9}$$

In the concrete, we change the phase of the packet at $t=T$,

$$\Psi(n,T) \to \exp\{ia\xi_n\}\Psi(n,T), \tag{10}$$

where ξ_n is a random number in a range $[-1,1]$ at each site n and a is the phase-change strength. As the a has the meaning in a range $-\pi \le a \le \pi$, we changed the value a in a range $a \le 10$.

We monitor the time dependence of the MSD for the whole process. Figure 10 shows the result for various change-strength a in binary periodic and disordered cases without perturbation($\epsilon=0$). In the case with small strength the state can almost return to the initial state concerning the MSD. However, it follows that in both cases as increase of the change-strength a the return to the initial state becomes difficult in the backward process. We follow that for same value of the $a=1.0$ in the disordered system the return to the initial state becomes more unstable when compared with the result in the periodic system.

4.2 Monochromatically perturbed case (L=1)

Figure 11 shows the results of the time reversal experiments for various phase-change strength a in periodic system with monochromatic perturbation ($L=1, \epsilon=0.2$). As increase of the value a, it become difficult to return to the initial state. The results of the time reversal experiments at a fixed value $a=0.1$ in disordered system with the monochromatic perturbation ($L=1$) is shown in Fig.12. The return to the initial state is much more difficult than periodic case in spite of the small spread of the wave packet. Different from the periodic system, even for small perturbation strength ϵ the irreversibility can be built in the disordered system.

Next we quantitatively characterize the sensitivity of the dynamically delocalized states to the phase-change by the following ratio,

$$\eta(T,\epsilon,a) = \frac{m_2(t=2T)}{m_2(t=T)}. \tag{11}$$

Note that if the phase-change fully breaks the quntum coherency in the state η becomes about two. Figure 13 shows ϵ dependence of the η for some a's. As ϵ increases η increases up to the saturation level (~ 2). Generally η in the disordered system is larger than one in the periodic system. Moreover, in the disordered system the larger ϵ is the faster the speed to reach the saturation level becomes. Figure 14 shows a dependence of the η for some ϵ's. In the unperturbed cases η of the disordered system is larger than periodic system and increases with fluctuation. In the perturbed case of $\epsilon = 0.1$, η reaches the saturation level (~ 2) even for $a = 0$. The value is almost consistent with "critical value" ϵ_c found in Sect.3.

Totally we can say that disordered system with coherent perturbation has much potential for irreversibility, and the disordered system has an ability to be entangled with the other quantum state.

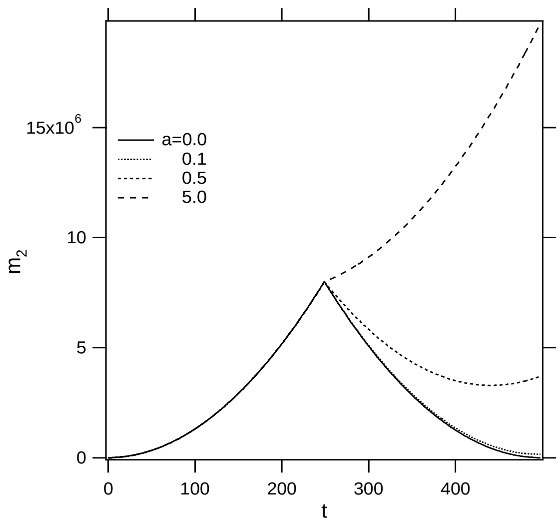

Figure 11: Time reversal experiments for various phase-change strength a in the periodic case with monochromatic perturbation ($L = 1, \epsilon = 0.2, T = 250$).

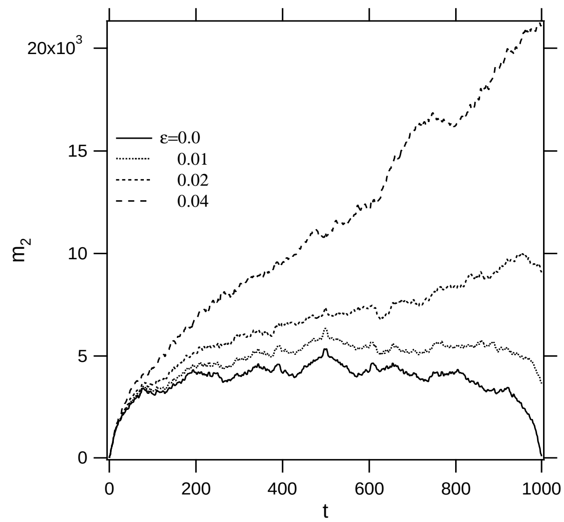

Figure 12: Time reversal experiments for various perturbation strength ϵ in the disordered case with monochromatic perturbation ($L = 1, a = 0.1, T = 500$).

5 Autonomous transformation and initial phase sensitivity

In this section, we consider the "classicalization" of the quantum wave packet in the monochromatically perturbed case ($L = 1$) by observing an effect of the initial phase change of the perturbation on the delocalized states. We reset $\omega_1 = \omega$ and $\phi_1 = \phi$ in the notation.

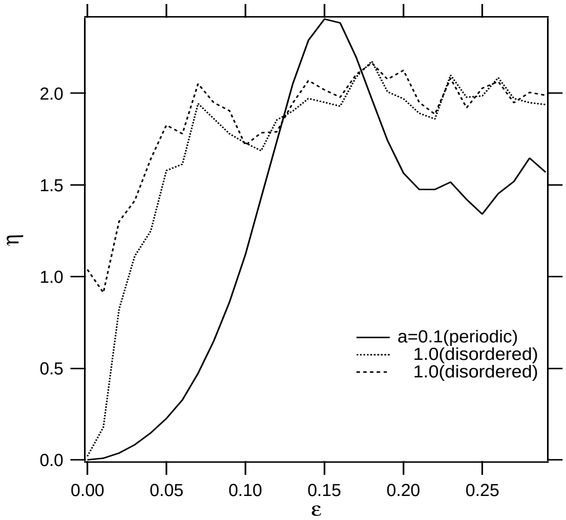

Figure 13: ϵ dependence of the η for some a's in the disordered and periodic systems ($L = 1, T = 500$).

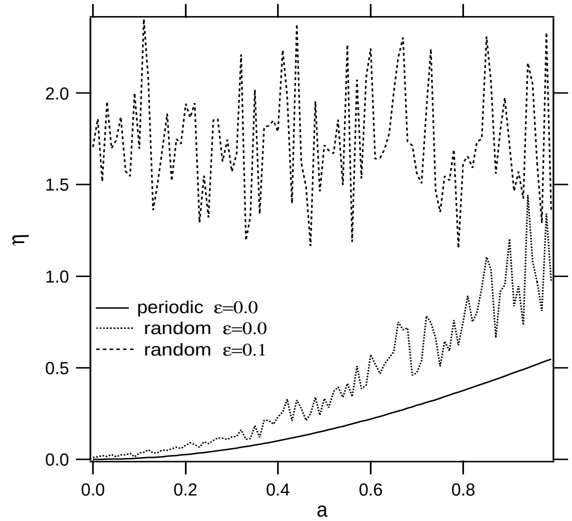

Figure 14: a dependence of the η for some ϵ's in the disordered and periodic systems ($L = 1, T = 500$).

5.1 Autonomous transformation

In the previous sections, we used the nonautonomous model. The nonautonomous model can be transformed into autonomous model. In this subsection we consider the delocalization though the autonomous expression for the time-dependent model. As the autonomous version of the $H^{osc}(t)$ with $L = 1$, we consider autonomous Hamiltonian consisting of three DOF as

$$H^{aut}_{L=1} = T(p) + 2\pi J + \omega I + V(n)\{1 + \epsilon \cos\phi\} \sum_k \cos(2\pi k \varphi), \tag{12}$$

where $J(\equiv -i\hbar\frac{\partial}{\partial\varphi})$ and $I(\equiv -i\hbar\frac{\partial}{\partial\phi})$ are action operators conjugate to angle variables φ and ϕ, respectively. The action-angle operators satisfy the commutation relations, $[\varphi, J] = [\phi, I] = i\hbar$. The action representation of the state $|\Psi(t) >$ is given by using the autonomous version of the time evolution as follows;

$$\begin{aligned} \Phi(n, m, t) &= \; < m| \exp\{-\frac{i}{\hbar} H^{aut} t\}|0 > \qquad (13) \\ &= \int_0^{2\pi} \frac{d\phi}{\sqrt{2\pi}} \exp\{i\omega m t\} < m|\phi >< \phi|T_+ \exp\{-\frac{i}{\hbar}\int_0^t ds H^{osc}(\omega s + \phi)\}|0 > \\ &= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\{im(\omega t + \phi)\}\Psi(n, \phi, t), \qquad (14) \end{aligned}$$

where $|0>$ is an initial state and T_+ is a time-ordering operator. We used the action eigenstate $|m>$ in the phase representation, $<\phi|m>=\frac{1}{\sqrt{2\pi}}\exp\{-im\phi\}$. See Ref.[7] for the more details. In our quantum dynamics the wave packet in action representation is given by Fourier transformation of $\Psi(n,\phi,t)$ with respect to the initial phase ϕ. Accordingly we can guess the classical dynamics given by the Hamiltonian H^{aut}, effect of the chaotic motion in a phase space makes a normal diffusion in action space of the linear oscillator (I,ϕ).

If there does not exist any correlation between the classical trajectories in the action space, the distribution function becomes Gaussian form in the action space [7]. We can expect that in the quantum dynamics once wave function is "classicalized " the diffusion of the wave packet in the action space continues with time.

5.2 Numerical result

We try to investigate the dissipative property of the dynamically delocalized state. Figure 15 shows the ϕ dependence of the real part of the wave function $\Psi^r(n^*,\phi,t)(\equiv Re\{\Psi(n^*,\phi,t)\})$ at the site $n^*=N/2$. It follows that the diffusion sensitively depends on the initial phase ϕ of the perturbation in the strongly perturbed case ($\epsilon=0.2$). Generally the structure of the phase dependence becomes complex as time elapses. Figure 16 shows the decay of the self-correlation function $C(\phi)\equiv<\Psi^r(t,\phi_0+\phi)\Psi^r(t,\phi_0)>_{\phi_0}$ at the initially localized site $n^*=n_0$. It decays rapidly and fluctuates around zero level in the case with strong perturbation ($\epsilon=0.2$). Even in the weakly perturbed case ($\epsilon=0.05$) the initial phase dependence becomes more complex as the strong perturbed case ($\epsilon=0.2$) as the time elapses. The larger time elapses the faster the correlation function $C(\phi)$ decays. It directly reflects to the phase sensitivity of the wave function. The same structure concerning the initial phase dependence exists in imaginary part and the probability distribution $|\Psi(n,\phi,t)|^2$ as shown in Fig.17. Such a structure can also be observed in the other site.(See Fig.17.) The growth of such a complex structure immediately corresponds to the diffusion property of the $\Phi(n,m,t)$ in action space through Fourier transformation. We can say the time-dependent perturbation bring about destructive of the interference of the quantum wave packet.

In the disordered system, when the packet spreads over the whole system the phase dependence becomes more complex. The number $N_n(t)$ of the nodes of the initial phase dependence increases obeying a rule, $N_n(t)\sim\sqrt{t}$ as time evolves. However, it should be noted that in the perturbed periodic system we can not observe the kinds of the growth of the complex structure in phase dependence as shown in Fig.18. The more details of the initial phase dependence will be given elsewhere [26].

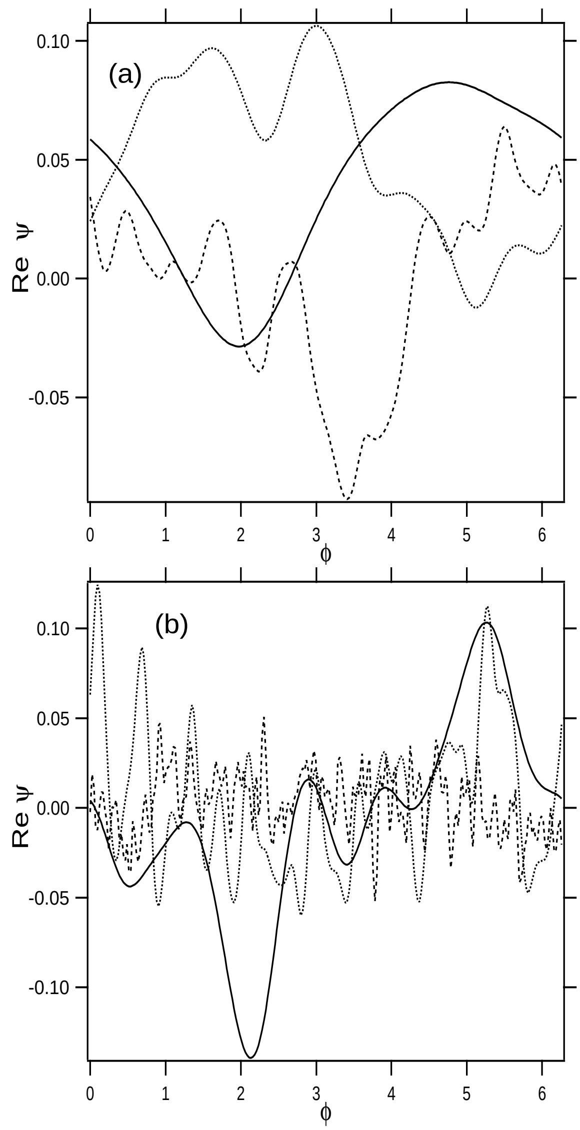

Figure 15: Some snapshots of the ϕ dependence of the real part of the wave packet $\Psi(n = n_0, t, \phi)$ in disordered system. We used one sample in this simulation. $(t = 4^3, 4^5, 4^7.)$ The parameters are (a)$L = 1$, $\epsilon = 0.05$ and (b)$L = 1$, $\epsilon = 0.2$.

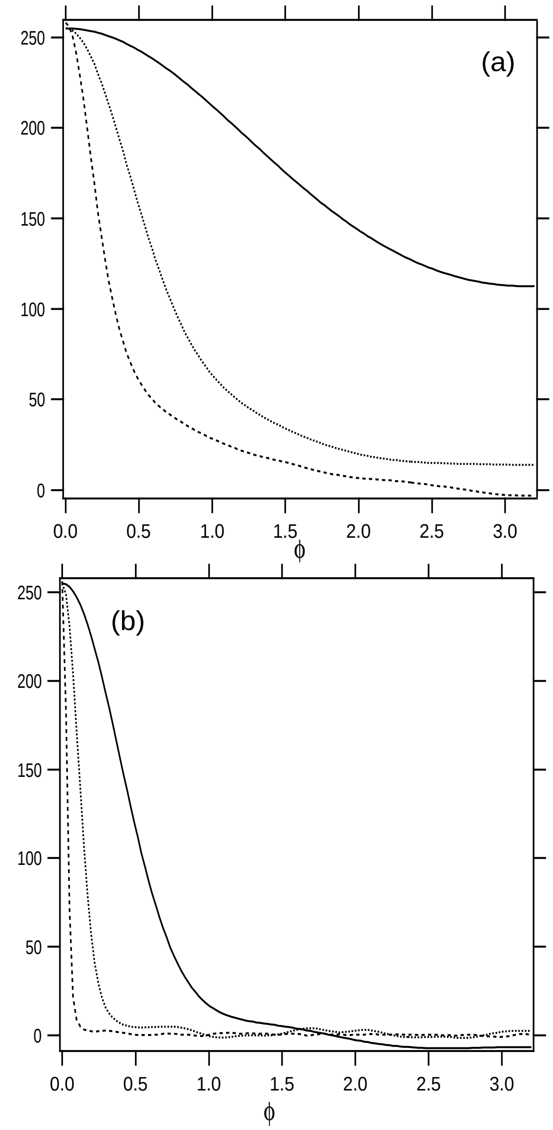

Figure 16: The correlation function $C(\phi)$ estimated by data in Fig.15. The parameters are (a)$L = 1$, $\epsilon = 0.05$ and (b)$L = 1$, $\epsilon = 0.4$. The ensemble average have done by shift of the initial phase ϕ_0.

5.3 Discussion

The quantum state we investigated in the perturbed kicked Anderson model is a pure state, i.e. $Tr\rho_{tot} = Tr\rho_{tot}^2 = 1$, where $\rho_{tot} = |\Psi(t) >< \Psi(t)|$ [27] . As a result, the quantum entropy

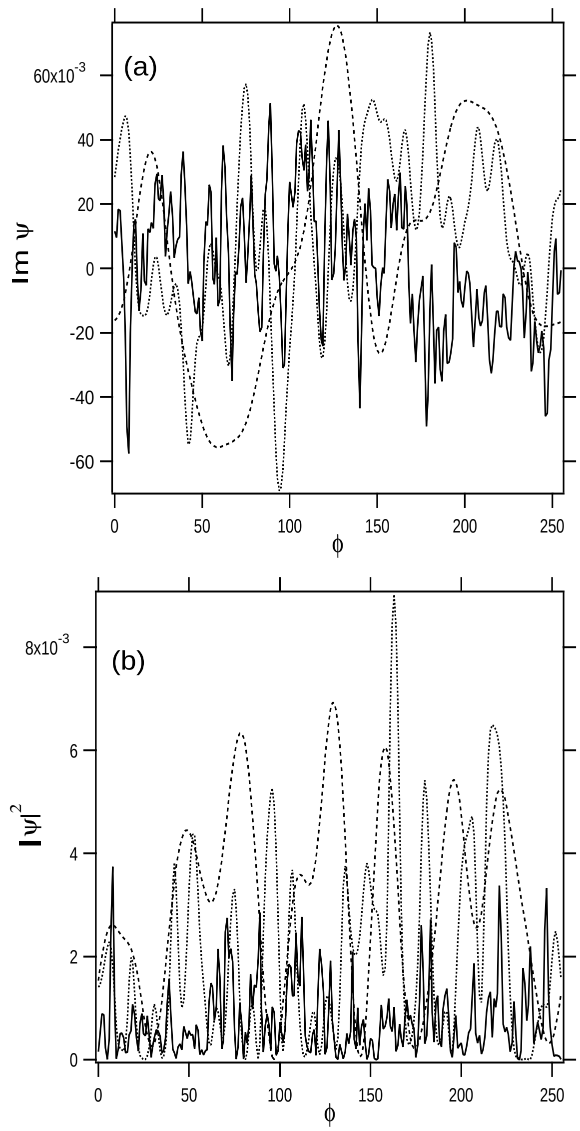

Figure 17: Some snapshots of the ϕ dependence of (a)the imaginary part of the wave packet $\Psi(n=n_0-5,t,\phi)$ and (b) the $|\Psi(n=n_0-5,t,\phi)|^2$ in the disordered system. We used one sample. The parameters are $L=1$, $\epsilon=0.2$.

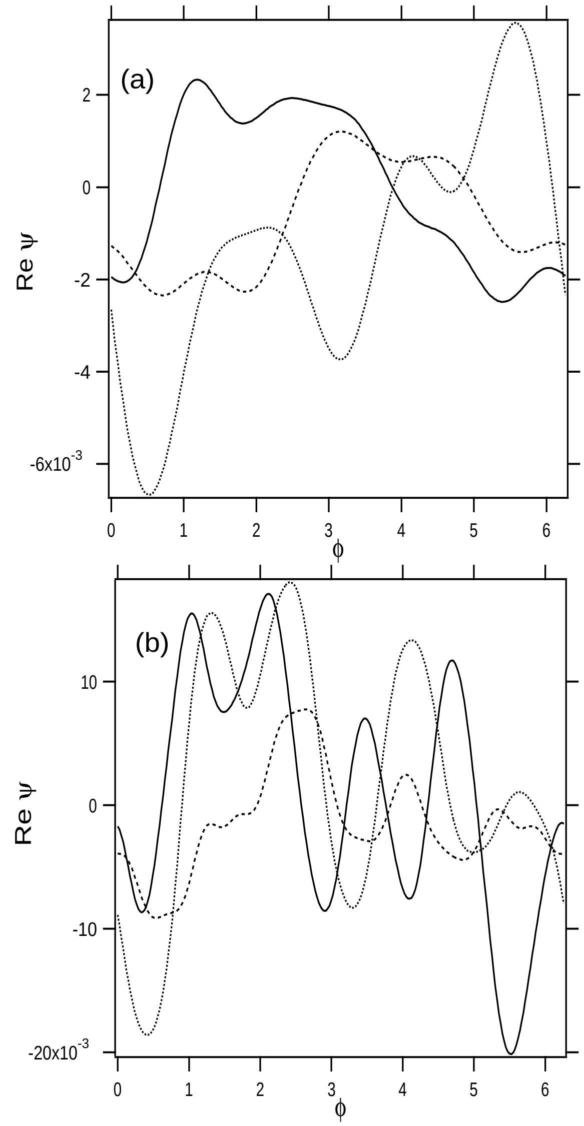

Figure 18: Some snapshots of the ϕ dependence of the real part of the wave packet $\Psi(n=n_0,t,\phi)$ in binary periodic system. ($t=4^3,4^4,4^5$.) The parameters are (a)$L=1$, $\epsilon=0.05$ and (b)$L=1$, $\epsilon=0.4$.

vanishes during the time-evolution, $S_{tot}=-Tr\rho_{tot}\log\rho_{tot}=0$ [28] . However, as seen in the last subsection, the chaotic behavior in the action space of the linear oscillator can be reflected on the kicked Anderson system. Then we can expect that the quantum state becomes an entangled state caused by the complex dynamical time-evolution. Accordingly, the reduced entropy of the partial trace $\rho_{red}(\equiv Tr_{osil}\rho_{tot}$ becomes positive, where Tr_{osil} means taking a trace concerning the linaer oscilator. The similar phenomena have been observed in some composite chaotic systems such as coupled kicked rotor [29].

6 Summary

We numerically investigated localization and delocalization of initially localized quantum wavepacket in kicked Anderson model with coherent time-dependent perturbation. The results we obtained in the present investigation are summarized as follows.

(1) In the kicked Anderson model($L = 0$), which a diffusion occurs in a early stage of time evolution is suppressed over a longer time scale, and MSD is eventually bounded by a certain finite level. The packet is exponentially localized.

(2) In the case of $L = 1$, diffusive behavior maintains within the time scale accessible by the numerical simulation. The diffusion process is anomalous diffusion for the small perturbation strength $\epsilon << 1$, in which the MSD increases as $m_2(t) \propto t^{\alpha}$ $(\alpha < 1)$. It seems that for larger than $\epsilon_c(= 0.1)$ the MSD grows obeying the normal diffusion rule as $m_2 \sim Dt$.

(3) In the case of $L \geq 2$, the diffusion process is normal diffusion $m_2(t) \sim Dt$ even for small ϵ comparatively.

(4) In the case of the subdiffusion, the two indexes (α, β) characterizing the ensemble averaged distribution function is estimated for several cases. The relation $\beta = 2/(2 - \alpha)$ between α and β is well fit in the diffusive side, while the data deviate from the relation in the localized side.

Moreover, we performed the time-reversal experiments after random phase-change of the wave packet in the unperturbed and the monochromatically perturbed kicked Anderson model.

(5) In the both unperturbed disordered and periodic system, the quantum state is robust against the random phase-change of the wave packet when the strength is small. As increase of the strength the state does not take the initial state by the backward process.

(6) In the case of $L = 1$, the dynamically delocalized state becomes sensitive to the random phase-change as increase of the perturbation strength ϵ. In this case, the difference between disordered and periodic system is much enhanced when compared to the unperturbed cases.

We also investigated the quantum diffusion in the monochromatically perturbed case ($L = 1$) by observing an effect of the initial phase change of the perturbation on the delocalized states.

(7) In the disordered system, the dynamically delocalized states are very sensitive to change of the initial phase of the perturbation, which is quite different from the case of the Bloch state in the periodic system. From autonomous picture, it follows that the fine structure of the initial

phase sensitivity related to diffusion of the wave packet in the action space.

We can regard the delocalization as a kind of "classicalization" by coupling with external small number DOF. The number of DOF of the total system is three. We can expect that once wave function is classicalized the diffusion of the wave packet in action space continues in Gaussian form. The more details about relation between the initial phase sensitivity and the delocalized state will be given in elsewhere [26].

Acknowledgments
This report is based on a collaboration with Professor K. S. Ikeda. The author would like to thank him for valuable suggestions and stimulating discussions about this study. This work is partially supported by a Grant-in-Aid for Scientific Research "Control of Molecules in Intense Laser Light Fields" provided by Ministry of Education, Science and Culture, Japan.

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[19] It should be noticed that appearance of the Gaussian distribution does not always mean classicalization of the quantum coherence. If we put a Gaussian wave packet in a periodic system, it spreads with keeping the Gaussian form. For example, in the binary periodic system, functional form of the ensemble averaged distribution function becomes almost Gaussian form as we can expect, however, the MSD grows in time as $m_2 \sim t^2$ with keeping the quantum coherence.

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