

Article

An Entropic Approach for Pair Trading in PSX

Laiba Amer and Tanweer Ul Islam * 

Department of Economics, National University of Sciences & Technology, Islamabad 44000, Pakistan

* Correspondence: tanweer@s3h.nust.edu.pk

Abstract: The perception in pair trading is to recognize that when two stocks move together, their prices will converge to a mean value in the future. However, finding the mean-reverted point at which the value of the pair will converge as well as the optimal boundaries of the trade is not easy, as uncertainty and model misspecifications may lead to losses. To cater to these problems, this study employed a novel entropic approach that utilizes entropy as a penalty function for the misspecification of the model. The use of entropy as a measure of risk in pair trading is a nascent idea, and this study utilized daily data for 64 companies listed on the PSX for the years 2017, 2018, and 2019 to compute their returns based on the entropic approach. The returns to these stocks were then evaluated and compared with the buy and hold strategy. The results show positive and significant returns from pair trading using an entropic approach. The entropic approach seems to have an edge to buy and hold, distance-based, and machine learning approaches in the context of the Pakistani market.

Keywords: pair trading; model uncertainty; model risk; optimal boundary; PSX

1. Introduction

According to quantitative models, pair trading involves a driving mechanism for mean reversion using a statistical arbitrage strategy. The perception is to recognize that when two stocks move together, their prices will converge to a mean value in the future [1]. Mean reversion allows traders to make a profit by matching a long position in one stock with an offsetting position in another stock [2]. Pair trading is an efficient method for the formation of portfolios or pair trading [3,4]; however, finding the accurate pairs and boundary points is not an easy task.

The profitability of pair trading decreased due to an increasing share of non-converging pairs [5]. To resolve the issue of non-converging pairs, several researchers contributed to the literature [6–9] and proposed cointegration as the most efficient solution for structuring pair trading [10].

After settling on how to find accurate pairs, the problem arose of how to find the mean-reverted point between them and how to identify the boundaries for when exactly the investors can buy or sell any asset. Yoshikawa [11] derived the entropy-based optimal boundary points for pair trading using Tokyo Stock Exchange 2015 data. The proposed approach for the optimal stopping problem is motivated by the work of Ekström et al. [12] and Suzuki [13]. This method is based on maximizing profit via pair trading and minimizing the relative entropy (risk). This is a robust method, as it directly tackles model misspecification [14] and provides a more persuasive solution. The choice of pairs is made through cointegration, the most effective way to identify stocks that move together [15]. Entropy has a wide application in finance as well [16–18].

In the context of Pakistan, there was a handful of studies conducted on pair trading [19,20], and interestingly, no one has yet considered the optimal stopping problem using stocks listed on the Pakistan Stock Exchange (PSX). This study employs the novel entropic approach proposed by Yoshikawa [11] to explore the optimal boundary points that yield the maximum profit for 64 companies listed on PSX for the period 2017–2019.



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The concept of maximizing the profit in pair trading based on relative entropy is a nascent idea in the literature, and this study is the first attempt in the context of Pakistan. The performance of this entropic approach is compared with the buy and hold strategy in terms of returns.

2. Data & Methodology

As mentioned in the last section, this study utilized the daily data for 64 companies listed on PSX for the years 2017, 2018, and 2019. These companies cover the major sectors, including cement, chemical, automobile assembler, food and personal care products, oil and gas marketing companies, oil and gas exploration companies, power generation and distribution, refinery, and pharmaceuticals. The firms' selection criterion was based on year-wise price earnings ratios (PER); a firm with a PER lower than the sample median value was selected in the sample. The underlying idea is that the stock below the median PER is undervalued and signifies potential for higher returns [21,22]. The choice of pairs was made through Johansen cointegration, which is the most effective way to identify stocks that move together [15]. In each year, we formulated all pairs $((n^2 - n)/2)$ of the selected stocks and assessed each pair for cointegration.

Keeping in view the potential jumps/structural breaks in high-frequency financial data [23], the following breakpoint unit root test proposed by Bai and Perron [24] was employed.

$$\Delta y_t = \alpha_0 + \alpha_1 t + \delta y_{t-1} + \sum_{i=1}^p \beta_i \Delta y_{t-i} + \mu_t \quad (1)$$

where μ_t is white noise.

3. Ornstein–Uhlenbeck (OU) Process

Pair trading utilizes the mean reversion of the composite process of two stocks. Following Yoshikawa [11], we considered the Ornstein–Uhlenbeck (OU) process X_t such that

$$dX_t = -\mu(X_t - \alpha)dt + \sigma dB_t, \quad X_0 = \alpha \quad (2)$$

where μ and σ are the positive constants, α is the mean-reversion point, and B_t is the p -Brownian motion. Let $X_t - \alpha = \check{X}_t$. Then, Equation (2) implies:

$$d\check{X}_t = -\mu\check{X}_t dt + \sigma dB_t, \quad \check{X}_0 = 0 \quad (3)$$

The optimal stopping problem at time t for the process \check{X}_t is defined as follows:

$$v^0(t, x) = \sup_{\tau \in \mathfrak{S}} E_x^S [e^{-\rho(\tau-t)} \check{X}_\tau] \quad (4)$$

where \mathfrak{S} is the set of all stopping points of B , and ρ is the discount rate. The solution of Equation (4) gives us the trading strategy: we short pair X when it attains the highest value and liquidate it when X attains zero value. These values are specified by the above equation. Alternatively, we take the long position for X for zero value and liquidate it for the highest value. The superscript S in Equation (4) is the solution to the following:

$$\inf_{s \in \mathfrak{S}} \left\{ E_x^S [e^{-\rho(\tau-t)} \check{X}_\tau] + \lambda e^{-\rho(\tau-t)} H_{\check{X}} [S|P] \right\} \quad (5)$$

where λ is a positive constant and $H(\cdot)$ is a relative entropy defined as follows:

$$H_{\check{X}} = \begin{cases} E_x^S [\ln(\frac{dS}{dP})], & S \in \mathfrak{S} \\ \infty, & \text{otherwise} \end{cases} \quad (6)$$

Thus, the optimal boundary $b(t)$ for Equation (4) is given as:

$$\ln(b(t)) + \frac{1}{\sigma^2} \frac{\rho}{\rho - \mu} (g(t) - b(t))^2 = \ln(b^*) + \frac{1}{\sigma^2} \frac{\rho}{\rho - \mu} (b^*)^2 \quad (7)$$

where $g(t) = -\frac{\sigma^2}{\lambda} t e^{-\mu t}$ & $b(0) = b^*$. Any investor holding pair X should liquidate when X touches $b(t)$ and, if not holding X , should short their position when X touches $b(t)$ and liquidate it when it reverts to mean zero.

4. Results and Discussion

From the eight selected sectors, we found 64 active firms listed on PSX for the years 2017, 2018, and 2019. After applying the PER benchmark, we got 33, 34, and 40 companies, respectively. Having selected the companies, the unit root test was applied to the time series data of these stocks to find the order of integration. All the time series are integrated of order one. This led us to find the cointegrated pairs using the Johansen cointegration test at 0.05 level of significance. We found $79 = (28 + 29 + 23 = 80 - 1)$ unique cointegrated pairs (one pair was repeated) out of $1869 = (528 + 561 + 780)$ pairs of the selected stocks in the 3-year period.

Having found the pairs, we applied the maximum likelihood method to find the parameters of the Ornstein–Uhlenbeck processes, μ , α , and σ , as given in Equation (2). MATLAB R2021b was used for the coding and estimation of these parameters. However, to compute the optimal boundary points, we needed to find the parameters ρ and λ as well. The parameter ρ is the discount rate, and the parameter λ represents the level of confidence. The lower the value of λ , the lower the confidence of the agent on the reference measure as a true probability measure among the class of all probability measures and vice versa. We used $\rho = 0.08978, 0.1315$, and 0.1440 as per the annual report of State Bank of Pakistan for 2017, 2018, and 2019, respectively, and by following Yoshikawa [11], four cases for the parameter, $\lambda = 0.001, 0.01, 0.1$, and ∞ were considered. Tables 1 and 2 present the results for only five pairs of stocks in each year involving the top listed companies (see Appendix A, Tables A1–A7 for the results of other companies). After computing the values of μ , α , and σ as furnished in Table 1, we estimated the rate of returns for different values of λ for the selected companies (Table 2). On balance, pair trading yielded optimal returns for lower values of λ s, which is understandable, as the parameter λ is linked with the penalty function. All the estimated parameter values are presented in Figures 1–3 and in Table 2 for their respective years. From these figures, it is evident that the values of the mean reversion parameter differ when the stocks in the pair are selected within the sector in comparison to when the stocks are selected across the sectors.

For the real data sets, the pair trading strategy was to set the position when the pair value touches either the mean reverted point or the boundary. For example, in Figure 1 (pair: PSO and MPLF), the mean reversion point was 60.29 where we set the position, and we liquidated the position when the pair value touched the boundary $b(t)$. If the position was set when the pair value touches the boundary, then it was liquidated when it touched the mean reversion point α . In Figure 2 (pair: PSO and BYCO), if we set our position when the pair value touched the boundary then we would liquidate at the mean reversion point, $\alpha = 9.26$. The next position was set when the pair value touched either the boundary $b(t)$ or the mean reversion point α and liquidated following the same rule.

Following this trading strategy, we estimated the rate of returns for the 80 unique pairs of the companies for the years 2017, 2018, and 2019. Gatev et al. [6] highlighted the transaction fee as an obstacle in trading. Because the transaction cost in the Pakistan Stock Exchange is 0.15 percent and we are dealing with pair trading, we discounted our return values by 0.3 percent. Table 2 provides these return values for five pairs from each year. The return values ranged from 0.2 to 25.2 percent for the year 2017, 0.4 to 19.5 percent for the year 2018, and 1.5 to 15.7 percent for the year 2019. All positive returns confirm profitable trades, which is line with the findings in the literature [1,11]. For all cointegrated pairs (Appendix A), average return values ranged from 2.9% to 18.5% which are much

higher than the return values estimated in [25], which ranged from 0.1% to 1.71% using the distance-based approach for the stocks listed on the PSX during the period 2009–2016. Shaukat et al. (2021) employed the distance-based method to select the pairs and compute returns to pair trading for financial (banks) and non-financial (cement industries) sectors with a formation period of 12 months. Cement industries yielded higher returns, whereas the banks yielded lower returns. Sohail et al. [20] estimated the return on pair trading using 80 stocks from five different sectors: banking, chemicals, cement, textile, and food and care products, all of which were listed on the PSX from 2011 to 2019. Trading periods of two and one year were used for the machine learning algorithm (clustering algorithm) and distance-based methods, respectively. The study found a maximum return of 2.07 percent for the textile sector using the distance-based approach, whereas the clustering algorithm yielded a maximum return of 2.55 percent.

The distance-based approach relies on the average squared differences between the normalized prices of stocks, and principal component analysis (PCA) is used to generate the indices of the stocks that represent the weighted average prices of the stocks to be used in the machine algorithm. By construct, PCA indices resemble those generated with the cointegration technique; we found parameters α and β such that the linear combination of the two stock prices, $\alpha p_1 + \beta p_2$, yielded a stationary process, whereas the weights in PCA may not yield stationary indices. Further, both the studies [20,25] did not allow cross-sector pairing that might have caused their low returns in comparison with our study. The profitability of pair trading decreases due to non-convergence of the pairs [5], and cointegration is the most efficient method to explore converging pairs [10]. Thus, the entropic approach seems to have an edge over the distance-based and machine learning approaches in the context of the Pakistani market.

Table 1. Ornstein–Uhlenbeck Process parameter estimation.

Sr	Pair Name	μ	α	σ
2017				
1	Pak State Oil (PSO) and Maple Leaf Cement LTD (MPLF)	0.04	60.29	84.91
2	Thata Cement (THAT) and Gharibwal Cement (GHAR)	0.02	101.3	39.25
3	Pak Oil Fields (PKOL) and Ittehad Chemicals LTD (ITHD)	3.74	1361.7	474.9
4	Pioneer Cement (PION) and Lalipir Power LTD (LPLP)	10.52	14.54	7.70
5	Engro Polymer and Chemical (EPCL) and Lalipir Power (LPLP)	2.29	15.71	8.47
2018				
6	Engro Power Generation Qadirpur LTD and Thata Cement LTD	6.59	69.44	16.43
7	Gharibwal Cement LTD and Dewan Cement LTD	0.04	79.3	31.87
8	Pakistan State Oil Company LTD and Best Way Cement LTD	0.02	90.7	50.78
9	Pakistan State Oil Company LTD and Byco Petroleum Pak LTD	5.37	9.26	5.25
10	Pioneer Cement LTD and Dewan Cement LTD	0.13	40.14	19.14
2019				
11	Nishat Chunnian Power LTD and Engro Polymer and Chemical LTD	0.01	91.05	35.11
12	Nishat Chunnian Power LTD and Maple Leaf Cement Factory	0.004	94.76	39.29
13	Thata Cement LTD and Pakistan State Oil Company LTD	1.85	23.69	10.38
14	Thata Cement LTD and Pakistan Oilfields LTD	0.01	42.82	14.44
15	Pioneer Cement LTD and Al Shaheer Corporation LTD	0.06	29.87	13.36

Further, to evaluate our results, we contrasted our results against the buy and hold strategy with trading periods of one quarter, annually, 2 years, and 3 years (Table 3). A trading period of one year is in line with the literature [20,25]. The rate of returns for the alternative strategy is summarized in Table 3. In general, except for 2019–Q4, the top-performing stocks made a loss for this strategy, whereas Table 2 shows pair trading provided stable profits. The buy and hold strategy has a considerable risk of human error considering the pressure of all the wrong choices one can make [26]. The optimization of

the boundaries backed by the Ornstein–Uhlenbeck process allowed us to incorporate all risks, improve the profitability of pair trading, and receive maximum positive returns [27]. Therefore, we suggest the pair trading strategy while taking model uncertainty into account.

Table 2. Rate of returns for different values of λ .

Pair Name	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
2017				
PSO and MPLF	0.042 (62.8)	0.037 (62.5)	0.051 (63.4)	0.087 (65.6)
THAT and GHAR	0.132 (114.7)	0.12 (113.5)	0.122 (113.7)	0.065 (108.0)
PKOL and ITHD	0.252 (1704.5)	0.246 (1697.3)	0.206 (1642.5)	0.023 (1393.5)
PION and LPLP	0.186 (17.2)	0.18 (17.2)	0.114 (16.2)	0.013 (14.7)
EPCL and LPLP	0.002 (15.7)	0.006 (15.8)	0.208 (19.0)	0.08 (17.0)
2018				
ENGP and THAT	0.055 (73.3)	0.054 (73.2)	0.04 (72.2)	0.004 (70.0)
GHAR and DECE	0.05 (83.3)	0.038 (82.2)	0.064 (84.5)	0.006 (80.0)
PSO and BEST	0.177 (106.8)	0.192 (108.2)	0.175 (106.6)	0.103 (92.6)
PSO and BYCO	0.187 (11.0)	0.177 (11.0)	0.14 (10.6)	0.022 (9.5)
PION and DECE	0.187 (47.6)	0.195 (47.9)	0.178 (47.3)	0.131 (45.4)
2019				
NCPL and ENGRO	0.094 (99.6)	0.087 (99.0)	0.091 (99.3)	0.041 (95.0)
NCPL and MPLF	0.131 (107.2)	0.157 (109.7)	0.135 (107.5)	0.089 (103.3)
THAT and PSO	0.131 (26.8)	0.132 (26.8)	0.121 (26.6)	0.039 (24.6)
THAT and PKOIL	0.024 (43.9)	0.015 (43.4)	0.03 (44.1)	0.111 (47.8)
PION and ALSHAHEER	0.034 (30.9)	0.049 (31.3)	0.038 (31.0)	0.089 (32.5)

Estimated stock pair values for the given confidence levels of the agent are in parentheses.

Table 3. Rate of returns from the buy and hold strategy.

Returns with Trading Period												
Company Name	2017				2018				2019			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Pak State Oil	−4.7	−7.6	19.9	−20.4	8.8	−3.5	0.5	−15.1	−8.4	−20.3	−7.5	19.4
Thata Cement LTD	11.7	−10.5	−32.7	−14.1	13.7	−15.7	−14.3	−24.4	−8.7	−24.1	−24.2	56.1
Pioneer Cement LTD	0.8	−7.8	−27.4	−28.4	10.2	−35.6	−4.1	−5.2	−21.0	−33.8	−15.3	51.3
Nishat Chunnian Power	−14.8	−5.3	−10.1	−20.0	−6.9	−9.2	−8.6	−1.1	−4.7	−16.1	4.7	11.3
Gharibwal Cement LTD	17.3	−24.3	−23.3	−26.8	6.3	−22.8	−8.3	−20.0	−17.6	−21.5	−17.2	53.1
1-Year				2-Year				3-Year				
	2017		2018	2019	2017–2018		2018–2019	2017–2019				
Pak State Oil	−20.9		−8.6	−2.4	−27.0		−6.5	−25.4				
Thata Cement LTD	−44.9		−37.8	−17.8	−66.3		90.3	−71.6				
Pioneer Cement LTD	−55.6		−33.7	−30.8	−70.4		−51.9	−78.5				
Nishat Chunnian	−43.2		−28.5	−19.0	−58.3		−42.1	−66.2				
Gharibwal Cement LTD	−53.5		−36.2	−14.0	−52.0		−44.9	−73.6				

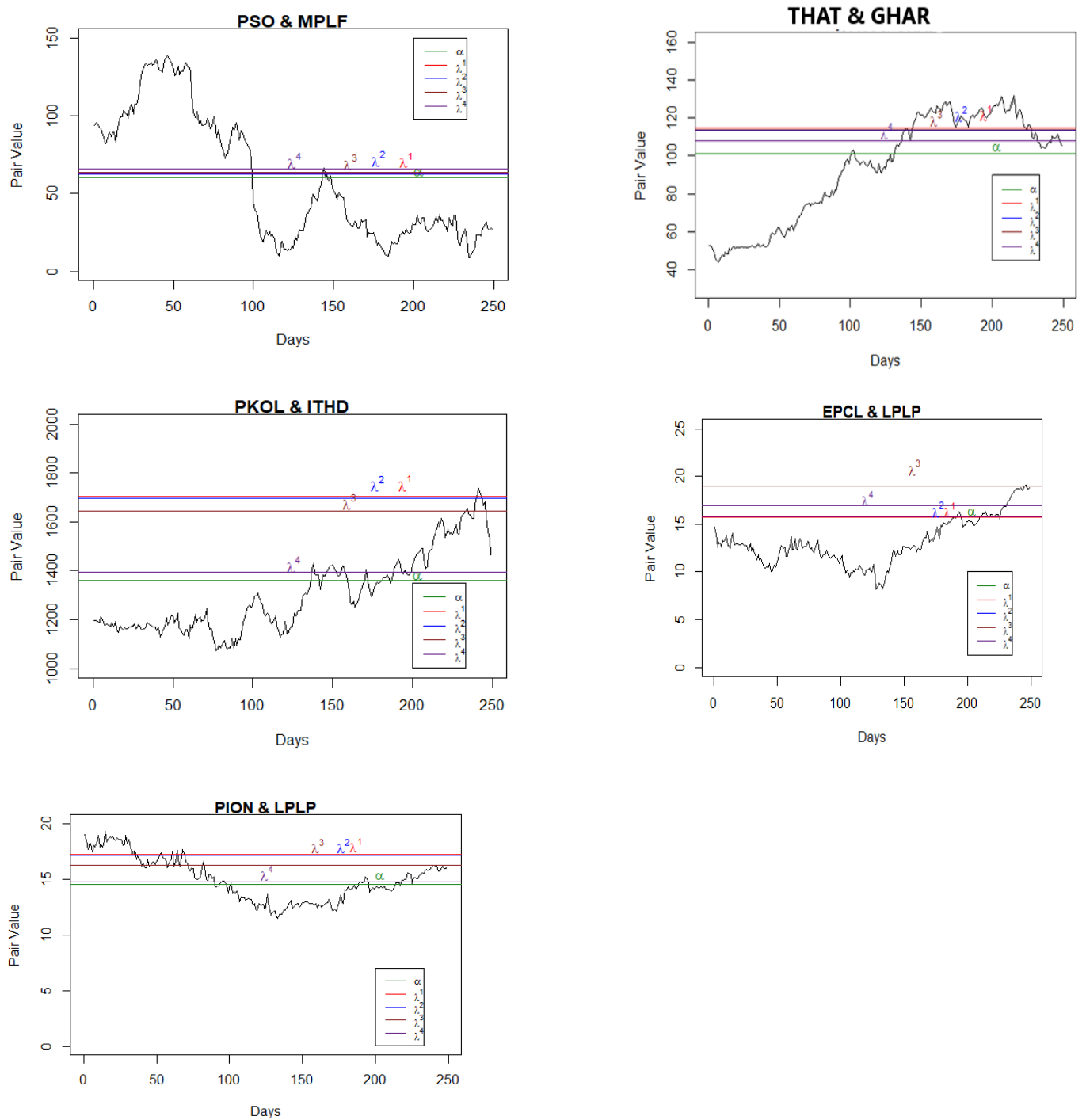


Figure 1. Pair values, boundaries, and mean values for the pairs (2017). Note: λ^1 , λ^2 , λ^3 , & λ^4 are the estimated paired stock values for the given confidence levels of the agent (see Table 2).

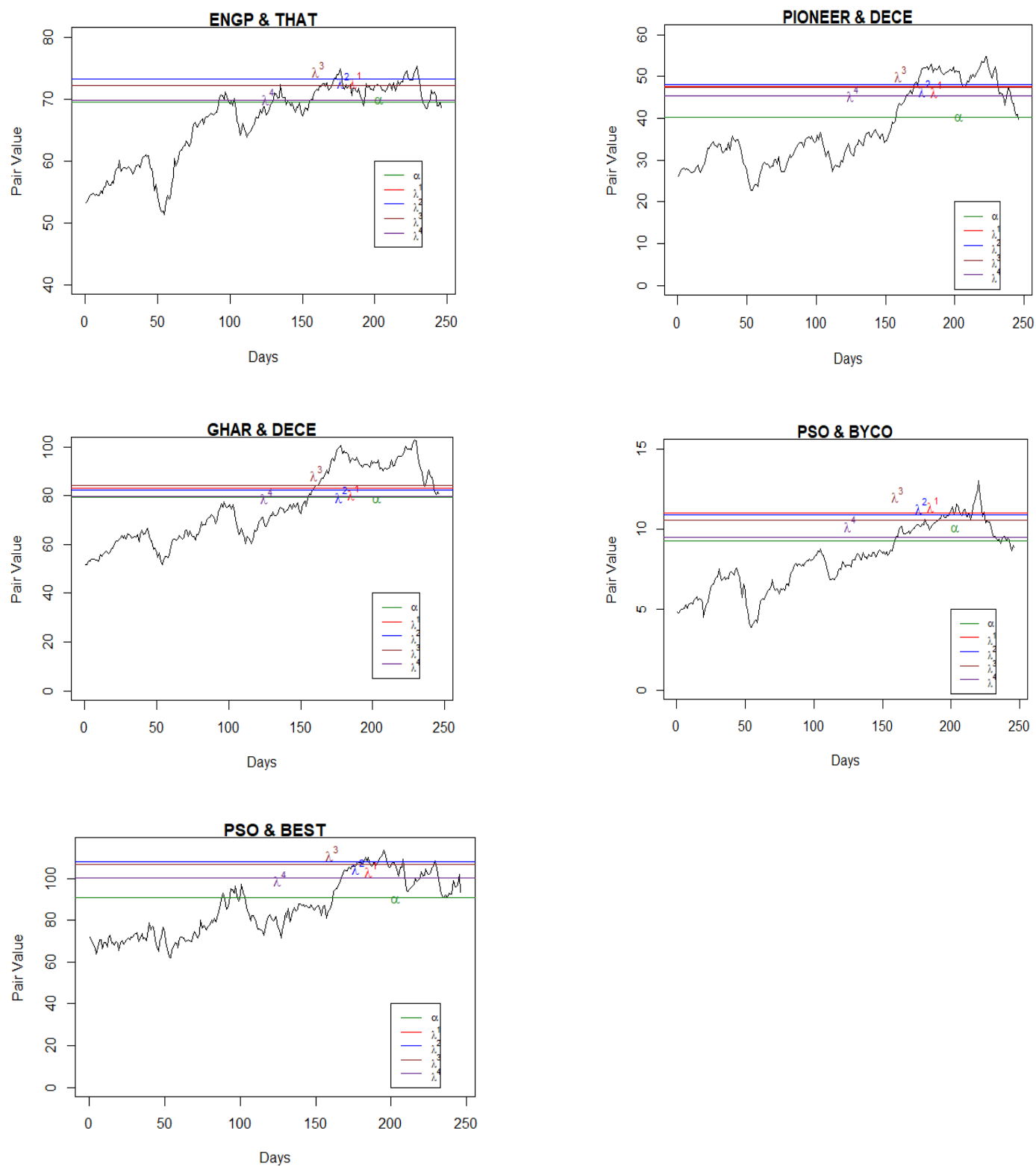


Figure 2. Pair values, boundaries, and mean values for the pairs (2018).

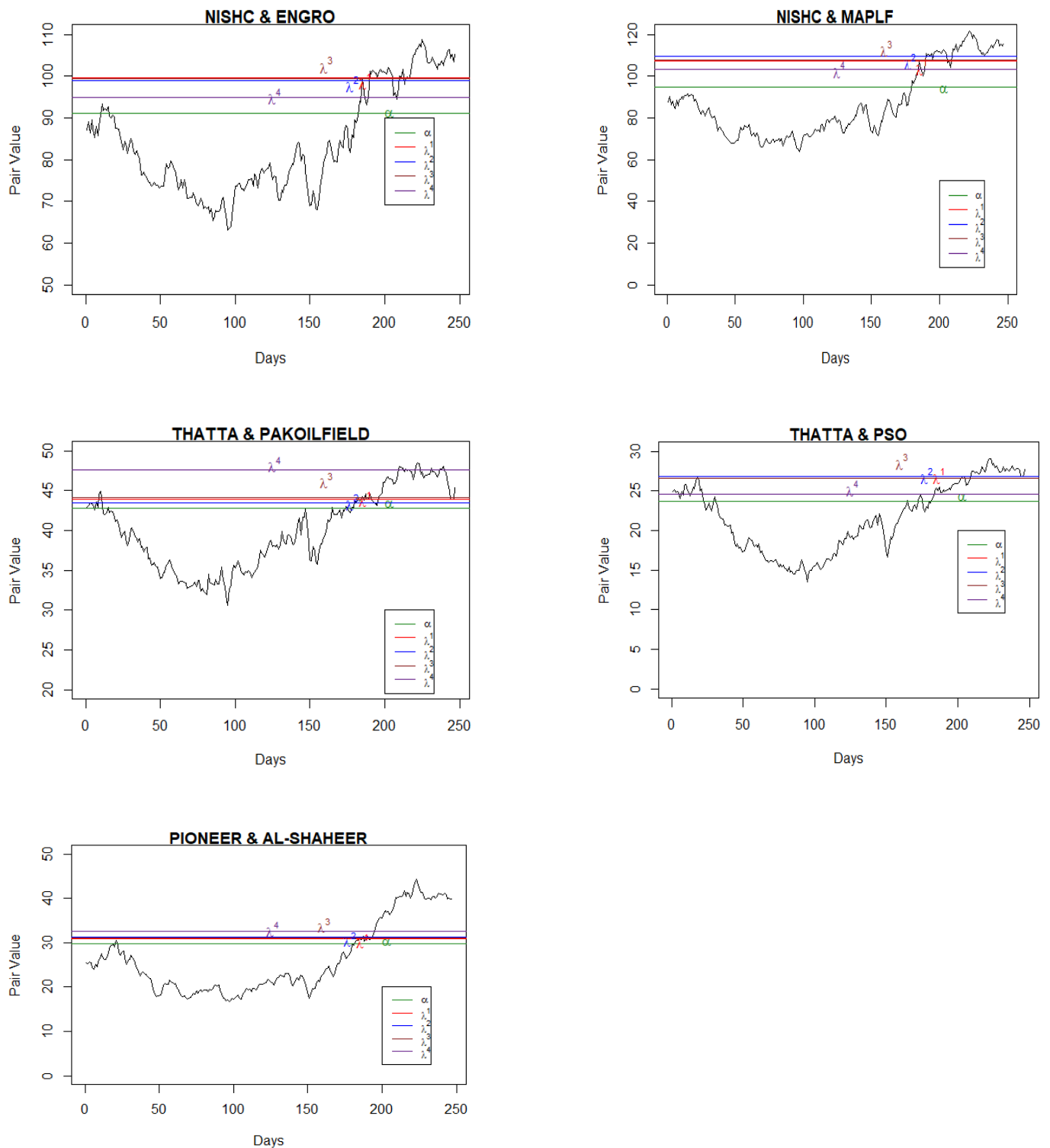


Figure 3. Pair values, boundaries, and mean values for the pairs (2019).

5. Conclusions

This study employed a novel entropic approach to explore the optimal boundary points that yield maximum profit for 64 companies listed on the Pakistan Stock Exchange (PSX) for the period 2017–2019. The concept of maximizing the profit in pair trading based on relative entropy is a nascent idea in literature, and this study is the first attempt to implement it in the context of Pakistan. The performance of this entropic approach is

contrasted with the buy and hold strategy in terms of returns. The following are the key findings of the study.

1. The values of the mean reversion parameter differ when the stocks in the pair are selected within the sector in comparison to when the stocks are selected across the sectors.
2. On balance, optimal returns are associated with lower values of λ s; approximately, 84 percent pairs yielded optimal returns for low values of λ ($\lambda = 0.001$ and 0.01).
3. The return values based on entropic pair trading approached ranges from 0.2 to 25.2 percent for the year 2017, 0.4 to 19.5 percent for the year 2018, and 1.5 to 15.7 percent for the year 2019. These values are much higher than the returns estimated in [20,25].
4. Based on the buy and hold strategy, all the top performing stocks make a loss.
5. The entropic approach seems to have an edge over the buy and hold, distance-based, and machine learning approaches in the context of the Pakistani market.

Pair trading is an efficient method that allows maximization of profitability by eliminating short-term price deviations in favor of long-term historical pricing relationships. The entropy-based pair trading method yielded positive returns for all the cointegrated pairs tested and confirmed their profits, which is line with the findings in literature [1,11]. According to the efficient market hypothesis (EMH), an active investor cannot be more effective than the one who buys and holds. Therefore, the returns estimated from the entropic approach were contrasted against the returns estimated through the buy and hold strategy. The buy and hold strategy yielded negative returns, except for a few cases implying losses. Consequently, we suggest the pair trading strategy while taking model uncertainty into account.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. 2017 Ornstein–Uhlenbeck process parameters.

Pair Name	μ	α	σ
Fauji Food (FAUJ) and Pak State Oil (PSO)	3.7	32.48	15.19
Fauji Food (FAUJ) and Gharibwal Cement (GHAR)	3.84	33.94	15.34
Fauji Food (FAUJ) and National Refinery (NATR)	3.11	24.63	14.63
Fauji Food (FAUJ) and Engro Power Qadirpur LTD (ENGP)	3.5	4.31	13.72
Fauji Food (FAUJ) and Bestway Cement (BEST)	3.29	41.38	15.74
Fauji Food (FAUJ) and Dewan Cement LTD(DECE)	3.8	36.58	15.89
Fauji Food (FAUJ) and Ghani Automobile Industries LTD (GAIL)	3.21	21.26	14.4
Fauji Food (FAUJ) and Ittehad Chemicals LTD (ITHD)	10.9	6.27	16.69
Fauji Food (FAUJ) and Ghandhara Industries LTD (GHAN)	4.06	32.23	15.1
Fauji Food (FAUJ) and Power Cement LTD (POWE)	3.99	30.8	14.84
Fauji Food (FAUJ) and Pakistan Petroleum LTD (PPL)	0.056	61.45	18.81
Fauji Food (FAUJ) and Lalpir Power LTD (LPLP)	3.6	56.36	18.76
Pak State Oil (PSO) and Maple Leaf Cement LTD (MPLF)	0.04	60.29	84.91
Thata Cement (THAT) and Gharibwal Cement (GHAR)	0.02	101.34	39.25
Pak Oil Fields (PKOL) and Ittehad Chemicals LTD (ITHD)	3.74	1361.72	474.89
Gharibwal Cement and Ghandhara Industries LTD(GHAN)	0.19	35.94	21.6
Gharibwal Cement and Power Cement LTD (POWE)	2.99	34.93	13.56

Table A1. Cont.

Pair Name	μ	α	σ
National Refinery (NATR) and Dewan Cement LTD (DECE)	3.22	72.22	24.43
National Refinery (NATR) and Ittehad Chemicals LTD (ITHD)	0.05	42.95	16.08
National Refinery (NATR) and Lalipir Power LTD (LPLP)	9.37	9.4	8.3
Pioneer Cement (PION) and Lalipir Power LTD (LPLP)	10.52	14.54	7.7
Dewan Cement LTD (DECE) and Ittehad Chemicals LTD (ITHD)	0.88	152.64	34.55
Dewan Cement LTD (DECE) and Power Cement LTD (POWE)	0	13.77	12.48
Ittehad Chemicals LTD (ITHD) and Ghandhara Industries (GHAN)	0.04	43.54	16.8
Ittehad Chemicals LTD (ITHD) and Power Cement LTD (POWE)	2.82	39.49	19.11
Ittehad Chemicals LTD (ITHD) and Pakistan Petroleum (PPL)	0.02	313.08	95.34
Ittehad Chemicals LTD (ITHD) and Lalipir Power LTD (LPLP)	0.01	55.73	18.22
Engro Polymer and Chemical (EPCL) and Lalipir Power (LPLP)	2.29	15.71	8.47

Table A2. 2017 rates of return at different λ .

Pair Name	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
FAUJ and PSO	0.060	0.059	0.048	0.007
FAUJ and GHAR	0.061	0.059	0.05	0.006
FAUJ and NATR	0.080	0.075	0.068	0.010
FAUJ and ENGP	1.522	1.525	1.294	0.358
FAUJ and BEST	0.001	0.001	0.000	0.010
FAUJ and DECE	0.055	0.055	0.043	0.004
FAUJ and GAIL	0.120	0.125	0.101	0.022
FAUJ and ITHD	1.046	0.910	0.630	0.075
FAUJ and GHAN	0.085	0.078	0.064	0.010
FAUJ and POWE	0.083	0.082	0.066	0.011
FAUJ and PPL	0.059	0.060	0.060	0.030
FAUJ and LPLP	0.051	0.047	0.041	0.006
PSO and MPLF	0.042	0.037	0.051	0.087
THAT and GHAR	0.132	0.12	0.122	0.065
PKOL and ITHD	0.252	0.246	0.206	0.023
GHAR and GHAN	0.044	0.045	0.031	0.048
GHAR and POWE	0.041	0.036	0.031	0.003
NATR and DECE	0.029	0.027	0.023	1.650
NATR and ITHD	0.216	0.218	0.201	0.172
NATR and LPLP	0.327	0.317	0.214	0.028
PION and LPLP	0.186	0.18	0.114	0.013
DECE and ITHD	0.041	0.04	0.043	0.011
DECE and POWE	0.077	0.086	0.081	0.020
ITHD and GHAN	0.216	0.234	0.232	0.191
ITHD and POWE	0.220	0.216	0.191	0.060
ITHD and PPL	0.051	0.044	0.034	0.032
ITHD and LPLP	0.079	0.095	0.077	0.065
EPCL and LPLP	0.002	0.006	0.208	0.080

Table A3. 2018 Ornstein–Uhlenbeck process parameters.

Pair Name	μ	α	σ
Nishat Chunnian Power LTD (NCPL) and Nishat Power LTD (NISH)	0.060	62.870	19.110
Nishat Chunnian Power LTD and Lotte Chemicals Pak LTD	14.340	38.550	13.150
Nishat Chunnian Power LTD and Dewan Cement LTD	0.045	71.740	24.950
Nishat Chunnian Power LTD and Byco Petroleum Pak LTD	4.080	36.460	10.500
Nishat Power LTD and Dewan Cement LTD	0.020	67.290	53.350
Nishat Power LTD and Byco Petroleum Pak LTD	38.400	31.070	21.260
Engro Power Generation QadirPur LTD (ENGP) and Thata Cement LTD	6.590	69.440	16.430
Engro Power Generation QadirPur LTD and Dewan Cement LTD	0.020	60.270	18.600

Table A3. Cont.

Pair Name	μ	α	σ
Attock Cement Pak LTD and Dewan Cement LTD	3.620	45.500	17.340
Honda Atlas Cars Pak LTD and Fauji Cement Company LTD	1.950	827.500	312.410
KOT Addu Power Company LTD and Bestway Cement LTD	7.920	100.470	26.610
KOT Addu Power Company LTD and Dewan Cement LTD	11.980	33.310	17.050
KOT Addu Power Company LTD and Byco Petroleum Pak LTD	6.850	22.350	7.080
Gharibwal Cement LTD and Dewan Cement LTD	0.040	79.300	31.870
Gharibwal Cement LTD and Fauji Cement Company LTD	0.010	66.230	24.540
Gharibwal Cement LTD and Byco Petroleum Pak LTD	5.030	26.280	8.920
Gharibwal Cement LTD and Quice Food Industries LTD	5.700	16.540	6.240
Ghandhara Nissan LTD (GHIN) and FAUJI Food LTD	0.010	60.720	30.090
Ghandhara Nissan LTD and Byco Petroleum Pak LTD	5.720	16.810	6.670
Pakistan State Oil Company LTD and Bestway Cement LTD	0.020	90.700	50.780
Pakistan State Oil Company LTD and Dewan Cement LTD	2.950	24.090	14.430
Pakistan State Oil Company LTD and Byco Petroleum Pak LTD	5.370	9.260	5.250
DYNEA Pak LTD and Dewan Cement LTD	14.920	39.560	53.730
Lotte Chemicals Pak LTD and Dewan Cement LTD	5.910	21.390	11.330
Lotte Chemicals Pak LTD and Byco Petroleum Pak LTD	0.120	32.490	13.350
Pioneer Cement LTD and Dewan Cement LTD	0.130	40.140	19.140
Millat Tractors LTD and Byco Petroleum Pak LTD	6.190	29.740	9.060
Dewan Cement LTD and Ghandhara Industries LTD	6.360	14.350	17.310
Ghandhara Industries LTD and Byco Petroleum Pak LTD	6.360	19.660	7.390

Table A4. 2018 rates of return at different λ .

Pair Name	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
NCPL and NISH	0.058	0.049	0.062	0.028
NCPL and LOTTE	0.161	0.153	0.085	0.008
NCPL and DECE	0.128	0.128	0.125	0.097
NCPL and BYCO	0.014	0.013	0.010	0.006
NISH and DECE	0.090	0.094	0.080	0.065
NISH and BYCO	0.228	0.193	0.057	0.004
ENGP and THAT	0.055	0.054	0.040	0.004
ENGP and DECE	0.016	0.007	0.005	0.033
ATTOC and DECE	0.024	0.020	0.015	0.008
HONDA and FAUJ	0.273	0.274	0.244	0.051
KOT and BEST	0.077	0.074	0.052	0.005
KOT and DECE	0.177	0.167	0.102	0.009
KOT and BYCO	0.092	0.087	0.065	0.008
GHAR and DECE	0.050	0.038	0.064	0.006
GHAR and FAUJ	0.095	0.055	0.048	0.013
GHAR and BYCO	0.068	0.065	0.051	0.006
GHAR and QUICE	0.103	0.098	0.075	0.010
GHIN and FAUJ	0.010	0.109	0.010	0.036
GHIN and BYCO	0.111	0.106	0.079	0.011
PSO and BEST	0.177	0.192	0.175	0.103
PSO and DECE	0.072	0.064	0.057	0.003
PSO and BYCO	0.187	0.177	0.140	0.022
DYNEA and DECE	0.477	0.446	0.245	0.020
LOTTE and DECE	0.347	0.341	0.264	0.049
LOTTE and BYCO	0.084	0.055	0.059	0.027
PION and DECE	0.187	0.195	0.178	0.131
MILLAT and BYCO	0.078	0.074	0.055	0.007
DECE and GHAN	0.454	0.438	0.335	0.049
GHAN AND BYCO	0.110	0.105	0.079	0.010

Table A5. 2019 Ornstein–Uhlenbeck process parameters.

Pair Name	μ	α	σ
Pakistan Refinery LTD and Oil & Gas Development CO LTD	0.020	96.400	42.740
Pakistan Refinery LTD and Ghani Automobile Industries LTD	6.840	41.610	39.760
National Refinery LTD and Pakistan Oilfields LTD	0.040	670.990	208.610
Nishat Chunnian Power LTD and Engro Polymer and Chemical LTD	0.010	91.050	35.110
Nishat Chunnian Power LTD and Pioneer Cement LTD	0.780	26.860	11.540
Nishat Chunnian Power LTD and Maple Leaf Cement Factory	0.004	94.760	39.290
Attock Refinery LTD and Attock Petroleum LTD	0.030	644.640	233.200
Dewan Farooque LTD and Descon Oxychem LTD	0.001	36.770	21.570
Dewan Farooque LTD and Cherat Cement Company LTD	0.040	100.360	61.410
Ittehad Chemicals LTD and Pak Suzuki Motors Company LTD	0.110	40.210	19.010
Thata Cement LTD and Pakistan State Oil Company LTD	1.850	23.690	10.380
Thata Cement LTD and Pakistan Oilfields LTD	0.010	42.820	14.440
Thata Cement LTD and Ghani Automobile Industries LTD	0.006	24.430	8.140
Descon Oxychem LTD and Pakistan Oilfields LTD	0.010	588.420	191.100
Cherat Cement Company LTD and Hi-Tech Lubricants LTD	0.008	68.540	29.140
Mari Petroleum Company LTD and Fauji Cement Company LTD	0.240	29.100	10.630
K Electric LTD and Fauji Cement Company LTD	1.320	15.310	5.080
Pakistan State Oil Company LTD and Pakistan Oilfields LTD	0.009	580.660	192.900
Pakistan Oilfields LTD and Honda Atlas Cars Pak LTD	4.310	787.640	261.200
Pakistan Oilfields LTD and Ghani Automobile Industries LTD	0.070	629.090	165.800
Fauji Cement Company LTD and Ghani Automobile Industries LTD	0.260	28.410	10.400
Pioneer Cement LTD and Al Shaheer Corporation LTD	0.060	29.870	13.360
Maple Leaf Cement Factory and Al Shaheer Corporation LTD	0.050	29.550	13.300

Table A6. 2019 rates of return at different λ .

Pair Name	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
PAKR and OG	0.082	0.087	0.095	0.025
PAKR and GAIL	0.343	0.333	0.241	0.029
NATR and PKOIL	0.084	0.077	0.096	0.065
NCPL and ENGRO	0.094	0.087	0.091	0.041
NCPL and PION	0.207	0.202	0.199	0.115
NCPL and MPLF	0.131	0.157	0.135	0.089
ATTOCR and ATTOCP	0.211	0.187	0.202	0.027
DEWAN and DESCON	0.186	0.192	0.197	0.124
DEWAN and CHERAT	0.257	0.232	0.222	0.117
ITHD and PAK SUZUKI	0.147	0.155	0.135	0.097
THAT and PSO	0.131	0.132	0.121	0.039
THAT and PKOIL	0.024	0.015	0.03	0.111
THAT and GAIL	0.037	0.045	0.041	0.071
DESCON and PKOIL	0.023	0.01	0.016	0.064
CHERAT and HITECH	0.068	0.031	0.041	0.008
MARI and FAUJ	0.027	0.029	0.015	0.123
KELEC and FAUJ	0.009	0.007	0.007	0.009
PSO and PKOIL	0.103	0.025	0.037	0.103
PKOIL and HONDA	0.073	0.072	0.056	0.002
PKOIL and GAIL	0.086	0.087	0.1	0.009
FAUJ and GAIL	0.194	0.189	0.193	0.147
PION and ALSHAHEER	0.034	0.049	0.038	0.089
MPLF and ALSHAHEER	0.042	0.036	0.035	0.104

Table A7. Returns based on buy and hold strategy.

2017		2018		2019	
Company Names	Returns	Company Names	Returns	Company Names	Returns
Attock Cement	−44.90	Attock Cement Pak LTD	−26.50	Al Shaheer Corporation LTD	−40.43
Attock Petroleum LTD	−24.60	Attock Petroleum LTD	−1.74	Attock Cement Pak LTD	−9.49
Attock Refinery LTD	−45.33	BestWay Cement LTD	−17.93	Attock Petroleum LTD	−16.85
Bestway cement	−52.19	Byco Petroleum Pak LTD	−33.59	Attock Refinery LTD	−23.76
Cherat Cement Company LTD	−40.97	Cherat Cement Company LTD	−35.48	BestWay Cement LTD	−3.36
Dera Ghazi Khan Cement	−39.93	Dera Ghazi Khan Cement LTD	−41.21	Cherat Cement Company LTD	−20.13
Descon Oxychem LTD	−24.34	Descon Oxychem LTD	119.92	Descon Oxychem LTD	−21.96
Dewan Cement LTD	−56.42	Dewan Cement LTD	−35.56	Dewan Farooque LTD	−56.18
DYNEA Pak LTD	65.35	DYNEA Pak LTD	−12.50	DYNEA Pak LTD	20.03
Engro Polymer and Chemical	54.77	Engro Polymer and Chemical LTD	48.95	Engro Polymer and Chemical LTD	−14.18
Engro Power Qadirpur	−5.91	Engro Power Generation QadirPur LTD	−15.31	Engro Power Generation QadirPur LTD	−11.56
Fauji Food LTD	−47.45	Fauji Cement Company LTD	−16.35	Fauji Cement Company LTD	−27.47
Ghandhara Industries LTD	−27.15	FAUJI Food LTD	83.63	FAUJI Food LTD	−54.14
Ghani Automobile Industries	7.79	Ghadhara Nissan LTD	−32.34	Ghani Automobile Industries LTD	−34.46
Gharibwal Cement	−53.49	Ghandhara Industries LTD	0.33	Gharibwal Cement LTD	−14.02
Indus Motor Company LTD	2.60	Gharibwal Cement LTD	−36.16	Hi Tech Lubricants LTD	−52.84
Ittehad Chemicals LTD	−31.67	Honda Atlas Cars Pak LTD	−64.27	Honda Atlas Cars Pak LTD	19.05
Kohat cement	−52.29	Indus Motor Company LTD	−29.46	Indus Motor Company LTD	−4.61
KOT ADDU Power	−31.49	Ittehad Chemicals LTD	15.57	Ittehad Chemicals LTD	−16.52
Lalipir Power LTD	−5.82	Kohat Cement LTD	−25.78	K Electric LTD	−27.48
Maple Leaf Cement Factory	−52.16	KOT Addu Power Company LTD	−11.16	Kohat Cement LTD	−7.11
National Refinery	−24.81	Lalipur Power LTD	−22.73	KOT Addu Power Company LTD	−36.57
Nishat Chunnian Power	−43.19	Lotte Chemicals Pak LTD	129.48	Lalipir Power LTD	−9.66
Nishat Power LTD	−45.95	Maple Leaf Cement Factory	−40.21	Lotte Chemicals Pak LTD	−20.80
Pak Oilfields	12.81	Mari Petroleum Company LTD	−5.47	Maple Leaf Cement Factory	−35.88
Pak State Oil	−20.88	Millat Tractors LTD	−28.84	Mari Petroleum Company LTD	13.55
Pakistan Petroleum LTD	10.17	Nishat Chunnian Power LTD	−28.53	Millat Tractors LTD	−4.01
Pakistan Refinery LTD	−9.83	Nishat Power LTD	−19.11	National Refinery LTD	−49.66
Pioneer Cement	−55.61	Oil & Gas Development CO LTD	−21.11	Nishat Chunnian Power LTD	−19.02
Power Cement LTD	−22.03	Pakistan Petroleum LTD	−16.89	Nishat Power LTD	3.42
Shell Pakistan LTD	−41.53	Pakistan State Oil Company LTD	−8.56	Oil & Gas Development CO LTD	5.80
Sitara Peroxide LTD	−55.80	Pioneer Cement LTD	−33.67	Pak Suzuki Motors Company LTD	27.62
Thata Cement LTD	−44.89	Quice Food Industries LTD	−8.66	Pakistan Oilfields LTD	0.13
		Thata Cement LTD	−37.78	Pakistan Petroleum LTD	5.57
				Pakistan Refinery LTD	−9.83
				Pakistan State Oil Company LTD	−2.38
				Pioneer Cement LTD	−30.80
				Quice Food Industries LTD	−19.56
				Sitara Peroxide LTD	−29.36
				Thata Cement LTD	−17.75

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